

**TARIK AL-SHEMMERI**

# **ENGINEERING THERMODYNAMICS**



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Tarik Al-Shemmeri

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## Preface

Thermodynamics is an essential subject taught to all science and engineering students. If the coverage of this subject is restricted to theoretical analysis, student will resort to memorising the facts in order to pass the examination. Therefore, this book is set out with the aim to present this subject from an angle of demonstration of how these laws are used in practical situation.

This book is designed for the virtual reader in mind, it is concise and easy to read, yet it presents all the basic laws of thermodynamics in a simplistic and straightforward manner.

The book deals with all four laws, the zeroth law and its application to temperature measurements. The first law of thermodynamics has large influence on so many applications around us, transport such as automotive, marine or aircrafts all rely on the steady flow energy equation which is a consequence of the first law of thermodynamics. The second law focuses on the irreversibilities of substances undergoing practical processes. It defines process efficiency and isentropic changes associated with frictional losses and thermal losses during the processes involved.

Finally the Third law is briefly outlined and some practical interpretation of it is discussed.

This book is well stocked with worked examples to demonstrate the various practical applications in real life, of the laws of thermodynamics. There are also a good section of unsolved tutorial problems at the end of the book.

This book is based on my experience of teaching at Univeristy level over the past 25 years, and my student input has been very valuable and has a direct impact on the format of this book, and therefore, I would welcome any feedback on the book, its coverage, accuracy or method of presentation.

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# 1. General Definitions

In this section general thermodynamic terms are briefly defined; most of these terms will be discussed in details in the following sections.

## 1.1 Thermodynamic System

Thermodynamics is the science relating heat and work transfers and the related changes in the properties of the working substance. The working substance is isolated from its surroundings in order to determine its properties.

**System** - Collection of matter within prescribed and identifiable boundaries. A system may be either an open one, or a closed one, referring to whether mass transfer or does not take place across the boundary.

**Surroundings** - Is usually restricted to those particles of matter external to the system which may be affected by changes within the system, and the surroundings themselves may form another system.

**Boundary** - A physical or imaginary surface, enveloping the system and separating it from the surroundings.

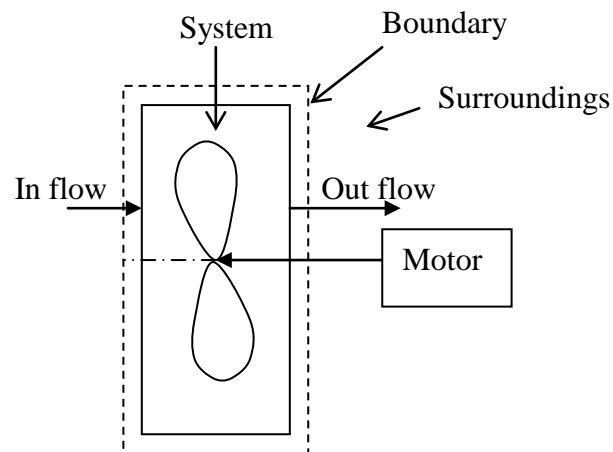


Figure 1.1: System/Boundary

## 1.2 Thermodynamic properties

**Property** - is any quantity whose changes are defined only by the end states and by the process. Examples of thermodynamic properties are the Pressure, Volume and Temperature of the working fluid in the system above.

**Pressure (P)** - The normal force exerted per unit area of the surface within the system. For engineering work, pressures are often measured with respect to atmospheric pressure rather than with respect to absolute vacuum.

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$$

In SI units the derived unit for pressure is the Pascal (Pa), where  $1 \text{ Pa} = 1 \text{ N/m}^2$ . This is very small for engineering purposes, so usually pressures are given in terms of kiloPascals ( $1 \text{ kPa} = 10^3 \text{ Pa}$ ), megaPascals ( $1 \text{ MPa} = 10^6 \text{ Pa}$ ), or bars ( $1 \text{ bar} = 10^5 \text{ Pa}$ ). The imperial unit for pressure are the pounds per square inch (Psi)  $1 \text{ Psi} = 6894.8 \text{ Pa}$ .

Specific Volume (V) and Density ( $\rho$ )

For a system, the specific volume is that of a unit mass, i.e.

$$v = \frac{\text{volume}}{\text{mass}} \quad \text{Units are m}^3/\text{kg}.$$

It represents the inverse of the density,  $v = \frac{1}{\rho}$ .

**Temperature (T)** - Temperature is the degree of hotness or coldness of the system. The absolute temperature of a body is defined relative to the temperature of ice; for SI units, the Kelvin scale. Another scale is the Celsius scale. Where

the ice temperature under standard ambient pressure at sea level is:  $0^\circ\text{C} \equiv 273.15 \text{ K}$

and the boiling point for water (steam) is:  $100^\circ\text{C} \equiv 373.15 \text{ K}$

The imperial units of temperature is the Fahrenheit where

$$T^\circ\text{F} = 1.8 \times T^\circ\text{C} + 32$$

**Internal Energy(u)** - The property of a system covering all forms of energy arising from the internal structure of the substance.

**Enthalpy (h)** - A property of the system conveniently defined as  $h = u + PV$  where u is the internal energy.

**Entropy (s)** - The microscopic disorder of the system. It is an extensive equilibrium property.

This will be discussed further later on.

### 1.3 Quality of the working Substance

A pure substance is one, which is homogeneous and chemically stable. Thus it can be a single substance which is present in more than one phase, for example liquid water and water vapour contained in a boiler in the absence of any air or dissolved gases.

**Phase** - is the State of the substance such as solid, liquid or gas.

**Mixed Phase** - It is possible that phases may be mixed, eg ice + water, water + vapour etc.

Quality of a Mixed Phase or Dryness Fraction (x)

The dryness fraction is defined as the ratio of the mass of pure vapour present to the total mass of the mixture (liquid and vapour; say 0.9 dry for example). The quality of the mixture may be defined as the percentage dryness of the mixture (ie, 90% dry).

**Saturated State** - A saturated liquid is a vapour whose dryness fraction is equal to zero. A saturated vapour has a quality of 100% or a dryness fraction of one.

**Superheated Vapour** - A gas is described as superheated when its temperature at a given pressure is greater than the saturated temperature at that pressure, ie the gas has been heated beyond its saturation temperature.

**Degree of Superheat** - The difference between the actual temperature of a given vapour and the saturation temperature of the vapour at a given pressure.

**Subcooled Liquid** - A liquid is described as undercooled when its temperature at a given pressure is lower than the saturated temperature at that pressure, ie the liquid has been cooled below its saturation temperature.

**Degree of Subcool** - The difference between the saturation temperature and the actual temperature of the liquid at a given pressure.

**Triple Point** - A state point in which all solid, liquid and vapour phases coexist in equilibrium.

**Critical Point** - A state point at which transitions between liquid and vapour phases are not clear. for  $H_2O$ :

- $P_{CR} = 22.09 \text{ MPa}$
- $T_{CR} = 374.14 \text{ }^{\circ}\text{C}$  (or  $647.3 \text{ }^{\circ}\text{K}$ )
- $v_{CR} = 0.003155 \text{ m}^3/\text{kg}$
- $u_f = u_g = 2014 \text{ kJ/kg}$
- $h_f = h_g = 2084 \text{ kJ/kg}$
- $s_f = s_g = 4.406 \text{ kJ/kgK}$

## 1.4 Thermodynamic Processes

A process is a path in which the state of the system change and some properties vary from their original values. There are six types of Processes associated with Thermodynamics:

**Adiabatic** : no heat transfer from or to the fluid

**Isothermal** : no change in temperature of the fluid

**Isobaric** : no change in pressure of the fluid

**Isochoric** : no change in volume of the fluid

**Isentropic** : no change of entropy of the fluid

**Isenthalpic** : no change of enthalpy of the fluid

## 2. Thermodynamics working fluids

Behaviour of the working substance is very essential factor in understanding thermodynamics. In this book, focus is given to pure substances such as gases and steam properties and how they are interrelated are important in the design and operation of thermal systems.

The ideal gas equation is very well known approximation in relating thermal properties for a state point, or during a process. However, not all gases are perfect, and even the same gas, may behave as an ideal gas under certain circumstances, then changes into non-ideal, or real, under different conditions. There are other equations, or procedures to deal with such conditions. Steam or water vapour is not governed by simple equations but properties of water and steam are found in steam tables or charts.

### 2.1 The Ideal Gas

Ideally, the behaviour of air is characterised by its mass, the volume it occupies, its temperature and the pressure condition in which it is kept. An ideal gas is governed by the perfect gas equation of state which relates the state pressure, volume and temperature of a fixed mass ( $m$  is constant) of a given gas ( $R$  is constant) as:

$$\frac{PV}{T} = mR \quad (1)$$

Where

- $P$  – Pressure (Pa)
- $V$  – Volume ( $\text{m}^3$ )
- $T$  – Absolute Temperature (K)
- $T(\text{K}) = 273 + t(\text{C})$
- $m$  – mass (kg)
- $R$  – gas constant (J/kgK)

The equation of state can be written in the following forms, depending on what is needed to be calculated

$$1. \text{ In terms of the pressure} \quad P = \frac{mRT}{V} \quad (2)$$

$$2. \text{ In terms of the volume} \quad V = \frac{mRT}{P} \quad (3)$$

$$3. \text{ In terms of the mass} \quad m = \frac{PV}{RT} \quad (4)$$

$$4. \text{ In terms of the temperature} \quad T = \frac{PV}{mR} \quad (5)$$



5. In terms of the gas constant  $R = \frac{PV}{mT}$  (6)

6. In terms of the density  $\rho = \frac{m}{V} = \frac{P}{RT}$  (7)

The specific gas constant  $R$ , is a property related to the molar mass ( $M$ ) in kg/kmol, of the gas and the Universal gas constant  $R_o$  as

$$R = R_o / M \quad (8)$$

where  $R_o = 8314.3 \text{ J/kgK}$

The ideal gas equation can also be written on time basis, relating the mass flow rate (kg/s) and the volumetric flow rate ( $\text{m}^3/\text{s}$ ) as follows:

$$P V_t = m_t R T \quad (9)$$

## 2.2 Alternative Gas Equation During A Change Of State:

The equation of state can be used to determine the behaviour of the gas during a process, i.e. what happens to its temperature, volume and pressure if any one property is changed. This is defined by a simple expression relating the initial and final states such as :

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad (10)$$

this can be rewritten in terms of the final condition, hence the following equations are generated :

$$\text{Final Pressure} \quad P_2 = P_1 \times \frac{T_2}{T_1} \times \frac{V_1}{V_2} \quad (11)$$

$$\text{Final Temperature} \quad T_2 = T_1 \times \frac{P_2}{P_1} \times \frac{V_2}{V_1} \quad (12)$$

$$\text{Final Volume} \quad V_2 = V_1 \times \frac{P_1}{P_2} \times \frac{T_2}{T_1} \quad (13)$$

## 2.3 Thermodynamic Processes for gases

There are four distinct processes which may be undertaken by a gas (see Figure 2.1):-

- a) Constant volume process, known as isochoric process; given by:-

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad (14)$$

- b) Constant pressure process; known as isobaric process, given by:-

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (15)$$

- c) Constant temperature process, known as isothermal process, given by:-

$$P_1 V_1 = P_2 V_2 \quad (16)$$

- d) Polytropic process given by:-

$$P_1 V_1^n = P_2 V_2^n \quad (17)$$

Note when  $n = C_p/C_v$ , the process is known as adiabatic process.

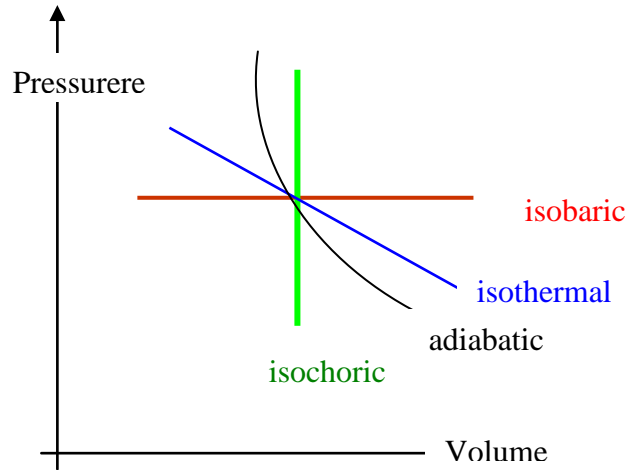


Figure 2.1: Process paths

## 2.4 Van der Waals gas Equation of state for gases

The perfect gas equation derived above assumed that the gas particles do not interact or collide with each other. In fact, this is not true. The simplest of the equations to try to treat real gases was developed by Johannes van der Waals. Based on experiments on pure gases in his laboratory, van der Waals recognized that the variation of each gas from ideal behavior could be treated by introducing two additional terms into the ideal gas equation. These terms account for the fact that real gas particles have some finite volume, and that they also have some measurable intermolecular force. The two equations are presented below:

$$PV = mRT$$

$$P = \frac{RT}{v - b} - \frac{a}{v^2} \quad (18)$$

where  $v$  is the specific volume (  $V/m$  ), and the Numerical values of  $a$  &  $b$  can be calculated as follows:

$$a = \frac{27 \cdot R^2 \cdot T_{critical}^2}{64 \cdot P_{critical}} \quad \text{and} \quad b = \frac{R \cdot T_{critical}}{8 \cdot P_{critical}} \quad (19)$$

Table 2.1, presents the various thermal properties of some gases and the values of the constants ( $a$ , and  $b$ ) in Van der Waals equation.

Substance	Chemical Formula	Molar Mass M (kg/kmol)	Gas constant R (J/kgK)	Critical Temp TC (K)	Critical Pressure PC (MPa)	Van der Waals Constants	
						a	b
Air	O <sub>2</sub> + 3.76 N <sub>2</sub>	28.97	286.997	132.41	3.774	161.427	0.00126
Ammonia	NH <sub>3</sub>	17.03	488.215	405.40	11.277	1465.479	0.00219
Carbon Dioxide	CO <sub>2</sub>	44.01	188.918	304.20	7.386	188.643	0.00097
Carbon Monoxide	CO	28.01	296.833	132.91	3.496	187.825	0.00141
Helium	He	4.003	2077.017	5.19	0.229	214.074	0.00588
Hydrogen	H <sub>2</sub>	2.016	4124.157	33.24	1.297	6112.744	0.01321
Methane	CH <sub>4</sub>	16.042	518.283	190.70	4.640	888.181	0.00266

Nitrogen	N <sub>2</sub>	28.016	296.769	126.20	3.398	174.148	0.00138
Oxygen	O <sub>2</sub>	32.00	259.822	154.78	5.080	134.308	0.00099
R12	CCl <sub>2</sub> F <sub>2</sub>	120.92	68.759	385	4.120	71.757	0.00080
Sulphur Dioxide	SO <sub>2</sub>	64.06	129.789	431	7.870	167.742	0.00089
Water Vapour	H <sub>2</sub> O	18.016	461.393	647.3	22.090	1704.250	0.00169

Table 2.1 Van Der Waals Constants for some gases

## 2.5 Compressibility of Gases

Compressibility factor,  $Z$ , is a measure of deviation from the ideal gas.

$$Z = \frac{P.v}{R.T} \quad (20)$$

Where  $v$  is the specific volume (  $V/m$  ),

Note:  $Z = 1$  for an ideal gas.

As  $Z$  approaches 1 for a gas at given conditions, the behavior of the gas approaches ideal gas behavior. Although, different gases have very different specific properties at various conditions; *all gases behave in a similar manner relative to their critical pressure,  $P_{cr}$  and critical temperature,  $T_{cr}$ .*

Hence, the gas pressures and temperatures are normalized by the critical values to obtain the *reduced pressure,  $P_R$  and temperature,  $T_R$ .* defined as

$$P_R = P/P_{cr} ;$$

$$T_R = T/T_{cr}$$

The reduced values can then be used to determine  $Z$  using the Generalized Compressibility Charts (Figure 2.2)

These charts show the conditions for which  $Z = 1$  and the gas behaves as an ideal gas:

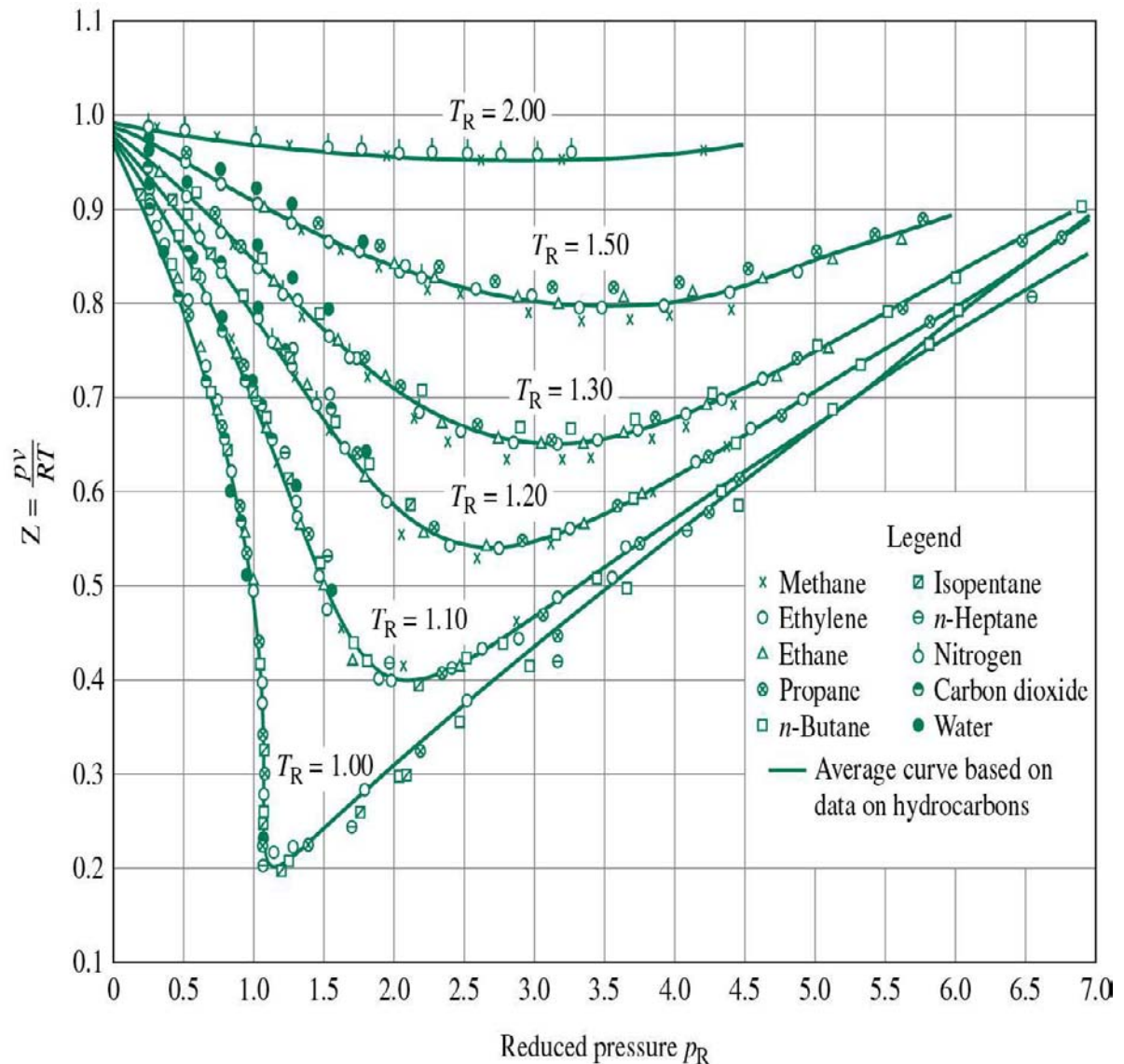


Figure 2.2: Compressibility Chart

## 2.6 The State Diagram – for Steam

Processes 1-2, 2-3, and 3-4 represent a typical constant pressure heating of water which initially is heated to its boiling point (1-2), upon continued heat input it starts to evaporate at point 2, it is incompletely liquid, then gradually some of the water becomes vapour till it reaches point 3, where all the water has evaporated, further heating will make the water vapour superheated (process 3-4).

