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Consistency among Differential Nuclear Data and Integral Observations: The ALVIN Code for Data Adjustment, for Sensitivity Calculations, and for Identification of Inconsistent Data

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ABSTRACT

Successful nuclear design requires adequate prediction of integral design param-
eters, and this in turn requires an adequate differential nuclear data base. Data
bases that apparently permit reduced biases and design margins have been developed by
a) least squares adjustment of differential data or b) trial-and-error selection from
alternative evaluated data sets. Criticisms and defenses of such procedures are dis-
cussed. We relate useful data adjustment to consistency of the combined differential-
integral data set, and we describe consistency tests related to least squares adjust-
ment procedures. We suggest an approach to data adjustment that is contingent on con-
sistency analysis. A FORTRAN code ALVIN has been developed to carry out the indicated
data consistency and adjustment calculations, and to compute required sensitivities of
integral parameters to nuclear data changes. The sensitivity modules of ALVIN are val-
idated by computing with two distinct methods the cross-section sensitivity profile for
neutron penetration through a thick iron shield. The data consistency and adjustment
modules DAFT2 (for arbitrary variance-covariance data) and DAFT3 (for differential data
base of arbitrary size uncorrelated with integral data) are validated by comparing their
results for a set of data for three ZPR criticals.

I. INTRODUCTION

Successful nuclear design requires adequate pre-
diction of a number of integral design parameters.
Among these are reactivity worths, energy deposition
rates, nuclide transmutation rates, radiation doses,
and Rossi-α. Adequate prediction of integral design
quantities requires, in turn, a recognizably ade-
quate nuclear data base. Massive programs for meas-
urement and analysis of differential and integral
nuclear data have been devoted to this objective,
and the end is not yet in sight.

In the meantime, "improved" nuclear data bases
have been achieved, apparently permitting reduced
design biases and margins, through a) data adjust-
ment by least squares techniques, or b) trial-and-
error selection among alternative data sets. Data
adjustment or selection is carried out so as to im-
prove agreement between calculations and measure-
ments made on as-built nuclear devices and on spe-
cial experiments devised to resemble them. Many

workers have contributed to this development.[1-26]
It is generally supposed that the data adjustment
or selection is not simply compensating for the sys-
tematic errors in computational technique; instead,
errors in calculation are assumed to be driven down
or allowed for as a result of numerical experimenta-
tion or as a result of comparison with very accurate
methods such as continuous energy Monte Carlo.

Critics of data adjustment and selection strat-
egies emphasize that forced but unphysical changes
in data may yield improved agreement with available
integral observations, but may worsen agreement with
unmeasured, and frequently more important, integral
design parameters. Advocates of data adjustment or
selection respond that differential data are not now
and possibly will never be measured to the accuracies
required and inferrable from good integral measure-
ments. Other arguments have been put forth on both
sides of the controversy. In any case, the strategy
chosen by a nuclear design organization to deal with

inadequacies in the nuclear data base is of suffi-
cient consequence that the choice is made deliber-
ately and the resulting adjusted or selected data
base often is protected.

Here we suggest an approach[27] having potential
not confined solely to the design organization, and
we describe a computer program, ALVIN, to implement
and test the approach. We assume the existence of
an evaluated nuclear data file, specifically ENDF/B,
containing carefully analyzed and selected integral
data as well as differential data, and containing
for these quantities evaluated variance and covari-
ance data as well. The combined first and second
moment differential and integral data set, or a par-
ticularly significant part of the data set, first
is tested for consistency in the statistical chi-
squared sense. (Consistency tests are discussed in
Sec. II-A.) If the data are consistent, there can
be no objection to adjusting the differential data
to improve agreement of predictions with the inte-
gral observations. (Data adjustment is discussed
in Sec. II-B.) When consistency has been establish-
ed, data adjustment is unlikely to distort unreal-
istically the differential data base. If, on
the other hand, the combined differential-integral
data set is clearly inconsistent, data adjustment
may be questionable and it might be more profitable
to use expedients such as design biases until the
source of the inconsistency is identified and re-
solved.

The next step is to identify the source of the
inconsistency, and this also is discussed in Sec.
II-A. If the inconsistency arises from over-opti-
mism as to the accuracy of integral data, this re-
sult is of great interest to designers in treating
design biases and margins. More likely there are a
number of sources of inconsistency. Consistency
tests applied to the unadjusted data set provide de-
tails of discrepancies only in particular integral
data. For corresponding details of inconsistencies
in particular differential data, the necessary tech-
niques are formally the same as data adjustment by
least squares. Intuitively, one is looking for
critical directions in the combined space of inte-
gral and differential parameters for which reduction
of the least squares functional forces the point rep-
resenting the data set to move far beyond reasonable
probability. Thus we are led to data adjustment for

analysis of consistency as well as for the achieve-
ment of an improved data base.

