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LAMS-861

Series A

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ELASTIC CONSTANTS AND SOUND VELOCITIES

IV. The Elastic Constants of Plutonium

Work done by:

G. Foster Evans (Appendix)
Henry L. Laquer
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Report written by:

Henry L. Laquer

[redacted]

ELASTIC CONSTANTS AND SOUND VELOCITIES

IV. The Elastic Constants of Plutonium

Summary

The elastic constants of two specimens of α-phase plutonium and of one specimen of δ-phase stabilized plutonium were obtained by methods described in an earlier report.[1] These data should be good to about one percent. Mathematical expressions are given to account for the effect of a coating of different material upon the observed resonance frequencies. The temperature coefficient of Young's modulus of α-phase plutonium has also been measured.

Introduction

This paper reports the elastic constant values obtained with three different nickel coated plutonium specimens. It is only a preliminary report, since work on specimens of different metallurgical history and treatment is in progress.

History of Samples

Unless noted differently, all fabrication work was done by CMR-11. The α-phase specimen, E-296, was vacuum cast. The highest temperature reached during the casting was just above 900°C. The highest pressure, 16 microns, occurred at about 700°C. After it had cooled to room

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[1] Henry L. Laquer and William E. McGee, LAMS-850

temperature, the sample was put into a press, heated in 1-3/4 hours from 30°C to 340°C, and extruded at 17.5 tons (28,000 psi) in the δ-phase. It was then machined, cleaned, and nickel coated at 62 ± 3°C.

The stabilized δ-phase specimen, E-295, containing three atom percent of gallium, was vacuum cast. The highest temperature reached in casting was also just above 900°C. The highest pressure, 12 microns, occurred at about 870°C. The casting was annealed for 1/4 hour at 500°C. After cooling, the sample was put in a press, heated in two hours from 30°C to 330°C, and extruded at 15 tons (24,000 psi). It was then machined, cleaned, and nickel coated at 100 ± 10°C.

The α-phase "thermal conductivity bar", Z-13, was vacuum cast by CMR-11. It was pressed to the highest possible density by CMR-5. A double acting die and a Riehle hydraulic testing machine were used. The die temperature was raised from room temperature to 160°C in two hours. After "soaking" the sample at 160°C for 30 minutes a load of 52,000 psi was applied. The temperature was held between 150 to 160°C with the specimen under load, before cooling under load to 38°C. The major part of the forming was done with the metal in the β-phase and some subsequent filling out to compensate for the β-α shrinkage. After this, the sample was stored under refrigeration for almost eight months and then nickel coated at about 70°C.

Table I lists the weights, densities, dimensions, and coating data for the three samples as reported by the groups doing the fabricating.

Technique of Measurement

The techniques used in this work are those described in a previous report[1]. The longitudinal and torsional vibrational frequencies of the samples E-295 and E-296 were obtained with Rochelle salt crystals. It had been our intention to re-measure these samples using the electrostatic method, but it was found that the coating had deteriorated to such an extent as to make the work unsafe without special dry-box instrumentation. With sample Z-13, the longitudinal resonances were obtained by the electrostatic method, and the torsional ones with torque bimorph crystals. The crystals were attached, at first, with strippable paint, and then, since this did not prove very satisfactory, with Amphenol "Coil Dope" #912. Phenyl salicylate was not available for the earlier work (E-295, E-296) and could not be used with the specimen Z-13, since its temperature would reach about 50°C when in equilibrium with the measuring equipment.

Experimental Results

Table II lists all the resonance frequencies observed with specimen E-296. The numbers listed in the

TABLE I

Plutonium Samples, Specifications

	E-296	E-295	Z-13
Composition	α-phase	δ-phase	α-phase
Machined Weight (gms)	431.523	364.996	1681.68
Machined Length (in.)	5.532	5.642	4.830
Machined Diameter (in.)	.560	.564	1.171_5
Density (bromobenzene) (gms/cc)	19.42_8	15.89_9	19.61
Density (calculated) (gms/cc)	19.33	15.80	19.71
Weight after cleaning (gms)	429.948	363.301	1679.80
Weight Loss (gms)	1.575	1.695	1.88
Coated Weight (gms)	432.466	367.967	1685.66
Coated Length (in.)	5.537	5.648	4.86_0
Coated Diameter (in.)	.565	.570	1.18_0
Nickel Weight (gms)	2.518	4.666	5.86
Nickel Thickness (calc.)(mils)	1.69	3.05	2.02

E-296, Observed Resonances (Kc)

	Expander Bars #85664						Torque Bisorghs #2JC184		
	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9
L-1	8.086	6.078	8.072	8.074	8.070	8.083			
L-2	16.067 16.062	16.064 16.057	16.057 16.053	16.040	16.033	(16.107)	16.101	16.060	(16.212)
L-3	24.025	24.014	24.007	23.959	23.936	24.040	23.773	23.675	24.021
L-4	32.000	32.011	31.984	31.971	31.980	(32.074)			
L-5	39.844	39.844	39.818	39.802	39.819	39.846			
L-6	47.565	47.565	47.544	47.516	47.529	(47.635)			
L-7	55.218	55.202	55.185	55.143	55.142	55.182			
L-8	62.612	62.597	62.560						
L-9	69.133	69.120	69.051						
L-10	75.170	76.155	76.057						
L-11	82.150	82.126	82.019						
T-1							5.381	5.379	5.378
T-2	10.853	10.855	10.856	10.812	10.810		10.875	10.848	10.867
T-3							15.871 15.877	15.861 15.857	15.856 15.884
T-4							21.184	21.181	21.302
T-5							26.469	26.454	26.494
T-6	31.667	31.553	31.645		31.605		31.647	31.634	31.762
T-7	36.750	36.824	36.882	36.799	36.852	36.864	36.896	36.931	36.880
T-8				42.128			42.131	42.182	42.276
T-9							47.416	47.394	47.459
T-10							52.580	52.567	52.812
T-11							57.944	57.918	57.986
T-12	63.224	63.202	63.142				63.191	53.170	63.281
T-13							68.434	68.416	68.525
							6.948	6.960	6.985
	15.125 15.122	15.132 15.122	15.126 15.123	15.126	15.120	15.763	15.135 15.141	15.134 15.136	15.277 15.275
	19.766	19.760	19.738	19.701	19.708	19.759	19.724	19.725	19.765
	22.145	22.420	22.625				24.286	24.202	
	24.557	24.570	24.568	24.561	24.521	24.669	24.589	24.576	24.520
	29.502	29.525	29.495	29.530	29.499	29.554	29.496	29.492	29.550
							33.904	33.922	33.632
	34.553 34.544	34.558 34.544	34.533 34.524	34.514	34.590	34.600	34.832	34.728	34.598
	39.620	39.610	39.578	39.539	39.675		39.564	39.663	39.599
	42.792	43.488	42.916						
	44.684	44.891	44.671	44.625	44.641	44.745	44.636	44.636	44.646
	49.682	49.689	49.667	49.627	49.635	49.720	49.649	49.647	49.738
	53.524	53.334							
	54.579	54.563	54.548	54.519	54.527	54.044	54.483	64.381	
	59.133	59.101	59.055	59.016	59.030	59.131	59.123	59.170	59.101
	62.076	52.068	62.023				61.186	61.157	61.856
	64.652	64.623	64.573						
	65.232	65.226	65.162						
		69.673	69.590						
	73.828	73.822	73.734						
	78.515	78.490	78.386						
	81.827	81.799	81.692						

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first six columns were obtained with 3/16 x 3/16 x 5/16 (inches) expander bars #85664, weighing .377 and .369 gms. Those in the last three columns were obtained with 3/16 x 3/16 x 5/16 (inches) torque bimorphs #2JC185, weighing .371 and .377 gms. It is obvious from the Table that many spurious, probably bending, vibrations are excited by the crystals. The use of the electrostatic method would thus have been very desirable, and we intend to do this as soon as our dry-box instrumentation is complete. A wire cradle support with the wires at the 1/4 points, i.e., the nodes for the second and sixth longitudinal and torsional harmonics was used in all runs except runs 6 and 9, where a rigid center clamp was used. Table III lists all the observed resonance frequencies for the stabilized δ-phase specimen E-295. Three runs were made using #85664 expander bars, weighing .389 and .377 gms. Two runs were made with #2JC185 torque bimorphs weighing .360 and .369 gms. Again a great number of bending vibrations are apparent. The center clamp was employed in runs 3 and 5. The longitudinal and torsional reduced frequencies for both bars are recorded in Table IV, and plotted in Figs. 1, 2, and 3.