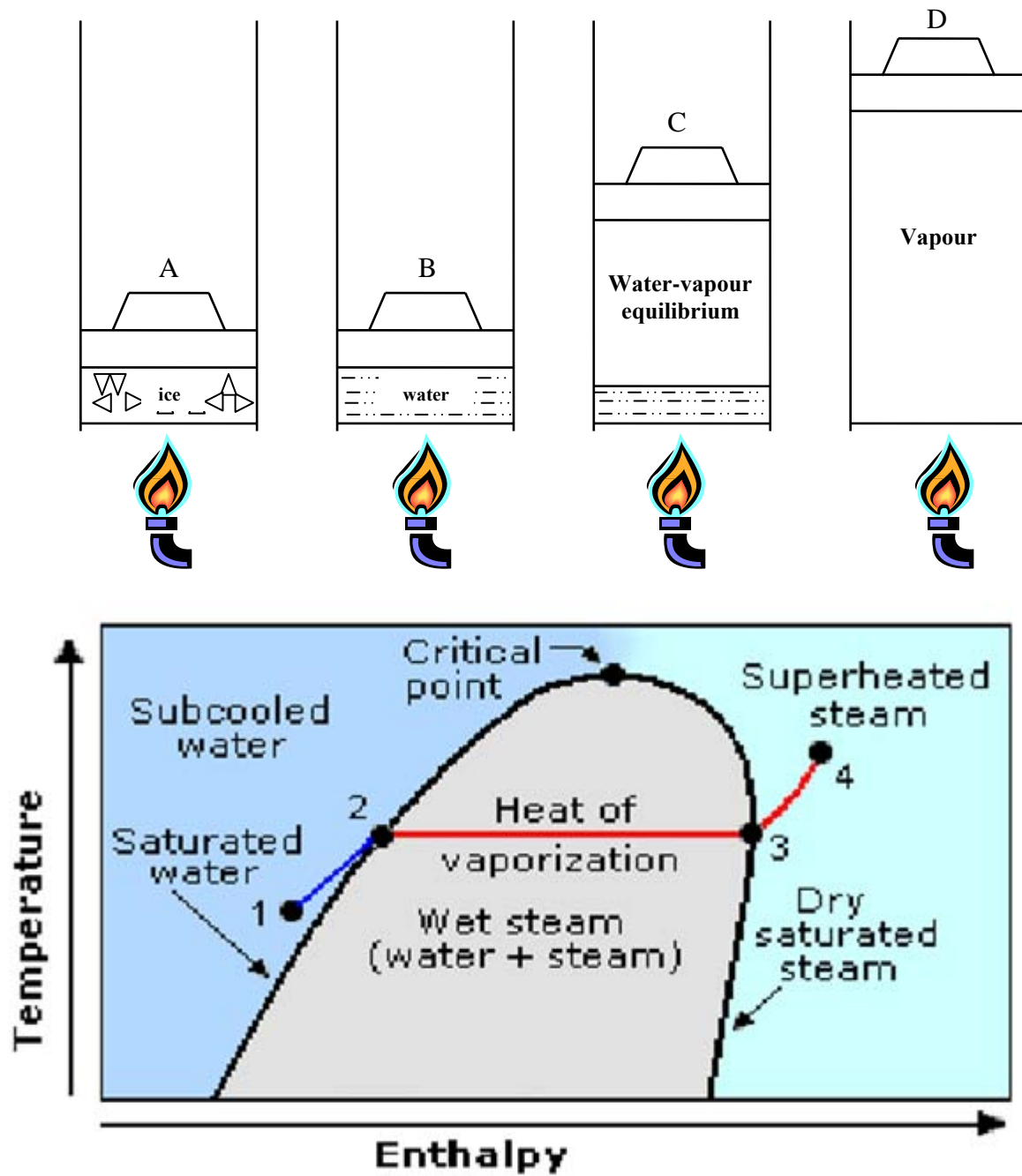


Figure 2.3: Formation of Vapour (Steam)



## 2.7 Property Tables And Charts For Vapours

Tables are normally available which give data for saturated liquid and saturated vapour, a listing for a given saturation pressure or temperature, some or all of the quantities  $v_f$ ,  $v_g$ ,  $u_f$ ,  $u_g$ ,  $h_f$ ,  $h_g$ ,  $s_f$ ,  $s_g$  and  $s_{fg}$ . The tables enable  $u$ ,  $h$  or  $s$  to be obtained for saturated liquid, wet vapour and dry saturated vapour. Charts for steam are readily available which show  $h$  or  $T$  as ordinate against  $s$  (specific entropy) as abscissa.

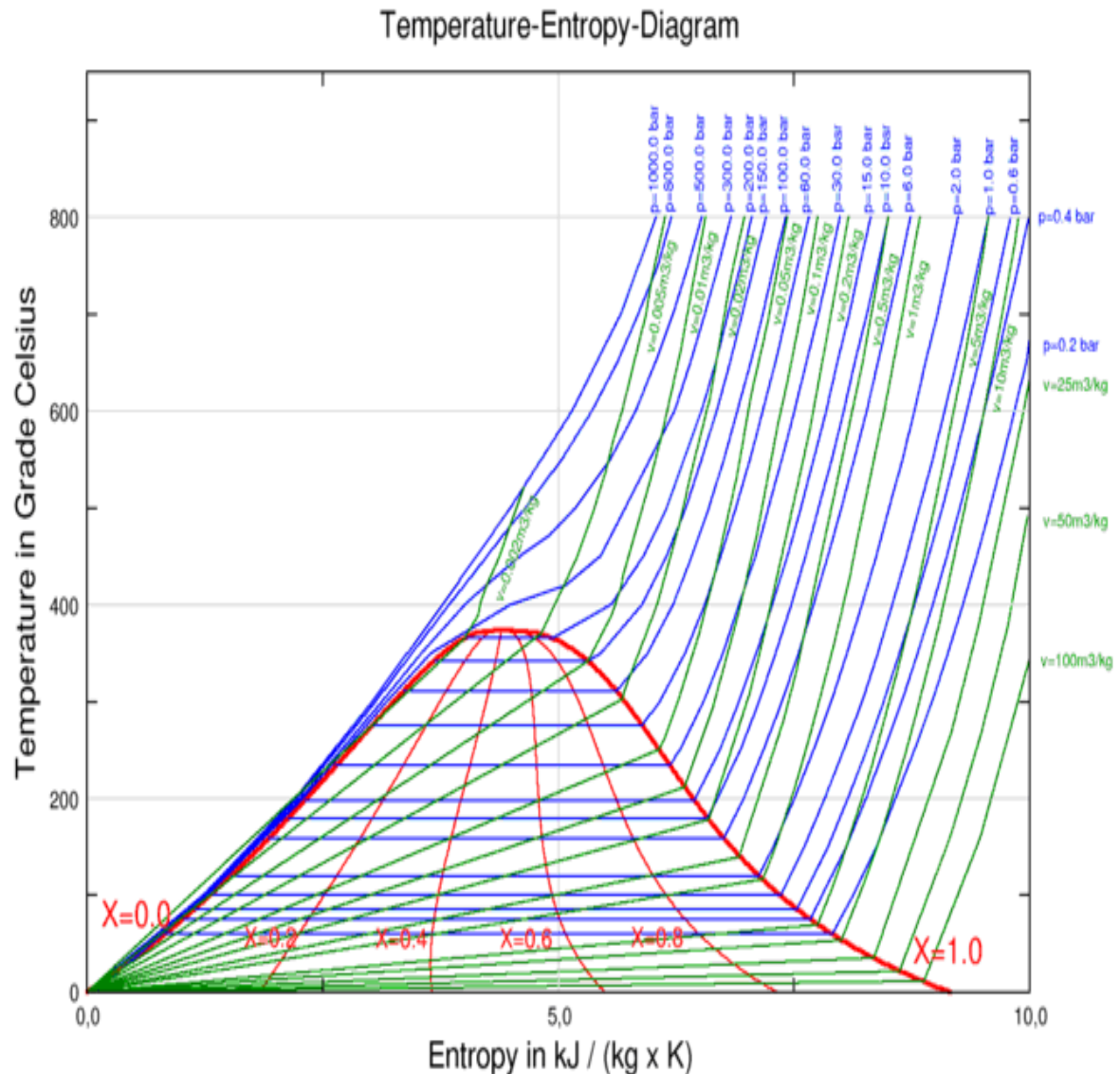


Figure 2.4: Temperature – Entropy chart for Water/Steam

Courtesy of: <http://en.wikipedia.org/>

### Calculations of steam properties in the mixed region:

The dryness fraction is an added property needed to define the mixture of water liquid and vapour during its transformation ( condensation or evaporation) and is defined as follows:-

$$x = \frac{\text{mass of vapour}}{\text{total mass of the system}} \quad (21)$$

The total mass = mass of vapour + mass of liquid; Hence the system volume along the two-phase, process 2-3 (Figure 2.3) is:

$$v = (1 - x) v_f + x v_g \quad (22)$$

At point state point 2,  $x = 0$

at state point 3,  $x = 1$  (Figure 2.3)

Values of  $v_f$  and  $v_g$  and other properties for real substances are normally given in tables. Suffix 'f' refers to the liquid; Suffix 'g' refers to the dry vapour; and Suffix 'fg' refers to the mixed phase.

$$v_{fg} = v_g - v_f$$

$$h_{fg} = h_g - h_f$$

$$S_{fg} = S_g - S_f$$

For wet steam of dryness fraction  $x$

$$v = (1 - x) \cdot v_f + x \cdot v_g$$

$$= v_f + x \cdot (v_g - v_f)$$

$$= v_f + x \cdot v_{fg}$$

Similar relations for  $u$ ,  $h$  and  $s$ .

$$v = v_f + x v_{fg}$$

$$u = u_f + x u_{fg}$$

$$h = h_f + x h_{fg}$$

$$s = s_f + x s_{fg} \quad (23)$$

or  $u = (1 - x) u_f + x u_g$

$$h = (1 - x) h_f + x h_g$$

$$s = (1 - x) s_f + x s_g \quad (24)$$

p	$t_s$	$v_f$	$v_g$	$h_f$	$h_g$	$s_f$	$s_g$
(kPa)	(°C)	(m <sup>3</sup> /kg)	(m <sup>3</sup> /kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)
10	45.81	0.001	14.674	191.83	2,585	0.6493	8.1502
100	99.63	0.00104	1.694	417.46	2,676	1.3026	7.3594
200	120.23	0.00106	0.8857	504.7	2,707	1.5301	7.1271
500	151.86	0.00109	0.3749	640.23	2,749	1.8607	6.8213

1000	179.91	0.00112	0.1944	762.81	2,778	2.1387	6.5865
2000	212.42	0.00117	0.0996	908.79	2,800	2.4474	6.3409
10000	311.06	0.00145	0.01803	1407.56	2,725	3.3596	5.6141
20000	365.81	0.00236	0.0058	1,826	2,410	4.0139	4.9269
22120	374.15	0.00317	0.00317	2,084	2,084	4.43	4.43

Table 2.2 Saturated Steam table at selected pressures

Enthalpy of Superheated Steam (kJ/kg)								
Pressure (bar)	Saturation Temperature (°C)	Temperature (°C)						
		Saturation	200	250	300	350	400	450
15	198.3	2792	2796	2925	3039	3148	3256	3364
20	212.4	2799		2904	3025	3138	3248	3357
30	233.8	2809		2858	2995	3117	3231	3343
40	250.3	2801			2963	3094	3214	3330

Table 2.3 Superheated Steam table at selected pressures

**Worked Example 2.1**

Self ignition would occur in the engine using certain brand of petrol if the temperature due to compression reached 350°C.

Calculate the highest ratio of compression that may be used to avoid pre-ignition if the law of compression is

$$PV^{1.3} = c$$

$$PV^{1.4} = c$$

Calculate the final pressure in each case. Assume inlet condition of 27°C and 1 bar.

**Solution**

$$(a) \quad \frac{V_1}{V_2} = \left( \frac{T_2}{T_1} \right)^{\frac{1}{n-1}}$$

$$\left( \frac{V_1}{V_2} \right)_i = \left( \frac{349 + 273}{300} \right)^{\frac{1}{0.3}} = 11.36$$

$$\left( \frac{V_1}{V_2} \right)_{ii} = \left( \frac{349 + 273}{300} \right)^{\frac{1}{0.4}} = 6.19$$

$$(b) \quad P_2 = P_1 \left( \frac{V_1}{V_2} \right)^n$$

$$P_{2i} = 1(11.36)^{1.3} = 23.5 \text{ bar}$$

$$P_{2ii} = 1(6.19)^{1.4} = 12.8 \text{ bar}$$

### Worked Example 2.2

Calculate the density of Ethane at 171 bar and 458K; assume for Ethane:

$$T_c = 305 \text{ K}$$

$$P_c = 48.80 \text{ bar}$$

$$R = 319.3 \text{ J/kgK}$$

- a) assuming it behaves as a perfect gas
- b) using the compressibility chart.

### Solution:

- a) for a perfect gas

$$\rho = \frac{P}{R.T} = \frac{171 \times 10^5}{319.3 \times 458} = 117 \text{ kg/m}^3$$

- b) using the compressibility chart:

$$T_R = T / T_{CR} = 458 / 305.4 = 1.5$$

$$P_R = P / P_{CR} = 171 / 48.8 = 3.527$$

READ Z from the chart ( Z = 0.8 )

ie 80% less dense compared with the perfect gas behaviour.

Or density = 146 kg/m<sup>3</sup>

**Worked Example 2.3**

Find the specific volume for H<sub>2</sub>O at 10 MPa and 500°C using:

- a) Compressibility chart;
- b) Steam tables (below)

<i>T</i>	<i>p</i> = 10.0 MPa (311.06 deg-C)			
	<i>v</i>	<i>u</i>	<i>H</i>	<i>s</i>
Sat.	0.018026	2544.4	2724.7	5.6141
325	0.019861	2610.4	2809.1	5.7568
350	0.02242	2699.2	2923.4	5.9443
400	0.02641	2832.4	3096.5	6.2120
450	0.02975	2943.4	3240.9	6.4190
500	0.03279	3045.8	3373.7	6.5966
550	0.03564	3144.6	3500.9	6.7561
600	0.03837	3241.7	3625.3	6.9029

Source: <http://www.sfsb.hr/>



**Solution:**

a) Compressibility Chart

$$P_R = \frac{P}{P_c} = \frac{100}{221.2} = 0.47$$

$$T_R = \frac{T}{T_c} = \frac{500 + 273}{374.15 + 273} = 1.19$$

$$\text{but } R = 8.3144/18.015 = 0.4615 \text{ kJ/kgK}$$

$$\text{Using Figure 2.2, } Z = 0.9 \quad \therefore \frac{PV}{RT} = 0.9$$

$$v = \frac{R \times T \times Z}{P} = \frac{461.5 \times 773}{100 \times 10^5} \times 0.9 = 0.032 \text{ m}^3 / \text{kg}$$

b) From Steam Tables:

The steam is superheated

$$\text{At } 100 \text{ bar and } 500^\circ\text{C}, v = 0.03279 \text{ m}^3 / \text{kg}$$

Both results are similar within to the 3<sup>rd</sup> decimal place.**Worked Example 2.4**

Determine the pressure of water vapour at 300°C, if its specific volume is 0.2579 m<sup>3</sup>/kg, using the following methods:

- a) Ideal gas equation
- b) Van-der-Waals equations

**Solution:**a)  $Pv = RT$ 

$$P = \frac{RT}{v} = \frac{461.5 \times 573}{0.2579} = 1.025 \text{ MPa}$$

$$\text{b) } a = \frac{27R^2T_c^2}{64P_c} = \frac{27 \times 461.5^2 \times 647.3^2}{64 \times 22.09 \times 10^6} = 1704$$

$$b = \frac{RT_c}{8P_c} = \frac{461.5 \times 647.3}{8 \times 22.09 \times 10^6} = 1.69 \times 10^{-3}$$

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$= \frac{461.5 \times 573}{0.2579 - 1.69 \times 10^{-3}} - \frac{1704}{0.2579^2}$$

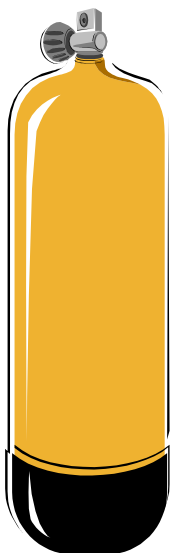
$$= 1032120.1 - 25619$$

$$= 1.006 \text{ MPa}$$

### Worked Example 2.5

An unknown gas has a mass of 1.5 kg contained in a bottle of volume  $1.17 \text{ m}^3$  while at a temperature of 300 K, and a pressure of 200 kPa. Determine the ideal gas constant and deduce the gas?

**Solution:**



Assuming perfect gas behaviour:

$$PV = mRT$$

$$200 \times 10^3 \times 1.17 = 1.5 \times R \times 300$$

$$\therefore R = 520 \text{ J/kgK}$$

$$\text{but } R = \frac{R_o}{M}$$

$$\text{hence } M = R_o/R = 8314.3/520 = 15.99$$

The nearest gas with

The small difference may be attributed to measurements errors.

**Worked Example 2.6**

A  $6 \text{ m}^3$  tank contains helium at  $400\text{K}$  is evacuated from atmospheric pressure of  $100\text{kPa}$  to a final pressure of  $2.5\text{kPa}$ .

Determine

- a) the mass of helium remaining in the tank;
- b) the mass of helium pumped out;
- c) if the temperature of the remaining helium falls to  $10^\circ\text{C}$ , what is the pressure in  $\text{kPa}$ ?

