Conf-950420--

LA-UR- 94-3154

Title: Asymptotic Analysis of The Several Competitive Equations To Solve The Time-Dependent Neutron Transport Equation

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Submitted to: Meeting: International Conference on Mathematics and Computations, Reactor Physics, and Environmental Analyses, April 30-May 4, 1995, Portland, Oregon

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ASYMPTOTIC ANALYSIS OF THE SEVERAL COMPETITIVE EQUATIONS TO SOLVE THE TIME-DEPENDENT NEUTRON TRANSPORT EQUATION

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ABSTRACT

Using conventional diffusion limit analysis, we asymptotically compare three competitive time-dependent equations (the telegrapher's equation, the time-dependent Simplified P_2 (SP_2) equation, and the time-dependent Simplified Even-Parity (SEP) equation). The time-dependent SP_2 equation contains higher order asymptotic approximations of the time-dependent transport equation than the other equations in a physical regime in which the time-dependent diffusion equation is the leading order approximation. In addition, we derive the multigroup *modified* time-dependent SP_2 equation from the multigroup time-dependent transport equation by means of an asymptotic expansion in which the multigroup time-dependent diffusion equation is the leading order approximation. Numerical comparisons of the time-dependent diffusion, the telegrapher's, the time-dependent SP_2, and S_4 solutions in 2-D X-Y geometry show that, in most cases, the SP_2 solutions contain most of the transport corrections for the diffusion approximation.

I. INTRODUCTION

A more or less complete model of the transport of neutrally-charged sub-atomic particles, as well as charged particles in some important physics regimes, is given by Bolzmann Equation.[1] The Discrete Ordinates (S_N) method has been conventionally used to solve this equation numerically. The S_N method provides great accuracy but for many applications, especially time-dependent, multidimensional cases, such as reactor-core-disruptive problems, requires too much computational time. To overcome the high-cost of the conventional S_N equations, several competitive simplified equations, such as the telegrapher's equation[2] and the time-dependent Simplified P_2 (SP_2) equation[3-4] which could be solved with almost the same computational effort as the time-dependent diffusion equation, can be used to obtain relatively inexpensive, but approximate solutions to the transport solutions which are more accurate than the diffusion solutions.

An asymptotic analysis can be performed to show that, for an important class of problems, the diffusion equation is an asymptotic limit of the transport equation.[7] A recent paper by the authors[4] provides a broadened view of the result discussed above. It shows that the time-dependent SP_2 equation has high-order asymptotic approximations of the time-dependent transport equation in a physical regime in which the conventional time-dependent diffusion equation is the leading-order approximation. In this paper, we compare the telegrapher's equation and the time-dependent SP_2 equation using the conventional asymptotic approaches. As a result, we find that the time-dependent SP_2 equation has the same asymptotic approximations as the time-dependent transport equation up to the third order in a physical regime in which the time-dependent diffusion equation is the leading order approximation, whereas the telegrapher's equation has the same asymptotic approximations only up to the second order. This implies an advantage of the former over the latter.

We formulate the telegrapher's equation and the time-dependent SP_2 equation for one-group, 2-D X-Y geometry problems and compare numerical time-dependent diffusion, SP_2, S_n, and telegrapher's solutions in various classes of problems. As we have shown previously for slab geometry problems,[6] the results show that, in most cases, the time-dependent SP_2 solutions are significantly more accurate than the diffusion solutions and can be obtained with a very small fraction of the computational effort of an S_n calculation. And even in the optically-thin regimes, in which both the time-dependent diffusion and the telegrapher's equation are no longer good approximations to the time-dependent transport equation, the time-dependent SP_2 solutions are quite close to the S_n solutions.

In addition, as we have previously shown in a one-group problem,[5] we also derive here the multigroup *modified* time-dependent SP_2 equation from the multigroup time-dependent transport equation by means of an asymptotic expansion[4] in which the multigroup time-dependent diffusion equation is the leading order approximation.

According to the recent paper by Noh et al.,[9] in 2-D X-Y geometry, there is another competitive equation called the Simplified Even-Parity (SEP) equation which is much more computationally efficient than the even-parity equation. We derive here the time-dependent SEP equation in 2-D X-Y geometry and analyze this equation asymptotically. The results show that the time-dependent SEP equation also has the same asymptotic approximations as the time-dependent transport equation only up to the second order.