With the pure plutonium specimen Z-13, the assignment of the higher longitudinal harmonics, although obtained by the electrostatic method, presented some difficulty, due to

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TABLE III

E-295, Observed Resonances (Kc)

	Expander Bars #85664			Torque Bimorphs 2JC185	
	Run 1	Run 2	Run 3	Run 4	Run 5
L-1	(5.333)	6.103	6.112	(5.328)	
L-2	12.129	12.141			
L-3	18.165	18.181	18.202		
L-4	24.089	24.117	(24.255)		
L-5	29.895	29.923	29.973		
L-6	35.561	35.592	(35.722)		
L-7	40.998		41.086		
L-8	46.165		(46.296)		
L-9	50.973				
T-1				4.013	4.002
T-2				7.912	7.829
T-3				11.853	11.850
T-4	15.787			15.794 15.795	
T-5				19.733	19.754
T-6				23.686	23.739
T-7				27.629	27.655
T-8	31.536			31.565	32.009
T-9	35.445			35.482	35.524
				2.885	2.884
	8.225		8.217	8.229	8.242
	8.249			8.253	
	11.447			11.451	11.446
	11.473		11.827	11.476	
	14.875			14.869	14.952
	18.415			18.411	18.608
	21.923	22.2			
	23.552		23.964		
	25.038			24.570	
	26.80				
	30.195			30.184	30.291
	33.777		33.765		
	37.370		37.440		
	39.382				
	40.834		40.814		
	43.399		43.529		
	44.409		44.637		
	47.284				
	47.485		47.593		
	50.434				
	50.680				

[redacted]

 [redacted]

TABLE IV

E-296, E-295, Reduced Frequencies

	E-296						E-295		
	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 1	Run 2	Run 3
L-1	8.086	8.078	8.072	8.074	8.070	8.083	?	6.103	6.112
L-2	8.034 8.031	8.032 8.029	8.029 8.027	8.020	8.017	(8.054)	6.065	6.071	
L-3	8.008	8.005	8.002	7.986	7.979	8.013	6.055	6.060	6.067
L-4	8.000	8.003	7.996	7.993	7.995	(8.019)	6.022	6.029	(6.064)
L-5	7.969	7.969	7.964	7.960	7.964	7.969	5.979	5.985	5.995
L-6	7.927	7.927	7.924	7.919	7.922	(7.939)	5.927	5.932	(5.954)
L-7	7.888	7.886	7.884	7.878	7.878	7.883	5.857		5.869
L-8	7.827	7.825	7.820				5.771		(5.787)
L-9	7.681	7.680	7.672				5.664		
L-10	7.617	7.616	7.606						
L-11	7.468	7.466	7.456						
	Run 7	Run 8	Run 9				Run 4	Run 5	
T-1	5.381	5.379	5.378				4.013	4.002	
T-2	5.438	5.424	5.434				3.956	3.915	
T-3	5.290 5.292	5.287 5.286	5.285 5.295				3.951	3.950	
T-4	5.296	5.295	5.326				3.949		
T-5	5.294	5.291	5.299				3.947	3.951	
T-6	5.275	5.272	5.294				3.948	3.957	
T-7	5.271	5.276	5.269				3.947	3.951	
T-8	5.266	5.273	5.285				3.946	4.001	
T-9	5.269	5.266	5.273				3.942	3.947	
T-10	5.258	5.257	5.281						
T-11	5.268	5.265	5.271						
T-12	5.266	5.264	5.273						
T-13	5.264	5.263	5.271						

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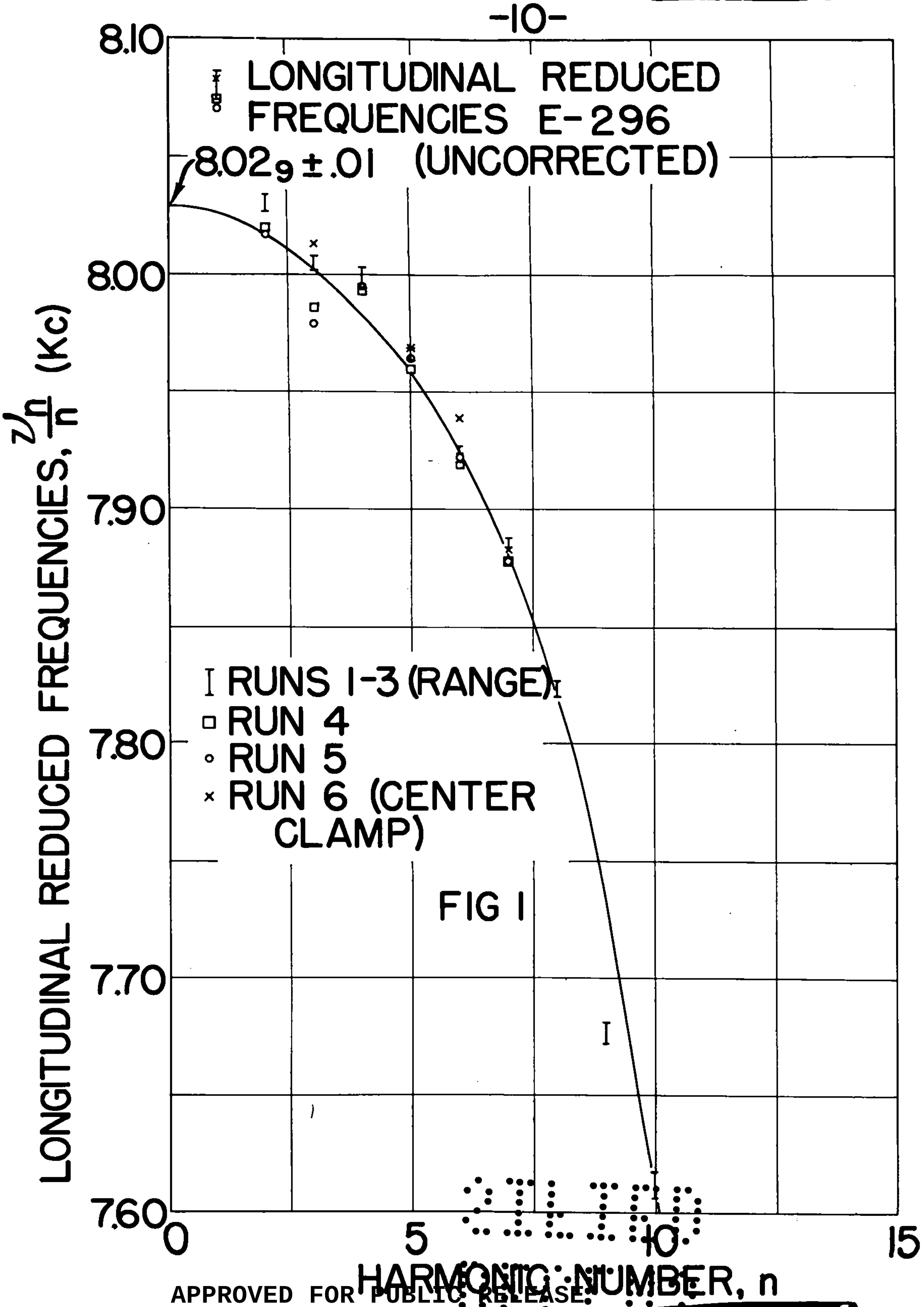

FIG 1

[redacted]

