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MULTI-MEDIUM CRITICAL-MASS PROBLEMS

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ABSTRACT

By use of a theory which has given excellent results for the critical mass of spheres both untamped and with infinite tampers, formulas are developed for more complicated configurations. Finite tampers, several successive tampers, air spaces between core and tamper, and holes in the active material are treated. The formulas obtained are found to be much simpler than those given by any other theory of similar accuracy. Numerical results and graphs are given for the various cases treated.

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MULTI-MEDIUM CRITICAL-MASS PROBLEMS

Introduction

Recently a method of treating critical-mass problems in the one-velocity-group approximation has been developed by Serber[1). This treatment yields formulas that are much simpler than those obtained by the spherical-harmonic method and yet gives critical radii with an error of less than 1 per cent (after application of a small correction factor[1)). The main obstacle in applying the spherical-harmonic method in its higher approximations to more complicated critical-mass problems is the size of the determinants which must be solved. Since this new method eliminates the prohibitive amount of computation involved in the polynomial method, it would seem to be particularly useful in obtaining critical masses for complicated cases. In this report applications of the method are made to the following problems:

a) Active material surrounded by a tamper of finite thickness.

b) Active material surrounded by a tamper of finite thickness which in turn is surrounded by another tamper, which may be of either finite or infinite thickness.

c) Active material is separated from an infinite tamper by an air space.

d) Active material is arranged in the form of a spherical shell with an empty hole inside. The case of a tamper extending to infinity is treated as well as the untamped case.

Actually our formulas will not be developed by fitting the integral

1) See LA-234

equation but will use a procedure suggested by A. H. Wilson in report BM 441. However, instead of using Wilson's argument (which obtains the results from the Boltzmann equation) we will justify the method using the integral equation.

Method

For complicated critical-mass problems the straight-forward method of fitting the integral equation must be modified in order to avoid a certain ambiguity which arises with respect to the proper choice of neutron density. To see the ambiguity let us examine the method. The problem of an active core embedded in an infinite tamper is attacked by first assuming the neutrons in each medium to have the same spatial dependence as in an infinite medium of that material. (For the core this is $(\sin k_1\sigma_1 r)/\sigma_1 r$ and for the tamper $Ae^{-k_2\sigma_2 r}/\sigma_2 r$. Here the σ's are the reciprocal transport mean free paths. Subscripts 1 and 2 refer to core and tamper respectively.

k_1 is given by $1 + f_1 = k_1/\tan^{-1}k_1$

$$\text{where } f_1 = \left[(\nu - 1)(\sigma_{fission})_1 - (\sigma_{capture})_1\right]/\sigma_1$$

and ν = number of neutrons coming off per fission.

k_2 is given by $1 + f_2 = k_2/\tanh^{-1}k_2$

$$\text{where } f_2 = -(\sigma_{capture})_2/\sigma_2$$

These solutions are then required to satisfy the integral equation for the neutron density at the center of the core.

$$n(o) = \int_{\substack{\text{all}\\ \text{space}}} \left[\sigma(r)\big(1 + f(r)\big) / 4\pi r^2\right] n(r) \left(\exp - \int_0^r \sigma(x)\,dx\right) dV \qquad (1)$$

This yields one equation connecting A and the critical radius (a) of the core. To obtain a second equation the solutions are required to satisfy the law of

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conservation of neutrons. This says that the number of neutrons absorbed in the tamper must equal the net number produced in the core. Stated mathematically this is:

$$\sigma_1 f_1 \int_{core} n\,dV = \sigma_2 f_2 \int_{tamper} n\,dV \qquad (2)$$

Combining the two equations gives an equation for a. Ambiguity arises when we have, for example, a sequence of one finite tamper followed by an infinite one. Using the above procedure the core density is assumed as:

$$n_1 = (\sin k_1 \sigma_1 r)/\sigma_1 r$$

in the finite tamper the density is

$$n_2 = \left[Ae^{-k_2\sigma_2 r} + Be^{k_2\sigma_2 r}\right]/\sigma_2 r$$

and in the infinite tamper.

$$n_3 = Ce^{-k_3\sigma_3 r}/\sigma_3 r$$

Fitting the integral equation and using the conservation law then gives us only two equations for the four quantities, A, B, C, and a. Hence some more assumptions would have to be made about the neutron densities. The ambiguity is contained in what these assumptions should be. Another example of the trouble one runs into is the problem of the central hole. To fit the integral equation at r = 0 would require some ad hoc hypotheses about the neutron density in the hole.

These troubles can be avoided, however, by reformulating what actually has been done in the case where they do not occur. First let us examine equation (2). Since for the steady state the net number of neutrons produced in the core must equal that flowing out of the core, the left-hand side of the

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equation must be the total flux of neutrons across the surface of the core. Similarly, since the number of neutrons absorbed in the tamper must be equal to that flowing into it, the right-hand side must be the total flux into the tamper.

Thus equation (2) merely states that the total flux is continuous across the core-tamper interface. Using the fact that the assumed solutions satisfy the equations:

$$\Delta n_1 + k_1^2 \sigma_1^2 n_1 = 0; \qquad \Delta n_2 - k_2^2 \sigma_2^2 n_2 = 0 \tag{3}$$

and the divergence theorem, we transform equation (2) into:

$$\left[-f_1 \operatorname{grad} n_1)_a / k_1^2 \sigma_1\right] \int_{\substack{\text{core}\\\text{surface}}} dS = \left[f_2 \operatorname{grad} n_2)_a / k_2^2 \sigma_2\right] \int_{\substack{\text{core}\\\text{surface}}} dS \tag{4}$$

From the above discussion and this equation we see that (2) amounts to equating the flux per unit area (F) in the two regions at the boundary surface. We also see that F for a medium is given by:

$$F = (-|f| \operatorname{grad} n)/k^2\sigma$$

Generalizing it can be said that at the boundary between two media the flux should be equated.