The ALVIN code provides the computational cap-
ability for this approach. Consistency and adjust-
ment procedures used in the code are described in
Sec. II, while necessary sensitivity calculations
are described in Sec. III. Programming techniques
are outlined in Sec. IV, and a code abstract is in-
cluded as Appendix A. The data consistency and ad-
justment module, DAFT2, is adapted from a previous
code, DAFT1.[28] Sample problems are described in
Sec. V and Appendixes C, D, E, and F. Code valida-
tion is carried out in terms of the sample problems
by carrying out required calculations in distinct
ways, then comparing the results. Data for three
ZPR assemblies provide the sample problem for the
data consistency and adjustment portions of ALVIN.
The sample problem for the sensitivity parts of the
code is one for which a biological dose equivalent
is produced by D-Be neutrons penetrating a thick
iron shield. Input and output are described in Sec.
VI. Capabilities and limitations of ALVIN are sum-
marized in Sec. VII.

II. CONSISTENCY TESTS AND DATA ADJUSTMENT

Group cross sections, particle and photon yield
data, and other differential data[*] in a nuclear data
library will be represented by $x_1, x_2, \ldots, x_{\hat{J}}$, where $\hat{J}$,
the number of differential parameters, may be of or-
der 10^3 to 10^5. Integral parameters $y_1, y_2, \ldots, y_{\hat{I}}$
such as reactivity worths and reaction rate ratios
are computed as functions of the primary parameters
$y_i(x_1, x_2, \ldots, x_{\hat{J}})$ for $i=1, 2, \ldots, \hat{I}$, or $y_i(x)$ in a con-
venient notation. Here $\hat{I}$, the number of integral
parameters, usually is of order 10^1 or 10^2. From a
combination of measurements, corrections, and analy-
ses one arrives at "evaluated" observed values x_1^e,
$x_2^e, \ldots, x_{\hat{J}}^e$ and $y_1^e, y_2^e, \ldots, y_{\hat{I}}^e$. Usually the evaluated
observed values of the integral parameters do not
equal the values of the integral parameters calcu-
lated using the evaluated data base x^e. That is, y_i^e

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[*]Other data might include x_j representing a nuclear
temperature characteristic of inelastic neutron
emission as suggested by D. W. Muir. As another
example, x_j might be a mixing parameter such that
the cross section is $x_j \sigma_a + (1-x_j)\sigma_b$, where σ_a and
σ_b are alternative physically reasonable evaluations
of the cross section.

differs from $y_i(x^e)$. In dealing with this discrepancy, we assume that numerical experimentation has shown it to be a result of errors in differential data, or integral data, or both, and not a result of inadequate computational techniques.

A. Tests for Consistency among Differential and Integral Data

The data may be discrepant in that $y_i^e \neq y_i(x^e)$, but at the same time they may be consistent in view of uncertainties in measurement and evaluation. The combined set of differential and integral parameters may be consistent in that their values could reasonably have been drawn from an assigned joint probability distribution. To test for consistency we frame the so-called simple null hypothesis[29] that the populations are normal with the evaluated means, variances, and covariances. We then draw a sample from the population and compute values for one or more statistics. The values thus obtained permit a decision to accept or reject the hypothesis at a certain level of confidence.

Consider initially the quadratic form

$$S_1 = \sum_{i=1}^{\hat{1}} \sum_{i'=1}^{\hat{1}} [y_i(x^e) - y_i^e] {}^{yy}V_{ii'}^{-1}$$

$$[y_{i'}(x^e) - y_{i'}^e] \quad , \qquad (1)$$

where ${}^{yy}V$ is the evaluated matrix of variances and covariances among the evaluated integral parameters. That is, ${}^{yy}V_{ii'}$ represents the evaluated variance of y_i^e if $i = i'$ and the evaluated covariance of y_i^e with $y_{i'}^e$ if $i \neq i'$. Under the simple null hypothesis we expect S_1 to be distributed as a $\chi_{\hat{1}}^2$ distribution with $\hat{1}$ degrees of freedom, symbolized as $\chi_{\hat{1}}^2$. Here the values $y_i(x^e)$ are calculated outside ALVIN and are regarded as sample values from the multivariate normal distribution with means y_i^e and evaluated variance-covariance matrix ${}^{yy}V$. For example, for the case described in Sec. V-A, $\hat{1}$ is 24 and S_1 is found to be 503. For the χ_{24}^2 distribution the value of 503 is far outside the 1% probability limits (there is a 1% probability that χ_{24}^2 is less than 10.7 and a 1% probability that χ_{24}^2 exceeds 43.0). Thus the simple null hypothesis is rejected at the "highly significant" level of 1%. We take this to imply that the combined set of means, variances, and co-

variances for the evaluated parameter set is highly inconsistent.