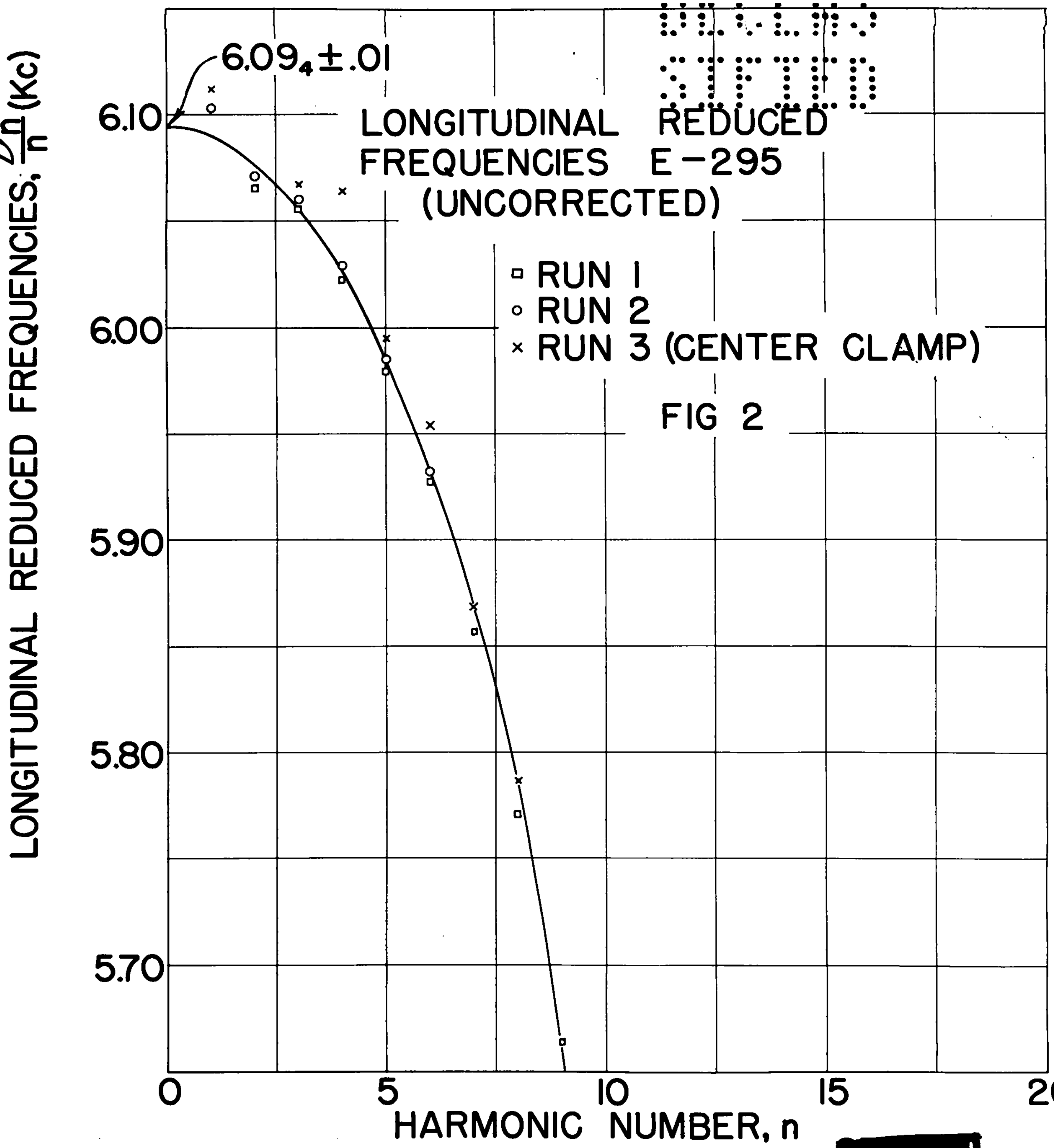

FIG 2

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[redacted]

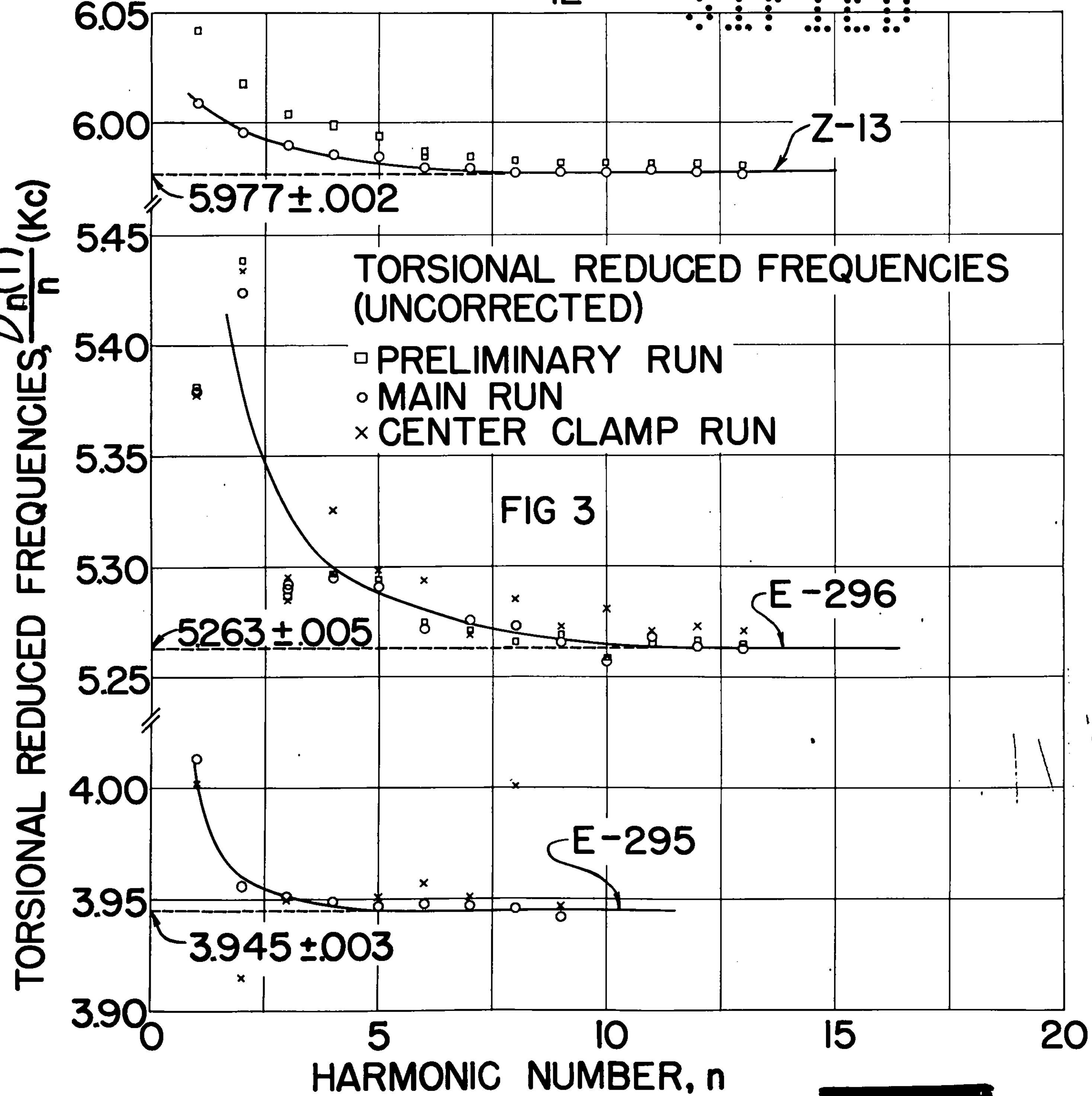

FIG 3

[redacted]

the rapid velocity dispersion caused by the large diameter of the bar. Table V lists the resonance frequencies observed in three runs with the electrostatic method, and those observed with #2JC185 torque bimorph crystals weighing .350 and .352 gms. The reduced frequencies, Table VI, for Z-13 are plotted in Figs. 3 and 4.

An inspection of the Figures shows that the scattering of the reduced frequencies from smooth curves during any one run is quite small; however, there are larger systematic discrepancies between successive runs. These are probably due to the fact that some measurements were made before the temperature of the specimen had reached a steady state value. The longitudinal $\nu_0(L)$'s, i.e., the frequencies for an infinitesimally thin bar, are obtained as the extrapolated intercept with the ordinate ($n = 0$). Since it is known from previous work[1] that the torsional frequencies obtained with crystals exhibit a pronounced anomalous dispersion, the torsional $\nu_0(T)$'s were taken as a horizontal asymptote to the observed frequencies. We note from Fig. 3 that, with the larger specimen (Z-13), this dispersion is less pronounced than with the smaller ones.

Table VII lists these completely uncorrected ν_0's together with probable errors, estimated from the scattering of the points in the Figures.

TABLE V

Z-13, Observed Resonances (Kc)

	Electrostatic Drive			Torque Bimorphs #2JC185	
	Run 1	Run 2	Run 3	Run 4	Run 5
L-1	9.041_5	8.993_4	9.024_1		
L-2	17.987	17.912	17.975_7		
L-3	26.729	26.626	26.725_5		
L-4	34.890 34.884	34.776 34.772	34.914		
L-5	41.440	41.327	41.491		
L-6		45.493	45.679		
L-7			48.314		
L-8	51.767	51.641	51.820		
L-9			54.518		
L-10	58.173	58.025	58.229		
L-11			62.252		
L-12	66.296	66.159	66.360		
L-13			70.384		
L-14	74.927	74.792	75.018		
L-15			78.235		
L-16		83.466	83.760		
T-1				6.042	6.009
T-2				12.037	11.992
T-3				18.011	17.969
T-4				23.994	23.946
T-5				29.970	29.925
T-6				35.920 35.913	35.882
T-7				41.894	41.859
T-8				47.865	47.825
T-9				53.842	53.801
T-10				59.819	59.776
T-11				65.803	65.766
T-12				71.784	71.741
T-13				77.748	77.696
		8.110		8.167	
		8.116			
		13.546		13.589	
		19.143			
		24.587		24.706	
		28.585		28.754	
				31.607	
				32.320	
				33.460	
				33.554	
				36.5	
			47.191		
			47.295		
			50.388		
			51.611		
			59.056		
			61.299		
			65.249		
			74.584		
			80.999	80.480	

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TABLE VI

Z-13, Reduced Frequencies

	Run 1	Run 2	Run 3		Run 4	Run 5
L-1	9.041_5	8.993_4	9.024_1	T-1	6.042	6.009
L-2	8.993_5	8.956_0	8.987_9	T-2	6.018	5.996
L-3	8.909_7	8.875_2	8.908_5	T-3	6.004	5.990
L-4	8.721	8.693	8.728_5	T-4	5.999	5.986
L-5	8.288	8.265	8.298_2	T-5	5.994	5.985
L-6		7.582	7.613_2	T-6	5.985	5.980
L-7			6.902_0	T-7	5.985	5.980
L-8	6.471	6.455	6.477_5	T-8	5.983	5.978
L-9			6.057_5	T-9	5.982	5.978
L-10	5.817	5.803	5.822_9	T-10	5.982	5.978
L-11			5.659_3	T-11	5.982	5.979
L-12	5.525	5.513	5.530_0	T-12	5.982	5.978
L-13			5.414_2	T-13	5.981	5.977
L-14	5.352	5.342	5.358_4			
L-15			(5.215_7)			
L-16		5.217	5.235_0			