An outline of the remainder of this paper is as follows. In Sec. II, we carry out an asymptotic analysis of the telegrapher's equation and the time-dependent SP_2 equation in general geometry. In Sec. III, we present numerical comparisons of the time-dependent diffusion, SP_2, S_n, and the telegrapher's solutions. In Sec. IV, we asymptotically derive the multigroup *modified* time-dependent SP_2 equation from the multigroup time-dependent transport equation. In Sec. V, we carry out an asymptotic analysis of the time-dependent SEP equation in 2-D X-Y geometry. In Sec. VI, we conclude with a summary and recommend for future work

II. ASYMPTOTIC ANALYSIS OF THE TELEGRAPHER'S EQUATION AND THE TIME-DEPENDENT SP_2 EQUATION IN GENERAL GEOMETRY

In this section, we shall consider the one-group time-dependent transport problem in multidimensional, multiplying medium with delayed neutrons and isotropic scattering. Multigroup problems require a more complicated asymptotic analysis that we will discuss in section IV.

The one-group, multidimensional time-dependent transport equation with delayed neutrons is

$$\frac{1}{v}\frac{\partial\psi}{\partial t}+\Omega\cdot\nabla\psi+\sigma_t\psi=\{(1-\beta)\nu\sigma_f+\sigma_s\}\phi+\lambda C+Q,$$

and

$$\frac{\partial C}{\partial t}=\beta\nu\sigma_f\phi-\lambda C,$$

where the notations are standard (see Ref. 1).

If we consider the asymptotic scaling;[4]

$$\sigma_t \Rightarrow \frac{\sigma_t}{\varepsilon}\,,\ \sigma_a \Rightarrow \varepsilon\sigma_a\,,\ \sigma_f \Rightarrow \varepsilon\sigma_f\,,\ \sigma_s \Rightarrow \frac{\sigma_t}{\varepsilon} - \varepsilon\sigma_a\,,\ Q \Rightarrow \varepsilon Q\,,\ C \Rightarrow \varepsilon C\,,\ v \Rightarrow \frac{v}{\varepsilon}\,.$$

the scaling defined above has been known to be one in which the diffusion equation is an asymptotic limit of the transport equation as $\varepsilon \to 0$.

We expand,

$$\psi = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + \cdots\,,\ \phi = \phi_0 + \varepsilon\phi_1 + \varepsilon^2\phi_2 + \cdots,\ \text{and}\ C = C_0 + \varepsilon C_1 + \varepsilon^2 C_2 + \cdots.$$

Applying these to the time-dependent transport equation and equating the coefficients of different powers of ε, we obtain the following equations:

$O(\varepsilon^{-1})$
$$\psi_0 = \phi_0\,,$$

$O(1)$
$$\psi_1 = \phi_1 - \frac{1}{\sigma_t}\Omega\cdot\nabla\phi_0\,,$$

$O(\varepsilon)$
$$\frac{1}{v}\frac{\partial\phi_0}{\partial t} - D\nabla^2\phi_0 + \sigma_a\phi_0 = (1-\beta)v\sigma_f\phi_0 + \lambda C_0 + Q\,,$$

and
$$\frac{\partial C_0}{\partial t} = \beta v\sigma_f\phi_0 - \lambda C_0\,, \qquad \text{where} \quad D = \frac{1}{3\sigma_t}\,,$$

$O(\varepsilon^2)$
$$\frac{1}{v}\frac{\partial\phi_1}{\partial t} - D\nabla^2\phi_1 + \sigma_a\phi_1 = (1-\beta)v\sigma_f\phi_1 + \lambda C_1\,,$$

and
$$\frac{\partial C_1}{\partial t} = \beta v\sigma_f\phi_1 - \lambda C_1\,,$$

$O(\varepsilon^3)$
$$\left(\frac{1}{v\sigma_t}\frac{\partial}{\partial t} - \frac{4}{15\sigma_t^2}\nabla^2\right)D\nabla^2\phi_0 + \frac{1}{v}\frac{\partial\phi_2}{\partial t} - D\nabla^2\phi_2 + \sigma_a\phi_2 = (1-\beta)v\sigma_f\phi_2 + \lambda C_2\,,$$

and
$$\frac{\partial C_2}{\partial t} = \beta v\sigma_f\phi_2 - \lambda C_2.$$

A. The Telegrapher's Equation

The one-group, multidimensional telegrapher's equation with delayed neutrons is

$$\frac{1}{\sigma_t v^2}\frac{\partial^2\phi}{\partial t^2} + \frac{1}{v}\left(1 + \frac{\sigma_a}{\sigma_t}\right)\frac{\partial\phi}{\partial t} - \frac{1}{\sigma_t v}\frac{\partial Q}{\partial t} - D\nabla^2\phi + \sigma_a\phi = (1-\beta)v\sigma_f\phi + \lambda C + Q\,,$$

and
$$\frac{\partial C}{\partial t} = \beta v\sigma_f\phi - \lambda C.$$

Applying the same scaling and expansion to these equations, we obtain the following equations;