Substituting the assumed neutron density for the two-medium problem in (1) yields:

$$k_1 = \sigma_1(1 + f_1) \int_0^a e^{-\sigma_1 r} \left[(\sin k_1\sigma_1 r)/\sigma_1 r)/\sigma_1 r\right] dr + \left\{\sigma_2(1 + f_2) e^{-\sigma_1 a}\right\}$$
$$\left\{\int_a^\infty e^{-\sigma_2(r-a)} n_2(r) \, dr\right\} \tag{5}$$

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The first integral on the right splits into

$$\sigma_1(1 + f_1)\left\{\int_0^\infty e^{-\sigma_1 r}\left[(\sin k_1\sigma_1 r)/\sigma_1 r\right]dr - \int_a^\infty e^{-\sigma_1 r}\left[(\sin k_1\sigma_1 r)/\sigma_1 r\right]dr\right\}$$

But $\displaystyle\int_0^\infty e^{-\sigma_1 r}\left[(\sin k_1\sigma_1 r)/\sigma_1 r\right]dr = (\tan^{-1}k_1)/\sigma_1$

From our definition of k_1 we see that $\sigma_1(1 + f_1)\displaystyle\int_0^\infty e^{-\sigma_1 r}\left[(\sin k_1\sigma_1 r)/\sigma_1 r\right]dr = k_1$ which just cancels the k_1 on the left of equation (5). Performing this operation, multiplying through by $e^{\sigma_1 a}$ and transposing terms puts (5) in the form:

$$\sigma_1(1 + f_1)\int_a^\infty e^{-\sigma_1(r-a)}\left[(\sin k_1\ \sigma_1 r)/\sigma_1 r\right]dr = \sigma_2(1 + f_2)\int_a^\infty e^{-\sigma_2(r-a)}n_2(r)\,dr$$

or (6)

$$\sigma_1(1 + f_1)\int_a^\infty e^{-\sigma_1(r-a)}n_1(r)\,dr = \sigma_2(1 + f_2)\int_a^\infty e^{-\sigma_2(r-a)}n_2(r)\,dr$$

Let $N(r,\mu)$ be the number of neutrons per unit volume whose velocity vector makes an angle $\cos^{-1}\mu$ with the radius vector corresponding to a density distribution $n(r)$. Now consider the expression: $\sigma(1 + f)\int_a^\infty e^{-\sigma(r'-r)}n(r')\,dr'$. Except for the lack of a factor representing the attenuation of the neutron beam from r to 0 this is the density of neutrons at the origin coming from distances greater than r away. Since the density at the origin is uniform in angle, that coming from any one solid angle is

$$\left[\sigma(1 + f)/4\pi\right]\int_r^\infty e^{-\sigma(r'-r)}\ n(r')\,dr'$$

But the density at the origin coming in from a unit solid angle from all distances greater than r and not attenuated between r and 0 is just $N(r,-1)$.

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Thus

$$N(r,-1) = \left[\sigma(1+f)/4\pi\right]\int_r^{\infty} e^{-\sigma(r'-r)}\, n(r')\, dr'$$

Comparing this with equation (6) we see that it merely says:

$$4\pi N_1(a_0,-1) = 4\pi N_2(a_0,-1)$$
$$\text{or} \qquad N_1(a_0,-1) = N_2(a_0,-1) \qquad (7)$$

The condition which we have used (and will henceforth use at all boundaries between two media) is that the flux coming in radially be continuous.

The integrals for the $N(r,-1)$ can be written in fairly simple form. Here we include a table of this function for all the density distributions we shall need in this report[2]:

$\underline{n(r)}$	$\underline{N(r,-1)}$
$(\sin k_1\sigma_1 r)/\sigma_1 r$	$\left[\sigma(1+f)/4\pi\right] I\ Ei(\sigma r + i\sigma k r)\ e^{\sigma_1 r}$
$(\cos k_1\sigma_1 r)/\sigma_1 r$	$\left[(1+f)/4\pi\right] Re\ Ei(\sigma r + i\sigma k r)\ e^{\sigma_1 r}$
$e^{-k\sigma r}/\sigma r$	$\left[(1+f)/4\pi\right] e^{\sigma r}\ Ei(\sigma r + k\sigma r)$
$e^{k\sigma r}/\sigma r$	$\left[(1+f)/4\pi\right] e^{\sigma r}\ Ei(\sigma r - k\sigma r)$

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2) The difficulty Wilson encountered in treating the cosine term arises from failing to notice that this is an asymptotic, not an exact, solution of the Boltzmann equation. If the Boltzmann equation with a point source (to which the problems in which the cosine occur more or less correspond) is solved one obtains the $(\sin k_1\sigma_1 r)/\sigma_1 r$ term, and in addition, a linear combination of $(\cos k_2\sigma_2 r)/\sigma_2 r$ and

$$\frac{1}{2\pi r}\int_1^{\infty} \frac{e^{-rK}\, dK}{\left\{1 - \left[(1+f)/K\right]\tanh^{-1}(1/K)\right\}^2 + (1+f)^2\, \pi^2/4K^2}$$

Asymptotically this gives the $\cos k_1\sigma_1 r/\sigma_1 r$ term.

Here Ei(x) stands for $\int_x^{\infty} e^{-y}/y \, dy$, and I means "imaginary part of" and Re means "real part of".

To see what to do with a free surface let us examine how the untamped sphere is treated by fitting the integral equation. On substituting $n_1(r) = (\sin k_1\sigma_1 r)/\sigma_1 r$ in the integral equation and going through the same manipulations as before one arrives at equation (6) except that the right hand integral is not present. Thus the formula is

$$\sigma_1(1 + f_1) \int_a^{\infty} e^{-\sigma_1(r-a)} n_1(r)\,dr = 0 \tag{8}$$

as we saw above this means $N_1(a,-1) = 0$; i.e. there is no flux coming radially inward.

The procedure for attacking more complicated problems should now be clear. In each of the regions of different neutron properties the neutron density is assumed to be that corresponding to an infinite medium of the material. Thus in a solid active core a neutron density $n = (\sin k_1\sigma_1 r)/\sigma_1 r$ is used. For an infinite tamper the solution is taken as:

$$n \sim e^{-k_2\sigma_2 r}/\sigma_2 r$$

In case the active material does not extend to the center the neutron density used in the active material is:

$$n = A(\sin k_1\sigma_1 r)/\sigma_1 r + B(\cos k_1\sigma_1 r)/\sigma_1 r$$

For a finite tamper solutions of the form:

$$n = \left[Ae^{-k_2\sigma_2 r} + Be^{k_2\sigma_2 r}\right] / \sigma_2 r$$

are taken.