**Solution:**



a)  $P_2 V_2 = m_2 R T_2$

$$R = \frac{R_o}{M} = \frac{8314.3}{4.003} = 2077 \text{ J / kgK}$$

The mass remaining is:

$$m_2 = \frac{2500 \times 6}{2077 \times 400} = 0.018 \text{ kg}$$

b) initial mass is:  $m_1 = \frac{100000 \times 6}{2077 \times 400} = 0.722 \text{ kg}$

$$\therefore \text{mass pumped out} = m_1 - m_2 = 0.704 \text{ kg}$$

since  $m_3 = m_2$ , then

c)  $P_3 = \frac{m_3 \cdot R \cdot T_3}{V_3} = \frac{0.018 \times 2077 \times 283}{6} = 1763 \text{ Pa}$

### Worked Example 2.7

A motorist equips his automobile tyres with a relief-type valve so that the pressure inside the tyre will never exceed 220 kPa (gauge). He starts the trip with a pressure of 200 kPa (gauge) and a temperature of 23°C in the tyres. During the long drive the temperature of the air in the tyres reaches 83°C. Each tyre contains 0.11 kg of air. Determine:

- the mass of air escaping each tyre,
- the pressure of the air inside the tyre when the temperature returns to 23°C.

**Solution:**



a)  $P_1 V_1 = m_1 R T_1$

$$m_1 = \frac{P_1 V_1}{R T_1} = \frac{200 \times 10^3 \times V_1}{287 \times (273 + 23)} = 0.11 \text{ kg}$$

$$V_1 = \frac{0.11 \times 287 \times 296}{200 \times 10^3} = 0.04672 \text{ m}^3$$

$$V_1 = V_2 = \text{constant}$$

$$P_2 V_2 = m_2 R T_2$$

$$220 \times 10^3 \times 0.04672 = m_2 \times 287 \times (273 + 83)$$

$$\therefore m_2 = 0.1006 \text{ kg}$$

$$\therefore dm = m_1 - m_2 = 0.11 - 0.1006 = 0.0094 \text{ kg}$$

b)  $V_3 = V_2 = V_1$  and  $m_3 = m_2$

$$P_3 = \frac{m R T_3}{V_4} = \frac{0.1006 \times 287 \times 296}{0.04672} = 183 \text{ kPa}$$

### Worked Example 2.8

300 kg/minute of steam at 3 MPa and 400°C is supplied to a steam turbine. determine the potential heat released from steam if it is condensed at constant pressure. Can you deduce the specific heat of the steam under this condition?

	$p = 3.00 \text{ MPa (233.90 C)}$			
$T$	$v$	$u$	$h$	$s$

Sat.	0.06668	2604.1	2804.2	6.1869
225				
250	0.07058	2644.0	2855.8	6.2872
300	0.08114	2750.1	2993.5	6.5390
350	0.09053	2843.7	3115.3	6.7428
400	0.09936	2932.8	3230.9	6.9212
450	0.10787	3020.4	3344.0	7.0834
500	0.11619	3108.0	3456.5	7.2338

Source: <http://www.sfsb.hr/>

**Solution:**

a) Constant pressure process

$$h_1 = 3230.9$$

$$h_2 = 2804.2$$

$$\text{Thermal energy available } Q = m \times (h_2 - h_1) = (300/60) \times (3230.9 - 2804.2) = 2133.5 \text{ kW}$$

- b) It can be seen from the table that the temperature at saturation is 233.90 C, so if the equation for heat exchange is used, this is the same heat found above:

$$Q = m.C_p.(T_2 - T_1)$$

$$\text{Hence } C_p = Q / (m \times (T_2 - T_1)) = 2133.5 / (2 \times (500 - 233.9)) = 4.009 \text{ kJ/kg K.}$$

Which is lower than that at lower pressures, at 1 bar  $C_p$  for water is about 4.18 kJ/kgK.

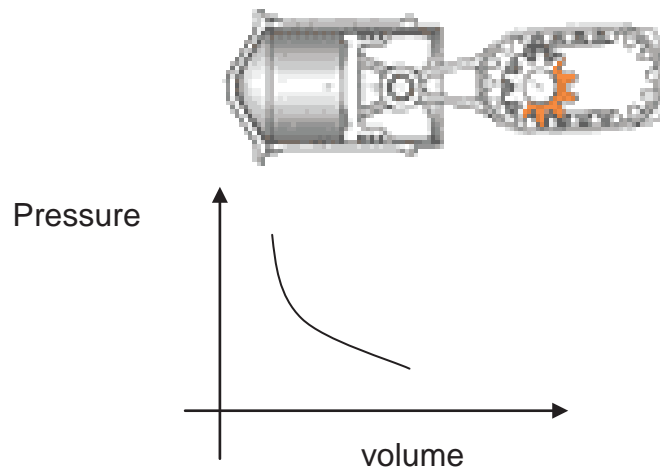
### Worked Example 2.9

Self-ignition would occur in an engine using certain brand of petrol if the temperature due to compression reaches 350 °C; when the inlet condition is 1 bar, 27°C.

Calculate the highest compression ratio possible in order to avoid self-ignition, if the compression is according to

- adiabatic, with index of 1.4; and
- polytropic, with index of 1.3

**Solution:**



The compression ratio is calculated as follows:

When  $n = 1.4$ , the volume ratio is :

$$\frac{V_1}{V_2} = \left( \frac{T_2}{T_1} \right)^{1/(n-1)} = \left( \frac{349.9 + 273}{27 + 273} \right)^{1/0.4} = 6.2$$



and when  $n = 1.3$ , the volume ratio is :

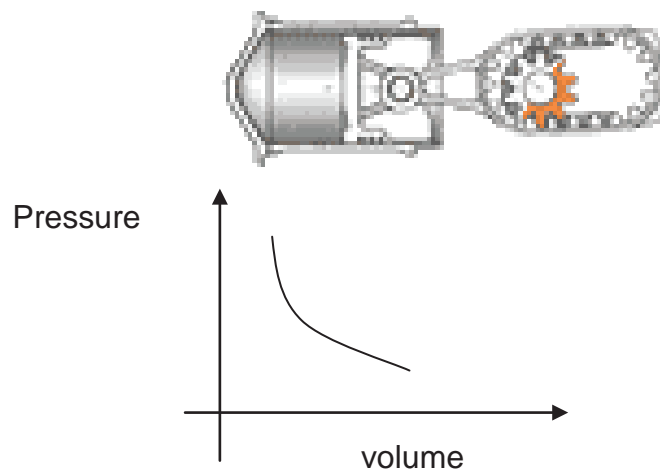
$$\frac{V_1}{V_2} = \left( \frac{T_2}{T_1} \right)^{1/(n-1)} = \left( \frac{349.9 + 273}{27 + 273} \right)^{1/0.3} = 11.4$$

### Worked Example 2.10

The gas in an internal combustion engine, initially at a temperature of  $1270^\circ\text{C}$ ; expands polytropically to five times its initial volume and one-eighths its initial pressure. Calculate:

- the index of expansion,  $n$ , and
- the final temperature.

**Solution:**



a) Since 
$$\frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^n$$

$n$  can be found by taking log of both sides, then rearranging the above equation

$$n = \frac{\frac{\ln(P_2/P_1)}{\ln(V_1/V_2)}}{\frac{\ln(1/8)}{\ln(1/5)}} = \frac{\ln(1/8)}{\ln(1/5)} = 1.292$$

- b) the final temperature is now evaluated:

$$\begin{aligned}T_2 &= T_1 \left( \frac{P_2}{P_1} \right)^{(n-1)/n} \\&= (1270 + 273) \left( \frac{1}{8} \right)^{0.2921/1.292} \\&= 698\text{K}\end{aligned}$$

**Worked Example 2.11**

Determine using Steam Tables, the volume occupied by 2 kg of steam at 500 kPa, under the following conditions and specify the state of steam.

- a) pure liquid state
- b) when it is in a pure vapour state
- c) 20% moisture content
- d) 20% dry.

$p$	$t_s$	$v_f$	$v_g$	$h_f$	$h_g$	$s_f$	$s_g$
(kPa)	(°C)	(m <sup>3</sup> /kg)	(m <sup>3</sup> /kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)
500	151.86	0.00109	0.3749	640.23	2,749	1.8607	6.8213

**Solution:**

From Steam Table:-  $v_f = 0.00109 \text{ m}^3/\text{kg}$  and  $v_g = 0.3749 \text{ m}^3/\text{kg}$

a) when pure liquid

$$v = 0.00109 \text{ m}^3/\text{kg}$$

$$\therefore V = 2 \times 0.00109 = 0.00218 \text{ m}^3$$

b) when it is saturated vapour

$$v = v_g = 0.3749 \text{ m}^3/\text{kg}$$

$$\therefore V = 2 \times 0.3749 = 0.7498 \text{ m}^3$$

c) The steam is obviously in its wet phase.  $X=0.8$

$$\therefore v = (1 - x) v_f + x v_g = 0.2 \times 0.00109 + 0.8 \times 0.3749 = 0.2785 \text{ m}^3/\text{kg}$$

$$\therefore V = 2 \times 0.2785 = 0.557 \text{ m}^3$$

d) The steam is obviously in its wet phase.  $X=0.2$

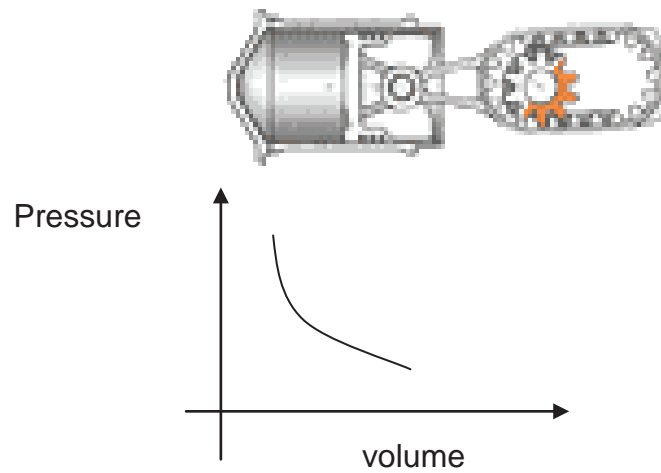
$$\therefore v = (1 - x) v_f + x v_g = 0.8 \times 0.00109 + 0.2 \times 0.3749 = 0.0758 \text{ m}^3/\text{kg}$$

$$\therefore V = 2 \times 0.0758 = 0.152 \text{ m}^3$$

**Worked Example 2.12**

The model '6SE-TCA3 Perkins' diesel engine have a stroke of 190 mm and a bore of 160 mm. If its clearance volume is 5% of the swept volume, determine the pressure and temperature at the end of compression when the inlet condition is 1 bar, 27°C.

Assume  $n = 1.38$



**Solution:**

The swept volume is

$$V_1 = V_2 = \frac{\pi}{4} D^2 L = \frac{\pi}{4} (0.160)^2 \times 0.190 = 0.00382 \text{ m}^3$$

The clearance volume is

$$V_c = \frac{5}{100} \times 0.00382 = 0.00019 \text{ m}^3$$

or 0.19 litre

$$\text{hence } V_1 = V_s + V_c = 3.82 + 0.19 = 4.01 \text{ litres}$$

The final pressure is:

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^n = 1 \left( \frac{4.01}{0.19} \right)^{1.38} = 67.2 \text{ bar}$$

The final temperature is:

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{n-1} = 300 \left( \frac{4.01}{0.19} \right)^{0.38} = 956 \text{ K}$$

## 3. Laws of Thermodynamics

There are four laws which relates the thermodynamics of substances.

### 3.1 Zeroth Law of Thermodynamics

If an object with a higher temperature comes in contact with a lower temperature object, it will transfer heat to the lower temperature object. The objects will approach the same temperature, and in the absence of loss to other objects, they will maintain a single constant temperature. Therefore, thermal equilibrium is attained.

The "zeroth law" states that if two systems are at the same time in thermal equilibrium with a third system, they are in thermal equilibrium with each other.

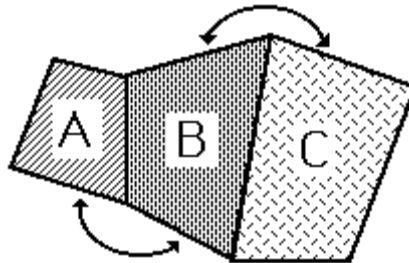


Figure 3.1: Analogy of the Zeroth Law of Thermodynamics.

If A and C are in thermal equilibrium with B, then A is in thermal equilibrium with B. Practically this means that all three are at the same temperature, and it forms the basis for comparison of temperatures. The Zeroth Law states that:-

*“two systems which are equal in temperature to a third system are equal in temperature to each other”.*

#### 3.1.1 Methods of Measuring Temperature

The Zeroth Law means that a thermometer can be used to assign a label to any system, the label being the value shown on the thermometer when it is thermal equilibrium with the system. We call the label “temperature”. Temperature is a function of state which determines whether a system will be in equilibrium with another.

To put this into practice we need a thermometer, which is a device that has some easily measured property,  $X$ , that varies with temperature. This might be the length of the mercury column in a mercury-in-glass thermometer for instance, or the pressure of a constant volume gas thermometer. We then require easily reproduced calibration temperatures. For instance, the *Centigrade*

temperature scale assigns temperatures of 0 and 100, to the temperature of ice in equilibrium with water (known as the ice point) and to the temperature of boiling water (the steam point). Letting the values of  $X$  at these two points be  $X_0$  and  $X_{100}$ , then the temperature in Centigrade,

Therefore, measuring temperatures may be based on one of the following properties:-

- a) Expansion of materials due temperature variations, eg gas thermometer, liquid-in-glass thermometer, bi-metal strip.
- b) Electrical resistance of materials.
- c) Electro-motive force induced within a circuit made up of two dissimilar materials.
- d) Radiative properties of surfaces.

### 3.1.2 International Temperature Scale

This scale is used to calibrate temperature measuring devices. It consists of a number of fixed points of known temperature which can be reproduced accurately (Table 3.1).

### Units of Temperature

The Kelvin (SI units) is the fraction(1/273.16) of the thermodynamic temperature of the triple point of water. Generally, conversion of Celsius to Kelvin :  $T(K) = T(^{\circ}C) + 273.15$

Fixed Point	Temperature (deg C)
Ice Point	0.01
Steam Point	100.00
Solidification of Antimony	630.74
Solidification of Gold	1064.43

Table 3.1 Example of Thermodynamic fixed temperatures

## 3.2 First Law of Thermodynamics

The first law of thermodynamics is the application of the conservation of energy principle.

### 3.2.1 First Law of Thermodynamics Applied to closed Systems

consider a closed system where there is no flow into or out of the system, and the fluid mass remains constant. For such system, the first law statement is known as the Non-Flow Energy Equation, or NFEE abbreviated, it can be summerised as follows:

$$\Delta U = Q - W \quad (25)$$

The first law makes use of the key concepts of internal energy ( $\Delta U$ ), heat ( $Q$ ), and system work ( $W$ ).

### 3.2.2 Internal Energy

Internal energy is defined as the energy associated with the random, disordered motion of molecules. It is separated in scale from the macroscopic ordered energy associated with moving objects; it refers to the invisible microscopic energy on the atomic and molecular scale. For example, a room temperature glass of water sitting on a table has no apparent energy, either potential or kinetic . But on the microscopic scale it is a seething mass of high speed molecules traveling at hundreds of meters per second. If the water were tossed across the room, this microscopic energy would not necessarily be changed when we superimpose an ordered large scale motion on the water as a whole.

During a non flow process the change in internal energy is calculated assuming the closed's system volume remains constant, the following equation is used



$$\Delta U = m.C_v.\Delta T \quad (26)$$

Where  $C_v$  is the specific heat capacity of the fluid, and  $\Delta T$  is the temperature difference during the process

### 3.2.3 Specific Heat

Heat may be defined as energy in transit from a high temperature object to a lower temperature object. An object does not possess "heat"; the appropriate term for the microscopic energy in an object is internal energy. The internal energy may be increased by transferring energy to the object from a higher temperature (hotter) object - this is properly called heating.

In order to heat or cool a given quantity of a gas in a given time, the following equation is used:

Quantity of Heat ( $Q$ ) = mass( $m$ ) x specific heat capacity( $C$ ) x temperature difference

Since this heat exchange may take place

Either at constant pressure:  $Q = \dot{m} C_p (T_2 - T_1)$  (27)

Or at constant volume:  $Q = \dot{m} C_v (T_2 - T_1)$  (28)

Where:

$C_p$	specific heat at constant pressure (kJ/kg K), see Table 3.2
$C_v$	specific heat at constant volume (kJ/kg K), see Table 3.2

Note that for a perfect gas  $C_p = C_v + R$  and  $\gamma = C_p / C_v$  (29)

### Specific Heat at Constant Volume $C_v$

Consider a closed system of unit mass, the first law of thermodynamics applied to this system is:-

$$q - w = du$$

If the volume is constant, then  $w = 0$ , it follows that  $q = du$

But  $q = C_v dT$

Hence  $du = C_v dT$

Or  $C_v = du/dT$

This is known as Joule's Law of internal energy which states that "the internal energy of a perfect gas depends upon its temperature only".

### Specific Heat at Constant Pressure $C_p$

Consider a constant pressure non-flow process, the first law:-

$$q - w = du$$

For a constant pressure process

$$W = p(v_2 - v_1) = (p_2 v_2 - p_1 v_1)$$

$$\text{hence } q - (p_2 v_2 - p_1 v_1) = u_2 - u_1$$

$$\text{or } q = (u_2 + p_2 v_2) - (u_1 + p_1 v_1) = h_2 - h_1$$

$$\text{but } q = C_p dT$$

$$\text{hence } h_2 - h_1 = C_p (T_2 - T_1)$$

$$\text{or } C_p = dh/dT$$

### Relationship Between Specific Heats

Since  $H = U + PV$

$$dH = dU + d(PV) \quad ; \quad d(PV) = d(mRT) = mRdT$$

$$m C_p dT = m C_v dT + m R dT$$

Therefore,  $C_p = C_v + R$       I.e       $C_p > C_v$

The ratio  $C_p/C_v = n$  is called the adiabatic index.

The reason for the differences between  $C_p$  and  $C_v$  is that in the constant pressure process part of the heat transferred is used in doing work against the moving system boundary and all heat therefore is not available for raising the gas temperature.