In practice the integral observations usually are considered to be independent (when common quantities like delayed neutron fraction are removed from quantities like reactivity worth observation). The quadratic form then becomes

$$S_1 = \sum_{i=1}^{\hat{1}} [y_i(x_i^e) - y_i^e]^2 / {}^{yy}V_{ii} \quad , \qquad (2)$$

where ${}^{yy}V_{ii}$ represents the evaluated variance for the evaluated integral parameter y_i^e. The individual contribution of each integral parameter to the value of S_1 is evident and is distributed as a χ_1^2 distribution with one degree of freedom according to the hypothesis. Thus deviant integral data can be identified under the initial hypothesis. However, no information has been obtained as to the consistency of particular differential data with this procedure.

To examine the consistency of the combined differential and integral data, consider the quadratic form

$$S_2 = \sum_{i=1}^{\hat{1}} \sum_{i'=1}^{\hat{1}} [y_i(x) - y_i^e] {}^{yy}W_{ii'} [y_{i'}(x) - y_{i'}^e]$$

$$+ \sum_{j=1}^{\hat{j}} \sum_{j'=1}^{\hat{j}} [x_j - x_j^e] {}^{xx}W_{jj'} [x_{j'} - x_{j'}^e]$$

$$+ 2 \sum_{i=1}^{\hat{1}} \sum_{j=1}^{\hat{j}} [y_i(x) - y_i^e] {}^{xy}W_{ij} [x_j - x_j^e] \quad , \qquad (3)$$

subject to the requirements that

$$y_i(x) = y_i(x^e) +$$

$$+ \sum_{j=1}^{\hat{j}} \left. \frac{\partial y_i}{\partial x_j} \right|_{x^e} [x_j - x_j^e] \quad , \quad i=1,2,\ldots,\hat{1}. \qquad (4)$$

The weight matrix W in the quadrature form will be taken to be the inverse of the matrix V of evaluated variances and covariances among the evaluated differential and integral parameters. ^{yy}W will represent the partition corresponding to the integral parameters, ^{xx}W will represent that for the differential parameters, and ^{xy}W will represent that for both. The matrices V and W are symmetric.

Different samples of differential and integral data will yield differing sample values of S_2. At the evaluated point $x = x^e$, used in the preceding initial tests, S_2 resembles S_1. If we minimize S_2 by adjusting the combined data set to $x = x^a$, then the sample value of S_2 is also distributed as $\chi_{\hat{i}}^2$. We can ask if the data set is consistent after adjustment by examining the new value of S_2. In the previous example S_2 is found to be 108 but remains improbably large. Data adjustment may be risky in such cases. Nevertheless, we can examine the contribution to S_2 of residuals $x_j^a-x_j^e$ in order to identify potentially deviant differential or integral data. The least squares adjustment process here is looked upon as a device to identify anomalies in the connected network of differential and integral data. Thus we are led to consider data adjustment, both for its role in the strategy of data improvement outlined in Sec. I, and for determination of data consistency.

Before discussing data adjustment it is useful to touch briefly on the normal approximation and on the linearity of Eq. (4).

Our consistency tests assume that the populations are normal, an assumption often made by evaluators in arriving at the evaluated variances and covariances. Hence, the assumed normality and the evaluated data values are related, and it is natural to have them appear together in the consistency tests. Nevertheless, the assumption of normality may be inadequate.

Linear expressions relate calculated integral parameters to differential parameters in Eq. (4) for convenience and because $\partial y_i/\partial x_j$ is easily calculated by perturbation techniques, while higher derivatives are not. The computation of $y_i(x)$ and $\partial y_i/\partial x_j$ is discussed in detail in Sec. III. We note here only that the computation of $\partial y_i/\partial x_j$ for $j=1,2,\dots,\hat{j}$ is carried out in a single calculation when, as in ALVIN, linear perturbation theory is used for the derivatives.