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These solutions are then required to satisfy certain boundary conditions. At a free surface the condition will be $N(r_s, -1) = 0$. At a surface separating two media $N(r_s, -1)$ and the flux F will be taken continuous. These conditions certainly hold for the correct solutions of the problem. However, it is not immediately obvious that merely requiring these of the approximate solution will yield good results. Experience with tamped and untamped spheres indicates that it does.

Finite Tampers

Here the problem is to find the critical radius for a sphere of active material embedded in a tamper of inner radius a and outer radius b. The core solution will be taken as $(\sin \sigma_1 k_1 r)/\sigma_1 r$ and the tamper solution as:

$$\left[A e^{-\sigma_2 k_2 r} + B e^{\sigma_2 k_2 r}\right] / \sigma_2 r$$

Subscripts 1 refer to core constants, 2 to tamper constants. Placing $N_2(b, -1) = 0$, equating $N_1(a, -1)$ to $N_2(a, -1)$ and setting

$$\left.(-f_1/k_1^2\sigma_1) \operatorname{grad} n_1\right)_a = \left.(f_2/k_2^2\sigma_2) \operatorname{grad} n_2\right)_a$$

gives us three equations for A, B, and a. Eliminating A and B gives us the following formula for the critical radius (a) in terms of b and the various nuclear constants:

$$\frac{-\sigma_1^2 k_1^2 (1 + f_1) e^{\sigma_1 a}\, I\, Ei(\sigma_1 a + i\, \sigma_1 k_1 a)}{f_1 \left[\sin \sigma_1 k_1 a - \sigma_1 k_1 a \cos k_1 \sigma_1 a\right]} \tag{9}$$

$$+ \frac{k_2^2 \sigma_2^2 (1+f_2) e^{\sigma_2 a} \left\{Ei(\sigma_2 a + \sigma_2 k_2 a) Ei(\sigma_2 b - \sigma_2 k_2 b) - Ei(\sigma_2 b + \sigma_2 k_2 b) Ei(\sigma_2 a - \sigma_2 k_2 a)\right\}}{f_2 \left\{Ei(\sigma_2 b + \sigma_2 k_2 b) e^{k_2 \sigma_2 a} (k_2 \sigma_2 a - 1) + (k_2 \sigma_2 a + 1) Ei(\sigma_2 b - \sigma_2 k_2 b) e^{-k_2 \sigma_2 a}\right\}}$$

$$= 0$$

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As a check on this method comparisons were made with some calculations made by Group T-4 on finite tampers. In the six cases checked the agreement was well within the accuracy to which the two sets of computation had been carried.

One case of practical interest has been calculated. This was to see what is lost by having a WC tamper of only 6" thickness with a core of 80 per cent 25. In units of the core mean free path the critical radius with an infinite WC tamper came out 1.5565. With 6" WC this was 1.5930 - a difference of 2.5 per cent in radius or 7.5 per cent in mass.

Double Tampers - One Infinite

By double tamper, one infinite we mean a configuration in which a spherical core of active material of radius a (region 1) is surrounded by tamper material extending to radius b (region 2); from b to ∞ there is some other tamper material (region 3).

The solutions taken in regions 1 and 2 are the same as those in the previous section. In region 3 we take $n(r) = Ce^{-\sigma_3 k_3 r}/\sigma_3 r$. To determine A, B, C, and a there are the four equations obtained by equating flux and N(r,-1) at a and b. Eliminating A, B, and C yields an equation for a:

$$\frac{-\sigma_1^2 k_1^2 (1+f_1) e^{\sigma_1 a} \, I\, Ei(\sigma_1 a + i\,\sigma_1 k_1 a)}{f_1 [\sin k_1 \sigma_1 a - \sigma_1 k_1 a \cos k_1 \sigma_1 a]} + \frac{k_2^2 \sigma_2^2 (1+f_2) e^{\sigma_2 a} \left[Ei(\sigma_2 a + \sigma_2 k_2 a) + M\, Ei(\sigma_2 a - \sigma_2 k_2 a)\right]}{f_2 \left\{(k_2 \sigma_2 a + 1) e^{-k_2 \sigma_2 a} - M e^{k_2 \sigma_2 a} (k_2 \sigma_2 a - 1)\right\}} = 0$$

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$$M = \frac{(k_2\sigma_2 b + 1)e^{-k_2\sigma_2 b} + K\ \mathrm{Ei}(\sigma_2 b + \sigma_2 k_2 b)}{e^{k_2\sigma_2 b}(k_2\sigma_2 b - 1) - K\ \mathrm{Ei}(\sigma_2 b - \sigma_2 k_2 b)}$$

$$K = \frac{-f_3 k_2^2 \sigma_2^2 (1 + f_2) e^{\sigma_2 b}(1 + \sigma_3 k_3 b)}{f_2 k_3^2 \sigma_3^2 (1 + f_3) e^{\sigma_3 b + \sigma_3 k_3 b}\ \mathrm{Ei}(\sigma_3 b + \sigma_3 k_3 b)} \qquad (10)$$

For the case of no absorption in region 3, K simplifies to:

$$K = \frac{1}{3}\ \frac{k_2^2 \sigma_2^2 (1 + f_2) e^{\sigma_2 b}}{f_2 \sigma_3^2\ e^{\sigma_3 b}\ \mathrm{Ei}(\sigma_3 b)}$$

Figure I shows the critical radius (for an 80 per cent 25 core followed by WC and then by infinite Fe) as a function of the WC thickness. After a thickness of about 3" no particular gain is achieved by increasing the amount of WC. The flat minimum at 5" is due to the effects of the favorable property of Fe, namely its lack of absorption, overcoming the bad effects caused by its small cross section.

Figure II is a similar curve with the Fe replaced by BeO.