$O(\varepsilon)$
$$\frac{1}{v}\frac{\partial\phi_0}{\partial t} - D\nabla^2\phi_0 + \sigma_a\phi_0 = (1-\beta)\nu\sigma_f\phi_0 + \lambda C_0 + Q \ .$$

and
$$\frac{\partial C_0}{\partial t} = \beta\nu\sigma_f\phi_0 - \lambda C_0 \ .$$

$O(\varepsilon^2)$
$$\frac{1}{v}\frac{\partial\phi_1}{\partial t} - D\nabla^2\phi_1 + \sigma_a\phi_1 = (1-\beta)\nu\sigma_f\phi_1 + \lambda C_1 \ .$$

and
$$\frac{\partial C_1}{\partial t} = \beta\nu\sigma_f\phi_1 - \lambda C_1 \ .$$

$O(\varepsilon^3)$
$$\frac{1}{\nu\sigma_t}\frac{\partial}{\partial t}(D\nabla^2\phi_0) + \frac{1}{v}\frac{\partial\phi_2}{\partial t} - D\nabla^2\phi_2 + \sigma_a\phi_2 = (1-\beta)\nu\sigma_f\phi_2 + \lambda C_2 \ .$$

and
$$\frac{\partial C_2}{\partial t} = \beta\nu\sigma_f\phi_2 - \lambda C_2 \ .$$

Comparing these asymptotic approximations with those of the time-dependent transport equation, we find that the telegrapher's equation has the same asymptotic approximations as the time-dependent transport equation up to the second order in a physical regime in which the time-dependent diffusion equation is the leading order approximation.

B. Time-Dependent SP_3 Equation

The one-group, multidimensional time-dependent SP_3 equations with delayed neutrons are

$$\frac{1}{v}\frac{\partial\psi_0}{\partial t} + \nabla\cdot\psi_1 + \sigma_a\psi_0 = (1-\beta)\nu\sigma_f\psi_0 + \lambda C + Q,$$

$$\frac{1}{v}\frac{\partial\psi_1}{\partial t} + \frac{1}{3}\nabla\psi_0 + \frac{2}{3}\nabla\psi_2 + \sigma_t\psi_1 = 0,$$

$$\frac{1}{v}\frac{\partial\psi_2}{\partial t} + \frac{2}{5}\nabla\psi_1 + \sigma_t\psi_2 = 0,$$

and
$$\frac{\partial C}{\partial t} = \beta\nu\sigma_f\phi - \lambda C.$$

Applying the same scaling and expansion to these equations, we obtain the following equations;

$O(\varepsilon)$
$$\frac{1}{v}\frac{\partial\phi_0}{\partial t} - D\nabla^2\phi_0 + \sigma_a\phi_0 = (1-\beta)\nu\sigma_f\phi_0 + \lambda C_0 + Q \ .$$

and
$$\frac{\partial C_0}{\partial t} = \beta\nu\sigma_f\phi_0 - \lambda C_0 \ .$$

$O(\varepsilon^2)$
$$\frac{1}{v}\frac{\partial\phi_1}{\partial t} - D\nabla^2\phi_1 + \sigma_a\phi_1 = (1-\beta)\nu\sigma_f\phi_1 + \lambda C_1$$

and

$$\frac{\partial C_2}{\partial t} = \beta \nu \sigma_f \phi_1 - \lambda C_2 \, .$$

$O(\varepsilon^3)$

$$\left(\frac{1}{\nu\sigma_t}\frac{\partial}{\partial t} - \frac{4}{15\sigma_t^2}\nabla^2\right) D\nabla^2\phi_0 + \frac{1}{\nu}\frac{\partial\phi_2}{\partial t} - D\nabla^2\phi_2 + \sigma_a\phi_2 = (1-\beta)\nu\sigma_f\phi_2 + \lambda C_2$$

and

$$\frac{\partial C_2}{\partial t} = \beta \nu \sigma_f \phi_2 - \lambda C_2 \, .$$

Thus, the time-dependent SP_3 equation has the same asymptotic approximations as the time-dependent transport equation up to the third order in a physical regime in which the time-dependent diffusion equation is the leading order approximation. Conclusively, we find that the time-dependent SP_3 equation contains higher-order asymptotic corrections to the time-dependent diffusion equation than the telegrapher's equation. Thus, one might expect the higher accuracy from the time-dependent SP_3 equation, especially in diffusive regimes.