Temperature K	$C_p$ J/kgK	$C_v$ J/kgK	$n$ $C_p / C_v$
250	1003	716	1.401
300	1005	718	1.400
350	1008	721	1.398
400	1013	726	1.395
450	1020	733	1.391
500	1029	742	1.387
550	1040	753	1.381
600	1051	764	1.376
650	1063	776	1.370
700	1075	788	1.364
750	1087	800	1.359
800	1099	812	1.354
900	1121	834	1.344
1000	1142	855	1.336

Table 3.2 Specific heat capacities for air at standard atmosphere

### 3.2.4 System Work

Work performed on or by the working fluid within a system's boundary is defined as the summation (or integration) of the product of pressure and volume of the fluid during a process.

$$W = \int P.dV \quad (30)$$

In calculating the process work, it is important to point out that for each process, the work done will be different, since there are four distinctly different processes, in the following sections, an expression for the work done will be evaluated for each process.

a. for a constant pressure process, the work during an isobaric process is simply :

$$W = p (V_2 - V_1) \quad (31)$$

b. for a constant volume process,  $dv = 0$ ; hence the work during an isochoric process is simply :

$$W = 0 \quad (32)$$

c. for a constant temperature ,isothermal process,  $PV = c$

hence 
$$W = \int \frac{c}{V} dV = c \ln \frac{V_2}{V_1} \quad (33)$$

but 
$$P_1 V_1 = P_2 V_2 \quad \therefore \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

hence the work done can be written in terms of pressure ratio:

$$\therefore W = mRT_1 \ln \frac{P_1}{P_2} \quad (34)$$

d. for an adiabatic ( polytropic ) process,  $PV^n = c$

integrating between states 1 and 2, the work done is derived :

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1} \quad (35)$$

and using the ideal gas definition, the work done can be written in terms of initial and final states temperatures:

$$W = \frac{mR(T_1 - T_2)}{n-1} \quad (36)$$

### 3.2.5 First Law of Thermodynamics Applied to Closed Systems (Cycle)

Since in a cycle the working fluid undergoing changes in its state will retain its initial conditions at any fixed point along the cycle. Hence the energy equation applied to a cycle is:-

$$\Sigma Q - \Sigma W = 0 \quad (37)$$

where  $\Sigma$  means the sum of heat or work around the cycle.

Practical Application of a Closed System (Cycle) - Assume compression and expansion to be adiabatic, from first law :-

$$Q_s - Q_r = W_e - W_c \quad (38)$$

Where  $Q_s$  heat supplied

$Q_r$  heat rejected

$W_c$  work of compression

$W_e$  work of expansion

### 3.2.6 First Law of Thermodynamics Applied to Open Systems

The first law of thermodynamics is based on the conservation of energy within a system. Open systems are associated with those, which have a steady flow, thus the first law applied to such systems is known as the Steady Flow Energy Equation (SFEE):-

$$Q - W = (\Delta H + \Delta KE + \Delta PE)$$

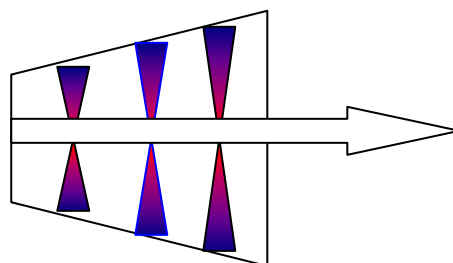
$$= m \left[ (h_2 - h_1) + \frac{1}{2} (V_2^2 - V_1^2) + g (Z_2 - Z_1) \right] \quad (39)$$

Variable	Symbol	Units
Heat transfer	Q	W
Work transfer	W	W
Mass flow rate	m	kg/s
Specific enthalpy	h	J/kg
Velocity	V	m/s
Gravitational acceleration	g	9.81 m/s <sup>2</sup>
Elevation above datum	z	m

### 3.2.6 Application of SFEE

#### a. Turbines or Compressors

if the SFEE is applied to the expansion of a fluid in a turbine as shown



$$Q - W = m (\Delta h + \Delta ke + \Delta Pe)$$

With the following simplifications are made

$$Q = 0 ,$$

$$\Delta ke = 0 ,$$

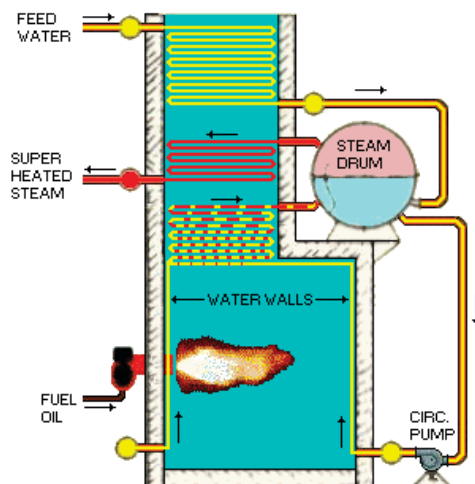
$$\Delta Pe = 0 \text{ are all neglected.}$$

$$\therefore W = m(h_1 - h_2) \quad (40)$$

hence for a turbine, the amount of energy produced “Work “ is equal to the enthalpy change between inlet and outlet.

### b. Boilers or Condensors

if the SFEE is applied to the heating or cooling ( evaporation or condensation ) of a fluid in a boiler or condensor



$$Q - W = m (\Delta h + \Delta ke + \Delta Pe)$$

With the following simplifications are made

There is no process work on the fluid  $W = 0 ,$

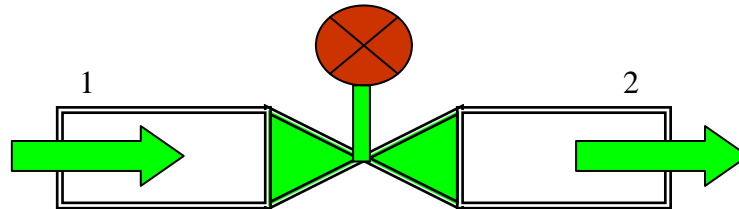
If Kinetic energy and Potential energy changes  $\Delta ke = 0 , \Delta Pe = 0$  are neglected. Then the SFEE reduces to :

$$\therefore Q = m(h_1 - h_2) \quad (41)$$

hence for a boiler or a condensor, the amount of energy supplied or extracted from the fluid “Heat” is equal to the enthalpy change for the fluid between inlet and outlet.

### c. Throttling valve

Consider the flow of fluid through a small valve as shown



if the SFEE is applied between sections 1 and 2 :

$$Q - W = m (\Delta h = \Delta ke + \Delta Pe)$$

$$Q = 0 \quad \text{Assuming adiabatic}$$



$W = 0$  No displacement work ( no work is inputted or extracted, ie no pump or turbine is attached )

$\Delta ke = 0$  Assumed ( inlet and exit velocities are similar or slow)

$\Delta Pe = 0$  Assumed ( entry and exit at the same or nearly the same elevation )

Hence, The SFEE, reduces to:

$$\therefore m(h_2 - h_1) = 0$$

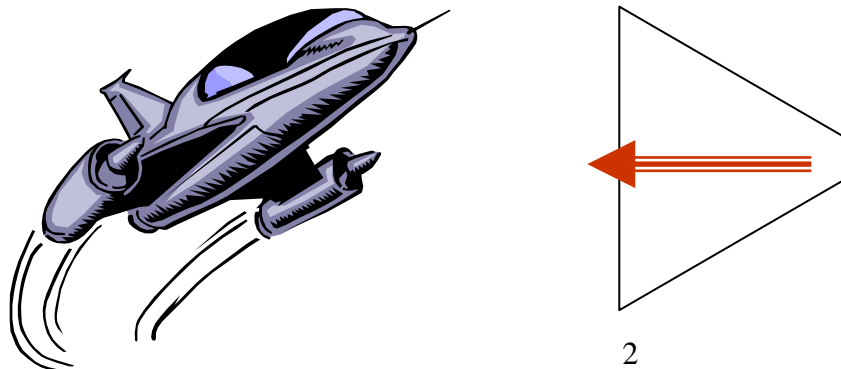
divide by the mass flow  $m$  to get:

$$\therefore h_2 = h_1 \quad (42)$$

hence for a control valve, the enthalpy of the fluid remains constant.

#### d. Diffuser

Consider the flow of fluid through a diffuser which is a device used in aircraft to reduce the kinetic energy of exhaust gases, as shown



if the SFEE is applied between sections 1 and 2:

$$Q - W = m(\Delta h = \Delta ke + \Delta Pe)$$

$Q = 0$  Assuming adiabatic

$W = 0$  No displacement work ( no work is inputted or extracted, ie no pump or turbine is attached )

$\Delta Pe = 0$  Assumed ( entry and exit at the same or nearly the same elevation )

Hence, The SFEE, reduces to:

$$\therefore h_2 - h_1 = \frac{V_2^2 - V_1^2}{2} \quad (43)$$

### 3.3 The Second Law of Thermodynamics

The second law of thermodynamics is a general principle which places constraints upon the direction of heat transfer and the attainable efficiencies of heat engines. It's implications may be visualized in terms of the waterfall analogy.

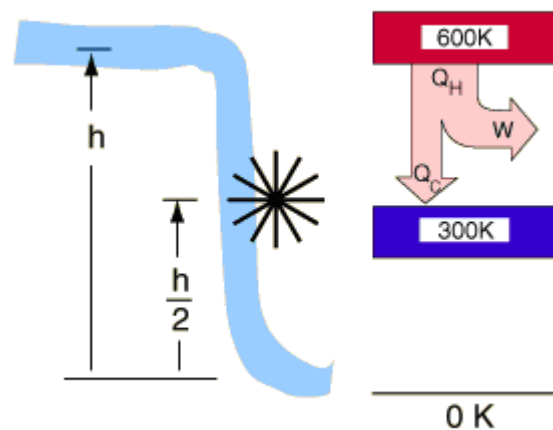


Figure 3.2: Analogy of the 2<sup>nd</sup> Law of Thermodynamics

#### 3.3.1 Second Law of Thermodynamics – statements:

The second law, indicates that, although the net heat supplied,  $Q_1 - Q_2$ , in a cycle is equal to the network done,  $W$ , the gross heat supplied,  $Q_1$  must be greater than the network done; some heat must always be rejected by the system.

$$Q_1 > W, \quad \text{or to be precise:-}$$

$$W = Q_1 - Q_2 \quad (44)$$

Where,  $Q_1$  is the heat supplied and  $Q_2$  is the heat rejected.

The ratio of network output to heat supplied is known as the thermal efficiency of the system.

There are two ways in which the second law is expressed:-

## a) Kelvin-Planck Statement

*“It is impossible to construct a system which when operating in a cycle will extract heat energy from a single source and do an equal amount of work on the surroundings”.*  
*It is never possible to achieve 100% thermal efficiency*

## b) Clausius Statement

*“It is impossible to construct a device which when operating in a cycle has no other effect than the transfer of heat energy from a cool to a hotter body”.*  
*It is not possible to do some work is done on or by the working fluid to the surroundings or vice versa.*

In 1865 Clausius observed that the amount  $dQ/T$  is proper to describe the thermodynamic phenomenon. This amount was named reduced heat or entropy. During a process, the change in entropy is defined as :

$$dS = Q/T \quad (45)$$

Definitions of Entropy :

1. is a state variable whose change is defined for a reversible process at  $T$  where  $Q$  is the heat absorbed.
2. a measure of the amount of energy which is unavailable to do work.
3. a measure of the disorder of a system.

### 3.3.2 Change of Entropy for a Perfect Gas Undergoing a Process

The First Law of Thermodynamics:-  $Q - W = \Delta U$

And the Second Law of Thermodynamics:-  $Q = Tds$

Therefore,

$$Tds - pdv = C_v dT, \quad \text{divide by } T$$

$$ds = C_v \frac{dT}{T} + R \frac{dV}{V}$$

$$s_2 - s_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

Since,  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$  then  $\frac{V_2}{V_1} = \frac{P_1 T_2}{P_2 T_1}$

Therefore,

$$\begin{aligned} s_2 - s_1 &= C_v \ln \frac{T_2}{T_1} + R \ln \left( \frac{P_1}{P_2} \right) \left( \frac{T_2}{T_1} \right) \\ &= C_p \ln \frac{T_2}{T_1} - R \ln \left( \frac{P_2}{P_1} \right) \end{aligned}$$

Since,  $C_v + R = C_p$

This can also be written in terms of volumes and pressures as:-

$$S_2 - S_1 = C_v \ln(P_2 / P_1) + C_p \ln(V_2 / V_1) \quad (46)$$

### 3.3.3 Implications of the Second Law of Thermodynamics

Since entropy is defined as a property which remains constant during a reversible adiabatic process; it follows that a temperature-entropy diagram would indicate a process by a straight line perpendicular to the entropy axis if the process is purely isentropic Figure (Figure 3.3). The friction in an irreversible process will cause the temperature of the fluid to be higher than it would have been in a frictionless (reversible) process. The entropy increased during an irreversible process.

**Isentropic Efficiency** – the entropy change in an irreversible adiabatic process leads to process efficiency. The ideal constant entropy process is termed isentropic and the ratio of the specific work transfer in the ideal process to that in the actual process is called the isentropic efficiency.

Since the majority of adiabatic machines are flow processes, isentropic efficiency is usually expressed in terms of the useful work  $W$ .

For the compression process, 
$$\eta_{ic} = \frac{W_{12'}}{W_{12}} \quad (47)$$

And for the expansion process 
$$\eta_{it} = \frac{W_{12}}{W_{12'}} \quad (48)$$

If changes in kinetic and potential energy are negligible, the SFEE may be used to rewrite these expressions in terms of specific enthalpy change and for a perfect gas, enthalpy change may be expressed by temperature change. Thus, for compression processes,(Figure 3.3):

$$\eta_{ic} = \frac{h_{2'} - h_1}{h_2 - h_1} \quad (49)$$

for a perfect gas it becomes

$$\eta_{ic} = \frac{T_{2'} - T_1}{T_2 - T_1} \quad (50)$$

The physical interpretation of this efficiency is that an irreversible compression process requires more work than an ideal process ( Figure 3.3) and in irreversible expansion process gives less work than an ideal process.

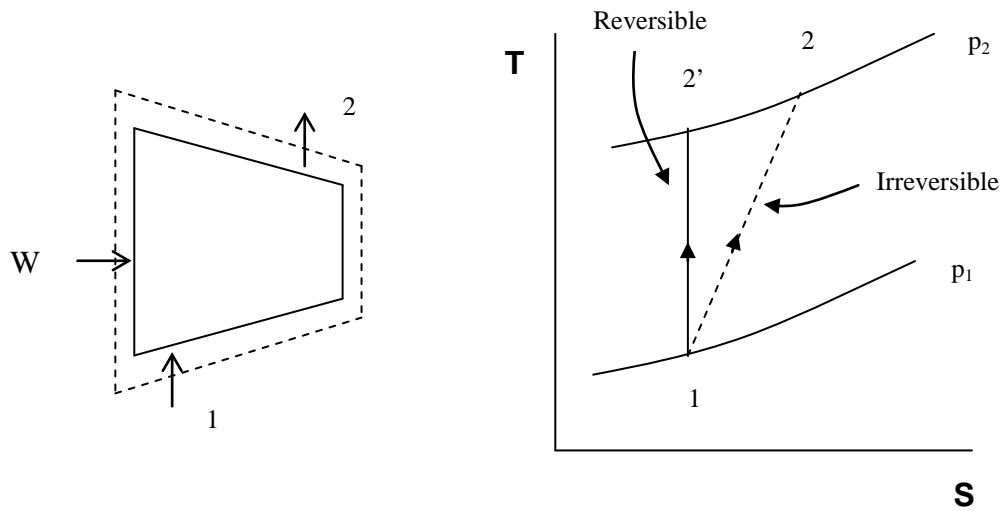


Figure 3.3 isentropic efficiency concept

### 3.4 Third Law

The entropy of a perfect crystal is zero when the temperature of a the crystal is equal to absolute zero (0 K).

- At 0 K, there is no thermal motion, and if the crystal is perfect, there will be no disorder
- Once the temperature begins to rise above 0, the particles begin to move and entropy gradually increases as the average kinetic energy of the particles increases.
- When temperature reaches the melting point of the substance ( $T_m$ ), there is an abrupt increase in entropy as the substance changes from a solid to a more disordered liquid.

Again the entropy increases gradually as the motion of the particles increases until the temperature reaches the boiling point of the substance ( $T_b$ ). At this point, there is another drastic increase in entropy as the substance changes from a confined liquid particles to radom motion gas particles.

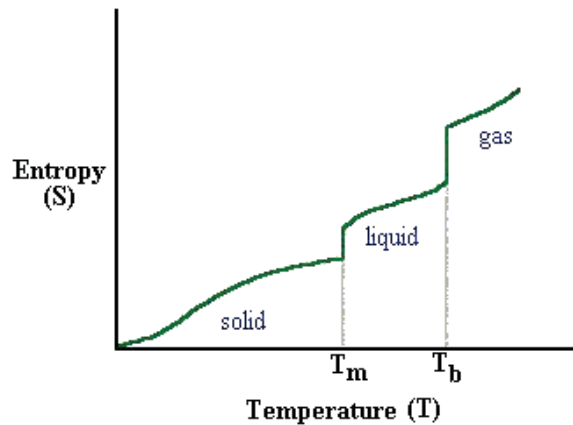


Figure 3.4 Temperature – Entropy relationship

### 3.4.1 The Third Law of Thermodynamics - Analysis

If we have sufficient heat capacity data (and the data on phase changes) we could write

$$S(T) = S(T = 0) + \int_0^T \frac{C_p}{T} dT. \quad (51)$$

(If there is a phase change between 0 K and T we would have to add the entropy of the phase change.)

Experimentally it appears that the entropy at absolute zero is the same for all substances. The third law of thermodynamics codifies this observation and sets

$$S(T = 0) = 0$$

for all elements and compounds in their most stable and perfect crystalline state at absolute zero and one atmosphere pressure. (All except for helium, which is a liquid at the lowest observable temperatures at one atmosphere.)