B. Nuclear Data Adjustment

Many groups have investigated and applied various adjustment techniques to nuclear data.[27] One might, as in other disciplines, introduce cost factors C_i, which represent the cost to the design of an error in predicting y_i in the sense that the overall design penalty is a function of the C_i and of the deviations $|y_i-y_i^e|$ from unknown true values y_i. In the absence of a more realistic penalty function, the nuclear designer might use differential data sets adjusted to minimize

$$C = \sum_{i=1}^{\hat{i}} C_i \left| y_i(x) - y_i^e \right| \quad . \tag{5}$$

This strategy, however, does not reflect uncertainties in determination of the differential and integral data. Moreover, it does not respond to the objection that to decrease one set of design biases may increase others for which integral observations are sparse. Most investigators have used a mixed strategy that attempts to improve the basic data set as well as the integral results, or at least does not seriously degrade the differential data. Barré, Chaudat, and others[3,5] have adjusted the differential data in multigroup form so as to minimize

$$S_1 = \sum_{i=1}^{\hat{i}} [y_i(x) - y_i^e]^2 / {}^{yy}V_{ii} \quad , \tag{6}$$

as in Eq. (2), but subject to constraints

$$\left| x_j - x_j^e \right| < b_j \quad , \quad j=1,2,\dots,\hat{j} \quad . \tag{7}$$

Conversely, Cecchini et al.[7] minimize the sum of squared residuals for the differential data,

$$S_2 = \sum_{j=1}^{\hat{j}} [x_j - x_j^e]^2 / {}^{xx}V_{jj} \quad , \tag{8}$$

subject to constraints

$$\left| y_i(x) - y_i^e \right| < a_i \quad , \quad i=1,2,3,\dots,\hat{i} \quad . \tag{9}$$

British, Israeli, and other groups[13,18,21] have minimized the general quadratic form, Eq. (3), assuming

correlations to be negligible, whereas Swedish[10,11,12] and Japanese[15,16] groups and others have included some correlations. It appears that correlations in the differential data[30,31] should be included.[16] Finally, linear programming techniques have been employed to minimize[25]

$$S_3 = \sum_{i=1}^{\hat{I}} \left| y_i(x) - y_i^e \right| / \mu_i + \sum_{j=1}^{\hat{J}} \left| x_j - x_j^e \right| / \nu_j \, , \quad (10)$$

subject to constraints

$$\left| y_i(x) - y_i^e \right| < \mu_i \quad , \quad i=1,2,\ldots,\hat{I} \quad , \quad (11)$$

and

$$\left| x_j - x_j^e \right| < \nu_j \quad , \quad j=1,2,\ldots,\hat{J} \quad . \quad (12)$$

Where bounds are required in Eqs. (6)-(12) they are usually taken to be one or two times standard deviations.

We choose to use the least squares approach, minimizing the quadratic form in Eg. (3), because of the connection of this technique with consistency tests. Bounds are not placed on changes of differential data during adjustment.[2,9] When the integral parameters are assumed to be linearly dependent on the differential data, as is expressed in Eq. (4), it is not necessary to iterate to convergence[32] in order to compute adjustments. Thus the data adjustment subroutines DAFT2 and DAFT3 (to be described) are simpler than DAFT1,[28] but their notation and coding techniques are otherwise similar. DAFT2 adjusts data and computes diagnostics in the general case when differential and integral parameters may be correlated; matrices of order at least $\hat{J} \times \hat{J}$ are inverted in this case. DAFT3 computes the same quantities, but requires inversion only of $\hat{I} \times \hat{I}$ matrices by use of a special technique described in Sec. II-B-2. This technique is only applicable when the differential data are uncorrelated with the integral data, but it permits simultaneous adjustment of arbitrarily large differential data libraries.

1. Data Adjustment with Full Correlations -- DAFT2. It is convenient conceptually and for coding purposes to form the union of the differential and integral parameters and to normalize them by dividing by their evaluated values. Let

$$Z_k(X) = \frac{z_k}{z_k^e} = \begin{cases} y_k(x)/y_k^e & \text{for } k=1,2,\ldots,\hat{I} \quad , \\[2ex] x_k/x_k^e & \text{for } k=\hat{I}+1,\hat{I}+2,\ldots,\hat{I}+\hat{J}=\hat{K} \, . \end{cases} \quad (13)$$

Similarly, the primary quantities to be adjusted are transformed to

$$X_j = x_j/x_j^e \quad \text{for } j=1,2,\ldots,\hat{J} \quad . \quad (14)$$

Let $^{ZZ'}V_{kk'}$ represent the evaluated variance for the evaluated parameter Z_k^e if $k=k'$, or the evaluated covariance between Z_k^e and $Z_{k'}^e$ if $k \neq k'$. Note that $^{ZZ'}V_{kk'} = ^{ZZ'}v_{kk'}/(z_k^e z_{k'}^e)$. Let $^{ZZ'}V$ represent the matrix with elements $^{ZZ'}V_{kk'}$, and form the weight matrix

$$^{ZZ'}W = ^{ZZ'}V^{-1} = \begin{pmatrix} ^{YY}W & ^{YX}W \\ ^{XY}W & ^{XX}W \end{pmatrix} \quad . \quad (15)$$

