LA-5699-MS
Informal Report

UC-20
Reporting Date: July 1974
Issued: August 1974

Magnetically Protected First Wall for a Laser-Induced Thermonuclear Reaction

by

Joseph J. Devaney

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG. 36

In the interest of prompt distribution, this LAMS report was not edited by the Technical Information staff.

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy $4.00 Microfiche $1.45

MAGNETICALLY PROTECTED FIRST WALL FOR A LASER-INDUCED THERMONUCLEAR REACTOR

by

Joseph J. Devaney

ABSTRACT

A modest magnetic field (3-kG) is found to protect a cylindrical wall (240-cm radius) against both the alpha pulse and the DT debris plasma from a laser-induced thermonuclear microexplosion (100-MJ). Our calculations indicate that the Debye lengths are sufficiently small compared to our minimum possible dimensions so that both the α particles and the debris behave collectively. Our explosion compresses the magnetic field to as much as 21 kG against a negligible-resistance coil or cylindrical sleeve. Expansions from 102 to 222 cm are possible depending on the residual gas density. The expanding plasmas are demonstrated to be weakly stable over a large part of their expansion. The instabilities that do develop are slow, allowing time for adequate pressure relief at the ends.

I. GENERAL

This report gives the reasoning and calculations supporting use of simple magnetic fields to protect the first or inner wall of the reaction cavity against the products of a reference laser-imploded DT pellet of 100-MJ yield. Briefly, the reason that simple field geometries are adequate for laser-induced controlled thermonuclear reactions, LCTR, but not for ordinary controlled thermonuclear reactions, CTR, (Sherwood) lies in the shorter plasma time of confinement needed. In fact for LCTR the magnetic field need not even confine the exploding plasma, but need only decelerate it sufficiently to prevent wall damage. However, our preliminary calculations here indicate actual cylindrical confinement, thus protecting the first wall for times beyond a plasma recoil back toward the axis.

Our geometry is the simplest, a microexplosion occurring on the axis of a long solenoid, see Fig. 1. Our objective is to protect the wall of the solenoid, but yet allow the plasma to stream toward the ends.

As in all magnetic confinement, the critical question is whether instability permits energetic plasma penetration to the wall, in this case before

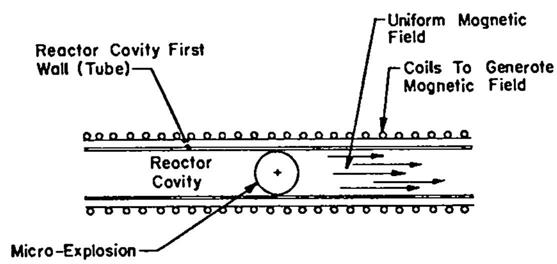

Fig. 1. Magnetically protected cavity wall.

the plasma rebounds from the compressed magnetic field. The answer for our reference design is negative—a conculsion we will support with the following three indicative calculations:

1. Calculation of flute instabilities[1,2] at pressure equilibrium by a method suggested by W. Riesenfeld. (The author is grateful to Dr. Riesenfeld for this "rule of thumb" and his continued advice.)
2. Flute instability criteria for an expanding superconducting plasma shell into an ambient uniform vacuum magnetic field.[3]

3. Differential Larmor radius stabilization of otherwise weakly unstable confined plasmas.[4]

Reference Initial Conditions[5,6]

We postulate 2.2×10^{19} outwardly directed α particles with an energy of 2 MeV, beginning in a shell of 0.0116-cm thickness at a radius of 0.13 cm for a total energy of 7 MJ. In addition, a fully ionized plasma of 50:50 DT mixture expands in (roughly) a spherical shell of 0.0116 cm with an initial outer radius of 0.13 cm, an initial momentum of 1.39×10^6 g·cm/s with a total energy of 15 MJ. Expansion in a shell is also a worst-case test of wall protection, because the magnetic field must then contain the highest pressure. These plasmas originate simultaneously on the axis of a solenoid of negligible resistance, of radius 240 cm, producing a uniform steady cylindrical field of 3 kG. The solenoid contains residual 50:50 DT gas of density ranging from 0 to 4.15×10^{-7} g/cm^3.

II. COLLECTIVE PLASMA BEHAVIOR

A whole plasma behaves collectively if its minimum dimension exceeds the Debye length, λ_D,[7] defined by

$$\lambda_D = \sqrt{kT/4\pi Ne^2} \tag{1}$$

where k is Boltzman's Constant, T is the absolute temperature, N is the electron density, and e is the charge.