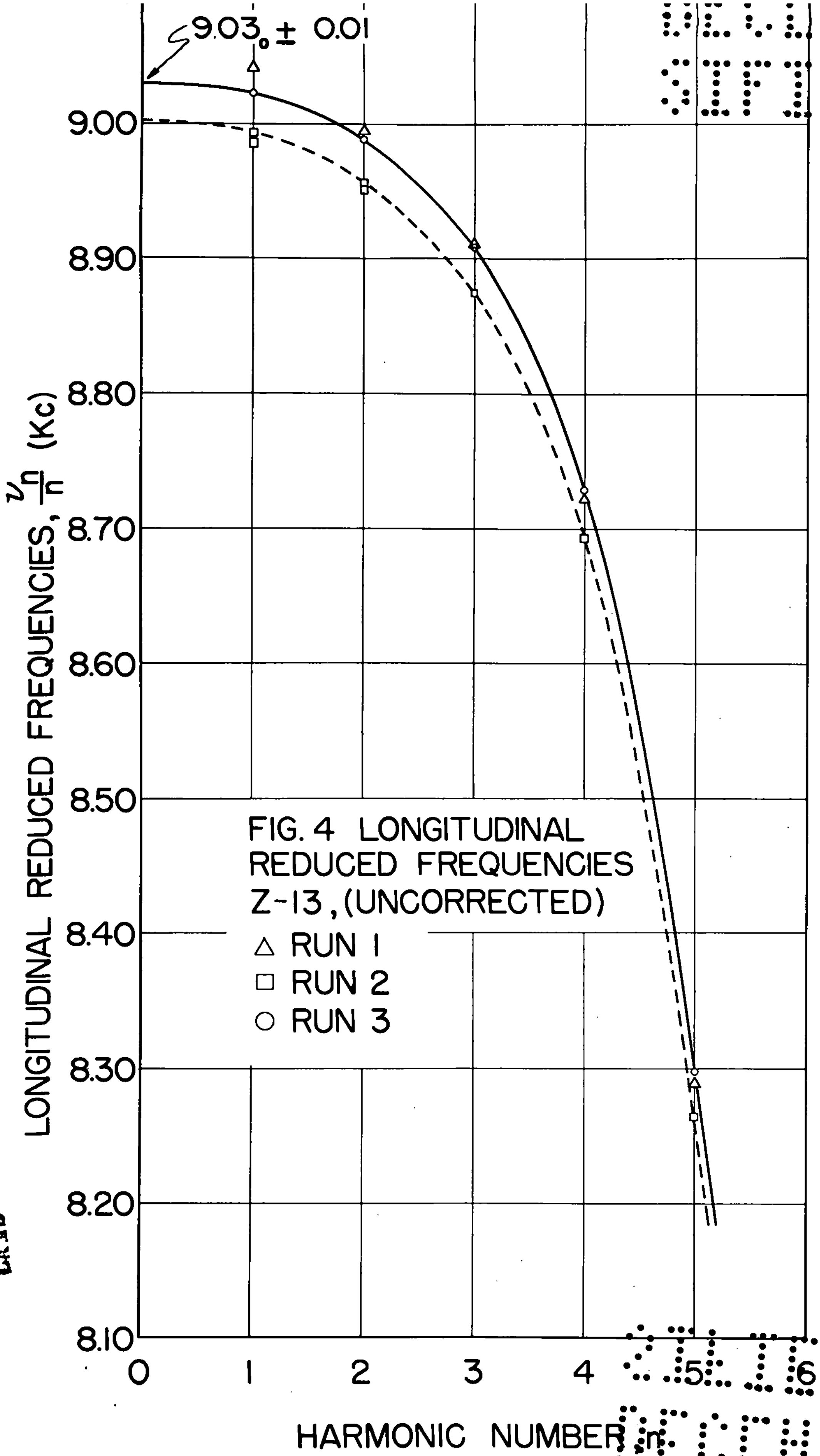

FIG. 4 LONGITUDINAL REDUCED FREQUENCIES Z-13, (UNCORRECTED)

TABLE VII

Uncorrected ν_0's (Kc)

	E-296	E-295	Z-13
$\nu_0(L)$	$8.02_9 \pm .01$	$6.09_4 \pm .01$	$9.030 \pm .01$
$\nu_0(T)$	$5.263 \pm .005$	$3.945 \pm .003$	$5.977 \pm .002$

Actually there is some ambiguity in extrapolating to $n = 0$ for the data obtained with crystals and it is necessary to be guided by the Bancroft dispersion plot (see p. 26 et seq) to arrive at the best value for $\nu_0(L)$. This is not necessary for data obtained by the electrostatic method.

Temperature Coefficient

The systematic discrepancies between successive runs led us to study the self-heating of one plutonium specimen (Z-13). This self-heating offers a very simple means to obtain approximate values for the temperature coefficient of the elastic moduli. The sample was cooled under tap water, placed on the electrostatic set-up as quickly as possible, and the first longitudinal resonance frequency was measured simultaneously with the surface temperature at a point near the middle of the specimen. An uncalibrated Chromel P - Alumel thermocouple was used. The time, temperature, and frequency values are given in

Table VIII. The temperature asymptotically approaches a value of about 51°C, as shown by Fig. 5. The frequency vs. temperature plot (Fig. 6) forms a straight line the slope of which is $(60 \pm 3) \times 10^{-5}/^{\circ}C$. This would be the temperature coefficient for the longitudinal sound velocity. The temperature coefficient of Young's modulus is then $(120 \pm 6) \times 10^{-5}/^{\circ}C$.

Calculation of Sound Velocities and Elastic Constants

In order to calculate the elastic constants of the plutonium specimens, we have to correct the ν_0's listed in Table VII in such a way as to compensate for the effect of the crystals and of the nickel coating.

a. Crystal Correction:

The relatively large masses of all the plutonium specimens make the simple m/M and i/I corrections appear sufficient to account for the loading effect of the crystals. Since all the crystals used had the same dimensions (3/16 x 3/16 x 5/16 inches), their moments of inertia about an axis perpendicular to and through the center of the square faces are equal to $m\,a^2/6$, or $m(0.476)^2/6 = 0.0378m$ gm-cm^2. Table IX summarizes the crystal corrections.

b. Coating Correction:

Whereas the crystals only add to the inertia (kinetic energy) of the vibrating system, the coating adds to the inertia and to the springiness (potential energy)

TABLE VIII

Temperature Coefficient Data

Time	Temperature	Frequency (L-1)
2:55 p.	20.5°C	---
3:05	37.8	9.091_1 Kc
3:08	39.0	9.085_4
3:12	41.0	9.077_0
3:15	42.0	9.071_6
3:20	43.9	9.061_6
3:32	47.1	9.040_9
3:47	49.5	9.030_0
4:00	49.8	9.026_2
4:05	50.0	9.024_1