This matrix is partitioned only to relate to the partitioned matrices appearing in Eq. (3); DAFT2 makes no assumptions requiring partitioning, such as that ^{YX}W vanishes. The Gauss-Markov theorem shows that for this choice of weight matrix, the variances are minimized for any linear combinations of the adjusted parameters; thus the variances on derived integral quantities are minimized as well. When normal distributions are assumed, as is done for our consistency tests, this is a maximum-likelihood estimate as well.[29,33]

The quadratic form to be minimized is, from Eq. (3),

$$S_2 = [Z(X) - Z^e]^{tr} \, ^{ZZ'}W[Z(X) - Z^e] \quad . \quad (16)$$

Here Z and Z^e are column vectors with elements Z_k and ones, respectively. The linear relations, Eq. (4), between differential and integral quantities are transformed to

$$Z_k(X) = Z_k(X^e) + \sum_{j=1}^{\hat{J}} A_{kj}[X_j - X_j^e] \, ,$$

$$k=1,2,\ldots,\hat{K} \quad , \quad (17)$$

where

$$Z_k(X^e) = \begin{cases} y_k(X^e)/y_k^e & \text{for } k=1,2,\dots,\hat{\imath} \\ 1 & \text{for } k=\hat{\imath}+1,\dots,\hat{k} \end{cases}, \quad (18)$$

and

$$A_{kj} = \begin{cases} Z_k(X^e)D_{kj} \\ \quad \text{for } k=1,2,\dots,\hat{\imath} \text{ and } j=1,2,\dots,\hat{\jmath} \\ \delta_{j,k-\hat{\imath}} \\ \quad \text{for } k=\hat{\imath}+1,\dots,\hat{k} \text{ and } j=1,2,\dots,\hat{\jmath} \end{cases}, \quad (19)$$

and

$$D_{kj} = \frac{x_j^e}{y_k(X^e)} \left. \frac{\delta y_k}{\delta x_j} \right|_{X^e}$$

$$\text{for } k=1,2,\dots,\hat{\imath} \text{ and } j=1,2,\dots,\hat{\jmath}. \quad (20)$$

The D matrix is the matrix of computed relative sensitivities. The normal equations to be solved are

$$\sum_{k=1}^{\hat{k}} \sum_{k'=1}^{\hat{k}} \left[Z_k(X^e) - z_k^e + \sum_{j'=1}^{\hat{\jmath}} A_{kj'}(X_{j'} - x_{j'}^e) \right]$$

$$\times {}^{ZZ'}W_{kk'}A_{k'j} = 0 \quad \text{for } j=1,2,\dots,\hat{\jmath}, \quad (21)$$

with solution

$$X_j^a = X_j^e + \sum_{j'=1}^{\hat{\jmath}} C_{jj'}^{-1}B_{j'} \quad \text{for } j=1,2,\dots,\hat{\jmath}. \quad (22)$$

Here

$$B_j = \sum_{k=1}^{\hat{k}} \sum_{k'=1}^{\hat{k}} [z_k^e - Z_k(X^e)]^{ZZ'}W_{kk'}A_{k'j}$$

$$\text{for } j=1,2,\dots,\hat{\jmath}, \quad (23)$$

and

$$C_{jj'} = \sum_{k=1}^{\hat{k}} \sum_{k'=1}^{\hat{k}} A_{kj'}{}^{ZZ'}W_{kk'}A_{k'j} \quad \text{for } j,j'=1,2,\dots,\hat{\jmath}. \quad (24)$$

This symbolism follows that in DAFT1 and is used in coding DAFT2. Now that the adjusted values of the differential parameters have been obtained, the adjusted values of the integral parameters develop from Eq. (17),

$$Y_i^a = y_i^a/y_i^e = y_i(X^e)/y_i^e$$

$$+ \sum_{j=1}^{\hat{\jmath}} A_{ij}[X_j^a - X_j^e]. \quad (25)$$

The matrix C^{-1} is the matrix of variances and covariances among the adjusted quantities as derived from the input evaluated uncertainty matrix ${}^{ZZ'}V$, but it does not reflect the actual dispersion in the data. Hence it is customary,[29,33] in obtaining the variance-covariance for the normalized adjusted data, to multiply by $S_2^a/\hat{\imath}$, a "dispersion multiplier," where S_2^a is computed after adjustment.