For a worst-case test of the α particles we take kT = 2 MeV and confine them to a shell of 0.116-mm thickness giving $\lambda_D < 6 \times 10^{-7}$ cm which is much less than the shell thickness 0.0116 cm at an outer radius R = 0.0116 cm. If we expand the alphas adiabatically as an ideal gas until stagnation at a radius of 209 cm, there $\lambda_o < 2 \times 10^{-5}$ cm, which is also much less than 0.0116 cm. Similarly, for a worst case, if we expand isothermally to 209 cm and take kT = 2 MeV we find $\lambda_D < 0.0178$ cm, which is of the order of 0.0116 cm, so that we can treat the reference α-particle burst as a directed plasma working against the magnetic field.

The Larmor radius of a 2-MeV α particle in a 3-kG field is 135.8 cm. The electron Larmor radius is ~ 1/8000th of that, but separation of 2.2×10^{19} 2-MeV alphas from their electrons would give rise to a potential of ~ 5×10^{10} V, so separation cannot occur and ambipolar or collective behavior is supported.

The DT debris remains a collective plasma because the Debye length, λ_D, varies from ~ 3×10^{-9} to 10^{-7} cm from a radius of 0.013 cm to a radius of 210 cm, respectively.

III. MAGNETIC PROTECTION

In keeping with the philosophy of worst-case investigation, we calculate the equatorial containment of the plasma by applying equatorial parameters to spherical geometry. In our case, the actual geometry is cylindrical, which will relieve pressure axially, thus imposing a lesser containment burden than indicated from our spherical-geometry calculation.

Because we assume that solenoid resistance is negligible and that the coil windings are arbitrarily tight (or else use an internal conducting cylindrical sleeve), a compressed magnetic field cannot penetrate the coil windings; we may, by conservation of flux, calculate the compressed field, B, to be that of the uncompressed field, B_o, times the inverse ratio of cross-sectional areas, thus:

$$B = B_o/[1 - (R/R_o)^2], \tag{2}$$

which gives rise to a magnetic pressure, p_m, resisting plasma expansion:

$$P_m = \frac{B^2}{8\pi} \text{ (Gaussian units)}. \tag{3}$$

The work done by the expanding plasma, W, is that done on the magnetic field, W_m, plus that compressing the residual gas, W_{gas}. The pressure of residual gas we take to be a power law in the Taylor-Sedov region,[6]

$$P_{gas} = 4.4 \times 10^{13} R^{-3} \text{ (cgs units)}, \tag{4}$$

leading to

$$W_g = \int_{R_1}^{R} P_g dV = \int_{R_1}^{R} 4.4 \times 10^{13} R^{-3} 4\pi R^2 dR$$

$$= 5.53 \times 10^{14} \ln \left|\frac{R}{R_1}\right|. \tag{5}$$

At less than R_1 where Taylor-Sedov theory breaks down we assume that P_{gas} is roughly constant,[8] so in that range,

$$W_g = P_g \frac{4\pi}{3} R^3. \tag{6}$$

The magnetic work is

$$W_m = \int P_m dV = \int_0^R \frac{B_o^{\ 2}}{8\pi\left[1 - \left(\frac{R}{R_o}\right)^2\right]} 4\pi R^2 dR$$

$$W_m = \frac{B_o^{\ 2} R_o^{\ 3}}{4}\left[\frac{RR_o}{R_o^{\ 2} - R^2} - \frac{1}{2}\ln\left|\frac{R_o + R}{R_o - R}\right|\right] \qquad (7)$$

where R_o is the coil radius and B_o is the initial uniform field.

The range of cases important to LCTR is spanned by:

1. Zero-density residual gas, $\rho_o = 0$, at the time of explosion, leaving the magnetic field as the only restraining force;
2. Low-density residual gas, 10^{12} DT particles/cm^3;
3. High (relative) density residual gas, 10^{17} DT particles/cm^3. Higher densities place unacceptable constraints on the admission of laser light onto the pellet through the chamber atmosphere.

By equating the directed energy of 2-MeV alphas and the total energy (15-MJ) in the debris plasma to the work done in expansion we can calculate the maximum equatorial expansion of the alphas and of the plasma.

Then, for Case 1, ($\rho_o = 0$), the 2-MeV alpha pulse will be reversed at a radius of 209 cm giving a compressed field of 12.4 kG. The plasma pulse will be reversed at less than 222 cm for a maximum field of less than 20.9 kG.

For Case 2, ($\rho_o = 4.15 \times 10^{-12}$ g/cm^3 DT), the plasma expands to less than 222 cm with a maximum field less than 20.4 kG.