Replacing the WC by B_{10} and the Fe by WC gives the curves shown in Figures III and IV. This is the problem encountered in considering the effect of surrounding the plug of the gun with B_{10} for safety in fabrication or shipping. Comparison with the results described below for an empty shell between core and tamper shows that the insertion of the B_{10} greatly increases the safety factor. Thus in the case of active material followed by an air space of equal volume and then by an infinite WC tamper one can only insert 1.16 WC tamped crits of material before reaching the critical state. If B_{10} is present instead of the air about 1.8 crits can be present without being

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supercritical.

These boron calculations were made assuming 100 per cent B_{10} and rough values of the cross sections. When the scattering data on B_{10} has been evaluated another calculation will be made assuming the correct B_{10} purity.

Double Finite Tampers:

If the second tamper material (region 3) extends only to a radius c instead of to infinity, we have double finite tampers. In the region 3 we now take the neutron density to be: $n(r) = \left[Ce^{-\sigma_3 k_3 r} + De^{\sigma_3 k_3 r}\right]/\sigma_3 r$ while in regions 1 and 2 it is taken as before. At a and b the same boundary conditions are used. At c the condition $N_3(c_0-1) = 0$ is added. The resulting equation for a is the same as above except that K must be replaced by

$$K = \left\{\frac{-f_3 k_2^2 \sigma_2^2 (1+f_2) e^{\sigma_2 b}}{f_2 k_3^2 \sigma_3^2 (1+f_3) e^{\sigma_3 b}}\right\} \cdot \tag{11}$$

$$\left\{\frac{Ei(\sigma_3 c + \sigma_3 k_3 c)\, e^{k_3 \sigma_3 b}(k_3\sigma_3 b - 1) + Ei(\sigma_3 c - \sigma_3 k_3 c)(k_3 \sigma_3 b + 1)\, e^{-\sigma_3 k_3 b}}{Ei(\sigma_3 b + \sigma_3 k_3 b)\, Ei(\sigma_3 c - \sigma_3 k_3 c) - Ei(\sigma_3 b - \sigma_3 k_3 b)\, Ei(\sigma_3 c + \sigma_3 k_3 c)}\right\}$$

The single case calculated with this formula was that of 80 per cent 25 followed by 6" WC and then by 6" Fe. Here $a_{crit} = 1.5730$ mfp instead of 1.5565 with infinite WC, showing that we lose about 3.2 per cent in mass by having the WC tamper finite even if Fe is added.

Air Space between Core and Infinite Tamper

Here the core of active material is assumed to extend to a radius a and the infinite tamper to begin at radius b ($b \geq a$). The neutron density in

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the core is: $n_1(\sin \sigma_1 k_1 r)/\sigma_1 r$ and in the tamper $n_2 = Ae^{-\sigma_2 k_2 r}/\sigma_2 r$. Since the total flux of neutrons crossing the surface of the active material must equal that crossing the inner surface of the tamper we must have

$$\left(-4\pi a^2 f_1/k_1^2\sigma_1\right) \text{grad } n_1\Big)_a = \left(4\pi b^2 f_2/k_2^2\sigma_2\right) \text{grad } n_2\Big)_b$$

Equating $N_2(b,-1)$ to $N_1(a,-1)$ gives the second of the two equations necessary to determine a and A. a is given by:

$$\frac{-\sigma_1^2 k_1^2 (1 + f_1) e^{\sigma_1 a}\, I\, Ei(\sigma_1 a + i\sigma_1 k_1 a)}{f_1\left[\sin k_1\sigma_1 a - \sigma_1 k_1 a \cos \sigma_1 k_1 a\right]} + \frac{k_2^2 \sigma_2^2 (1 + f_2) e^{\sigma_2 b + \sigma_2 k_2 b}}{f_2(1 + k_2\sigma_2 b)} Ei(\sigma_2 b + \sigma_2 k_2 b) = 0 \qquad (12)$$

Figure V shows how the critical mass varies as a function of a/b. It is important for the gun to note that with a geometry of core, space of equal volume, and then tamper one can put in at least 1.16 crits and still be subcritical. Since the gun plug has a still less favorable configuration, one can safely put more than 1.2 crits into it.

Calculations of critical radii from the above formulas are very simple using the graphs of LA-234. These give plots of what are essentially the terms in the above equations. By successive guessing and reading from the graphs rapid computations can be made.

Comparison with results obtained by Glauber with the spherical harmonic method for $a/b \simeq 1$ shows essentially complete agreement with this method.

Central Spherical Hole in Active Material

For the sake of the implosion and the integral experiments it is desirable to know the effect of central holes on the critical mass of both tamped

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and untamped spheres.

Let us take the radius of the hole to be a and the outer radius of active material b. For the neutron density in the active material assume:

$$n_1(r) = \left\{\sin \sigma_1 k_1 r + A \cos \sigma_1 k_1 r\right\} / \sigma_1 r$$

At b the boundary conditions are $N_1(b, -1)$ and F continuous in the case of an infinite tamper. At the edge of the hole (a) the condition of zero flux is imposed, i.e.

$$\left.\operatorname{grad} n\right)_a = 0$$

For an untamped sphere the formula for b is:

$$-I\ Ei(\sigma_1 b + i\sigma_1 k_1 b) + \frac{(\sigma_1 k_1 a - \tan \sigma_1 k_1 a)}{1 + \sigma_1 k_1 a \tan \sigma_1 k_1 a} \operatorname{Re} Ei(\sigma_1 b + i\, \sigma_1 k_1 b) = 0 \qquad (13)$$

For a tamper extending to infinity the corresponding formula (assuming $n_2 = Be^{-k_2\sigma_2 r}/\sigma_2 r$) is

$$\left\{(1 + f_1)k_1^2 \sigma_1^2 e^{\sigma_1 b}/f_1\right\}$$

$$\left\{\frac{I\ Ei(\sigma_1 b + i\sigma_1 k_1 b) - \left(\dfrac{\sigma_1 k_1 a - \tan \sigma_1 k_1 a}{1 + \sigma_1 k_1 a \tan \sigma_1 k_1 a}\right) \operatorname{Re} Ei\ (\sigma_1 b + i\, \sigma_1 k_1 b)}{\left[\sin \sigma_1 k_1 b - \sigma_1 k_1 b \cos \sigma_1 k_1 b\right] + \left(\dfrac{\sigma_1 k_1 a - \tan \sigma_1 k_1 a}{1 + \sigma_1 k_1 a \tan \sigma_1 k_1 a}\right)\left[\cos \sigma_1 k_1 b + \sigma_1 k_1 b \sin \sigma_1 k_1 b\right]}\right\}$$

$$-\left[k_2^2 (1 + f_2)\sigma_2^2\, e^{\sigma_2 b + k_2 \sigma_2 b}/f_2(1 + k_2 \sigma_2 b)\right]\ Ei(\sigma_2 b + \sigma_2 k_2 b) = 0 \qquad (14)$$

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To check the validity of the formula for small holes a perturbation theory was used. For the untamped case with 100 per cent 25 and a hole of radius 0.1 mfp the perturbation theory gave an increase of the radius b over that which would be obtained with no hole of 0.00058 mfp. The above formula gave 0.0006 mfp a satisfactory check.