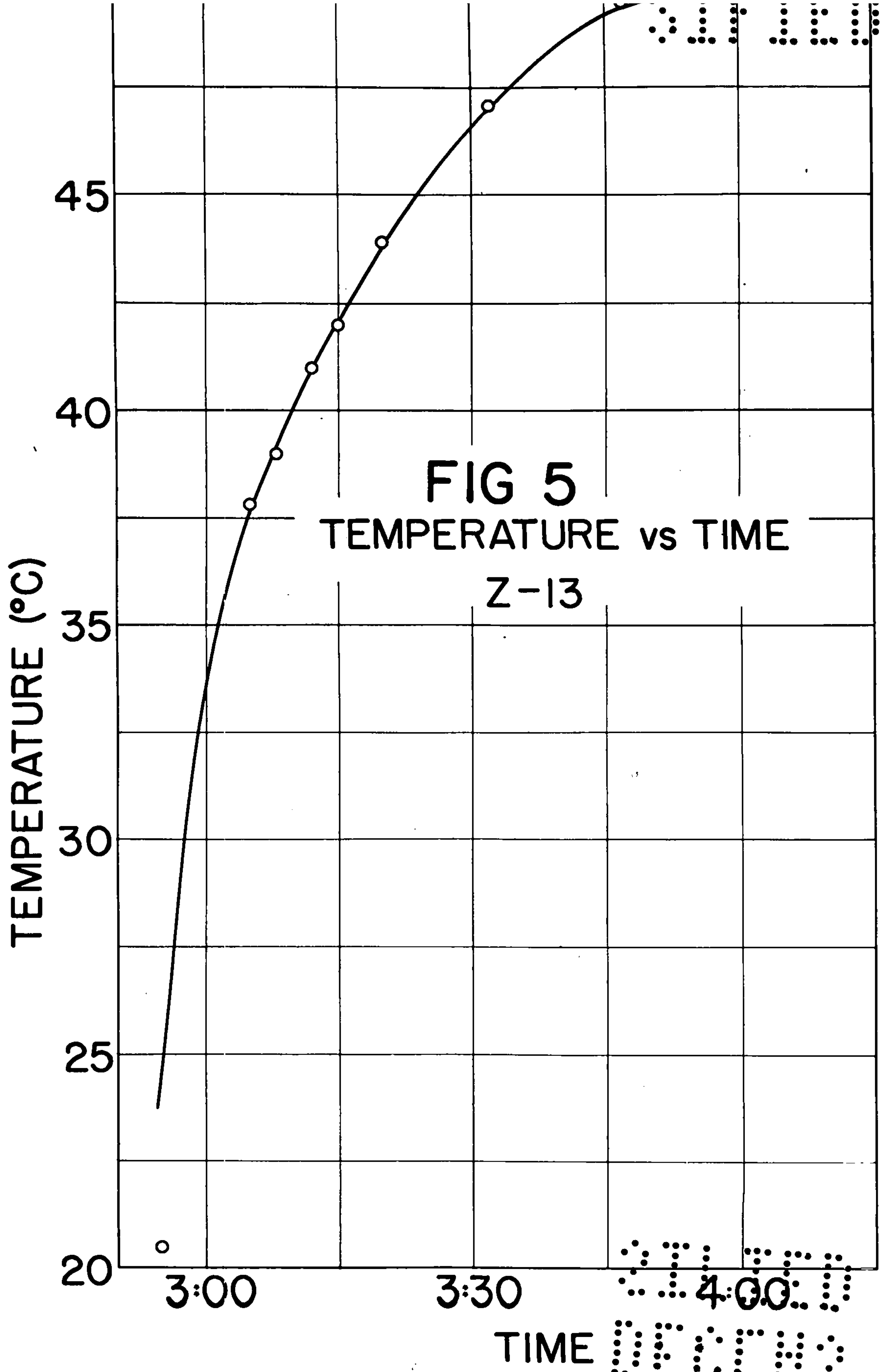

FIG 5
TEMPERATURE vs TIME
Z-13

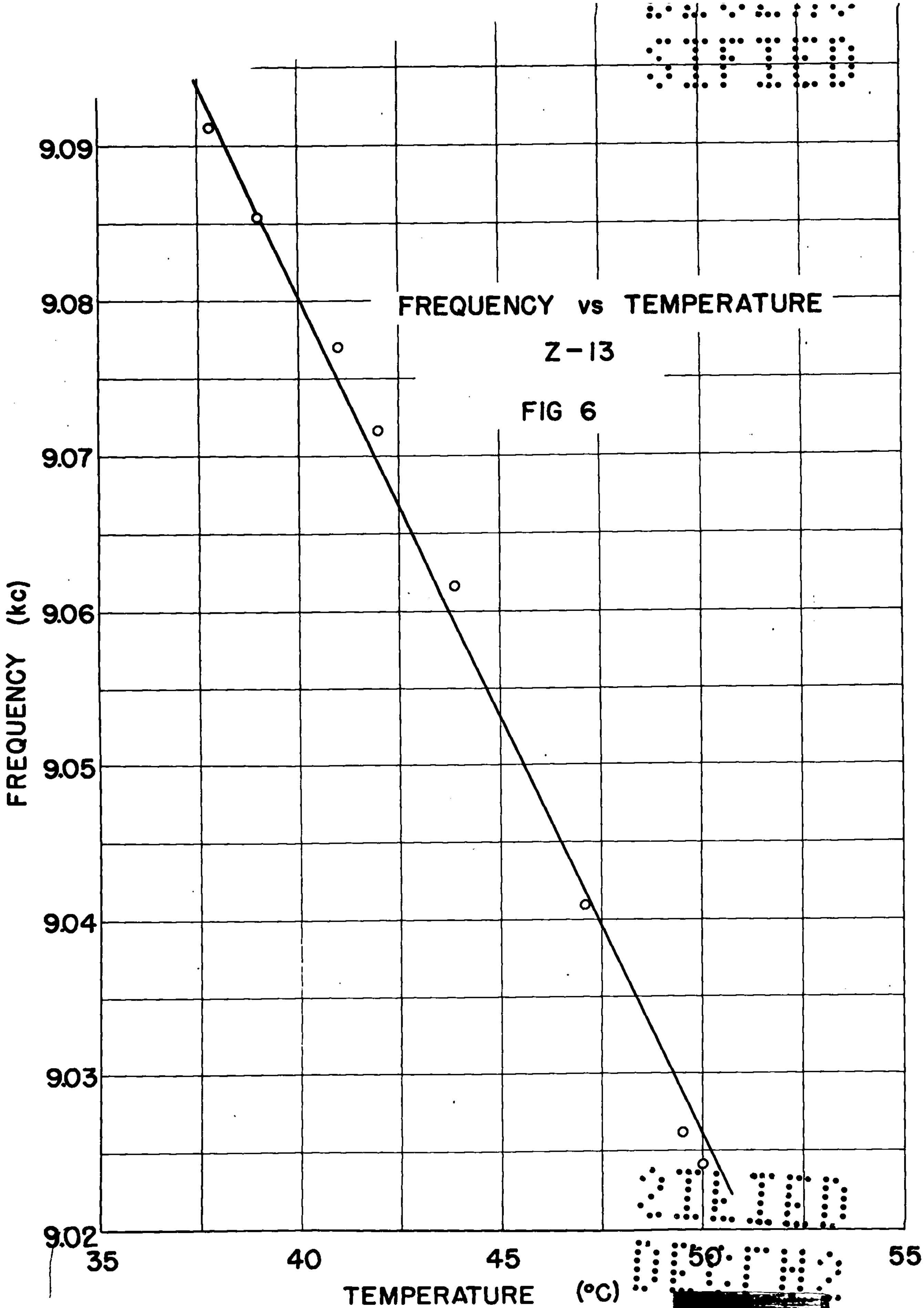
FREQUENCY vs TEMPERATURE
Z-13
FIG 6
FREQUENCY (kc)
9.09
9.08
9.07
9.06
9.05
9.04
9.03
9.02
35
40
45
50
55
TEMPERATURE (°C)

TABLE IX

Crystal Corrections

	E-296	E-295	Z-13
M (Bar)	429.948	363.301	1679.80
m (2 Crystals)	.746	.766	---
m/M (Longit.)	$.0017_4$	$.0021_1$	---
I (Bar)	108.735	93.202	1859.161
m (2 Crystals)	.748	.729	.702
i (2 Crystals)	.0283	.0276	.0265
i/I (Torsion)	$.0002_6$	$.0003_0$	$.0000_1$

 [redacted]

of the system. Birch and Bancroft[2] have solved the problem for the case of torsional vibrations, using the approximate methods developed by Rayleigh[3]. By the same methods, Foster Evans, (Alt. Group Leader, T-3) has obtained the corresponding relations for longitudinal vibrations. The derivation of these relations is given in the Appendix.

In general, if both kinetic and potential energy of a vibrating system are increased by a small amount, the frequency, ν_o, of the undisturbed system is related to the frequency, ν, of the actual system by

$$\nu_o = \nu(1 + a - c) \tag{1}$$

where a is a function of the additional kinetic energy and c a function of the additional potential energy.

For longitudinal vibrations, the coating of thickness ΔR at the sides gives

$$a = \Delta R/R \cdot \rho'/\rho \ , \tag{2}$$

$$c = \Delta R/R \cdot E'/E \ , \tag{3}$$

and the coating at the two ends

$$a' = 2\Delta R/L \cdot \rho'/\rho \ . \tag{4}$$

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2 Francis Birch and Dennison Bancroft, J. of Geol. 46, 59-87 (1938).

3 Lord Rayleigh, The Theory of Sound, 2nd ed. (reprint), Vol. 1, Ch. 4, Dover Publications, New York, (1945).

[redacted]

For torsional vibrations, the sides give

$$a = 2\Delta R/R \cdot \rho'/\rho \,, \qquad (5)$$

$$c = 2\Delta R/R \cdot \mu'/\mu \,, \qquad (6)$$

and the coating at the two ends

$$a' = 2\Delta R/L \cdot \rho'/\rho \,. \qquad (7)$$

To obtain numerical values for these corrections, we take the following values for nickel from the Handbook of Chemistry and Physics[4]:

$$E' = 21. \times 10^{11} \text{ dynes/cm}^2$$
$$\mu' = 7.3 \times 10^{11} \text{ dynes/cm}^2$$
$$\rho' = 8.90 \text{ gms/cm}^3.$$

The thin nickel layer deposited by carbonyl decomposition may well have properties different from the bulk properties just listed. This problem still needs to be investigated. All the coating corrections, as well as the quantities necessary to calculate them, are summarized in Table X. Table XI lists the sum of the crystal and coating corrections to be applied to the uncorrected ν_0's in Table VII, expressed both as percentage of frequency and as number of cycles.

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4 Handbook of Chemistry and Physics, 30th ed., Chemical Rubber Publishing Co., Cleveland, Ohio, 1947, pp. 451, 1708.