Case 3 ($\rho_o = 4.15 \times 10^{-7}$ g/cm^3 DT), the alpha pulse expands to 208-cm maximum radius with a corresponding maximum field of 11.9 kG. These values are little different from those for vacuum. We used an average stopping power in DT of 1.6×10^3 MeV-cm^2/g for the α particles. On the other hand, the original explosion debris plasma expands to less than 14.8 cm. The original plasma is thus easily decelerated by the residual gas. Of course, the residual gas is itself shocked and ionized. The total excursion of ionized material therefore exceeds 14.8 cm and must be otherwise determined, which we do from momentum considerations.

The residual gas can decelerate but not subtract momentum from the explosion; momentum can be subtracted only by the magnetic field (or the wall).

If such outward momentum is p, then

$$p = \int_0^R F(R)\ \frac{dt}{dR}\, dR \qquad (8)$$

where R is the radius at which the shocked ionized gas is brought to rest and F(R) is the decelerating force. For simplicity we take all impacted gas to be ionized. However, in fact the magnetic field cannot protect the first wall against a pressure pulse of unionized residual gas. Should the latter be strong, one must also mechanically strengthen the first wall.

Conservation of momentum gives dR/dt as a function of R:

$$p = mv = (m + dm)(v + dv) \rightarrow vdm + mdv = 0. \qquad (9)$$

Solving, the velocity, v, and moving mass, m, are related by

$$v = \frac{c}{m}, \qquad (10)$$

where c is a constant.

Thus,

$$v = \frac{dR}{dt} = \frac{c}{m_o + \rho_o(4\pi R^3/3)}\ . \qquad (11)$$

Initially, $v = 1.07 \times 10^8$ cm/s if the effective kinetic energy is half the total energy;[8] and $v = 1.97 \times 10^8$ cm/s if all the energy is kinetic. For our specific initial conditions the constant, c, equals 1.4×10^6 to 2×10^6 g·cm/s.

Our restraining force here is wholly magnetic:

$$F(R) = P_m A = \frac{B_o^{\ 2}}{8\pi\left[1 - \left(\frac{R}{R_o}\right)^2\right]^2} \cdot 4\pi R^2. \qquad (12)$$

The integral for the momentum, p, becomes:

$$p = \frac{B_o^{\ 2} R_o^{\ 4}}{2c}\int_o^R \frac{m_o + \rho_o(4\pi/3)\ R^3}{(R_o^{\ 2} - R^2)^2}\ R^2\, dR \qquad (13)$$

$$p = \frac{B_o{}^2R_o{}^2}{2c}\left(\frac{m_o}{2}\left\{\frac{R}{[1-(R/R_o)^2]} - \frac{R_o}{2}\ln\left|\frac{1+(R/R_o)}{1-(R/R_o)}\right|\right\}\right.$$

$$\left. + \frac{a}{2}R_o{}^4\left\{-\left[1-(R/R_o)^2\right] + \left[1-(R/R_o)^2\right]^{-1} + 2\ln\left|1-(R/R_o)^2\right|\right\}\right). \qquad (14)$$

Substituting our parameters, Case 3, all the (shocked) plasma is brought to rest between R = 102 cm, B = 3650 G (50% in kinetic energy) and R = 113 cm, B = 3850 G (all in kinetic energy).

IV. PLASMA STABILITY

1. Plasma Instability at Pressure Equilibrium

At pressure equilibrium between the plasma and the magnetic field, that is, at stagnation, we may expect flute irregularities to grow as:[1,2]

$$\exp\ (t/\tau)$$

where

$$\tau \sim 2\pi R/(V_A\sqrt{n}) \qquad (15)$$

and $V_A = B^2/4\pi\rho$ is the Alfven velocity: n is the number of flutes (asymmetric explosions, n = 1, are taken to be the most probable for LCTR); and ρ is the plasma mass density. As a figure of merit for the stability of expansion against our magnetic field we calculate the time constant, τ, times the effective axial velocity, v_a. This value, a distance, call it D_a, is a conservative estimate of the axial expansion that may occur before the onset of serious instabilities. The estimate is conservative because it is made at highest pressure (equilibrium, $\beta = 1$) which occurs only momentarily. We may expect, as we show later, less instability, even stability at $\beta < 1$.

Case 1, no residual gas - The alphas treated as a plasma have a time constant $\tau = 5.7 \times 10^{-5}$ s, with a velocity of 9.8×10^8 cm/s, giving $D_a \sim$ 34 000 cm. The plasma has a time constant $\tau = 3.2 \times 10^{-4}$ s; taking a velocity of 10^8 cm/s,[6] we get $D_a \sim$ 56 000 cm.