From this we see that this method gives good results for small holes. In this region it also agrees with volumetric theory. For larger holes we know that (14) will give too small a mass. Applying what in the light of experience (cf. LA-234) would seem to be reasonable corrections gives a curve which again agrees with volumetric theory. Moreover, checking volumetric theory in the limit of very large holes (which can be done by comparison with a properly chosen plane slab problem) again shows it to be correct. Thus, instead of (14) the following simple receipe can be used. To calculate the critical mass with any size central hole, merely assume the active material to be uniformly distributed in a sphere of radius the same as that of the outer radius of the shell of active material. Our experience indicates that this should give the critical mass correct to within a few per cent.

<u>Conclusions:</u>

In conclusion it can be said that this method serves to solve the more complicated critical-mass problems rather conveniently in an approximation which is probably fairly good as long as one stays away from extreme cases (such as very large holes and very thin shells or tampers). However, for holes we advocate the use of volumetric theory instead of this method. In the above work no use was made either in the formulas derived or in the results quoted of the empirical fact that over a quite wide range for which the correct answers are

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known this method gives critical radii which are about 2 per cent too low. If one adds this 2 per cent to all radii calculated from the formulas in this report results correct to 1 or 2 per cent should be obtainable. Moreover, relative effects such as the per cent increase in critical mass due to using a thick, but still not infinite, tamper should be given very accurately.

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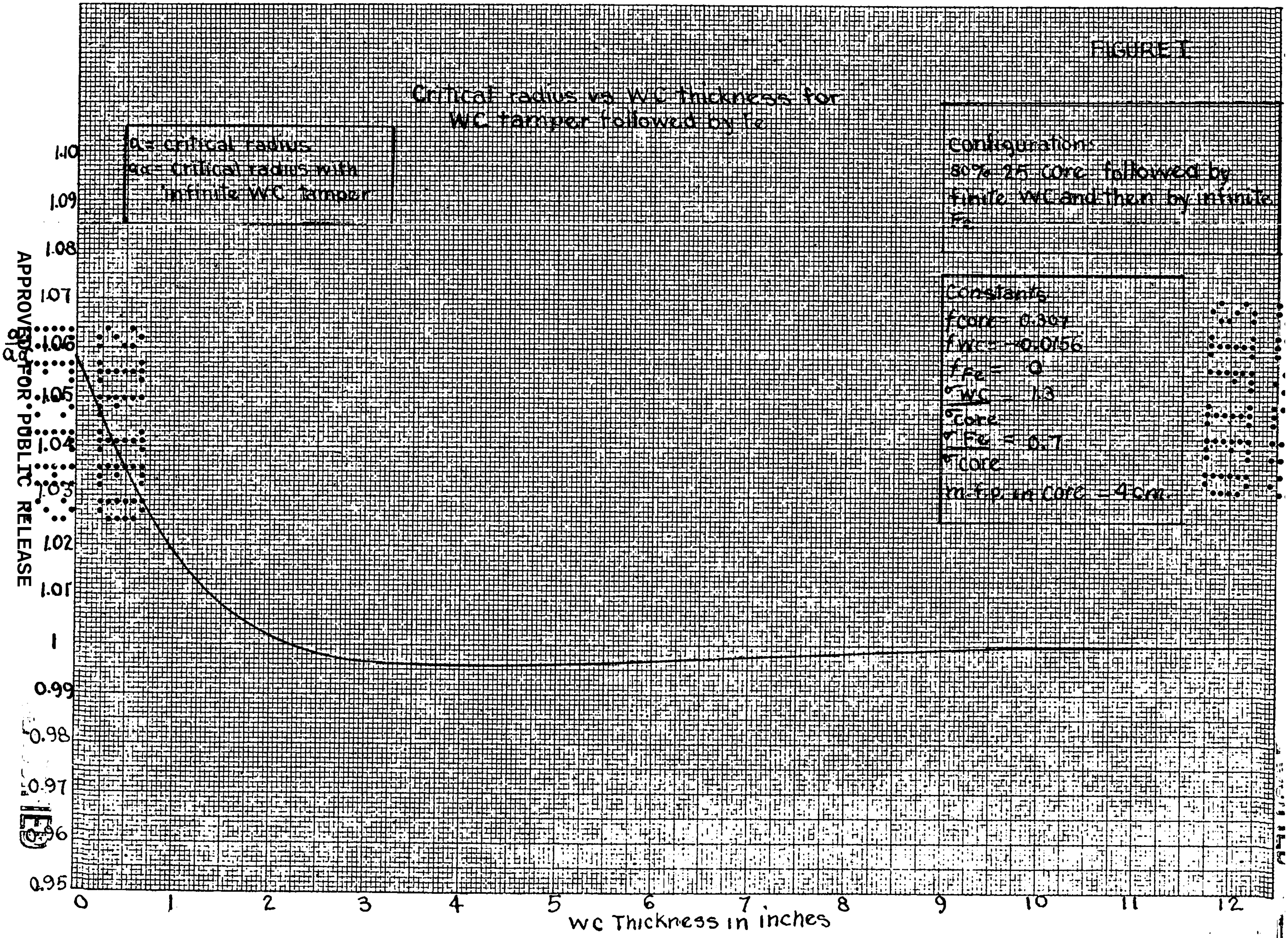
FIGURE I
Critical radius vs WC thickness for
WC tamper followed by Fe
a = critical radius
a₀ = critical radius with
infinite WC tamper
Configuration:
80% 25 core followed by
finite WC and then by infinite
Fe
Constants
f core = 0.307
f WC = -0.0156
f Fe = 0
σ WC / σ core = 1.3
σ Fe / σ core = 0.7
m.f.p. in core = 4 cm.
a/a₀
1.10
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
0.96
0.95
0
1
2
3
4
5
6
7
8
9
10
11
12
WC Thickness in inches