[redacted]

TABLE X

Coating Corrections

		E-296	E-295	Z-13
$\Delta R/R$		$.0060_4$	$.0108_2$	$.0034_5$
$2\Delta R/R$		$.0120_7$	$.0216_3$	$.0069_0$
$2\Delta R/L$		$.0006_1$	$.0010_8$	$.0008_4$
ρ'/ρ		$.4581_0$	$.5597_8$	$.4538_5$
E'/E		2.16	4.72	2.21
μ'/μ		1.77	4.20	1.76
Longitudinal	c	$.0130_5$	$.0510_7$	$.0076_2$
	a	$.0027_7$	$.0060_6$	$.0015_7$
	a'	$.0002_8$	$.0006_0$	$.0003_8$
Torsional	c	$.0213_6$	$.0908_5$	$.0121_4$
	a	$.0055_3$	$.0121_1$	$.0031_3$
	a'	$.0002_8$	$.0006_0$	$.0003_8$

[redacted]

TABLE XI

Overall Corrections

		E-296	E-295	Z-13
Longitudinal:	percent	-0.82_6	-4.23_0	-0.56_7
	cycles	-66.	-252.	-51.
Torsional :	percent	-1.52_9	-7.78_4	-0.86_2
	cycles	-80.	-295.	-51.

Fortunately, these corrections are small, and, assuming that the coating thickness is known, their magnitude is in doubt by no more than plus or minus two or three cycles, except for the δ-phase specimen E-295, for which the corrections are uncertain by plus or minus ten cycles. However, it should be remembered that all corrections are proportional to the coating thickness ΔR and, so far, little is known about its uniformity along the main axis of the bars or even about any circular cross section. Table XII lists the corrected ν_0's, the sound velocities v_ℓ and v_t, the elastic constants E, μ, and Poisson's ratio σ calculated therefrom. As a further check of the consistency of the data, the σ calculated from the intercepts is compared with the one obtainable from the velocity dispersion curves according to Bancroft[5]. In doing this, an assumption is

5 Dennison Bancroft, Phys. Rev. 59, 588-593 (1941)

[redacted]

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TABLE XII

PLUTONIUM, RESULTS

SPECIMEN	ν_0(L) (Kc)	ν_0(T) (Kc)	v_l	v_t	E	μ	σ FROM INTERCEPTS	σ FROM DISPERSIO
			cm/SEC x 10^{-5}		DYNES / cm^2 x 10^{-11}			
E-296 (α)	$7.96_3 \pm .01$	$5.18_3 \pm .01$	2.24	1.46	$9.73 \pm .03$	$4.12 \pm .02$	0.18	0.20
E-295 (δ)	$5.84_2 \pm .02$	$3.65_0 \pm .02$	1.67	1.05	$4.46 \pm .03$	$1.74 \pm .02$	0.28	0.33
Z-13 (α)	$8.97_9 \pm .01$	$5.92_6 \pm .01$	2.20	1.45	$9.52 \pm .02$	$4.15 \pm .02$	0.15	0.17

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made which is not necessarily correct for the higher harmonics; namely, that the percentage corrections given in Table XI are the same for all the harmonics studied. Plots of the normalized frequencies against the Bancroft parameter n d/2L (for the uncoated specimens) are given in Figs. 7, 8, and 9.

The α-phase specimen, E-296, shows some scattering from a smooth curve probably due to the crystals which were used here. The dispersion curve gives a value of Poisson's ratio of 0.20 $\pm$.01 against a calculated value of 0.18. The δ-phase specimen E-295 shows similar scattering. Here we obtain a Poisson's ratio of 0.33 $\pm$.01 against a calculated value of 0.28. The α-phase specimen Z-13, with the large diameter shows the most anomalous behavior in that the actual dispersion curve does not match any of Bancroft's calculated ones. We can estimate a value of Poisson's ratio of 0.17 $\pm$ .03 whereas the calculated value is 0.15.

Conclusions

Due to the uncertainty involved in measuring coated specimens, the data summarized in Table XII can only be considered as preliminary. However, it may be stated that pure α-phase plutonium has elastic moduli slightly smaller than palladium or copper and slightly larger than brass.[4] The gallium stabilized δ-phase alloy is more plastic (higher Poisson's ratio) and has elastic moduli about the same as magnesium or tin.[4]

[redacted]