III. NUMERICAL CALCULATIONS AND RESULTS

First, differencing the time variable in the time-dependent SP_3 equations using a fully-implicit scheme and formulating an equation with the scalar flux, $\psi_0 (\equiv \phi)$, only, we obtain;

$$-\nabla \cdot D'\nabla\{\phi^{n+1} + \frac{4}{5\sigma_t'}(\sigma_a'\phi^{n+1} - Q)\} + \sigma_a'\phi^{n+1} = S^n, \tag{1}$$

where

$$\sigma_t' = \sigma_t + \frac{1}{v\Delta t}, \quad \sigma_a' = \sigma_a + \frac{1}{v\Delta t}, \quad D' = \frac{1}{3\sigma_t'},$$

and

$$S^n = Q + \frac{1}{v\Delta t}\phi^n - \nabla \cdot D'\nabla \frac{4}{5\sigma_t'}\left(\frac{1}{v\Delta t}\phi^n - \frac{5}{2}\psi_2^n\right) - \frac{1}{\sigma_t'}\nabla \cdot \left(\frac{1}{v\Delta t}\psi_1^n\right).$$

Equation (1) is in the form of a conventional diffusion equation. Spatially differencing this equation in 2-D X-Y geometry, we obtain the matrix equation $\mathbf{A}\cdot\phi^{n+1}=\mathbf{S}^n$ at each time step, n, where $\mathbf{A}$ is a five-diagonal symmetric matrix. Since these equations are very similar to those obtained from differencing the time-dependent diffusion equation, solutions are obtained with almost the same computational effort and with a very small fraction of the time needed for the time-dependent S_N solutions.

We have compared the time-dependent diffusion, the telegrapher's, the time-dependent SP_3, and S_8 solutions in various classes of 2-D X-Y geometry problems with and without delayed neutrons and the numerical results show that, in most cases, the time-dependent SP_3 solutions are significantly more accurate than the conventional time-dependent diffusion and the telegrapher's solutions and can be obtained with a very small fraction of the computational effort of an S_8 calculation (see Table 1). And even in the optically-thin regimes, in which both the time-dependent diffusion and the telegrapher's equation are no longer good approximations to the time-dependent transport equation, the time-dependent SP_3 solutions are quite close to the S_8 solutions.

The configuration of a sample problem is shown in Fig. 1. In Fig. 1, σ is the total cross section with a unit of [1/cm], c is the scattering ratio defined as σ_s/σ, and Q is the internal source with a unit of [1/cm^3 /sec]. The left and bottom boundaries of the system are reflective and the right and top boundaries are vacuum. We start with zero initial fluxes everywhere and compare the time-dependent variations of the scalar fluxes which are calculated by the time-dependent diffusion, SP_2, S_8, and the telegrapher's equations at point 1 and 2.

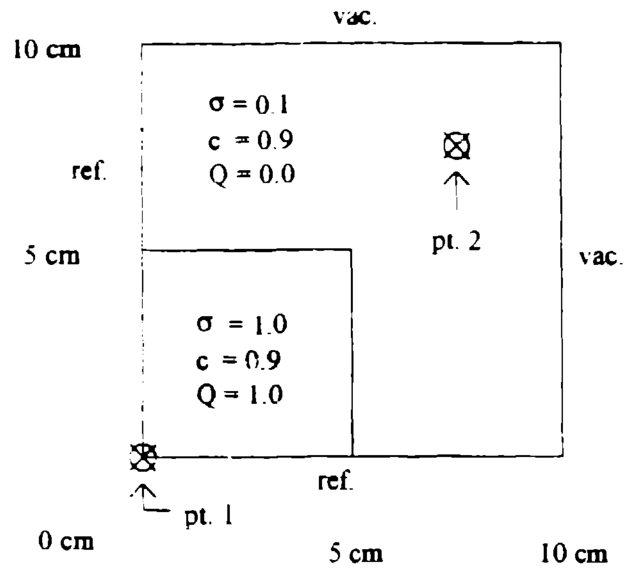

Fig. 1 Sample Problem

The results are shown in Fig. 2 and Fig. 3. At each point, the SP_2 solutions are much closer to the S_8 solutions than the diffusion and the telegrapher's solutions. Moreover, as shown in Fig. 3, SP_2 solutions are quite close to the S_8 solutions even in the optically-thin regimes in which both the diffusion and the telegrapher's equation are no longer good approximations.