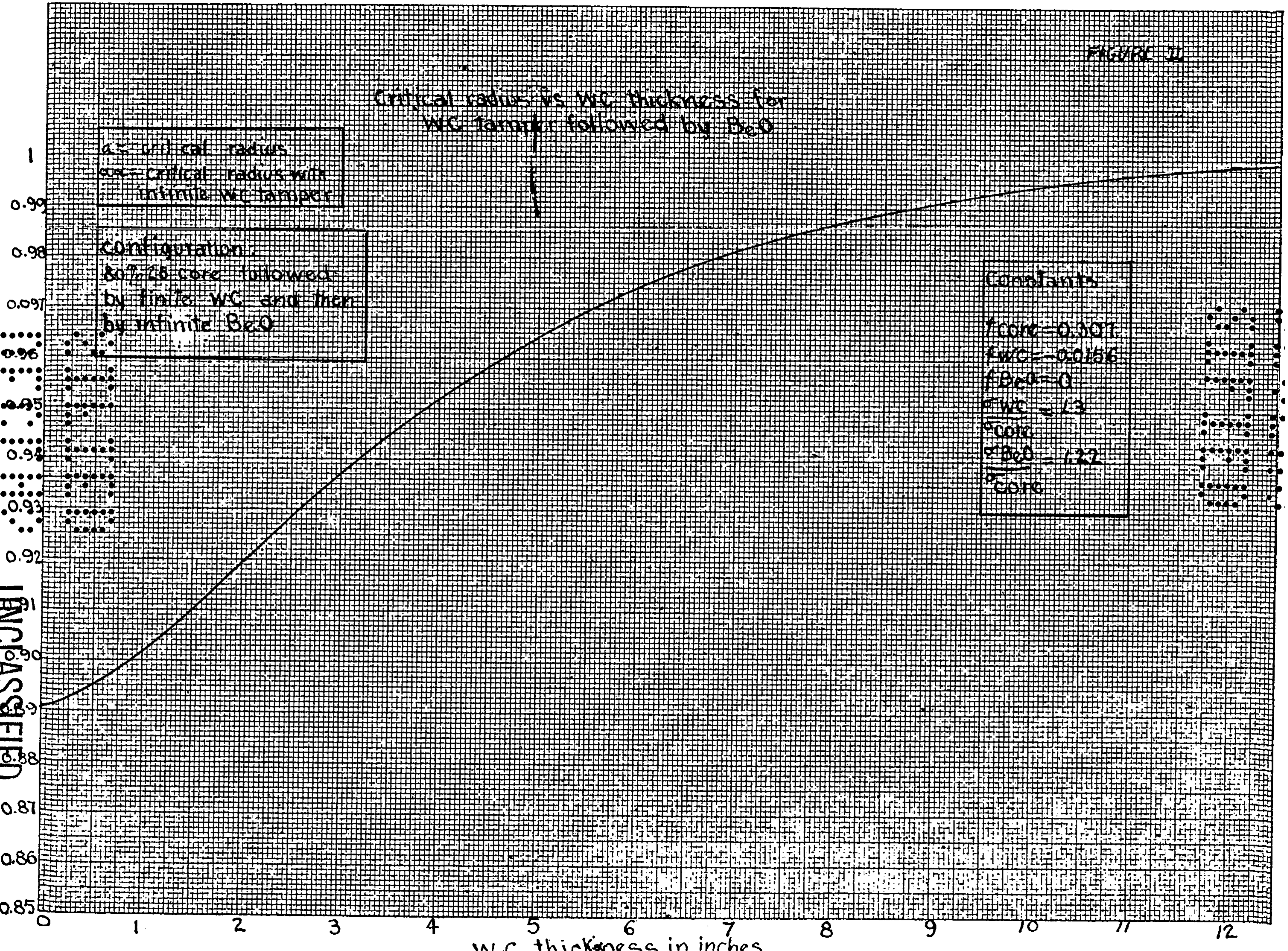
FIGURE II
Critical radius vs WC thickness for
WC tamper followed by BeO
a = critical radius
a∞ = critical radius with
infinite WC tamper
configuration:
80% 25 core followed
by finite WC and then
by infinite BeO
Constants
f core = 0.307
f WC = -0.0156
f BeO = 0
σ WC / σ core = 1.3
σ BeO / σ core = 1.22
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.90
0.89
0.88
0.87
0.86
0.85
0
1
2
3
4
5
6
7
8
9
10
11
12
W.C. thickness in inches