$$^{XX}M = \frac{S_Z^a}{\hat{\imath}} C^{-1}. \quad (26)$$

If $S_2^a/\hat{\imath}$ is less than unity, it is replaced by unity in ALVIN. Similarly, combining Eqs. (25) and (26), one obtains the variance-covariance matrix among the adjusted integral quantities,

$$^{YY}M = A \, {}^{XX}M \, A^{tr}, \quad (27)$$

and the covariance matrix between adjusted integral and differential quantities,

$$^{YX}M = A \, {}^{XX}M, \quad (28)$$

2. Adjustment of Arbitrarily Large Differential Data Sets -- DAFT3. Daft 2 must invert one matrix (V) of order $(\hat{\imath} + \hat{\jmath}) \times (\hat{\imath} + \hat{\jmath})$ and one matrix (C) of order $\hat{\jmath} \times \hat{\jmath}$. When $\hat{\jmath}$, the number of differential quantities in the library, is large, say 10^3 to 10^5, the matrix inversion becomes a problem which is frequently resolved by adjusting only part of the data set. Had the adjustment been applied to another

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part of the library, the result would be different. This matrix inversion problem can be circumvented,[8] and arbitrarily large differential data sets adjusted simultaneously when, as is assumed by most groups, the differential data are uncorrelated with the integral data.

Let ^{YY}V represent the evaluated variance-covariance matrix for the evaluated integral quantities, let ^{XX}V represent the evaluated variance-covariance matrix for the differential quantities, and suppose that the evaluated differential quantities are uncorrelated with the evaluated integral quantities. The weight matrix expressed by Eq. (15) then has partitions $^{YY}W = {}^{YY}V^{-1}$, $^{XX}W = {}^{XX}V^{-1}$, and $^{XY}W = 0$. Let ^{YX}A represent the upper $\hat{1} \times \hat{J}$ partition of the A matrix expressed by Eq. (19). That is, a component $^{YX}A_{kj}$ of this $\hat{1} \times \hat{J}$ matrix is $Z(x^e)D_{kj}$. The adjusted values of the differential quantities are obtained in this case from

$$x^a - x^e = {}^{XX}V \, {}^{YX}A^{tr} ({}^{YY}V + {}^{YX}A \, {}^{XX}V \, {}^{YX}A^{tr})^{-1}$$

$$\times \; [Y^e - Y(X^e)] \quad , \tag{29}$$

where the largest matrix to be inverted is only of order $\hat{1} \times \hat{1}$. The adjusted values of the integral quantities are obtained as before from Eq. (25). The variance-covariance matrix of the adjusted differential quantities is

$$^{XX}V^a = {}^{XX}V - {}^{XX}V \, {}^{YX}A^{tr} ({}^{YY}V + {}^{YX}A \, {}^{XX}V \, {}^{YX}A^{tr})^{-1}$$

$$\times \; {}^{YX}A \; {}^{XX}V \tag{30}$$

and again a matrix of order only $\hat{1} \times \hat{1}$ must be inverted. The variance-covariance matrix ^{XX}M is obtained from Eq. (26) by replacing C by $^{XX}V^a$, and ^{YY}M and ^{YX}M are computed as before from Eqs. (27) and (28), respectively.

Because $\hat{1}$ usually is much less than $\hat{J}$, this technique, coded as DAFT3, requires inversion of much smaller matrices than is the case for DAFT2.

III. SENSITIVITY CALCULATIONS

The calculations described in the previous section required derivatives $\delta y_i / \delta x_j$ of integral parameter i with respect to differential parameter j.

These derivatives can be expressed conveniently as relative sensitivities [see Eq. (20)],

$$D_{ij} = \frac{x_j^e}{y_i(x^e)} \left. \frac{\delta y_i}{\delta x_j} \right|_{x^e} \quad , \tag{31}$$

where both integral values $y_i(x^e)$ and derivatives are computed for a given evaluated differential data set x^e. Many workers also have contributed to this development.[34-45] We now describe computations of relative sensitivities and derived quantities, such as sensitivity profiles, from linear perturbation theory using SENSI and related modules in ALVIN.