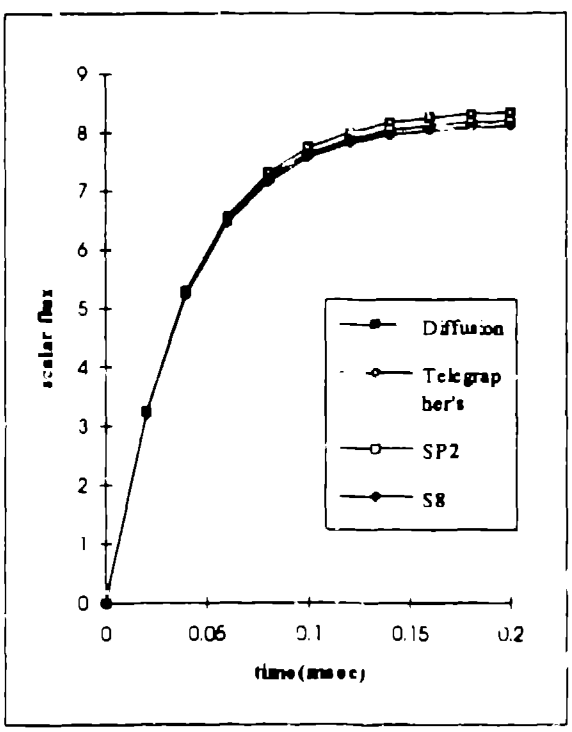

Fig. 2 Scalar Flux Variation at Pt. 1

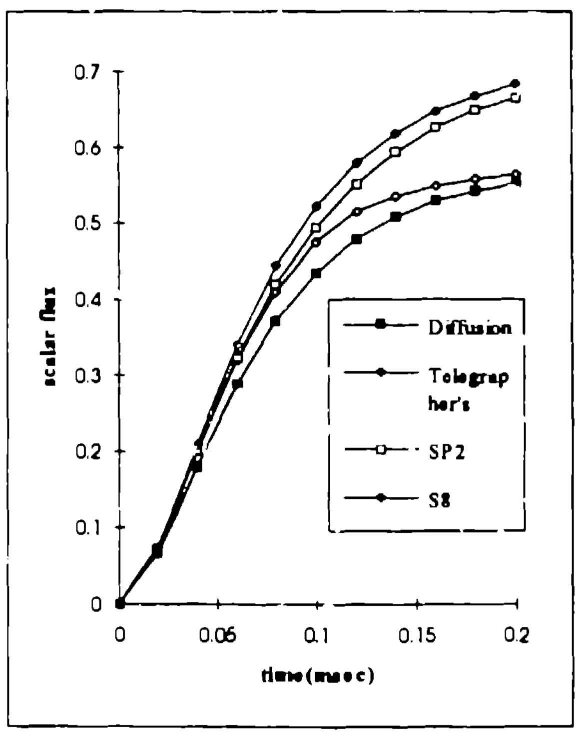

Fig. 3 Scalar Flux Variation at Pt. 2

Equation	Diffusion	Telegrapher's	SP2	S8
CPU time (sec)	0.235	0.264	0.291	6.138

Table 1 Elapsed CPU Time for Each Equation

IV. ASYMPTOTIC DERIVATION OF THE MULTIGROUP MODIFIED TIME-DEPENDENT SP_3 EQUATION

As we have shown in Section II. delayed neutrons do not alter the results of the asymptotic analyses and thus, for simplicity, we shall not consider delayed neutrons in this section (σ_f can be included in σ_s). For the same reason, we only present here the asymptotic analysis for planar geometry, however, we have performed the multidimensional analysis and obtained the same results.

The multigroup, planar geometry time-dependent transport equation with isotropic scattering is[8]

$$\frac{1}{v}\frac{\partial}{\partial t}\psi(x,\mu,t)+\mu\frac{\partial\psi}{\partial x}+\sigma_t\psi=\frac{\sigma_s}{2}\int_{-1}^{1}\psi(x,\mu',t)d\mu'+\frac{1}{2}Q(x,t), \tag{2}$$

where

$\psi(x,\mu,t) =$ angular flux (a G vector)

$Q(x,t) =$ source (a G vector)

$\sigma_t =$ total cross section (a G×G diagonal matrix)

$\sigma_s =$ scattering cross section (a G×G matrix)

G = number of energy groups.