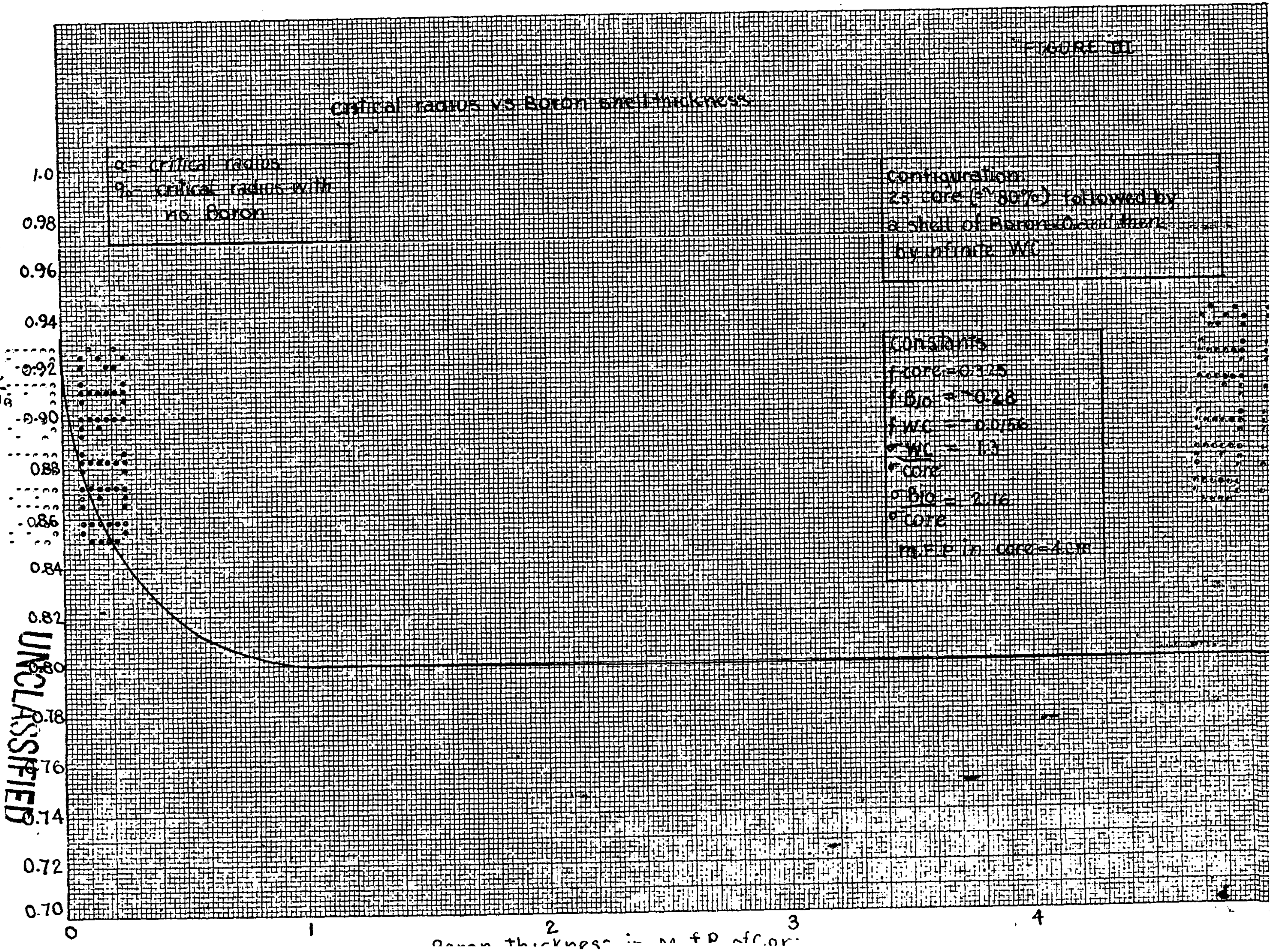
FIGURE III
Critical radius vs Boron shell thickness
a = critical radius
a₀ = critical radius with no Boron
Configuration:
25 core (~80%) followed by a shell of Boron 10 and then by infinite WC
Constants
f core = 0.325
f B10 = ~0.28
f WC = ~0.0156
σ WC / σ core = 1.3
σ B10 / σ core = 2.16
m.f.p. in core = 4 cm
a/a₀
1.0
0.98
0.96
0.94
0.92
0.90
0.88
0.86
0.84
0.82
0.80
0.78
0.76
0.74
0.72
0.70
0
1
2
3
4
Boron thickness in M.f.P. of Core
UNCLASSIFIED

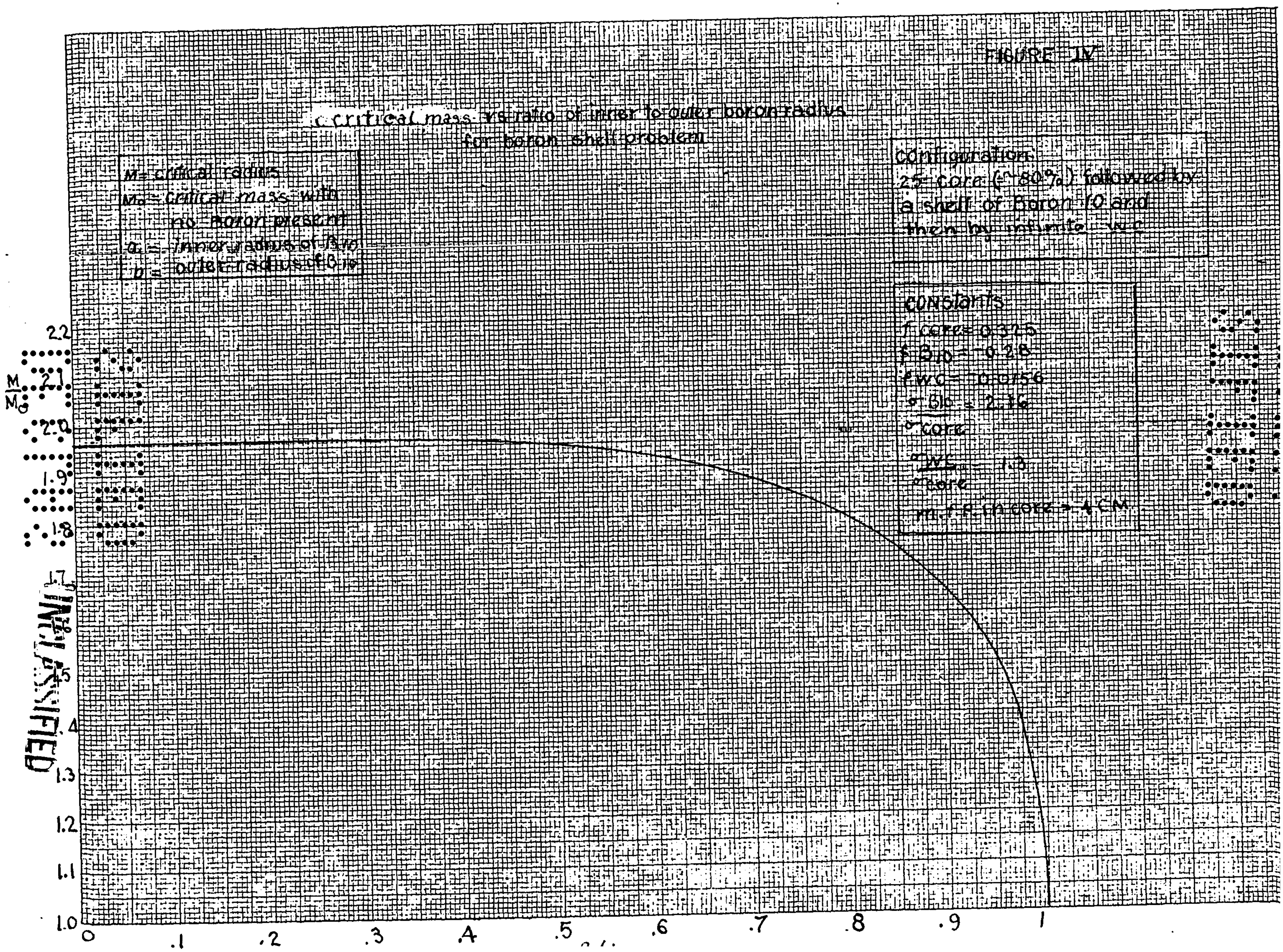
FIGURE IV
critical mass vs ratio of inner to outer boron radius
for boron shell problem
M = critical radius
M_0 = critical mass with
no boron present
a = inner radius of B_{10}
b = outer radius of B_{10}
configuration
25 core (~80%) followed by
a shell of Boron 10 and
then by infinite WC
constants
f core = 0.325
f B_{10} = -0.28
f WC = -0.0156
σB_{10} / σ core = 2.16
σ WC / σ core = 1.3
m.f.p. in core = 4 CM
$\frac{M}{M_0}$
2.2
2.1
2.0
1.9
1.8
1.7
1.5
1.4
1.3
1.2
1.1
1.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1