The advantage of this law is that it allows us to use experimental data to compute the absolute entropy of a substance. For example, suppose we want to calculate the absolute entropy of liquid water at 25° C. We would need to know the  $C_p$  of ice from 0 K to 273.15 K and the  $C_p$  of liquid water from 273.15 K to 298.15 K. The value of the Entropy is determined by the following equation:

$$S_{\text{H}_2\text{O}}^{\circ}(25^{\circ}\text{C}) = 0 + \int_0^{273.15} \frac{C_p(s)}{T} dT + \frac{\Delta H_{\text{fus}}}{273.15} + \int_{273.15}^{298.15} \frac{C_p(l)}{T} dT. \quad (52)$$

Process	Constant Volume	Constant Pressure	Constant Temperature	Polytropic	Reversible Adiabatic or Isentropic
Law	$\frac{P}{T} = \text{const}$	$\frac{V}{T} = \text{const}$	$PV = \text{const}$	$PV^n = \text{const}$	$PV^{\gamma} = \text{const}$
P, V, T. Relation	$\frac{P_1}{T_1} = \frac{P_2}{T_2}$	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$	$P_1 V_1 = P_2 V_2$	$P_1 V_1^n = P_2 V_2^n$ $\frac{P_1}{P_2} = \left( \frac{V_2}{V_1} \right)^n$ $= \left( \frac{T_1}{T_2} \right)^{\frac{n}{n-1}}$	$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$ $\frac{P_1}{P_2} = \left[ \frac{V_2}{V_1} \right]^{\gamma}$ $= \left[ \frac{T_1}{T_2} \right]^{\frac{\gamma}{\gamma-1}}$
Change in Internal Energy $\Delta U$	$mC_v(T_2 - T_1)$	$mC_v(T_2 - T_1)$	0	$mC_v(T_2 - T_1)$	$mC_v(T_2 - T_1)$
Work Transfer $W = \int p dv$	0	$P(V_2 - V_1)$ Or $mR(T_2 - T_1)$	$PV \ln \frac{V_2}{V_1}$ Or $mRT \ln \frac{V_2}{V_1}$	$\frac{P_1 V_1 - P_2 V_2}{(n-1)}$ $\frac{mR(T_1 - T_2)}{(n-1)}$	$\frac{P_1 V_1 - P_2 V_2}{(\gamma-1)}$ $\frac{mR(T_1 - T_2)}{(\gamma-1)}$ $mC_v(T_1 - T_2)$
Heat Transfer Q	$mC_v(T_1 - T_2)$	$mC_p(T_1 - T_2)$	$PV \ln \frac{V_2}{V_1}$ Or $mRT \ln \frac{V_2}{V_1}$	$W + (U_2 - U_1)$	0
Change in Entropy $\Delta S = S_2 - S_1$	$mC_v \ln \frac{T_2}{T_1}$ or $mC_v \ln \frac{P_2}{P_1}$	$mC_p \ln \frac{T_2}{T_1}$ or $mC_p \ln \frac{V_2}{V_1}$	$mR \ln \frac{V_2}{V_1}$ or $mR \ln \frac{P_1}{P_2}$	$m \left[ R \ln \frac{V_2}{V_1} + C_v \ln \frac{T_2}{T_1} \right]$ $m \left[ R \ln \frac{P_1}{P_2} + C_p \ln \frac{T_2}{T_1} \right]$ $m \left[ C_p \ln \frac{V_2}{V_1} + C_v \ln \frac{P_2}{P_1} \right]$	0

Table 3.3: Perfect Gas Process-Relation



**Worked Example 3.1**

A closed rigid container has a volume of  $1 \text{ m}^3$  and holds air at 345 kPa and  $20^\circ\text{C}$ . Heat is added until the temperature is  $327^\circ\text{C}$ . Determine the change in Internal Energy:-

- a) Using an average value of the specific heat.
- b) Taking into account the variation of specific heat with temperature.

**Solution:**

a)  $\Delta U = mC_v \Delta T$

$$C_v = \frac{764 + 718}{2} = 741 \text{ J/kgK}$$

$$m = \frac{PV}{RT} = \frac{345 \times 10^3 \times 1}{287 \times 293} = 4.1026 \text{ kg}$$

Therefore

$$\Delta U = 4.1026 \times 741 (327 - 20) = 932 \text{ kJ}$$

$$\text{b) } \Delta U = m \int_{T_1}^{T_2} C_v \cdot T$$

$$\Delta u = \int_{T_1}^{T_2} C_v T = C_{v_2} T_2 - C_{v_1} T_1$$

$$= 764 \times 600 - 718 \times 293$$

$$= 248,319 \text{ J/kg}$$

$$m = \frac{PV}{RT} = \frac{345 \times 10^3 \times 1}{287 \times 293} = 4.1026 \text{ kg}$$

Therefore,

$$\Delta U = m \times \Delta u = 1018.7 \text{ kJ}$$

**Worked Example 3.2**

An adiabatic steam turbine expands steam from a pressure of 6 MPa and a temperature of 500°C to a pressure of 10 KPa. The isentropic efficiency of the turbine is 0.82 and changes in kinetic and potential energy may be neglected. Determine the state of the steam at exit from the turbine and the specific work transfer.

$T$	$p = 6.0 \text{ MPa (257.64 deg-C)}$			
	$v$	$u$	$h$	$s$
Sat.	0.03244	2589.7	2784.3	5.8892
500	0.05665	3082.2	3422.2	6.8803

$p$	$t_s$	$v_f$	$v_g$	$h_f$	$h_g$	$s_f$	$s_g$
(kPa)	(°C)	(m³/kg)	(m³/kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)
10	45.81	0.001	14.674	191.83	2,585	0.6493	8.1502

**Solution:**

From steam tables at 6 MPa and 500°C,  $h_1 = 3422.2 \text{ kJ/kg}$ , and  $s_1 = 6.8803 \text{ kJ/kg K}$

At 10 KPa,  $s_f = 0.6493$ , and  $s_g = 8.1502$

$s_{2'} = s_1$  and  $x_{2'}$  is found using

Then  $6.8803 = 0.6493 + x_{2'} (8.1502 - 0.6493)$ , from which  $x_{2'} = 0.917$

This is the state of steam at exit from the turbine (mixed phase with dryness fraction,  $x=0.917$ )

Thus  $h_2 = h_f + x h_{fg} = 191.83 + 0.917 \times (2585 - 191.83) = 2387 \text{ kJ/kg}$

From

$$\eta_{it} = \frac{h_1 - h_2}{h_1 - h_{2'}}$$

ie,  $0.82 = \frac{3422.2 - h_2}{3422.2 - 2387}$

from which  $h_2 = 2573 \text{ kJ/kg}$

The turbine specific work ( $m=1$  kg/s) is  $=W = m \cdot (h_2 - h_1) = 1 \times (3422.2 - 2573) = 849$  kJ/kg.

### Worked Example 3.3

Show that for a polytropic process, the change in entropy may be expressed as :

$$S_2 - S_1 = \frac{\gamma - n}{n - 1} \cdot m \cdot C_v \cdot \ln \frac{T_1}{T_2}$$

**Solution:**

1<sup>st</sup> Law:  $Q = U + W$

with  $U = C_v \cdot dT$  and  $W = P \cdot dV$

from the 2<sup>nd</sup> law:  $Q = T \cdot dS$

hence

$$T \cdot dS = m C_v \cdot dT + P \cdot dV$$

With  $PV=RT$ , Then the above equation becomes:

$$T \cdot dS = m C_v \cdot dT + (mRT/V) \cdot dV$$

Divide by T

$$dS = m C_v \cdot dT/T + mR \cdot dV/V$$

integrate between states 1 and 2

$$S_2 - S_1 = m \left[ C_v \cdot \ln \frac{T_2}{T_1} + R \cdot \ln \frac{V_2}{V_1} \right]$$

for a polytropic process  $\frac{V_2}{V_1} = \left[ \frac{T_1}{T_2} \right]^{\frac{1}{n-1}}$

$$\text{hence } S_2 - S_1 = m \left[ C_v \cdot \ln \frac{T_2}{T_1} - \frac{R}{n-1} \cdot \ln \frac{T_2}{T_1} \right] = m \cdot \left[ C_v \left[ 1 - \frac{R}{n-1} \right] \cdot \ln \frac{T_2}{T_1} \right]$$

but  $C_p = C_v + R$

and  $C_p = \gamma \cdot C_v$

hence  $R = C_v \cdot (\gamma - 1)$

and thus

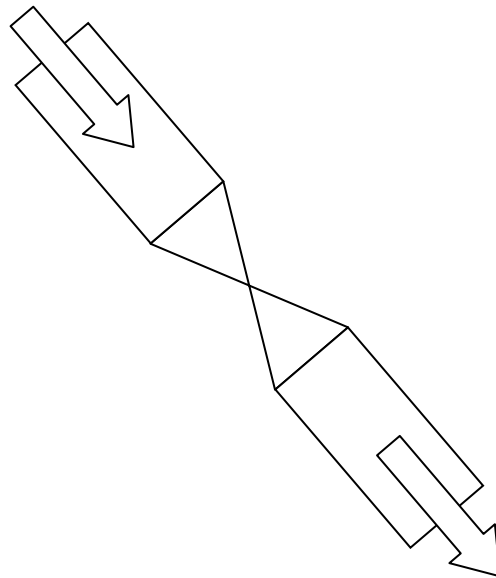
$$S_2 - S_1 = m \cdot C_v \left[ \frac{\gamma - n}{n-1} \right] \cdot \ln \frac{T_1}{T_2}$$

QED

**Worked Example 3.4**

Steam at 1.0 MPa, 0.95 dry is throttled to 10 kPa. What is the quality of the steam after throttling?

$p$ (kPa)	$t_s$ (°C)	$v_f$ (m <sup>3</sup> /kg)	$v_g$ (m <sup>3</sup> /kg)	$h_f$ (kJ/kg)	$h_g$ (kJ/kg)	$s_f$ (kJ/kg.K)	$s_g$ (kJ/kg.K)
10	45.81	0.001	14.674	191.83	2,585	0.6493	8.1502
1000	179.91	0.00112	0.1944	762.81	2,778	2.1387	6.5865



**Solution:**

SFEE

$$Q - W = m (\Delta h = \Delta ke + \Delta Pe)$$

The SFEE, reduces to:  $h_2 = h_1$

at 1.0 MPa ,  $h_f = 762.81$  kJ/kg and  $h_g = 2,778$  kJ/kg

$$h_1 = 762.81 + 0.95 \times (2778 - 762.81) = 2677.24 \text{ kJ/kg}$$

at 10 kPa with  $h_g = 2,585$  kJ/kg

since  $h_2 = h_1 = 2677.24$  kJ/kg , which is higher than  $h_g$  at 10 kPa, the steam will be in a superheated condition.

**Worked Example 3.5**

A steam turbine receives steam at 2 MPa and 250 °C, and exhausts at 0.1 MPa, 0.85 dry.

- Neglecting heat losses and changes in ke and Pe, estimate the work output per kg steam.
- If, when allowance is made for friction, radiation, and leakage losses, the actual work obtained is 80% of that estimated in (a), calculate the power output of the turbine when consuming 600 kg of steam per minute.

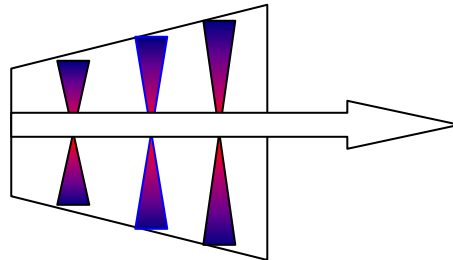
Saturated table extract for P = 100 KPa

$p$ (kPa)	$t_s$ (°C)	$v_f$ (m <sup>3</sup> /kg)	$v_g$ (m <sup>3</sup> /kg)	$h_f$ (kJ/kg)	$h_g$ (kJ/kg)	$s_f$ (kJ/kg.K)	$s_g$ (kJ/kg.K)
100	99.63	0.00104	1.694	417.46	2,676	1.3026	7.3594

Superheated table extract at P = 2 MPa

T (°C)	V (m <sup>3</sup> /kg)	h (kJ/kg)	S (kJ/kgK)
240	0.1084	2876	6.495

**Solution:**



$$a) \quad Q - W = m (\Delta h + \Delta ke + \Delta Pe)$$

$Q = 0$ ,  $\Delta ke = 0$ ,  $\Delta Pe = 0$  are all neglected.

$$\therefore W = m(h_1 - h_2)$$

$$h_1 = 2876 \text{ kJ/kg}$$

$$h_2 = 417.46 + 0.85 \times (2676 - 417.46) = 2337.2 \text{ kJ/kg}$$

$$\therefore W_{\text{ideal}} = 1 \times (2876 - 2337) = 539 \text{ kJ/kg}$$

$$b) \quad W_{\text{actual}} = 0.8 \times 539 = 431 \text{ kJ/kg}$$

$$\text{Power} = m \times W = (600/60) \times 431 = 4.31 \text{ MW}$$

**Worked Example 3.6**

A boiler receives feed water at 40°C and delivers steam at 2 MPa and 500°C. If the furnace is oil fired, the calorific value of oil being 42000 kJ/kg and 4000 kg oil are burned while 45000 kg of steam are produced, determine :

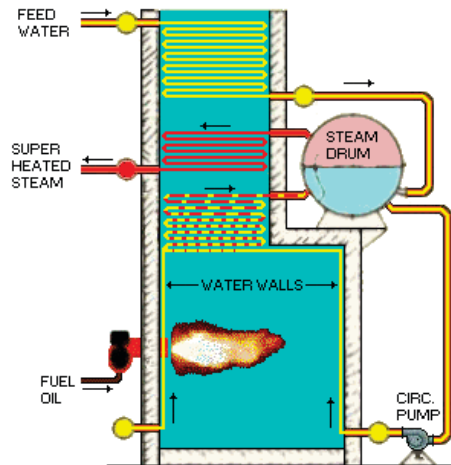
- a) the heat supplied in the boiler.
- b) the efficiency of the boiler.

Assume the values of enthalpies at the two state points as :

$$h_1 = h_{f@40^\circ\text{C}} = 169.33 \text{ kJ/kg} \quad \text{at 2 MPa, 500 C, } h_2 = 3467.6 \text{ kJ/kg}$$



**Solution:**



a) Constant pressure process.

$$h_1 = h_{f@40^\circ\text{C}} = 169.33 \text{ kJ/kg} \qquad h_2 = 3467.6 \text{ kJ/kg}$$

SFEE ignoring  $W$ ,  $\Delta ke$  and  $\Delta Pe$ :

$$Q_s = m_s (h_2 - h_1) = 45000 (3467.6 - 169.33) = 1.484 \times 10^8 \text{ kJ}$$

b) The heat generated by burning oil in the furnace is

= mass of oil burned x calorific value

$$= 4000 \times 42000 = 1.68 \times 10^8 \text{ kJ}$$

$$\therefore \text{Boiler efficiency} = \frac{\text{Energy Output}}{\text{Energy Input}} = \frac{1.484 \times 10^8}{1.68 \times 10^8} = 88\%$$

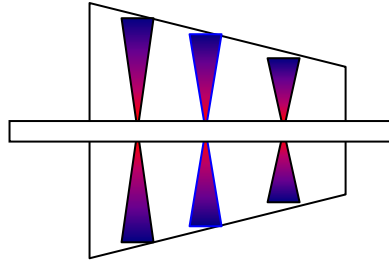
### Worked Example 3.7

An air compressor receives air at  $27^\circ\text{C}$  and delivers it to a receiver at the rate of  $0.5 \text{ kg/s}$ . It is driven by an electric motor which absorbs  $10 \text{ kW}$  and the efficiency of the drive is  $80\%$ .

Water jacket cooling is used at the rate of  $6 \text{ kg/min}$  while its temperature rises from  $10^\circ\text{C}$  to  $20^\circ\text{C}$ .

Estimate the temperature of the air delivered.

Data :  $C_{p_w} = 4.186$ , and  $C_{p_a} = 1.005 \text{ kJ/kgK}$

**Solution:**

SFEE neglecting  $\Delta KE$  and  $\Delta PE$  reduces to:

$$Q - W = \Delta H + \Delta KE + \Delta PE$$

Neglecting changes in velocity and elevation between inlet and exit to the compressor, change in enthalpy is equal to

$$\Delta H = Q - W$$

Heat loss to cooling water is  $Q = -m_w C_{p_w} (T_{w2} - T_{w1})$

$$= - (6/60) \times 4.186 \times (20 - 10)$$

$$= - 4.186 \text{ kW}$$

Actual work taking efficiency into account is

$$W = - 10 / 0.80 = -12.5 \text{ kW}$$

$$\Delta H = m_a C_{p_{air}} (T_{a2} - T_{a1})$$

Hence back to the SFEE :

$$\Delta H = Q - W$$

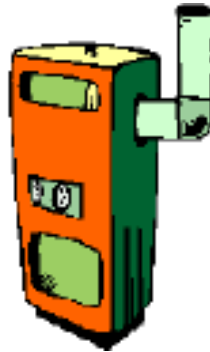
$$-4.186 + 12.5 = 0.5 \times 1.005 (T_{a2} - 27)$$

solving to find  $T_{a2} = 43.5^\circ\text{C}.$

**Worked Example 3.8**

Air at  $27^{\circ}\text{C}$  receives heat at constant volume until its temperature reaches  $927^{\circ}\text{C}$ . Determine the heat added per kilogram? Assume for air  $C_v = 0.718 \text{ kJ/kgK}$ .

**Solution:**



Closed system for which the first law of Thermodynamics applies,

$$Q - W = \Delta U$$

$W = 0$  No work transfer at constant volume process.

$$\therefore Q = \Delta U$$

$$\Delta U = Q = m.C_v.(T_2 - T_1)$$

$$= 1 \times 0.718 (927 - 27)$$

$$= 646.2 \text{ kJ/kg}$$

hence  $Q = \Delta U = 646.2 \text{ kJ/kg}$ .

Note that the  $C_v$  used is an average value.

### Worked Example 3.9

An insulated, constant-volume system containing 1.36 kg of air receives 53 kJ of paddle work. The initial temperature is  $27^\circ\text{C}$ . Determine

- the change of internal energy.
- the final temperature.

Assume a mean value  $C_v = 0.718 \text{ kJ/kgK}$ .

**Solution:**

$$\text{a) } Q - W = \Delta U$$

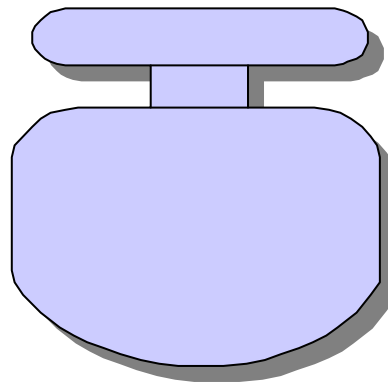
$$Q = 0 \text{ ( insulated system )}$$

$$W = -53 \text{ kJ ( externally inputted work )}$$

The change in internal energy  $\Delta U$  is

$$\Delta U = -W = +53 \text{ kJ} \quad \text{Since } Q = 0$$

$$\text{b) } \Delta U = m C_v \Delta T$$



$$\therefore 53 = 1.36 \times 0.718 (T_2 - 27)$$

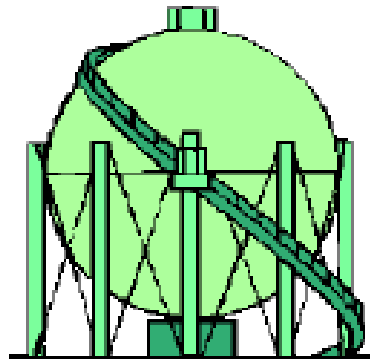
$$T_2 = 27 + \frac{53}{1.36 \times 0.718} = 81.3^\circ \text{C}$$

### Worked Example 3.10

An ideal gas occupies a volume of  $0.5 \text{ m}^3$  at a temperature of  $340 \text{ K}$  and a given pressure. The gas undergoes a constant pressure process until the temperature decreases to  $290 \text{ K}$ . Determine

- the final volume,
- the work if the pressure is  $120 \text{ kPa}$

**Solution :**



- Since  $P = \text{constant}$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$V_2 = V_1 \times \frac{T_2}{T_1} = 0.5 \times \frac{290}{340} = 0.426 \text{ m}^3$$

- $W = \int P.dV$

for a constant pressure process,

$$\begin{aligned}
 W &= p (V_2 - V_1) \\
 &= 120 (0.426 - 0.5) \\
 &= - 8.88 \text{ kJ}
 \end{aligned}$$

the negative sign indicates that work is imported from an external source on the system

### Worked Example 3.11

30 kg/s steam at 3 MPa, 300°C expands isentropically in a turbine to a pressure of 100 kPa. If the heat transfer from the casing to surrounding air represents 1 per cent of the overall change of enthalpy of the steam, calculate the power output of the turbine. Assume exit is 2 m above entry and that initial velocity of steam is 10 m/s whereas exit velocity is 1 m/s.