Integrating Eq. (2) over the whole angle, $\frac{1}{2}\int_{-1}^{1}Eq.(2)d\mu$, and introducing

$$\phi(x,t)=\int_{-1}^{1}\psi(x,\mu,t)d\mu = \text{scalar flux (a G vector)},$$

we obtain

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(x,t)+\frac{\partial}{\partial x}\cdot\int_{-1}^{1}\mu'\psi(x,\mu',t)d\mu'+(\sigma_t-\sigma_s)\phi=Q(x,t). \tag{3}$$

And from Eq. (2) and Eq. (3), we also obtain

$$\frac{1}{v}\frac{\partial}{\partial t}\psi(x,\mu,t)+\mu\frac{\partial\psi}{\partial x}+\sigma_t\psi=\frac{1}{2}\{\frac{1}{v}\frac{\partial}{\partial t}\phi(x,t)+\sigma_t\phi+\frac{\partial}{\partial x}\int_{-1}^{1}\mu'\psi(x,\mu',t)d\mu'\}. \tag{4}$$

Introducing into Eq. (3) and Eq. (4) the asymptotic scaling;

$$\sigma_t\Rightarrow\frac{\sigma_t}{\varepsilon}\,,\ \sigma_s\Rightarrow\frac{\sigma_s}{\varepsilon}\,,\ Q\Rightarrow\varepsilon Q\,,\ v\Rightarrow\frac{v}{\varepsilon}\,,$$

we obtain

$$\frac{\varepsilon}{v}\frac{\partial}{\partial t}\phi(x,t)+\frac{\partial}{\partial x}\int_{-1}^{1}\mu'\psi(x,\mu',t)d\mu'+\frac{1}{\varepsilon}(\sigma_t-\sigma_s)\phi=\varepsilon Q(x,t), \tag{5}$$

and

$$\frac{\varepsilon}{v}\frac{\partial}{\partial t}\psi(x,\mu,t)+\mu\frac{\partial\psi}{\partial x}+\frac{\sigma_t}{\varepsilon}\psi=\frac{1}{2}\{\frac{\varepsilon}{v}\frac{\partial}{\partial t}\phi(x,t)+\frac{\sigma_t}{\varepsilon}\phi+\frac{\partial}{\partial x}\int_{-1}^{1}\mu'\psi(x,\mu',t)d\mu'\}. \tag{6}$$

And we introduce into Eq. (6) the expansion;

$$\psi=\psi_0+\varepsilon\psi_1+\varepsilon^2\psi_2+\cdots. \tag{7}$$

and, equate the coefficients of different powers of ε, we obtain the following equations:

$O(\varepsilon^{-1})$ $\qquad \psi_0=\frac{1}{2}\phi$,

$O(1)$ $\qquad \psi_1=(-\mu)(\frac{1}{\sigma_t}\frac{\partial}{\partial x})\frac{\phi}{2}$.

$O(\varepsilon)$ $\qquad \psi_2=(\mu^2-\frac{1}{3})(\frac{1}{\sigma_t}\frac{\partial}{\partial x})^2\frac{\phi}{2}$,

$O(\varepsilon^2)$ $\qquad \psi_3=\frac{1}{v\sigma_t}\frac{\partial}{\partial t}(\frac{\mu}{\sigma_t}\frac{\partial}{\partial x})\frac{\phi}{2}+(\frac{\mu}{3}-\mu^3)(\frac{1}{\sigma_t}\frac{\partial}{\partial x})^3\frac{\phi}{2}$,

$O(\varepsilon^3)$ $\qquad \psi_4=-\frac{1}{v\sigma_t}\frac{\partial}{\partial t}(\mu^2-\frac{1}{3})(\frac{1}{\sigma_t}\frac{\partial}{\partial x})^2\frac{\phi}{2}+(\mu^4-\frac{\mu^2}{3}-\frac{4}{45})(\frac{1}{\sigma_t}\frac{\partial}{\partial x})^4\frac{\phi}{2}$,

and so on. Introducing these moments into Eq.(7), we obtain an asymptotic expansion for ψ in terms of ϕ. Applying this expansion into Eq. (5), we finally obtain the equation for ϕ as follows;

$$\frac{\varepsilon}{v}\frac{\partial}{\partial t}\phi(x,t)-\frac{\partial}{\partial x}[\frac{\varepsilon}{3}(\frac{1}{\sigma_t}\frac{\partial}{\partial x})-\frac{\varepsilon^3}{3}(\frac{1}{\sigma_t}\frac{\partial}{\partial x})\{\frac{1}{v\sigma_t}\frac{\partial}{\partial t}-\frac{4}{15}(\frac{1}{\sigma_t}\frac{\partial}{\partial x})^2\}+O(\varepsilon^5)]\phi+\frac{1}{\varepsilon}(\sigma_t-\sigma_s)\phi=\varepsilon Q(x,t). \tag{8}$$

If we delete the terms of $O(\varepsilon^3)$ and higher in Eq. (8) and reapply the scaling reversely, we obtain,

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(x,t)-\frac{\partial}{\partial x}\frac{1}{3\sigma_t}\frac{\partial}{\partial x}\phi+(\sigma_t-\sigma_s)\phi=Q(x,t). \tag{9}$$

These are the conventional multigroup time-dependent diffusion equations.