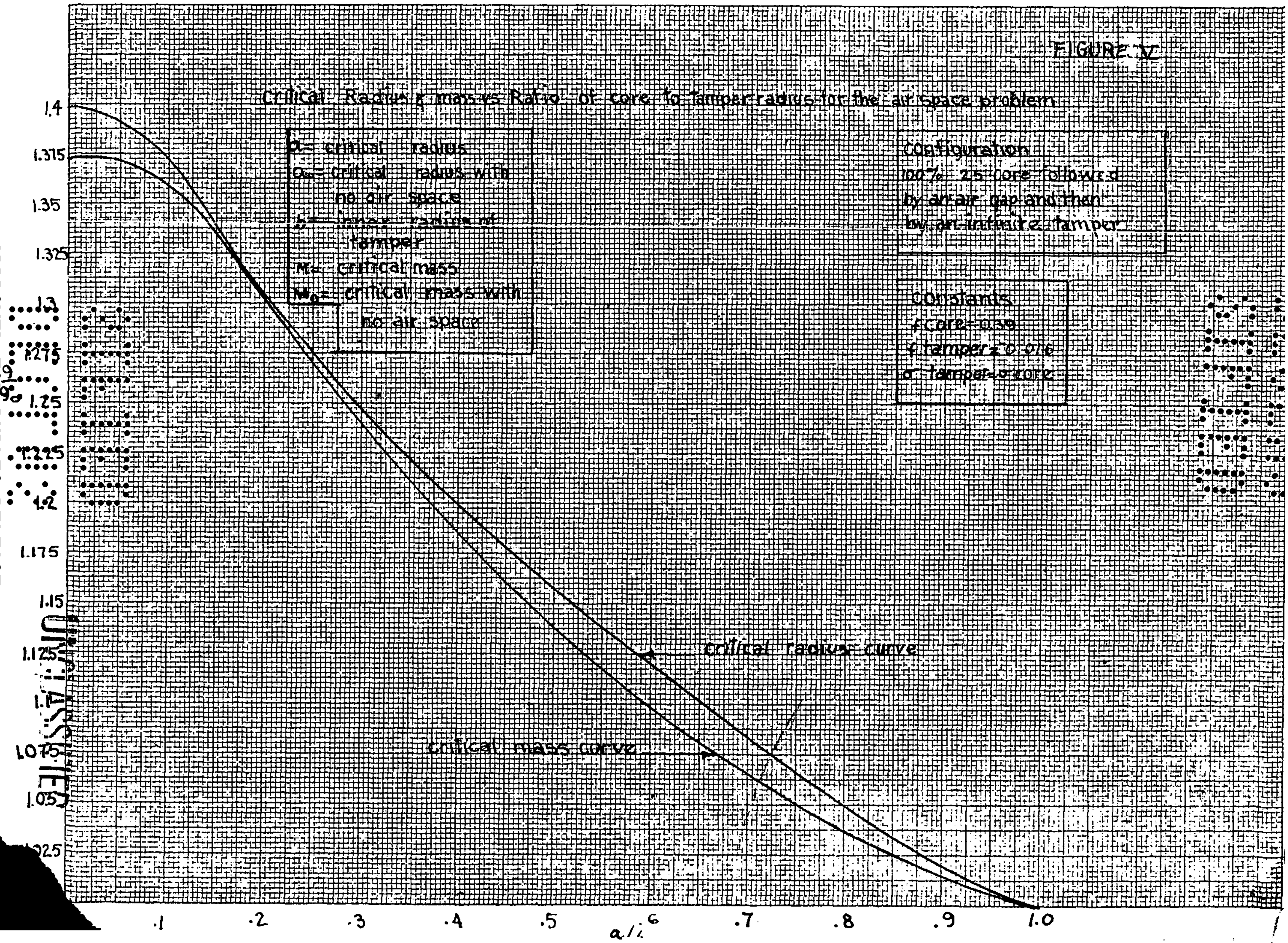
FIGURE V
Critical Radius & mass vs Ratio of core to tamper radius for the air space problem
a = critical radius
a₀ = critical radius with no air space
b = inner radius of tamper
M = critical mass
M₀ = critical mass with no air space
configuration
100% 25 core followed by an air gap and than by an infinite tamper
constants
f core = 0.39
f tamper = 0.916
σ tamper = σ core
critical radius curve
critical mass curve
1.4
1.315
1.35
1.325
1.3
1.275
1.25
1.225
1.2
1.175
1.15
1.125
1.1
1.075
1.05
1.025
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
a/b