Case 2, residual DT gas; density, 10^{12} particles/cm^3 - The plasma has an equilibrium flute-instability time constant of $\tau = 3 \times 10^{-4}$ s which, integrating the velocity profile of Ref. 8, gives $D_a \sim$ 4600 cm.

Case 3, residual DT gas; density, 10^{17} particles/cm^3 - The alpha plasma has a time constant $\tau = 5.9 \times 10^{-5}$ s, which is long enough for the axially directed alphas to be stopped in the gas at D_a = 3000 cm. We used an average stopping power of 1.6×10^3 MeV-cm^2/g for alphas in DT, treating the alphas in this regime as particles, not as gas. The plasma has a time constant $\tau = 5.2 \times 10^{-4}$ s, which allows the shocked plasma to move axially for a distance $D_a \simeq$ 600 cm.

In every instance the magnetic field has persisted uniformly enough until the explosive debris is well on its way outward along the axis of the solenoid. We now turn to a more complete description of the expansion of a plasma shell into a vacuum magnetic field.

2. Plasma Stability of a Spherical Shell

Flute-instability growth of an initially spherical superconducting plasma shell into a large vacuum magnetic field is given by formulae proportional to the growth term $e^{t/\tau}$ where the time constant, τ, is (from Poukey[3]):

$$\tau = (2/3)(n\alpha)^{-1/2} \quad n\alpha >> 1 \qquad (16a)$$

$$\tau = \left(\frac{32}{81}\right)^{2/3}(n\alpha)^{-2/3} \quad n\alpha << 1, \qquad (16b)$$

n being the number of flutes and

$$\alpha \equiv \frac{B_o{}^2R_o{}^3}{2\,MV_o{}^2} = \frac{B_o{}^2R_o{}^3}{4\,E_o} \quad \text{(Gaussian units)}\ . \qquad (17)$$

Here B_o is the initial magnetic field, uniform throughout space; R_o is the radius of the sphere at t = 0 expanding outward with an initial velocity V_o, a total mass M, and a total initial kinetic energy E_o. As before we take n = 1, since the asymmetry of implosion is most likely a simple off-center (n = 1) type.

For a worst-case calculation we take E_o as only half the total plasma energy, $E_o = 7.5 \times 10^{13}$ erg, and we take B_o to be the largest of all cases, B_o = 21 kG; we then get, for both the alphas and the plasma:

$$n\alpha \sim 3.4 \times 10^{-10} << 1,$$

and the second time-constant formula yields:

$$\tau \sim 2.4 \times 10^5\ \text{s},$$

adequately long indeed for all gases to exit any reasonably sized chamber.

This calculation should be a lower bound for instability time constants of spherical shell expansions beginning at a magnetic field of 3 kG and compressing to a pressure equilibrium of 6 to 21 kG. Poukey[3] did not study expansions into non-vacuums.

3. Finite Larmor Radius Stabilization

Because the Larmor radii of ions and electrons are finite and different, otherwise weakly unstable confined plasmas actually are stable.[4] The different electron and ion Larmor radii may build up a charge separation out of phase with particle drift separation. Because the latter drives the flute instability, the result can be stable oscillation if:

$$(ka_i)^2 > \omega_H/\Omega_i, \tag{18}$$

where k is the wave number, which we have taken as n/R, with n being the number of flutes; a_i is the ion Larmor radius (gyromagnetic), $a_i = m_i v_i c/e_i B$; Ω_i is the ion Larmor angular frequency (cyclotron frequency), $\Omega_i = e_i B/m_i c$; and ω_H is the hydrodynamic growth rate (Taylor instability).

The growth rate for Taylor instability under gravity is:[9]

$$\omega_H{}^2 = kg\,\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \tag{19}$$

for two fluids of density ρ_1 and ρ_2, k here is the wave number of the instability, g is the gravitational acceleration.

A magnetic field behaves as $\rho_1 \equiv 0$, so $\omega_H{}^2 = kg$ and $g \rightarrow (\vec{R}/R^2)(V_{||}{}^2 + \frac{1}{2}V_{\perp}{}^2)$ for equilibrium.[10] Because R is the radius of curvature of the B-field, this radius is identical to our R; $V_{||}$, $V_{\perp}$ are the velocities parallel and perpendicular to the surface. Because our fluid is not in equilibrium, we must add $\ddot{R}$; also, we are confining our study to the equatorial region where $V_{||} \equiv 0$, thus $g \rightarrow \ddot{R} + (\dot{R}^2/2R)$.