If we delete the terms of $O(\varepsilon^5)$ and higher in Eq. (8) and introduce Eq. (9) into the resultant equation, we obtain,

$$\left(\frac{1}{v\sigma_t}\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\frac{4}{15\sigma_t}\frac{\partial}{\partial x}\frac{1}{\sigma_t}\right)\left\{\frac{1}{v}\frac{\partial}{\partial t}\phi+(\sigma_t-\sigma_s)\phi-Q\right\}+\frac{1}{v}\frac{\partial}{\partial t}\phi(x,t)-\frac{\partial}{\partial x}\frac{1}{3\sigma_t}\frac{\partial}{\partial x}\phi+(\sigma_t-\sigma_s)\phi=Q(x,t).\quad(10)$$

These are the **multigroup *modified* time-dependent SP_3 equations**[4] which could be obtained by neglecting the time derivative term of the second moment of angular flux, $\partial\psi_2/\partial t$, in the multigroup time-dependent SP_3 equations. We carry out the same asymptotic analysis on this equation as we do in Sec. II, and the results show that the *modified* time-dependent SP_3 equation has also the same asymptotic approximations as the time-dependent transport equation up to the third order in a physical regime in which the time-dependent diffusion equation is the leading order approximation. The advantages of using *modified* time-dependent SP_3 equation for the one-group case are discussed in Ref. 6.

V. ASYMPTOTIC ANALYSIS OF THE TIME-DEPENDENT SEP EQUATION IN 2-D X-Y GEOMETRY

In the recent paper by Noh et al.,[7] the authors introduced the Simplified Even-Parity (SEP) equation which could be obtained from the even-parity equation using the assumption, $\chi(\mu,\eta)=\chi(\mu,-\eta)$, in 2-D X-Y geometry ($\chi$ is even-parity flux). With the resulting elimination of the cross-derivative terms of the even-parity equation, SEP equation is much more computationally efficient than the original even-parity equation. All the work in that paper deals with the steady-state case.

In this paper, we derive the time-dependent SEP equation from the time-dependent transport equation using the same assumption. This derivation is straightforward, and we will not show it here for brevity.

The time-dependent SEP equation in 2-D X-Y geometry is

$$\frac{1}{\sigma_t v^2}\frac{\partial^2\chi}{\partial t^2}+\frac{2}{v}\frac{\partial\chi}{\partial t}-\frac{\mu^2}{\sigma_t}\frac{\partial^2\chi}{\partial x^2}-\frac{\eta^2}{\sigma_t}\frac{\partial^2\chi}{\partial y^2}+\sigma_t\chi=\left(\frac{1}{v\sigma_t}\frac{\partial}{\partial t}+1\right)(\sigma_s\phi+Q).\quad(11)$$

Applying the same scaling and expansion to this equation as in Sec. II, we obtain the following equations;

$O(\varepsilon)$
$$\frac{1}{v}\frac{\partial\phi_0}{\partial t}-D\nabla^2\phi_0+\sigma_a\phi_0=Q\,,$$

$O(\varepsilon^2)$
$$\frac{1}{v}\frac{\partial\phi_1}{\partial t}-D\nabla^2\phi_1+\sigma_a\phi_1=0\,,$$

$O(\varepsilon^3)$
$$\left(\frac{1}{v\sigma_t}\frac{\partial}{\partial t}-\frac{4}{15\sigma_t^2}\nabla^2\right)D\nabla^2\phi_0+\frac{4}{15\sigma_t^2}\frac{\partial^4\phi_0}{\partial x^2\partial y^2}+\frac{1}{v}\frac{\partial\phi_2}{\partial t}-D\nabla^2\phi_2+\sigma_a\phi_2=0\,.$$

Comparing these asymptotic approximations with those of the time-dependent transport equation in Sec. II, we find that the time-dependent SEP equation also has the same asymptotic approximations as the time-dependent transport equation only up to the second order in a physical regime in which the time-dependent diffusion equation is the leading order approximation.