$p = 3.00 \text{ MPa (233.90 °C)}$				
$T$	$v$	$u$	$h$	$s$
300	0.08114	2750.1	2993.5	6.5390

$p$	$t_s$	$v_f$	$v_g$	$h_f$	$h_g$	$s_f$	$s_g$
(kPa)	(°C)	(m³/kg)	(m³/kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)
100	99.63	0.00104	1.694	417.46	2,676	1.3026	7.3594

**Solution:**

At 3 MPa, 300 °C

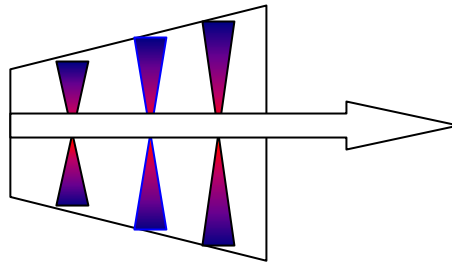
$$h_1 = 2993.5 \text{ kJ/kg}$$

$$s_1 = 6.539 \text{ kJ/kgK}$$

at 100 kPa,

$$s_f = 1.3026, s_g = 7.3594 \text{ kJ/kgK}$$

$$h_f = 417.46, h_g = 2676 \text{ kJ/kg}$$



$$x_2 = \frac{6.539 - 1.3026}{7.3594 - 1.3026} = 0.864$$

$$h_2 = 417.46 + 0.864 \times (2676 - 417.46) = 2370 \text{ kJ/kg}$$

The Steady Flow Energy Equation applies to this situation:

$$Q - W = m[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)]$$

with  $Q = -0.01(h_2 - h_1)$  Heat loss ( negative sign )

$$\begin{aligned}
 W &= -m[1.01 \times (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)] \\
 &= -30[(2993.5 - 2370) + \frac{1^2 - 10^2}{2 \times 1000} + 9.81 \times 2 / 1000] = +19 \text{ kW}
 \end{aligned}$$

+ indicate useful power output

**Worked Example 3.12**

A piston and cylinder mechanism contains 2 kg of a perfect gas. The gas expands reversibly and isothermally from a pressure of 10 bar and a temperature of 327 °C to a pressure of 1.8 bar.

Calculate:

- the work transfer,
- the heat transfer; and
- the specific change in enthalpy of the gas.

Take  $R=0.3 \text{ kJ/kg K}$  and  $n=1.4$

**Solution:**

- The work done is given by

$$W = \int P dV$$

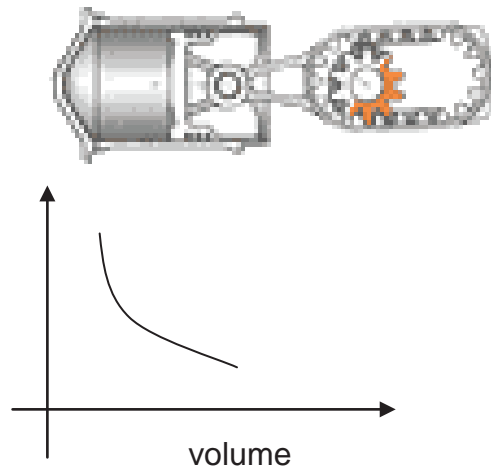
$$PV = c \text{ or } P = c/V$$

$$W = \int \frac{c}{V} dV = c \ln \frac{V_2}{V_1}$$

$$P_1 V_1 = P_2 V_2 \quad \therefore \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$\therefore W = mRT_1 \ln \frac{P_1}{P_2}$$

$$= 2 \times 0.3 \times 600 \times \ln\left(\frac{10}{1.8}\right) = 617 \text{ kJ}$$



- This is a closed system, hence Non-flow Energy Equation applies

$$Q - W = \Delta U$$

$$\Delta U = C_v \cdot \Delta T$$



since for isothermal process isotherma, ie temperature difference is zero, then the internal energy = 0, and hence  $Q = W = 617 \text{ kJ}$

c)  $\Delta h = C_p \cdot \Delta T = 0$  since for isothermal process .

### Worked Example 3.13

The gas expanding in the combustion space of a reciprocating engine has an initial pressure of 50 bar and an initial temperature of  $1623^\circ \text{C}$ . The initial volume is  $50000 \text{ mm}^3$  and the gas expands through a volume ratio of 20 according to the law  $pV^{1.25} = \text{constant}$ . Calculate

- a) the work transfer and
- b) heat transfer in the expansion process.

Take  $R = 270 \text{ J/Kg K}$  and  $C_v = 800 \text{ J/Kg K}$ .

**Solution:**

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^{1.25} = 50 \left( \frac{1}{20} \right)^{1.25} = 1.182 \text{ bar}$$

$$T_2 = T \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = (1623 + 273) \left( \frac{1.182}{50} \right)^{0.25} = 896.6 \text{ K}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

$$= \frac{50 \times 10^5 \times 50,000 \times 10^{-9} - 1.182 \times 50,000 \times 10^{-9} \times 20}{1.25 - 1.0} = 527 \text{ J}$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{50 \times 10^5 \times 50,000 \times 10^{-9}}{270 \times (1623 + 273)} = 4.88 \times 10^{-4} \text{ kg}$$

$$\Delta U = m C_v \Delta T = 4.88 \times 10^{-4} \times 800 (1623 + 273 - 896.6) = -1390 \text{ J}$$

$$Q - W = \Delta U$$

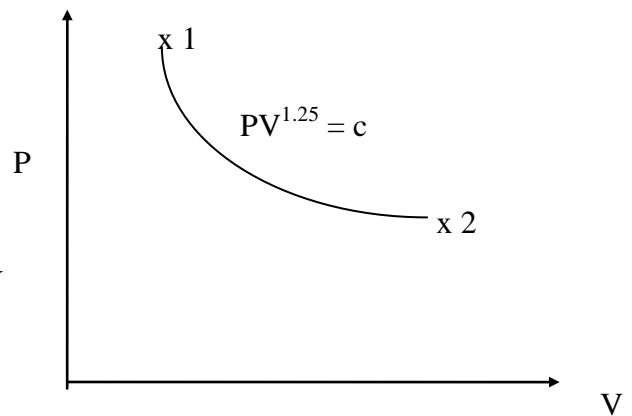
$$\therefore Q = \Delta U + W = -390 + 527 = 136 \text{ J}$$

**Worked Example 3.14**

A reciprocating steam engine cylinder contains 2kg of steam at a pressure of 30 bar and a temperature of 300 °C. The steam expands reversibly to a final pressure of 2 bar, according to the law  $p v^{1.2} = c$ . Calculate

- the final state of the steam,
- the work transfer and
- the heat transfer in the process.

$p = 3.00 \text{ MPa (233.90 °C)}$				
$T$	$V$	$u$	$h$	$s$
300	0.08114	2750.1	2993.5	6.5390



$p$	$t_s$	$v_f$	$v_g$	$h_f$	$h_g$	$s_f$	$s_g$
(kPa)	(°C)	(m³/kg)	(m³/kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)
200	120.23	0.00106	0.8857	504.7	2,707	1.5301	7.1271

**Solution:**

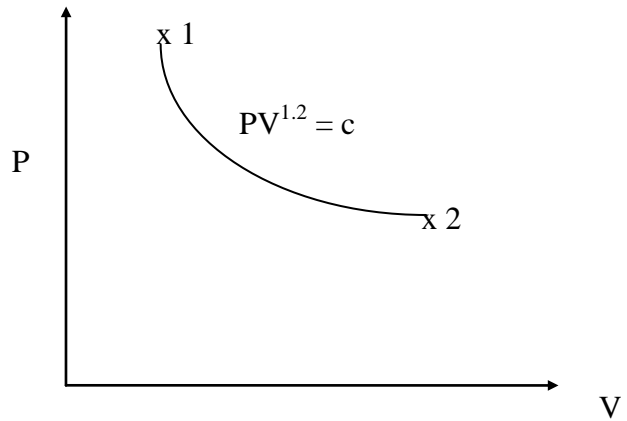
$$a) \quad V_1 = m \cdot v_1 = 2 \times 0.08114 = 0.16228 \text{ m}^3$$

$$V_2 = V_1 \left( \frac{P_1}{P_2} \right)^{\frac{1}{1.2}} = 0.16228 \left( \frac{30}{2} \right)^{\frac{1}{1.2}} = 1.55 \text{ m}^3$$

$$v_2 = \frac{V_2}{m} = \frac{1.55}{2} = 0.775 \text{ m}^3 / \text{kg}$$

$$v_2 = v_{f2} + x_2 v_{fg2}$$

$$\therefore x_2 = \frac{0.775 - 0.0010605}{0.8857 - 0.0010605} = 0.875$$



$$b) \quad W = \frac{P_1 V_1 - P_2 V_2}{n - 1} = \frac{30 \times 10^5 \times 0.16228 - 2 \times 10^5 \times 1.55}{1.2 - 1} = 884 \text{ kJ}$$

$$c) \quad u_2 = 504.49 + 0.875 \times 2025 = 2276.4 \text{ kJ / kg}$$

$$\Delta u = 2276.4 - 2750.1 = -473.7 \text{ kJ / kg} = -947.4 \text{ kJ},$$

$$\therefore Q = \Delta U + W = -947.4 + 884 = -63.4 \text{ kJ / kg}$$

**Worked Example 3.15**

Steam at a pressure of 6 MPa and a temperature of 500 °C enters an adiabatic turbine with a velocity of 20 m/s and expands to a pressure of 50 kPa, and a dryness fraction of 0.98. The steam leaves with a velocity of 200 m/s. The turbine is required to develop 1MW. Determine:

- a) the mass flow rate of steam required, when KE is neglected, and  
 b) What is the effect of KE on the answer?

$T$	$p = 6.0 \text{ MPa (257.64 deg-C)}$			
	$v$	$U$	$h$	$s$
500	0.05665	3082.2	3422.2	6.8803

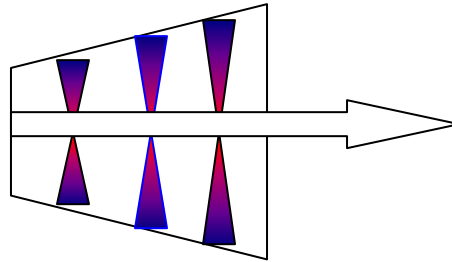
$p$	$t_s$	$v_f$	$v_g$	$h_f$	$h_g$	$s_f$	$s_g$
(kPa)	(°C)	(m³/kg)	(m³/kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)
100	99.63	0.00104	1.694	417.46	2,676	1.3026	7.3594

**Solution:**

a)  $h_1 = 3422.2 \text{ kJ/kg}$

$$h_2 = 340.49 + 0.98 \times (2676 - 417.46) = 2599.8 \text{ kJ/kg}$$

This situation is governed by the Steady Flow Energy Equation



$$Q - W = m \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(Z_2 - Z_1) \right]$$

Neglecting the changes in Kinetic and Potential energies,  $Q = 0$  for an adiabatic process

Approximately  $W = (h_1 - h_2)$

$$1 \times 10^3 = m(342.2 - 2599.8) \quad \text{hence } m = 1.216 \text{ kg/s}$$

(b) taking KE into consideration:  $W = m \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} \right]$

$$1 \times 10^3 = m \left( 3422.2 - 2599.8 + \frac{200^2 - 20^2}{2} \right)$$

$$m = 1.187 \text{ kg/s or an error of 2.4\%}$$

### Worked Example 3.16

Air, which may be considered a perfect gas, enters an adiabatic nozzle with negligible velocity. The entry pressure is 6 bar and the exit pressure is 1 bar; the entry temperature is 760 K. The flow throughout the nozzle is reversible and the mass flow rate is 2 kg/s. Calculate the exit velocity.

Take  $C_p = 1004.5 \text{ J/kg K}$  and  $n = 1.4$

**Solution:**

This situation is an open system for which the SFEE applies

$$Q - W = m \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(Z_2 - Z_1) \right]$$

$Q = 0$  adiabatic

$W = 0$  no work transfer in the system

For a perfect gas

$$h_2 - h_1 = C_p (T_2 - T_1)$$

$$g(Z_2 - Z_1) = 0 \text{ negligible}$$

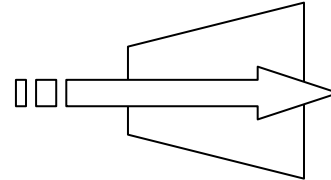
$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 760 \left( \frac{1}{6} \right)^{\frac{0.4}{1.4}} = 455.5 \text{ K}$$

The SFEE reduces to (dividing by the mass)

$$\therefore 0 = C_p (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$0 = 1004.5(455.5 - 760) + \frac{V_2^2}{2}$$

$$\therefore V_2 = 782 \text{ m/s}$$

**Worked Example 3.17**

3 kg/s of steam enters an adiabatic condenser at a pressure of 100 kPa with dryness fraction 0.80, and the condensate leaves the condenser at a temperature of 30 °C. The condenser is cooled by water which enters at a temperature of 5 °C and leaves at a temperature of 25 °C. Calculate the mass flow rate of cooling water required if all changes in kinetic and potential energy may be neglected. Assume  $C_p = 4.2 \text{ kJ/kgK}$ , and the enthalpy at 100 kPa & 30°C = 125 kJ/kg.

$p$	$t_s$	$v_f$	$v_g$	$h_f$	$h_g$	$s_f$	$s_g$
(kPa)	(°C)	(m <sup>3</sup> /kg)	(m <sup>3</sup> /kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)

100	99.63	0.00104	1.694	417.46	2,676	1.3026	7.3594
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**Solution:**

At 100 kPa

$$h_1 = 340.49 + 0.8x(2676 - 417.46) = 2224.3 \text{ kJ/kg}$$

at 100 kPa & 30 °C

$$h_2 = 125 \text{ kJ/kg}$$





## Energy Balance

$$m_s(h_1 - h_2) = m_w C_{pw}(t_{w2} - t_{w1})$$

$$3(2224.3 - 125) = m_w \times 4.2(25 - 5)$$

$$\therefore m_w = 75 \text{ kg/s}$$

Note that this represents a ratio of nearly 25 kg of water to every kg of steam

**Worked Example 3.18**

A reciprocating compressor delivers 0.1 kg/s of air at a pressure of 12 bar. The air enters the compressor at a pressure of 1 bar and a temperature of 15 °C. Calculate the delivery temperature of the air, the work transfer rate and the heat transfer rate in the compression process for:

- i. reversible polytropic compression,  $PV^{1.2} = \text{constant}$ ;
- ii. reversible adiabatic compression;
- iii. reversible isothermal compression.

Air -  $R = 0.287 \text{ kJ/kgK}$ ,  $C_p = 1.005 \text{ kJ/kg K}$ ,  $C_v = 0.718 \text{ kJ/kgK}$  and  $n = 1.4$ .

**Solution:**

$$\text{i. } T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = (288) \left( \frac{12}{1} \right)^{\frac{0.2}{1.2}} = 435.7 \text{ K}$$

$$W = \frac{mR(T_1 - T_2)}{n - 1} = \frac{0.1 \times 287(288 - 435.7)}{0.2} = -21.2 \text{ kW}$$

$$\Delta U = mC_v(T_2 - T_1) = 0.1 \times 0.718(435.7 - 288) = 10.6 \text{ kW}$$

$$Q - W = \Delta U \quad \therefore Q = \Delta U + W = 10.6 - 21.2 = -10.6 \text{ kW}$$

$$\text{ii. } T_2 = 288 \left( \frac{12}{1} \right)^{\frac{0.4}{1.4}} = 585.8 \text{ K}$$

$$W = \frac{0.1 \times 0.287(288 - 585.8)}{0.4} = -21.36 \text{ kW}$$

$$\Delta U = 0.1 \times 0.718(585.8 - 288) = 21.36 \text{ kW}$$

$$\therefore Q = \Delta U + W = 21.36 - 21.36 = 0$$

$$\text{iii. } W = mRT_1 \ln \frac{P_1}{P_2} = 0.1 \times 0.287 \times 288 \ln \frac{1}{12} = -20.5 \text{ kW}$$

$$\Delta U = mC_v \Delta T = 0$$

$$\therefore Q = W = -20.5 \text{ kW} \quad \text{ie loss}$$

### Worked Example 3.19

A reciprocating internal combustion engine has a clearance volume of  $0.0001 \text{ m}^3$  and a compression ratio (volume ratio) of 10. The pressure and temperature of the combustion gases when the piston is at top dead centre are  $4000 \text{ kN/m}^2$  and  $1800^\circ\text{C}$  respectively.

Assuming that the expansion process follows  $PV^{1.3} = \text{constant}$ , calculate:

- the work transfer in this process, and
- the temperature of the gases at the end of the process.

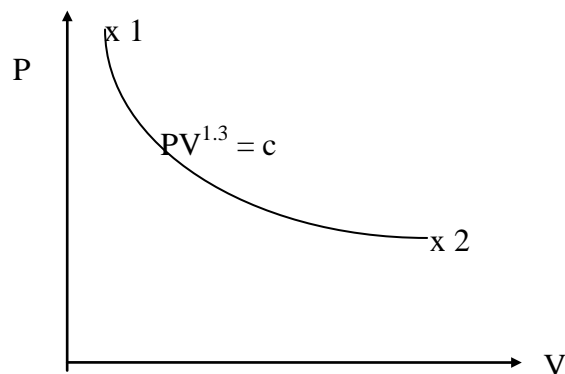
**Solution:**

- since  $V_1 = 0.0001 \text{ m}^3$

$$\frac{V_2}{V_1} = 10$$

$$\therefore V_2 = 0.001 \text{ m}^3$$

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^n$$



$$= 4000 \left( \frac{1}{10} \right)^{1.3} = 200 \text{ kN} / \text{m}^2$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1} = \frac{4000 \times 10^3 \times 0.0001 - 200 \times 10^3 \times 0.001}{1.3 - 1} = 66.7 \text{ J}$$

$$\text{b) } T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{n-1}$$

$$= (1800 + 273) \left( \frac{1}{10} \right)^{0.3} = 1039 \text{ K}$$

**Worked Example 3.20**

A reciprocating steam motor is supplied with dry saturated steam at a pressure of 1.6MPa ( specific volume = 0.1238 m<sup>3</sup>/kg). The stroke of the motor is 0.8m and the bore is 0.3m. The clearance volume is negligible. The steam enters the cylinder, expands at constant pressure for ¼ of the stroke and then expands reversibly according to a law  $PV = \text{constant}$ , til the end of the stroke. Calculate

- the mass of the steam,
- the work transfer and the heat transfer in the process?