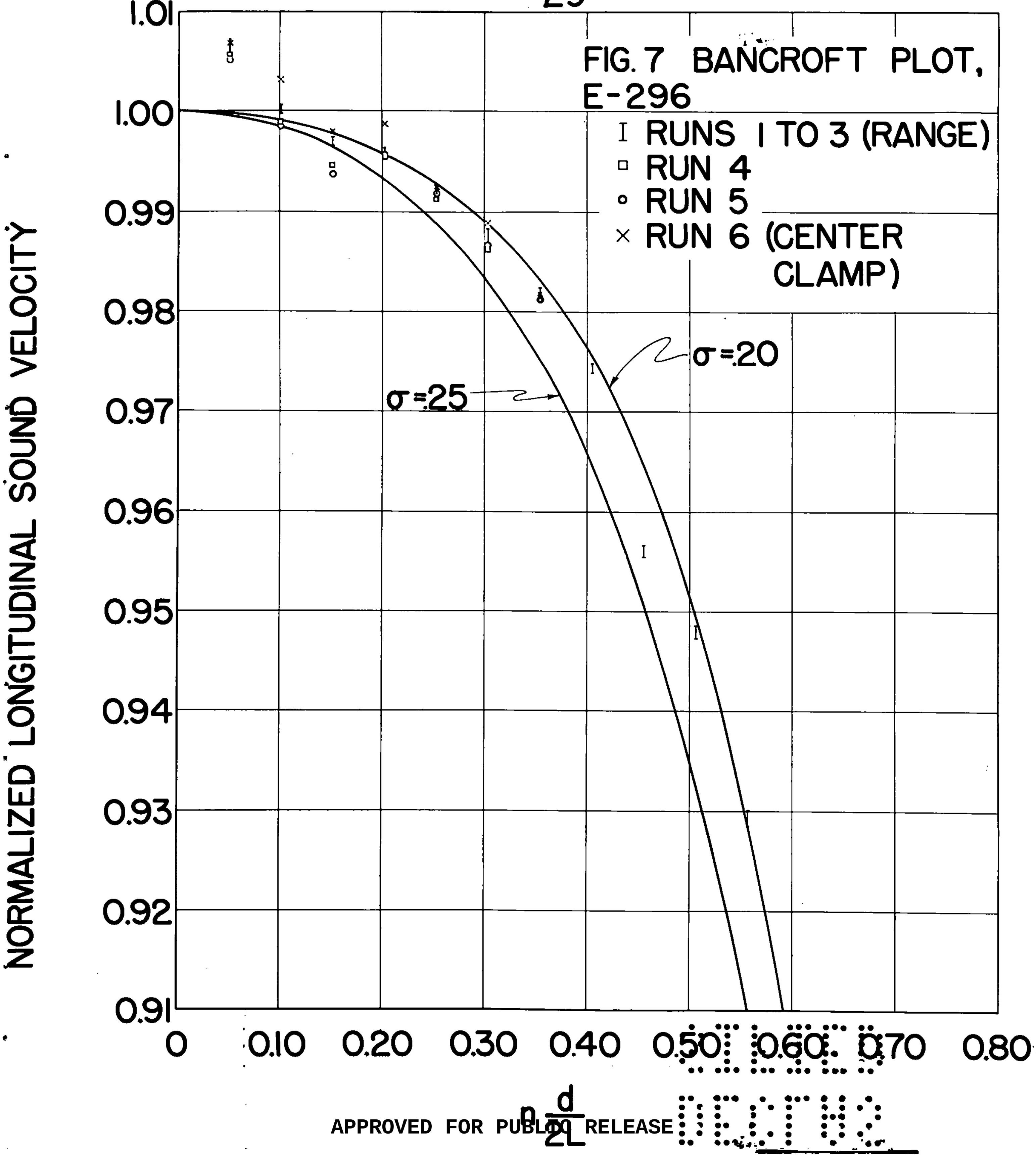

FIG. 7 BANCROFT PLOT, E-296

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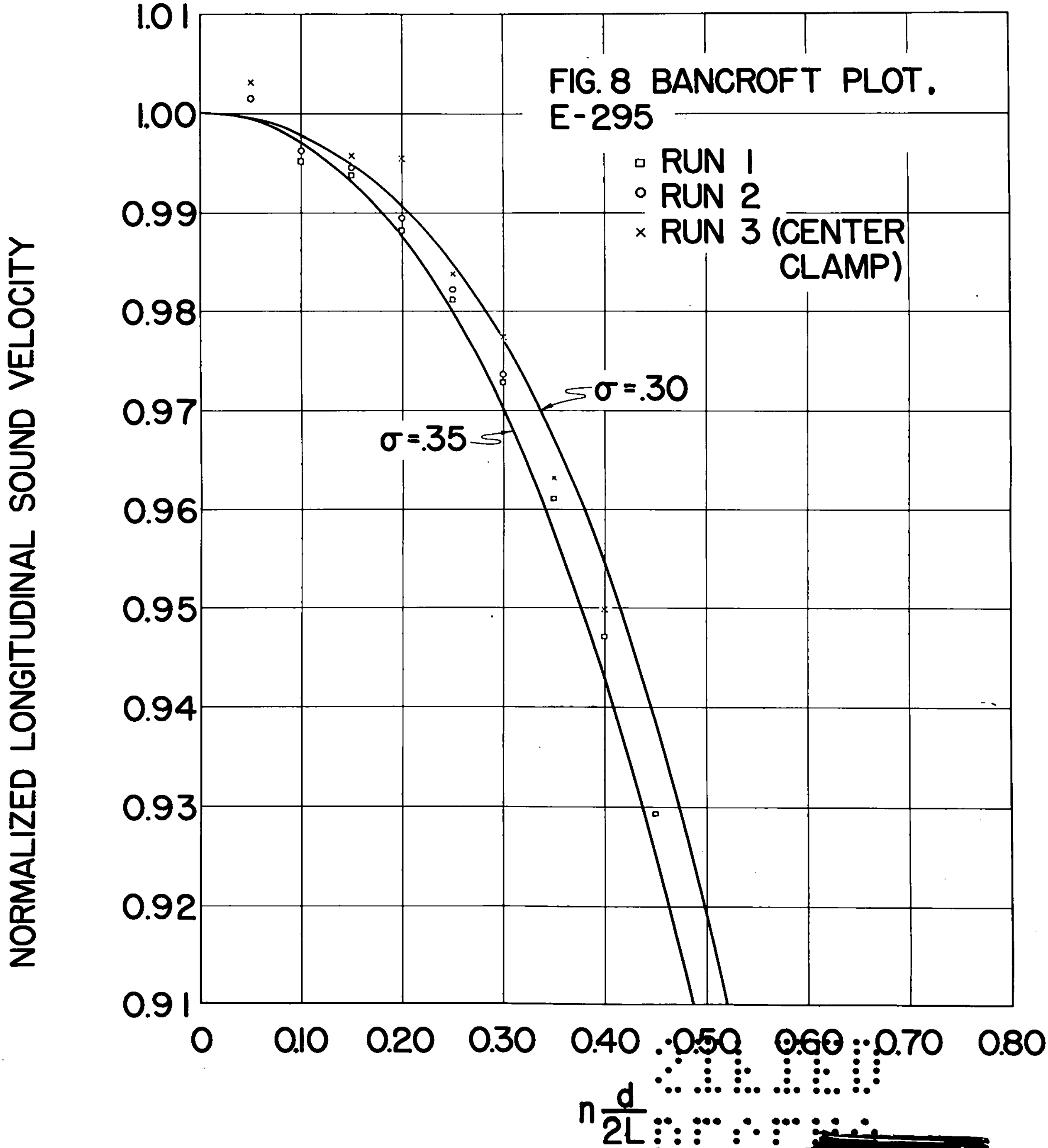
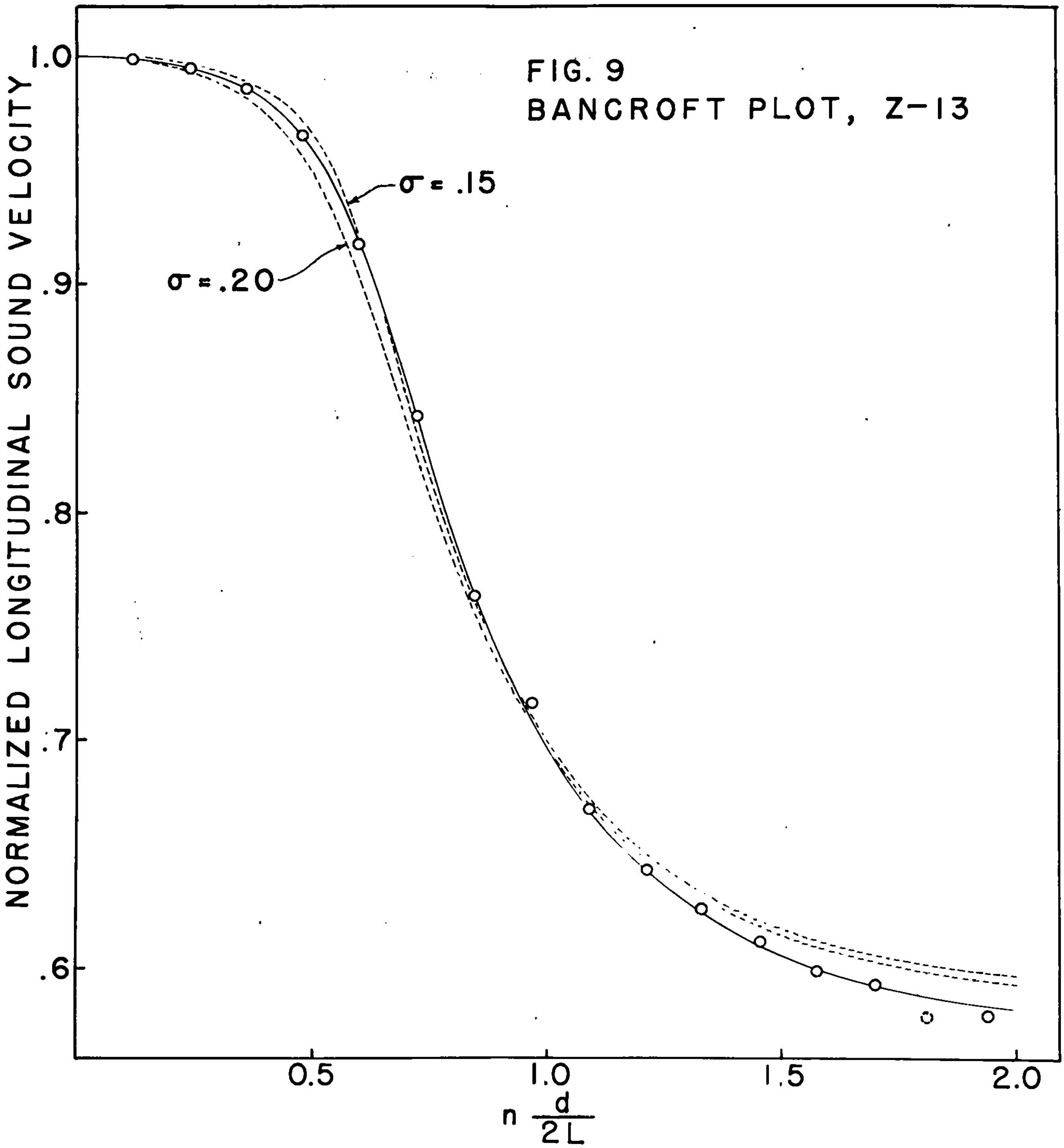


FIG. 8 BANCROFT PLOT, E-295

[redacted]



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APPENDIX

The Effect of a Uniform Coating on the Resonance Frequencies of Metal Rods

In the following, the effect of a uniform coating upon the observed resonance frequencies of a homogeneous metal rod is treated by the approximation methods developed by Rayleigh[3]. The rod has a density, ρ, and elastic moduli E and μ. The coating of thickness, ΔR, has a different density, ρ', and elastic moduli E' and μ'.

In a conservative system, having one degree of freedom, the kinetic energy, T, is, as long as the displacements, q, are small

$$T = 1/2\ a\ \dot{q}^2 \ . \qquad \text{(A-1)}$$

The gradient of the potential energy, U, will be a force which for elastic materials will be proportional to the displacement, if the coordinates are chosen such that

U = 0 for q = 0. Thus

$$dU/dq = cq \qquad \text{(A-2)}$$

$$U = 1/2\ cq^2 \ . \qquad \text{(A-3)}$$

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3 cf. page 23.

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Since the system is conservative:

$$T + U = \text{const.} \tag{A-4}$$

$$1/2\, a\dot{q}^2 + 1/2\, cq^2 = \text{const.} \tag{A-5}$$

Differentiate:

$$a\dot{q}\ddot{q} + cq\dot{q} = 0\ , \tag{A-6}$$

$$\ddot{q} + c/a\ q = 0\ . \tag{A-7}$$

This is the wave equation and its solution is:

$$q = A \cos\ (n\omega_o t + B), \tag{A-8}$$

where

$$\omega_o = 2\pi\nu_o = \sqrt{c/a.} \tag{A-9}$$

If now T and U of the system are perturbed in such a way that

$$c \longrightarrow c + \Delta c\ ,$$

and $$a \longrightarrow a + \Delta a\ ,$$

the resonance frequencies of the system will be given by

$$\omega^2 = \frac{c + \Delta c}{a + \Delta a} = \frac{c}{a}\ \frac{(1 + \Delta c/c)}{(1 + \Delta a/a)}\ , \tag{A-10}$$

and one calculates ω_o (or ν_o) from ω (or ν) according to

$$\omega_o^2/\omega^2 = (1 + \Delta a/a)/(1 + \Delta c/c) \cong 1 + \Delta a/a - \Delta c/c\ , \tag{A-11}$$

$$\omega_o \cong \omega\ (1 + \Delta a/2a\ -\ \Delta c/2c)\ . \tag{A-12}$$

(a) Longitudinal Vibrations

Longitudinal vibrations are described by the linear displacement ξ. For the unperturbed case, we have

$$d^2\xi/dt^2 = E/\rho \cdot d^2\xi/dx^2 \tag{A-13}$$

with the solution for a free-free bar

$$\xi = s(n,t) \cos n\pi x/L \ , \tag{A-14}$$

the s being the displacement from the equilibrium position of one end at any time, t.