In addition, we derive here the *modified* time-dependent SEP equation which could be obtained by neglecting the time derivative term of the odd-parity flux, $\partial\beta/\partial t$, in the time-dependent transport equation. The *modified* time-dependent SEP equation in 2-D X-Y geometry is

$$\frac{1}{v}\frac{\partial\chi}{\partial t}-\frac{\mu^2}{\sigma_t}\frac{\partial^2\chi}{\partial x^2}-\frac{\eta^2}{\sigma_t}\frac{\partial^2\chi}{\partial y^2}+\sigma_t\chi=\sigma_s\phi+Q \qquad (12)$$

Compared with Eq. (11), Eq. (12) contains only the first order time derivative term of χ. Accordingly, solutions can be obtained more computationally efficiently than the original time-dependent SEP equation. We carry out the same asymptotic analysis on this equation and the results are as follows;

$O(\varepsilon)$
$$\frac{1}{v}\frac{\partial\phi_0}{\partial t}-D\nabla^2\phi_0+\sigma_a\phi_0=Q\ ,$$

$O(\varepsilon^2)$
$$\frac{1}{v}\frac{\partial\phi_1}{\partial t}-D\nabla^2\phi_1+\sigma_a\phi_1=0\ ,$$

$O(\varepsilon^3)$
$$\left(-\frac{4}{15\sigma_t^2}\nabla^2\right)D\nabla^2\phi_0+\frac{4}{15\sigma_t^3}\frac{\partial^4\phi_0}{\partial x^2\partial y^2}+\frac{1}{v}\frac{\partial\phi_2}{\partial t}-D\nabla^2\phi_2+\sigma_a\phi_2=0\ .$$

Thus, the time-dependent *modified* SEP equation also has the same asymptotic approximations as the time-dependent transport equation up to the second order.

VI. CONCLUSIONS

We have shown that the time-dependent SP_3 equation contains higher order asymptotic approximations to the time-dependent transport equation than the other competitive time-dependent equations in a physical regime in which the time-dependent diffusion equation is the leading order approximation. And numerical results show that, in most of cases in 2-D X-Y geometry, the time-dependent SP_3 solutions are a lot more accurate than the time-dependent diffusion and the telegrapher's solutions. In addition, we derive the multigroup *modified* time-dependent SP_3 equation from the multigroup time-dependent transport equation by an asymptotic expansion. These results imply that, in many problems in which the conventional time-dependent diffusion equation is not sufficiently accurate, the time-dependent SP_3 equation could give a lot more accurate solutions than the time-dependent diffusion equation with almost the same computational efforts. A numerical test for the time-dependent SP_3 equation in more general cases, such as in 2-D R-Z geometry, with a multigroup energy treatment, is being carried out by the authors to validate the advantages of the time-dependent SP_3 equation in more realistic problems. Those conclusions and future work mentioned above for the time-dependent SP_3 equation are also applicable to the *modified* time-dependent SP_3 equation.

ACKNOWLEDGMENTS

The authors acknowledge helpful discussions with E. W. Larsen of the University of Michigan and T. Noh of the Los Alamos National Laboratory. This work was conducted under the auspices of the U. S. Department of Energy.

REFERENCES

1. E. E. Lewis and W. F. Miller, Jr., *Computational Methods of Neutron Transport*, John Wiley and Sons, Inc., New York, 1984, *rev.* in 1993.

2. M. Ash, *Nuclear Reactor Kinetics*, McGraw-Hill, Inc., New York, 1965.

3. E. M. Gelbard, "Simplified Spherical Harmonics Equations and Their Use in Shielding Problems," *WAPD-T-1182 (Rev. 1)* (February, 1961)

4. D. Tomasevic and E. W. Larsen, "The Simplified P_3 Correction to the Multidimensional Diffusion Equation," *Trans. Am. Nucl. Soc.*, **66**, 232 (1992).

5. E. W. Larsen, J. M. Morel, and J. M. McGhee, "Asymptotic Derivation of the Simplified P_N Equations," *Proc. Topl. Mtg. Mathematical Methods and Supercomputing in Nuclear Applications, M&C + SNA '93*, April 19-23, 1993, Karlsruhe, Germany, Vol. 1, p. 718 (1993).

6. U. Shin, W. F. Miller, Jr., and J. E. Morel, "Asymptotic Derivation of the Time-Dependent SP_3 Equations and Numerical Calculations," *Trans. Am. Nucl. Soc.*, **69**, 207 (1993).

7. E. W. Larsen, J. E. Morel, and W. F. Miller, Jr., "Asymptotic Solutions of Numerical Transport Problems in Diffusive Regimes," *Comp. Phys.*, **69**(#2), 283-324 (April, 1987).

8. E. W. Larsen, "Asymptotic Derivation of the Multigroup P_1 and SP_N Equations," *Trans. Am. Nucl. Soc.*, **69**, 209 (1993).

9. T. Noh, W. F. Miller, Jr., and J. E. Morel, "Improved Approximations Applied to the S_N Even-Parity Equation," *Trans. Am. Nucl. Soc.*, **69**, 214 (1993).