Using the instantaneous total energy $E \equiv \frac{1}{2} M\dot{R}^2$,

$$g = \ddot{R} + E/MR; \text{ hence,} \tag{20}$$

$$\omega_H = \sqrt{k[\ddot{R} + (E/MR)]}\ . \tag{21}$$

By substituting Eq. (21) into Eq. (18) our stability criterion reduces to:

$$(n/R)^{3/2}\,\frac{2E_i C}{e_i B} > \sqrt{\ddot{R} + (E/MR)} = \sqrt{\ddot{R} + (\dot{R}^2/2R)} \tag{22}$$

where E_i is the individual ion energy. When this inequality holds we may expect flute stabilization.

For our parameters we find for all cases that the alpha plasma fulfills the inequality to more than 120-cm radius, and the debris plasma to within 3 cm of the turnaround radius.

We conclude then, by two separate lines of reasoning (Numbers 1+3 and 2, of pp. 1,2 and pp. 4,5) that explosions into a magnetic field are either stable or slowly growing unstable. Based on our parameters for a <u>single</u> explosion, these stability calculations substantiate beyond reasonable doubt that magnetic wall protection is feasible. For the detailed study of <u>multiple</u> explosive impacts on stability, as in our case where the alpha pulse is followed by the expanding debris, detailed study must be deferred. We do note, however, three reassuring points, namely that:

1. Actual plasma reversal times at maximum radius and magnetic field are short compared to flute-instability time constants. (For example, in Case 1, the start and return of the plasma to 3 cm of turnaround radius takes ~ 6×10^{-8} s for alphas versus an alpha instability time constant, $\tau_\alpha \sim 6 \times 10^{-5}$s. For the explosive debris the 3-cm return time is ~ 2×10^{-7} s versus a plasma instability time constant, $\tau_{plasma} \sim 3 \times 10^{-4}$ s.)
2. The alpha-particle flute-instability time constant is sufficiently long for the debris plasma to strike the magnetic field before such growth has gone far, especially for low-density residual gas.
3. Larger pellets, especially those made of higher Z materials, will attenuate the alpha pulse with the result that our single-pulse calculations become more applicable to the remaining dominant, if not exclusive, debris pulse.

V. CONCLUSIONS

The calculations of this note strongly support the possibility of magnetic protection of a cylindrical cavity wall against energetic alpha particles and plasma debris from microexplosions. Our reference design was based on a 10^{15} erg (100-MJ) DT explosion in an initially uniform 3-kG cylindrical field of 240-cm radius. For more details of a magnetically protected laser fusion reactor concept see the work of Frank, Freiwald, Merson, and Devaney.[11]

ACKNOWLEDGEMENTS

The early thoughts of this paper were developed during active collaboration on an ANS paper[11] with T. G. Frank, D. Freiwald, and T. Merson. It is a pleasure to acknowledge helpful discussions with W. Riesenfeld and C. M. Gillespie, all of LASL.

REFERENCES

1. C. L. Longmire, Elementary Plasma Physics, (Interscience, New York, 1967), pp. 241-246.

2. M. N. Rosenbluth and C. L. Longmire, "Stability of Plasmas Confined by Magnetic Fields," Ann. of Phys. 1, 120 (1951).

3a. J. W. Poukey, "Expansion of a Plasma Shell into a Vacuum Magnetic Field," Phys. Fluids, 12, 1452 (1969).

3b. J. R. Albritton, "A Snowplow Model for the Dynamics of Expanding Plasmas," Yale University Thesis (1973), to be published.

4. M. N. Rosenbluth, N. A. Krall, and N. Rostoker, "Finite Larmor Radius Stabilization of 'Weakly' Unstable Confined Plasmas," Nuclear Fusion Supplement, Pt. 1 (1962), p. 143.

5. T. G. Frank, LASL, private communication (1974).

6. D. Friewald, LASL, unpublished work (1974).

7. D. J. Rose and M. Clark, Plasmas and Controlled Fusion, (MIT Press, Cambridge, MA, 1969), pp. 161,162.

8. D. Freiwald, LASL, private communication (1974).

9. F. H. Harlow and A. A. Amsden, "Fluid Dynamics," Los Alamos Scientific Laboratory Report LA-4700 (1971).

10. M. N. Rosenbluth and C. Longmire, Ann. of Phys. 1, 120 (1957).

11. T. Frank, D. Freiwald, T. Merson, and J. Devaney, "A Laser Fusion Reactor Concept Utilizing Magnetic Fields for Cavity Wall Protection," presented at First Topical Meeting on the Technology of Controlled Nuclear Fusion, San Diego, California (American Nuclear Society) April 16-18, 1974.