A. Inhomogeneous Transport

The particle or photon flux $\psi(\xi)$ at a point ξ in phase-time space satisfies the inhomogeneous linear Boltzmann equation

$$L\psi = S \quad , \tag{32}$$

where $S(\xi)$ is the local source density. The adjoint flux $\psi^+(\xi)$ satisfies the adjoint equation

$$L^+\psi^+ = S^+ \quad , \tag{33}$$

where $S^+(\xi)$ is an adjoint source, and where L^+ is an operator adjoint to the Boltzmann operator L. The adjoint operator and boundary conditions on ψ^+ are defined such that

$$(\psi^+, L\psi) = (L^+\psi^+, \psi) \quad . \tag{34}$$

Here (ϕ, χ) symbolizes an inner product, in this case just the integral of $\phi(\xi)\chi(\xi)$ over the relevant region of phase-time space.

Choose $S^+(\xi)$ so that (S^+, ξ) is an integral quantity of interest. Suppose that the operators and sources change as might occur, for example, if differential nuclear data are changed. Then

$$(L + \delta L)(\psi + \delta\psi) = S + \delta S \quad , \tag{35}$$

and

$$(L^+ + \delta L^+)(\psi^+ + \delta\psi^+) = S^+ + \delta S^+ \quad . \tag{36}$$

When these results are combined, it follows that the exact change $\delta(s^+, \psi)$ in the integral quantity, even for large changes in operators and sources, is

$$\delta(s^+, \psi) = -(\psi^+, \delta L[\psi + \delta\psi]) + (\psi^+, \delta S) + (\delta s^+, \psi + \delta\psi) \quad , \tag{37}$$

or

$$\delta(s^+, \psi) = -(\delta L^+[\psi^+ + \delta\psi^+], \psi) + (\psi^+ + \delta\psi^+, \delta S) + (\delta s^+, \psi) \quad , \tag{38}$$

in addition to two other equivalent forms.

We now confine ourselves to the case of fixed sources and small perturbations, i.e., to linear perturbation theory. In this case, from Eqs. (37) and (38),

$$\delta(s^+, \psi) = -(\psi^+, \delta L\psi) = (-\delta L^+ \psi^+, \psi) \quad . \tag{39}$$

Further, consider the time-independent, one-space-dimensional, multigroup situation for which the Boltzmann equation and its adjoint are

$$(\Omega \cdot \nabla + \Sigma_g)\psi_g(r, \Omega)$$

$$- \sum_{g'=1}^{\hat{g}} \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \Sigma_{g' \to g \, j}^j \psi_{g'j}(r) P_j(\mu)$$

$$= S_g(r, \Omega) \quad , \tag{40}$$

and

$$(-\Omega \cdot \nabla + \Sigma_g)\psi_g^+(r, \Omega)$$

$$- \sum_{g'=1}^{\hat{g}} \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \Sigma_{g \to g' \, j}^j \psi_{g'j}^+(r) P_j(\mu)$$

$$= S_g^+(r, \Omega) \quad . \tag{41}$$

for each multigroup $g = 1, 2, \ldots, \hat{g}$. Here $\psi_g(r, \Omega)$ and $\psi^+(r, \Omega)$ are the adjoint flux, respectively, at posi-

tion r in multigroup g and in direction Ω, with Legendre expansions

$$\psi_g(r, \Omega) = \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \psi_{gj}(r) P_j(\mu) \quad , \tag{42}$$

$$\psi_g^+(r, \Omega) = \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \psi_{gj}^+(r) P_j(\mu) \quad , \tag{43}$$

in terms of the Legendre polynomials, $P_j(\mu)$, of the polar angle cosine μ. The Legendre coefficients of flux, ψ_{gj}, and adjoint flux ψ_{gj}^+, are readily computed from S_N, multigroup Monte Carlo, P_N, and diffusion theory solutions, and their use eliminates the necessity for determining consistent quadrature techniques for the inner product, so Legendre fluxes and adjoint fluxes are utilized henceforth. In this case the inner product expressions of Eqs. (37) and (38) become

$$\delta(s^+, \psi) = -(\psi^+, \delta L\psi)$$

$$= \int d\underline{r} \sum_g \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \psi_{gj}^+$$

$$\times [-\delta\Sigma_g \psi_{gj} + \sum_{g'} \delta\Sigma_{g' \to g}^j \psi_{g'j}] \quad , \tag{44}$$

and

$$\delta(s^+, \psi) = -(\delta L^+ \psi^+, \psi)$$

$$= \int d\underline{r} \sum_g \sum_{j=0}^{\infty} \frac{2j+1}{4\pi}$$

$$\times [-\delta\Sigma_g \psi_{gj}^+ + \sum_{g'} \delta\Sigma_{g \to g'}^j \psi_{g'j}^+]\psi_{gj} \quad , \tag{45}$$

for the change in the integral quantity consequent to changes in group cross sections. Thus for changes in

particular cross sections we obtain from either Eq. (14) or (15) the results

$$\frac{\partial(s^+,\psi)}{\partial\Sigma_g} = -\int d\underline{r}\sum_{j=0}^{\infty}\frac{2j+1}{4\pi}\psi_{gj}^+\psi_{gj} \quad , \qquad (46)$$

$$\frac{\partial(s^+,\psi)}{\partial\Sigma_{g'\rightarrow g}^j} = \int d\underline{r}\frac{2j+1}{4\pi}\psi_{gj}^+\psi_{g'j} \quad . \qquad (47)$$