We calculate

$$T = \int_0^L 1/2 \ (\pi R^2 \rho) \ dx \, \dot{\xi}^2 \tag{A-15}$$

$$= \pi R^2 \rho/2 \int_0^L \dot{\xi}^2 \, dx \tag{A-16}$$

$$= \pi R^2 \rho/2 \ \dot{s}^2 \int_0^L \cos^2 \ n\pi x/L \ dx \tag{A-17}$$

$$= \pi R^2 \rho/2 \ \dot{s}^2 \ L/n\pi \left[(n\pi x/2L) - 0 \right]_0^L \tag{A-18}$$

$$= \pi r^2 L\rho/4 \ \dot{s}^2 = M/4 \ \dot{s}^2 = 1/2(M/2) \ \dot{s}^2 \tag{A-19}$$

To obtain the potential energy U, we have to integrate the net force on any elementary disc:

$$\underbrace{(\pi R^2)}_{\text{area}} \ \underbrace{\left(E \frac{d^2\xi}{dx^2} dx\right)}_{\text{force/unit area}} = \pi R^2 E \ s \ (-n^2\pi^2/L^2) \cos (n\pi x/L) \ dx \tag{A-20}$$

$$= - \underbrace{(n^2\pi^3 R^2 E/L^2)}_{k} \ \xi \ dx \tag{A-21}$$

$$U = -\int_0^L \int_0^{\xi} (-\,k\,\xi\,dx)\,d\xi \tag{A-22}$$

$$= ks^2/2 \int_0^L \cos^2\ n\pi x/L\ \ dx \tag{A-23}$$

$$U = ks^2/2\quad L/n\pi \quad \Big[(n\pi x/2L) - 0\Big]_0^L \tag{A-24}$$

$$= \frac{ks^2L}{4} = \frac{1}{2}\left(\frac{kL}{2}\right) s^2 = \frac{1}{2}\left(\frac{n^2\pi^3R^2E}{2L}\right) s^2 \tag{A-25}$$

From equations (1), (3), (19), and (25)

$$a = M/2 \tag{A-26}$$

$$c = n^2\pi^3R^2E/2L \tag{A-27}$$

We first consider the effect of the coating at the sides of the bar:

$$\Delta T_1 = \int_0^L 1/2\ (2\pi R\Delta R\rho' dx)\,\dot{\xi}^2\ , \tag{A-28}$$

$$= \pi R\Delta R\rho' \int_0^L \dot{\xi}^2\ dx \tag{A-29}$$

$$\Delta T_1/T = \Delta a_1/a = 2\pi R\Delta R\rho'/\pi R^2\rho \tag{A-30}$$

$$= 2\Delta R/R \cdot \rho'/\rho = m/M \tag{A-31}$$

The net force on a ring of thickness dx is:

$$(2\pi R\Delta R)\ \left(E'\,\frac{d^2\xi}{dx^2}\,dx\right) = 2\pi R\Delta RE'\quad s\left(\frac{-n^2\pi^2}{L^2}\right)\cos\frac{n\pi x}{L}\,dx \tag{A-32}$$

$$= \frac{-2n^2\pi^3R\Delta R}{L^2}\ E'\ \ \xi\,dx \tag{A-33}$$

Since equation (33) will be integrated exactly as equation

(21) we can write immediately

$$\Delta U_1/U = \Delta a_1/a = 2\Delta R/R \cdot E'/E \tag{A-34}$$

The coating at the ends adds only to the kinetic energy of the system

$$\Delta T_2 = 1/2 \ (\pi R^2 \Delta R \rho') \quad \dot{s}^2 \tag{A-35}$$

$$\Delta T_2/T = \Delta a_2/a = 2\Delta R\rho'/L\rho = 2m/M \ . \tag{A-36}$$

If there is a coating of thickness ΔR at each end, the combined effect will be just twice that given in equation (36).

(b) Torsional Vibrations

Torsional vibrations can be fully described by considering θ, the angular displacement. The wave equation in this case is

$$d^2\theta/dt^2 = \mu/\rho \quad d^2\theta/dx^2 \ , \tag{A-37}$$

with the solution for a free-free bar

$$\theta = \phi(n,t) \cos n\pi x/L \ , \tag{A-38}$$

the ϕ being the angular displacement of one end of the bar at any time, t.

$$T = \int_0^L 1/2 \ (\pi R^2 \rho \ \frac{R^2}{2} \ dx) \ \dot{\theta}^2 \tag{A-39}$$

$$= 1/4 \ \pi R^4 \rho \ \ \dot{\phi}^2 \int_0^L \cos^2 \ n\pi x/L \ \ dx \tag{A-40}$$

$$= (\pi R^4 L\rho/8) \ \dot{\phi}^2 = (MR^2/8) \ \dot{\phi}^2 \ . \tag{A-41}$$

To obtain the potential energy U, we must consider a ring of thickness dr. The net torque on this ring is:

$$\underbrace{2\pi r dr}_{\text{area}} \cdot \underbrace{r}_{\text{arm}} \cdot \underbrace{\frac{d^2(r\theta)}{dx^2}\,\mu dx}_{\text{shear/unit area}} = 2\pi\mu r^3 dr \;\; \frac{d^2\theta}{dx^2} \;\; dx \qquad \text{(A-42)}$$

The net torque on an elementary disc is:

$$\int_0^R \left(2\pi\mu \frac{d^2\theta}{dx^2}\, dx\right) r^3\, dr = \frac{\pi\mu}{2} R^4 \;\; \frac{d^2\theta}{dx^2} \;\; dx \qquad \text{(A-43)}$$

$$= \frac{\pi\mu R^4}{2} \;\; \phi\left(\frac{-n^2\pi^2}{L^2}\right) \;\; \cos\frac{n\pi x}{L} \;\; dx \qquad \text{(A-44)}$$

$$= -\underbrace{\frac{n^2\pi^3R^4\mu}{2L^2}}_{k'} \;\; \theta\, dx \qquad \text{(A-45)}$$

$$U = -\int_0^L \int_0^\theta -(k'\theta\, dx) \;\; d\theta \qquad \text{(A-46)}$$

$$= (k'\phi^2/2) \int_0^L \cos^2 \;\; (n\pi x/L) \;\; dx \qquad \text{(A-47)}$$

$$= \frac{k'\phi^2 L}{4} = \frac{1}{2} \left(\frac{k'L}{2}\right)\phi^2 = \frac{1}{2}\left(\frac{n^2\pi^3R^4\mu}{4L}\right)\phi^2 \qquad \text{(A-48)}$$

From equations (1), (3), (41), and (48)

$$a = MR^2/4 \qquad \text{(A-49)}$$

$$c = n^2\pi^3R^4\mu/4L \qquad \text{(A-50)}$$

Again we consider first the effect of the coating at the sides of the bar.

$$\Delta T_1 = \int_0^L 1/2 \;\; (2\pi R\Delta R\rho'\, dx)\; R^2 \;\; \dot\theta^{\,2} \qquad \text{(A-51)}$$

$$= \pi R^3 \Delta R \rho' \int_0^L \dot\theta^{\,2}\, dx \qquad \text{(A-52)}$$

$$\Delta T_1/T = \Delta a_1/a = 4\Delta R/R \cdot \rho'/\rho = 2m/M = i/I \qquad \text{(A-53)}$$

$$\Delta U_1 = -\int_0^L \int_0^\theta \underbrace{(2\pi R\Delta R)}_{\text{area}} \cdot \underbrace{R}_{\text{arm}} \cdot \underbrace{\left(\frac{d^2\theta}{dx^2}\, dx\, R\mu'\right)}_{\text{shear/unit area}} d\theta \,, \qquad \text{(A-54)}$$

$$= 2\pi R^3\Delta R\mu' \int_0^L \int_0^\theta - (d^2\theta/dx^2)\, dx\, d\theta \,, \qquad \text{(A-55)}$$

$$\Delta U_1/U = \Delta c_1/c = 4\Delta R/R \cdot \mu'/\mu \,. \qquad \text{(A-56)}$$

The coating at the ends, again adds kinetic energy only

$$\Delta T_2 = 1/2\ (\pi R^2\Delta R\rho')\ R^2/2\ \ \dot{\phi}^2 \,, \qquad \text{(A-57)}$$

$$\Delta T_2/T = \Delta a_2/a = 2\Delta R/L \cdot \rho'/\rho = 2m/M = 2i/I \,. \qquad \text{(A-58)}$$

According to these simple approximations the corrections should thus have the same fractional value for all harmonics of a given type of vibration.

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