**Solution:**

$$\text{a) } V_2 = \frac{\pi}{4} \times 0.3^2 \times 0.8 = 0.0565 \text{ m}^3;$$

$$V_1 = \frac{V_2}{4} = 0.0141 \text{ m}^3$$

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right) = 1.6 \times \frac{1}{4} = 0.4 \text{ MPa}$$

$$\therefore m = V_1 / v_1 = \frac{0.0141}{0.1238} = 0.114 \text{ kg}$$

- The work Transfer is

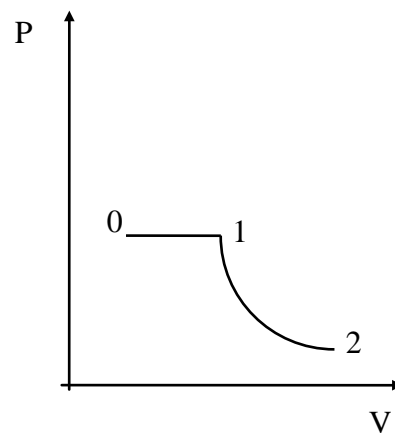
$$W = P_1 V_1 \ln \frac{V_2}{V_1} = 1.6 \times 10^6 \times 0.0141 \ln 4 = 31.3 \text{ kJ}$$

This system can be considered as a Closed system for which the NFEE applies

$$Q - W = \Delta U$$

but  $\Delta U = 0$  , since isothermal process, hence

$$Q = W = 31.3 \text{ kJ} \quad \text{i.e. gain}$$



**Worked Example 3.21**

A piston and cylinder mechanism has its piston fixed so that the volume contained is  $0.0025\text{m}^3$ . The mechanism is filled with wet steam at a pressure of 2 bar. The steam is heated until it reaches the critical point. The piston is released and the steam expands adiabatically to a pressure of 2 bar and a volume of  $0.5\text{m}^3$ . Calculate:

- the mass of steam in the mechanism,
- the dryness fraction of the steam after expansion.

$p$ (kPa)	$t_s$ (°C)	$v_f$ ( $\text{m}^3/\text{kg}$ )	$v_g$ ( $\text{m}^3/\text{kg}$ )	$h_f$ (kJ/kg)	$h_g$ (kJ/kg)	$s_f$ (kJ/kg.K)	$s_g$ (kJ/kg.K)
200	120.23	0.00106	0.8857	504.7	2,707	1.5301	7.1271
22120	374.15	0.00317	0.00317	2,084	2,084	4.43	4.43

**Solution:**

$$\text{a) } P_1 = 2\text{bar}, V_1 = 0.0025\text{m}^3$$

$$P_2 = 221\text{bar}, V_2 = 0.0025\text{m}^3$$

$$v_2 = v_c = 0.00317\text{m}^3/\text{kg}$$

as it is in a critical state

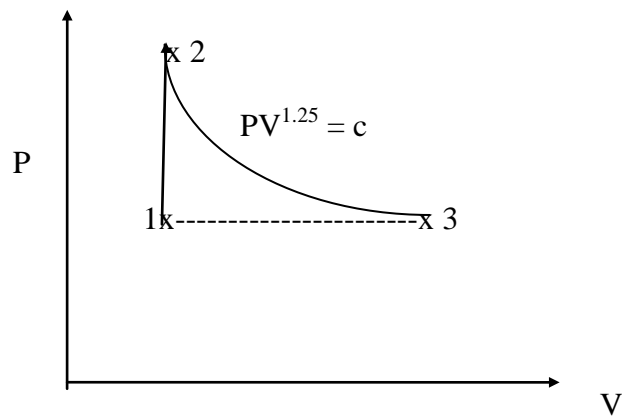
$$P_3 = 2\text{bar}, V_3 = 0.5\text{m}^3$$

$$m = \frac{V_2}{v_2} = \frac{0.0025}{0.00317} = 0.788\text{kg}$$

$$\text{b) } v_3 = \frac{V_3}{m} = \frac{0.5}{0.788} = 0.634\text{m}^3/\text{kg}$$

$$v = v_f + X.(v_g - v_f)$$

$$0.634 = 1.0605 \times 10^{-3} + x_3(0.8857 - 0.00106) \quad \therefore x_3 = 0.715$$



**Worked Example 3.22**

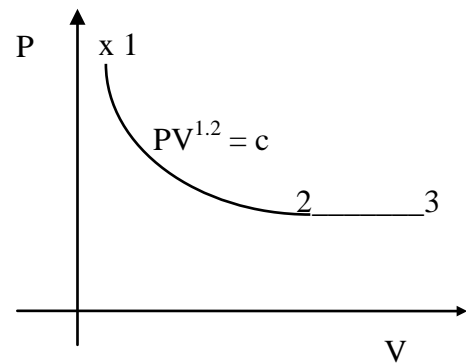
A mass of gas occupying  $0.08\text{m}^3$  at  $6\text{kN/m}^2$  and  $80^\circ\text{C}$  is expanded reversibly in a non-flow process according to a law  $PV^{1.2} = \text{constant}$ . The pressure at the end of expansion is  $0.7\text{kN/m}^2$ . The gas is then heated at constant pressure to the original temperature. The specific heat capacities at constant pressure and constant volume are  $1.00$  and  $0.74 \text{ kJ/kg K}$  respectively. Determine:-

- the work transfer in the expansion process;
- the heat transfer in the expansion process;
- the volume at the end of the heating process;
- the change in internal energy during the heating process.

**Solution**

$$\text{a) } R = C_p - C_v = 1.00 - 0.74 = 0.26 \text{ kJ / kgK}$$

$$\therefore m = \frac{P_1 V_1}{RT_1} = \frac{6 \times 10^3 \times 0.08}{260 \times (80 + 273)} = 5.23 \times 10^{-3} \text{ kg}$$



$$V_2 = V_1 \left( \frac{P_1}{P_2} \right)^{1/n} = 0.08 \left( \frac{6}{0.7} \right)^{1/1.2} = 0.479 m^3$$

$$W = \left( \frac{P_1 V_1 - P_2 V_2}{n-1} \right) = \left( \frac{6 \times 10^3 \times 0.08 - 0.7 \times 10^3 \times 0.479}{1.2-1} \right) = +0.723 kJ$$

$$b) \quad T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = (80 + 273) \left( \frac{0.7}{6} \right)^{\frac{0.2}{1.2}} = 246.7 K$$

$$\Delta U = m C_v (T_2 - T_1) = 5.23 \times 10^{-3} \times 740 (-26 \times 80) = -0.411 kJ$$

$$\Delta U = Q - W$$

$$\therefore Q = \Delta U + W = -0.411 + 0.723 = +0.312 kJ \quad \text{i.e. heat gain}$$

$$c) \quad V_3 = V_2 \frac{T_3}{T_2} \quad \therefore V_3 = 0.479 \times \frac{353}{246.7} = 0.685 m^3$$

$$d) \quad \Delta U_{23} = m C_v (T_3 - T_2)$$

$$= 5.23 \times 10^{-3} \times 0.740 (246.7 - 353) = -0.411 kJ$$

### Worked Example 3.23

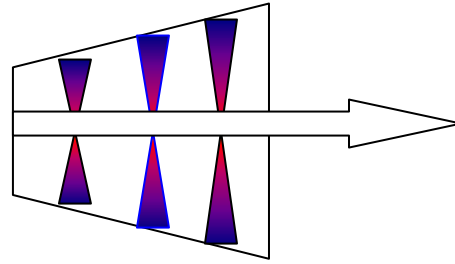
An ideal centrifugal air compressor takes in air at 1 bar, 15°C and compresses it reversibly and adiabatically to a pressure of 4 bar.

- (Calculate the delivery temperature of the gas.
- (If kinetic energy and potential energy changes are negligible calculate the specific work transfer in the compression process.

Air may be assumed to be a perfect gas with specific heat capacity at constant pressure  $C_p = 1.005 kJ/kg \text{ K}$  and  $n = 1.4$ .

**Solution:**

$$\begin{aligned} \text{a) } T_2 &= T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \\ &= (15 + 273) \left( \frac{4}{1} \right)^{\frac{0.4}{1.4}} = 428 \text{ K} \end{aligned}$$



b) For the present system, the SFEE applies

$$Q - W = m \left[ C_p (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} + g(Z_2 - Z_1) \right]$$

$Q = 0$  adiabatic

$\Delta PE = 0$  neglected (assumed)

$\Delta KE = 0$  neglected (assumed)

hence:

$$W = m C_p (T_1 - T_2)$$

$$= 1 \times 1.005 (288 - 428) = -140.7 \text{ kJ/kg}$$

### Worked Example 3.24

A reversible adiabatic air turbine drives a small generator which requires a power of 2kW. The air supply for the turbine is provided by a reservoir and the pressure and temperature at turbine entry may be considered constant at 9 bar, 20°C respectively. The velocity of the air at inlet to the turbine is small and may be neglected but at exit the velocity is 55m/s. The exit pressure is 1.2 bar. Calculate:

- the air temperature at exit from the turbine, and
- the mass flow rate of air stating any assumptions made.

Air may be considered a perfect gas for which the specific heat capacity at constant pressure  $C_p = 1.005 \text{ kJ/kg K}$  and  $n = 1.4$ .



**Solution:**

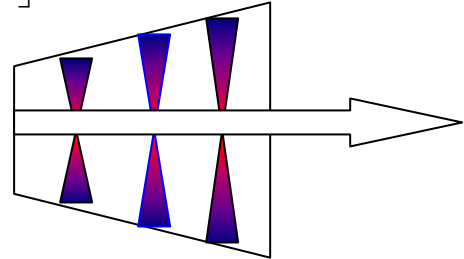
- a) The situation is an open system for which the SFEE applies:

$$Q - W = m \left[ C_p (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} + g(Z_2 - Z_1) \right]$$

$$Q = 0 \text{ adiabatic}$$

$$g(Z_2 - Z_1) = 0 \text{ (assumed)}$$

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = 293 \left( \frac{1.2}{9} \right)^{\frac{0.4}{1.4}} = 164.76 K$$



b) The SFEE can now be used to determine the mass flow rate

$$-W = m \left[ C_p (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2000} \right]$$

$$-2 = m \left[ 1.005(164.7 - 293) + \frac{55^2 - 0}{2000} \right]$$

$$\therefore m = \frac{-2}{-128.88 + 1.51} = +0.0157 \text{ kg/s}$$

### Worked Example 3.25

Steam at a pressure of 2 MPa and a temperature of 240°C enters a nozzle with a velocity of 15m/s. The steam expands reversibly and adiabatically in the nozzle to a pressure of 100 kPa and a dryness fraction of 0.9. Calculate the velocity of the steam at exit from the nozzle,

Saturated table extract for P = 100 KPa

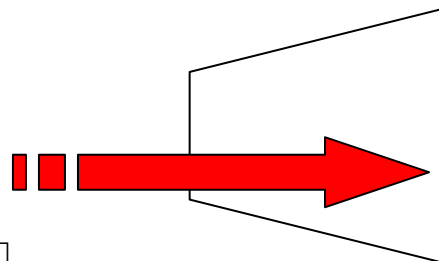
$p$	$t_s$	$v_f$	$v_g$	$h_f$	$h_g$	$s_f$	$s_g$
(kPa)	(°C)	(m³/kg)	(m³/kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)
100	99.63	0.00104	1.694	417.46	2,676	1.3026	7.3594

Superheated table extract at P = 2 MPa

T (°C)	V (m³/kg)	h (kJ/kg)	S (kJ/kgK)
240	0.1084	2876	6.495

**Solution:**

a) SFEE applies,



$$Q - W = m \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(Z_2 - Z_1) \right]$$

$Q = 0$  adiabatic,  $W=0$ , No work transfer, and the Potential Energy change is negligible

Hence the energy equation can now be rearranged as follows:

$$V_2 = \sqrt{V_1^2 + 2m(h_1 - h_2)}$$

from the steam tables,  $h_1 = 2876 \text{ kJ/kg}$

$$h_2 = 417.46 + 0.9 \times (2676 - 417.46) = 2450 \text{ kJ/kg}$$

$$V_2 = \sqrt{V_1^2 + 2m(h_1 - h_2)} = \sqrt{15^2 + 2 \times 0.6 \times (2876 - 2450) \times 10^3} = 715 \text{ m/s}$$

### Worked Example 3.26

A one pass steam generator receives saturated water at 20 bar, and converting it into steam at 400°C. The mass flow rate of steam is 1200 kg/h, calculate the heat transfer in the generator.

Enthalpy of Superheated Steam ( $\text{kJ/kg}$ )								
Pressure (bar)	Saturation Temperature (°C)	Temperature (°C)						
		Saturation	200	250	300	350	400	450
20	212.4	2799		2904	3025	3138	3248	3357

### Solution:

SFEE applies,

$$Q - W = m \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(Z_2 - Z_1) \right]$$

Neglecting the changes in Kinetic and Potential energies,

also No work done  $W = 0$

hence the SFEE reduces to:  $Q = m (h_2 - h_1)$

from the steam table, we find:

$$h_1 = 2799$$

$$h_2 = 3248 \text{ kJ/kg}$$

hence the boiler capacity ( heat gained by the steam):

$$Q = \frac{1200}{3600}(3248 - 2799) = 149.667 \text{ kW}$$

### Worked Example 3.27

1kg of gas occupies a volume of  $0.4\text{m}^3$  at a pressure of  $100 \text{ kN/m}^2$ . The gas is compressed isothermally to a pressure of  $450 \text{ kN/m}^2$ . Determine the work of compression, and the change in entropy of the gas during the compression. Assume for the gas  $R = 300 \text{ J/kgK}$

**Solution:**

- a) For an isothermal process, the work is given by

$$W = PV \ln \frac{V_2}{V_1}$$

Since isothermal,  $PV = c$

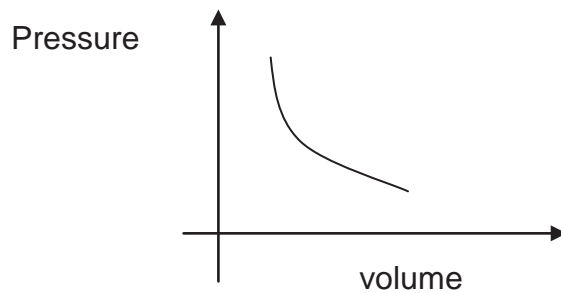
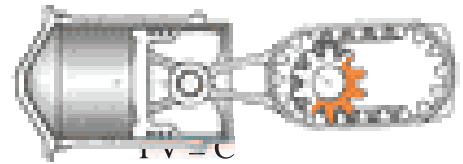
$$\therefore \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$W = PV \ln \frac{P_1}{P_2}$$

$$\therefore W = 100 \times 10^3 \times 0.4 \ln \frac{100}{450} = -60 \text{ kJ} \quad (-\text{ve indicates compression})$$

- b) the change of Entropy is given by:

$$\Delta s = m.R.\ln \frac{P_1}{P_2} = 1 \times 0.300 \times \ln \frac{100}{450} = -0.451 \text{ kJ/kgK}$$



**Worked Example 3.28**

A gas contained in a closed system at a pressure of 1 bar and temperature of 15°C. A mass of 0.9kg of the gas is heated at constant pressure to raise its temperature from 15°C to 250°C. Determine the work done, and during the process.  $R = 0.185 \text{ kJ/kgK}$

**Solution:**

Using the ideal gas equation:

$$V_1 = \frac{mRT_1}{P_1} = \frac{0.9 \times 0.185 \times 288 \times 10^3}{1 \times 10^5} = 0.4795 \text{ m}^3$$

For a constant pressure:

$$V_2 = V_1 \frac{T_2}{T_1} = 0.4795 \left( \frac{250 + 273}{15 + 273} \right) = 0.871 \text{ m}^3$$

the work during an isobaric process is given by:

$$W = P(V_2 - V_1)$$

$$W = 10^5 (0.871 - 0.4795) = 39.1 \text{ kJ}$$

+ ve implies that the gas is expanding.

### Worked Example 3.29

The cylinder of an engine has a stroke of 300mm and a bore of 250mm. The volume ratio of compression is 14:1. Air in the cylinder at the beginning of compression has a pressure of 96 kN/m<sup>2</sup> and a temperature of 93 °C. The air is compressed for the full stroke according to the law  $PV^{1.3} = C$ . Determine the work transfer per unit mass of air. Assume air  $R = 287 \text{ J/kgK}$ .

#### Solution:

compression work is given by:

$$V_1 = \frac{300}{1000} \times \frac{\pi}{4} \left( \frac{250}{1000} \right)^2 = 0.0147 \text{ m}^3$$

$$V_2 = V_1 / 14 = 0.00105 \text{ m}^3$$

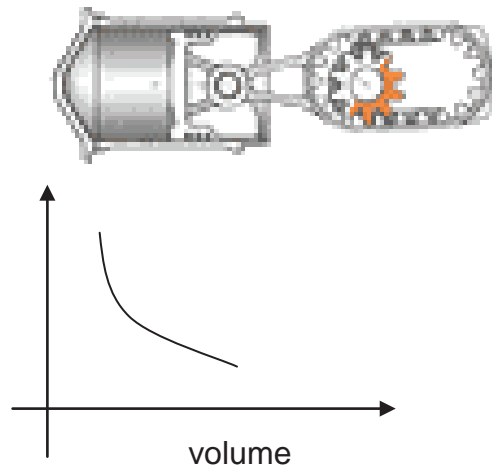
$$P_1 = 96 \times 10^3 \text{ N/m}^2$$

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^{1.3} = 96(14)^{1.3} = 2966.45 \text{ kN/m}^2$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

$$\therefore W = \frac{96 \times 10^3 \times 0.0147 - 2966.45 \times 10^3 \times 0.00105}{1.3 - 1} = -5.678 \text{ kJ/stroke}$$

$$\therefore m = \frac{p \cdot v}{R \cdot T} = \frac{96 \times 10^3 \times 0.00147}{287 \times (273 + 93)} = 0.0134 \text{ kg/stroke}$$



hence  $W = (-5.6785 \text{ kJ/stroke}) / (0.0134 \text{ kg/stroke}) = -424 \text{ kJ/kg}$

-ve sign indicates compression.

### Worked Example 3.30

A mass of air at 330°C, contained in a cylinder expanded polytropically to five times its initial volume and  $1/8^{\text{th}}$  its initial pressure which is 1 bar. Calculate:

- the value of the expansion index,
- the work transfer per unit mass.

**Solution:**

- for a polytropic process,

$$\frac{P_1}{P_2} = \left( \frac{V_2}{V_1} \right)^n$$

$$n = \frac{\ln(P_2 / P_1)}{\ln(V_2 / V_1)} = \frac{\ln(8)}{\ln(5)} = 1.292$$

- The work done for a polytropic process is,

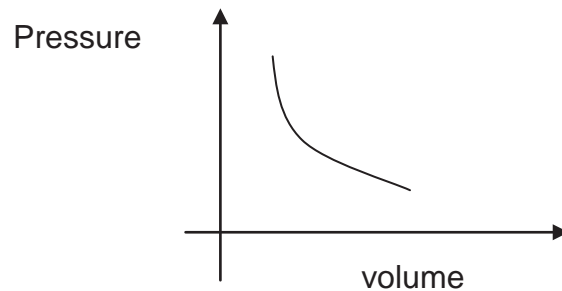
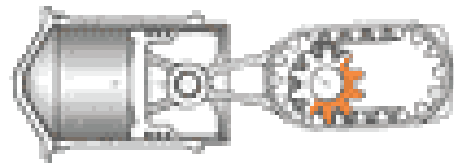
$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

$$V_1 = mRT_1 / P_1$$

$$= 1 \times 287 \times 603 / 10^5$$

$$= 1.7306 \text{ m}^3$$

$$V_2 = 5V_1 = 8.653 \text{ m}^3$$



$$\text{Therefore } W = \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{1 \times 10^5 \times 1.7306 - \frac{1}{8} \times 10^5 \times 8.653}{1.292 - 1} = +222 \text{ kJ}$$

### Worked Example 3.31

Steam at a pressure of 10 bar and dryness fraction of 0.96 expands adiabatically to a pressure of 2 bar according to  $PV^{1.12} = \text{constant}$ . Determine the work done during expansion per unit mass of steam.

$p$	$t_s$	$v_f$	$v_g$	$h_f$	$h_g$	$s_f$	$s_g$
(kPa)	(°C)	(m³/kg)	(m³/kg)	(kJ/kg)	(kJ/kg)	(kJ/kg.K)	(kJ/kg.K)
1000	179.91	0.00112	0.1944	762.81	2,778	2.1387	6.5865



**Solution:**

At 10 bar saturated steam, the specific volume is:

$$\begin{aligned} v &= v_f + x (v_g - v_f) \\ &= 0.00112 + 0.96 \times (0.1944 - 0.00112) \\ &= 0.18667 \text{ m}^3/\text{kg} \end{aligned}$$

for the process

$$\begin{aligned} v_2 &= v_1 \left( \frac{P_1}{P_2} \right)^{(1/n)} \\ V_2 &= 0.18667 \times \left( \frac{10}{2} \right)^{(1/1.12)} = 0.7855 \text{ m}^3 / \text{kg} \end{aligned}$$

Work done is  $W = \frac{P_1 V_1 - P_2 V_2}{n - 1}$

$$\therefore W = \frac{10 \times 10^5 \times 0.18667 - 2 \times 10^5 \times 0.7855}{1.12 - 1} = 246 \text{ kJ} / \text{kg}$$

**Worked Example 3.32**

A nuclear reactor generates 3000 MW of heat. The heat is transferred in a heat exchanger of energy transfer efficiency 75% into steam which is expanded in a turbine in order to produce a power output. The steam is condensed in a condenser, releasing 1800 MW of heat, and pumped back through the heat exchanger by a feed pump which requires 3% of the power output from the turbine. Determine:

- The net power output from the plant.
- The power output from the turbine.
- The overall thermal efficiency of the plant.

**Solution:**

a) Consider the first law of thermodynamics for a cycle:

$$\sum Q - \sum w = 0$$

$$(3000 \times 0.75 - 1800) - (W_t - 0.03W_t) = 0$$

$$450 - 0.97W_t = 0$$

$$W_t = \frac{450}{0.97} = 463.9 \text{ MW}$$

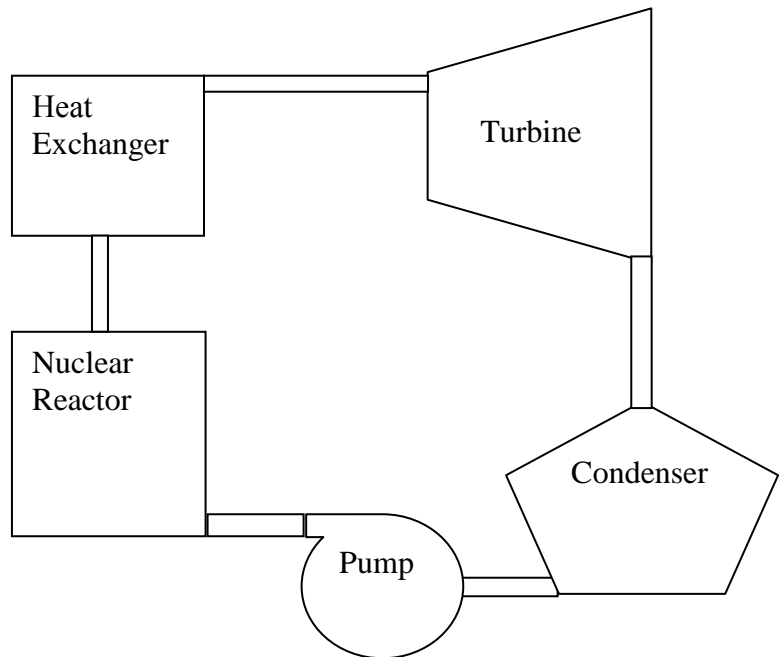
$$W_{net} = W_t - W_p = 0.97W_t$$

$$= 450 \text{ MW}$$

This is the net power output

The cycle efficiency is:

$$\eta = \frac{W_{net}}{Q_s} = \frac{450}{3000} \times 100 = 15\%$$

**Worked Example 3.33**

Milk initially at 30°C is to be kept in a chilled tank at 5°C. If the total volume of milk is 100 litres, its density is 1100 kg/m<sup>3</sup> and the specific heat capacity of 4.2 kJ/kgK.

- a) Determine the heat extraction rate assuming the chiller to be perfectly insulated
- b) What would be the chiller consumption if heat transfer through the chiller body is?
  - i. + 5 kW gain in summer, ii) -5 kW loss in winter

**Solution:**

- a) The Energy balance of the system implies that heat absorbed by the refrigerant is taken away from the milk in order to keep it cool.

$$Q = mC_p \Delta T$$

$$\text{but } m = \rho V$$

$$\text{i.e. } Q = \rho V C_p \Delta T$$

$$= \frac{100}{1000} = 1100 \times 4.2 \times (30 - 5) = 11.55 \text{ kW}$$

- b) In this case there is another source of heat exchange, hence;

$$Q_{\text{chiller}} = mC_p \Delta T (+ \text{or} -) \Delta Q$$

The sign (+ or -) depends on whether heat is lost as in winter or gained as in summer.

hence;

$$Q_{chiller} = 11.5 + 5 = 16.55kW \text{ in summer}$$

and

$$Q_{chiller} = 11.55 - 5 = 6.55kW \text{ in winter}$$

### Worked Example 3.34

You have a 200 gram cup of coffee at 100 C, too hot to drink.

- How much will you cool it by adding 50 gm of water at 0 C?
- How much will you cool it by adding 50 gm of ice at 0 C?  
for ice assume  $h_i = -333.5$  and  $h_f = 417 \text{ kJ/kgK}$

**Solution:**

- (a) Heat lost by coffee = heat gained by cold water

$$m_c \times C_{p_c} \times (100 - t_{c2}) = m_w \times C_{p_w} \times (t_{c2} - 0)$$

$$0.200 \times 4200 \times (100 - t_{c2}) = 0.050 \times 4200 \times (t_{c2} - 0)$$

$$\text{solve to get } t_{c2} = 80^\circ\text{C}$$



- (b) Heat lost by coffee = heat gained by ice

$$m_c \times C_{p_c} \times (100 - t_{c2}) = m_w \times h_{ig} + m_w \times C_{p_w} \times (t_{c2} - 0)$$

$$0.200 \times 4200 \times (100 - t_{c2}) = 0.05 \times (417 - 333.5) + 0.050 \times 4200 \times (t_{c2} - 0)$$

$$\text{solve to get } t_{c2} = 64^\circ\text{C}$$



**Worked Example 3.35**

Determine for a unit mass of air, the change in enthalpy when heated from zero C to 100 °C if:

- i.  $C_p = 1 \text{ kJ/kgK}$  constant
- ii.  $C_p = 0.95 + 0.00002 * T - 0.03 \times 10^{-6} * T^2$

**Solution**

- i. change in enthalpy

$$dh = m C_p (T_2 - T_1)$$

$$= 1 \times 1 \times (100 - 0)$$

$$= 100 \text{ kJ/kg}$$

- ii.  $\Delta h = \int C_p dT$

Using the definition of specific heat in terms of temperature, the change in enthalpy:

$$\Delta h = \int_0^{100} (0.95 + 0.02 \times 10^{-3} T - 0.03 \times 10^{-6} T^2)$$

Integrating between the limits of temperatures from zero to 100:

$$\Delta h = \left[ 0.95T + 0.01 \times 10^{-3} T^2 - 0.01 \times 10^{-6} T^3 \right]_0^{100}$$

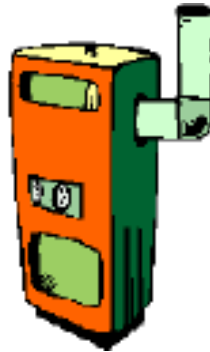
$$\Delta h = 96.09 \text{ kJ/kg}$$

The difference between the calculated values in (i) and (ii) is about 4% which is due to the fact that the specific heat capacity is not strictly constant for different temperatures.

**Worked Example 3.36**

A burner heats air from 20 to 40°C at constant pressure. Determine the change in entropy for a unit mass of air going through the heater, assuming that for air  $C_p = 1 \text{ kJ/kgK}$

**Solution:**



Constant pressure process, the change in entropy is calculated as:

$$\Delta S = m.C_p.\ln\frac{T_2}{T_1}$$

$$= 1 \times 1 \times \ln\frac{303}{293}$$

$$= 0.03356 \text{ kJ/kgK}$$

## 4. Thermodynamics Tutorial problems

### 4.1 First Law of Thermodynamics N.F.E.E Applications

1. In a non-flow process there is heat transfer loss of 1055 kJ and an internal energy increase of 210 kJ. Determine the work transfer and state whether the process is an expansion or compression.

[Ans: -1265 kJ, compression]

2. In a non-flow process carried out on 5.4 kg of a substance, there was a specific internal energy decrease of 50 kJ/kg and a work transfer from the substance of 85 kJ/kg. Determine the heat transfer and state whether it is gain or loss.

[Ans: 189 kJ, gain]

3. During the working stroke of an engine the heat transferred out of the system was 150 kJ/kg of the working substance. If the work done by the engine is 250 kJ/kg, determine the change in internal energy and state whether it is decrease or increase.

[Ans: -400 kJ/kg, decrease]

4. Steam enters a cylinder fitted with a piston at a pressure of  $20 \text{ MN/m}^2$  and a temperature of  $500^\circ\text{C}$ . The steam expands to a pressure of  $200 \text{ kN/m}^2$  and a temperature of  $200^\circ\text{C}$ . During the expansion there is a net heat loss from the steam through the walls of the cylinder and piston of  $120 \text{ kJ/kg}$ . Determine the displacement work done by one kg of steam during this expansion.

[Ans:  $168.6 \text{ kJ/kg}$ ]

5. A closed rigid system has a volume of  $85 \text{ litres}$  contains steam at  $2 \text{ bar}$  and dryness fraction of  $0.9$ . Calculate the quantity of heat which must be removed from the system in order to reduce the pressure to  $1.6 \text{ bar}$ . Also determine the change in enthalpy and entropy per unit mass of the system.

[Ans:  $-38 \text{ kJ}$ ]

6.  $2 \text{ kg}$  of air is heated at constant pressure of  $2 \text{ bar}$  to  $500^\circ\text{C}$ . Determine the change in its entropy if the initial volume is  $0.8 \text{ m}^3$ .

[Ans:  $2.04 \text{ kJ/K}$ ]

## 4.2 First Law of Thermodynamics S.F.E.E Applications

1. A boiler is designed to work at  $14 \text{ bar}$  and evaporate  $8 \text{ kg/s}$  of water. The inlet water to the boiler has a temperature of  $40^\circ\text{C}$  and at exit the steam is  $0.95$  dry. The flow velocity at inlet is  $10 \text{ m/s}$  and at exit  $5 \text{ m/s}$  and the exit is  $5 \text{ m}$  above the elevation at entrance. Determine the quantity of heat required. What is the significance of changes in kinetic and potential energy on the result?

[Ans:  $20.186 \text{ MW}$ ]

2. Steam flows along a horizontal duct. At one point in the duct the pressure of the steam is  $1 \text{ bar}$  and the temperature is  $400^\circ\text{C}$ . At a second point, some distance from the first, the pressure is  $1.5 \text{ bar}$  and the temperature is  $500^\circ\text{C}$ . Assuming the flow to be frictionless and adiabatic, determine whether the flow is accelerating or decelerating.

[Ans: Decelerating]

3. Steam is expanded isentropically in a turbine from  $30 \text{ bar}$  and  $400^\circ\text{C}$  to  $4 \text{ bar}$ . Calculate the work done per unit mass flow of steam. Neglect changes in Kinetic and Potential energies.

[Ans:  $476 \text{ kJ/kg}$ ]

4. A compressor takes in air at  $1 \text{ bar}$  and  $20^\circ\text{C}$  and discharges into a line. The average air velocity in the line at a point close to the discharge is  $7 \text{ m/s}$  and the discharge pressure is  $3.5 \text{ bar}$ . Assuming that the compression occurs isentropically, calculate the work input to the compressor. Assume that the air inlet velocity is very small.



[Ans: -126.6 kW/kg]

5. Air is expanded isentropically in a nozzle from 13.8 bar and 150°C to a pressure of 6.9 bar. The inlet velocity to the nozzle is very small and the process occurs under steady-flow, steady-state conditions. Calculate the exit velocity from the nozzle knowing that the nozzle is laid in a horizontal plane and that the inlet velocity is 10 m/s.

[Ans: 390.9 m/s]

### 4.3 General Thermodynamics Systems

1. A rotary air compressor takes in air (which may be treated as a perfect gas) at a pressure of 1 bar and a temperature of 20°C and compress it adiabatically to a pressure of 6 bar. The isentropic efficiency of the processes is 0.85 and changes in kinetic and potential energy may be neglected. Calculate the specific entropy change of the air. Take  $R = 0.287$  kJ/kg K and  $C_p = 1.006$  kJ/kg K.

[Ans: 0.07 kJ/kg K]

2. An air receiver has a capacity of  $0.86\text{ m}^3$  and contains air at a temperature of 15°C and a pressure of  $275\text{ kN/m}^2$ . An additional mass of 1.7 kg is pumped into the receiver. It is then left until the temperature becomes 15°C once again. Determine,
- the new pressure of the air in the receiver, and
  - the specific enthalpy of the air at 15°C if it is assumed that the specific enthalpy of the air is zero at 0°C.

Take  $C_p = 1.005$  kJ/kg,  $C_v = 0.715$  kJ/kg K

[Ans:  $442\text{ kN/m}^2$ ,  $15.075\text{ kJ/kg}$ ]

3. Oxygen has a molecular weight of 32 and a specific heat at constant pressure of  $0.91\text{ kJ/kg K}$ .
- Determine the ratio of the specific heats.
  - Calculate the change in internal energy and enthalpy if the gas is heated from 300 to 400 K.

[Ans: 1.4, 65 kJ/kg, 91 kJ/kg]

4. A steam turbine inlet state is given by 6 MPa and 500°C. The outlet pressure is 10 kPa. Determine the work output per unit mass if the process:-
- is reversible and adiabatic (ie 100% isentropic),
  - such that the outlet condition is just dry saturated,

- c) such that the outlet condition is 90% dry.

[Ans: 1242.7 kJ/kg, 837.5 kJ/kg, 1076.8 kJ/kg]

5. Determine the volume for carbon dioxide contained inside a cylinder at 0.2 MPa, 27°C:-  
a) assuming it behaves as an ideal gas  
b) taking into account the pressure and volume associated with its molecules

[Ans: 0.2833 m<sup>3</sup>/kg]

6. A cylindrical storage tank having an internal volume of 0.465 m<sup>3</sup> contains methane at 20°C with a pressure of 137 bar. If the tank outlet valve is opened until the pressure in the cylinder is halved, determine the mass of gas which escapes from the tank assuming the tank temperature remains constant.

[Ans: 20.972 kg]

7. Find the specific volume for H<sub>2</sub>O at 1000 kN/m<sup>2</sup> and 300°C by using:-

- a) the ideal gas equation assuming  $R = 461.5 \text{ J/kg K}$   
b) steam tables

[Ans: 0.264 m<sup>3</sup>/kg, 0.258 m<sup>3</sup>/kg]

8. Determine the specific volume of steam at 6 MPa using the steam tables for the following conditions:-

- a) dryness fraction  $x = 0$
- b) dryness fraction  $x = 0.5$
- c) dryness fraction  $x = 1$
- d) its temperature is  $600^{\circ}\text{C}$

[Ans: 0.001319, 0.01688, 0.03244, 0.06525 m<sup>3</sup>/kg]

9. Steam at 4 MPa,  $400^{\circ}\text{C}$  expands at constant entropy till its pressure is 0.1 MPa. Determine:

- a) the energy liberated per kg of steam
- b) repeat if the process is 80% isentropic

[Ans: 758 kJ/kg, 606 kJ/kg]

10. Steam (1 kg) at 1.4 MPa is contained in a rigid vessel of volume 0.16350 m<sup>3</sup>. Determine its temperature.

- a) If the vessel is cooled, at what temperature will the steam be just dry saturated?
- b) If cooling is continued until the pressure in the vessel is 0.8 MPa; calculate the final dryness fraction of the steam, and the heat rejected between the initial and the final states.

[Ans:  $250^{\circ}\text{C}$ ,  $188^{\circ}\text{C}$ , 0.678; 8181 kJ]

11. Steam (0.05 kg) initially saturated liquid, is heated at constant pressure of 0.2 MPa until its volume becomes 0.0658 m<sup>3</sup>. Calculate the heat supplied during the process.

[Ans: 128.355 kJ]

12. Steam at 0.6 MPa and dryness fraction of 0.9 expands in a cylinder behind a piston isentropically to a pressure of 0.1 MPa. Calculate the changes in volume, enthalpy and temperature during the process.

[Ans: 1.1075 m<sup>3</sup>, -276 kJ/kg,  $-59^{\circ}\text{C}$ ]

13. The pressure in a steam main pipe is 1.2 MPa; a sample is drawn off and throttled where its pressure and temperature become 0.1 MPa,  $140^{\circ}\text{C}$  respectively. Determine the dryness fraction of the steam in the main stating reasonable assumptions made!

[Ans: 0.986, assuming constant enthalpy]

14. A boiler receives feed water at 20 kPa as saturated liquid and delivers steam at 2 MPa and 500°C. If the furnace of this boiler is oil fired, the calorific value of oil being 42000 kJ/kg determine the efficiency of the combustion when 4.2 tonnes of oil was required to process 42000 kg of steam.

[Ans: 96%]

15. 10 kg/s steam at 6 MPa and 500°C, expands isentropically in a turbine to a pressure of 100 kPa. If the heat transfer from the casing to surroundings represents 1 per cent of the overall change of enthalpy of the steam, calculate the power output of the turbine. Assume exit is 5 m above entry and that initial velocity of steam is 100 m/s whereas exit velocity is 10 m/s.

[Ans: 96%]