These derivatives, of the form $\partial y_i/\partial x_j$, completely characterize nuclear data sensitivities as computed by multigroup S_N, Monte Carlo, P_N, or diffusion theory codes. Fission neutron production cross sections can be thought of as being absorbed in the $\Sigma_{g'\rightarrow g}^j$ arrays. The expressions, Eqs. (46) and (47), do not require cross sections or values of (s^+,ψ), i.e., y_i, so they represent a discrete calculation which is carried out by a basic subroutine, SENSI, in ALVIN. Legendre fluxes and adjoint fluxes are read in (KFLUX=1) by subroutine REDFLX, or are computed (KFLUX=2) by REDFLX from discrete ordinate fluxes and adjoints. SENSI computes the inner product in slab, spherical, or cylindrical geometry according as KGEOM is 1,2, or 3, respectively. In ALVIN the derivative $\partial(s^+,\psi)/\partial\Sigma_g$ is labeled DYDG(IG), and $\partial(s^+,\psi)/\partial\Sigma_{g'\rightarrow g}^j$ is labeled DYDL(IJ,IGI,IG). These quantities can be normalized and stored into sensitivity arrays for use in data adjustment.

If the inhomogeneous problem is time-dependent, we merely add a time integral to the inner product, add $V_g^{-1}\partial\psi/\partial t$ to Eq. (40), add $-V_g^{-1}\partial\psi/\partial t$ to Eq. (41), and note that these do not change when differential multigroup data are changed. Thus Eqs. (46) and (47) apply also to the time-dependent case if the right-hand sides are integrated over time. If KTIMS = 1, the time integral is carried out in SENSI by reading in fluxes and adjoints at each time step, executing Eqs. (46) and (47), multiplying by the width of the time step, accumulating, and repeating the operation for all time steps.

Two aspects of the adjoint source s^+ are noteworthy. First, suppose (s^+,ψ) represents a detector response or dose such that a response or dose per unit monoenergetic flux is Σ_{det} over a volume V_{det}; then we wish (s^+,ψ) to equal $V_{det}\Sigma_{det}\psi_0$ if the angle integrated flux ψ_0 is uniform over the detector re-

gion. The inner product (s^+,ψ) then is $\int d\Omega\int d\underline{r}s^+(\Omega)\psi(\Omega)$ or $V_{det}s^+(\Omega)\psi_0$ in this case. Thus the directed adjoint source $s^+(\Omega)$ is Σ_{det}, but the angle integrated source frequently input into transport codes is $4\pi\Sigma_{det}$.

Second, consider a time-dependent problem for which the response of interest is the temperature $\theta(t_m)$ at a time t_m, of a material of volume V_m, with total heat capacity C, with a total heat transfer coefficient K to a reservoir, and subject to fission heating. Let q_g represent the local energy deposition per fission in group g. Then

$$C\frac{d\theta}{dt} + K\theta = \sum_g\int d\underline{r}\int d\underline{\Omega}\, q_g\Sigma_{fg}\psi_g \quad . \qquad (48)$$

The solution to Eq. (48) is

$$(s^+,\psi) = \theta(t)$$

$$= \int_0^t dt'\sum_g\int d\underline{\Omega}\int_{V_m} d\underline{r}\, e^{\frac{k}{c}(t-t')}\frac{q_g}{c}\Sigma_{fg}\psi(t') \qquad (49)$$

so the angle-integrated adjoint source is

$$s_g^+ = 4\pi q_g\Sigma_{fg}\frac{1}{c}e^{-\frac{k}{c}(t-t_m)}U(t-t_m) \quad . \qquad (50)$$

in V_m and zero elsewhere. Here we have used the unit function $U(\tau)$ which is unity for $\tau \geqslant 0$ and is zero otherwise. Adjoint sources generally can be constructed by this method.

B. Reactivity, Rossi-α, and Other Eigenproblems

The transport operator L can be broken up in many ways and for each there is an eigenproblem,

$$L'\psi_a = aL''\psi_a \quad . \qquad (51)$$

Here ψ_a is the eigenfunction corresponding to eigenvalue a. For example, L" might be the fission neutron production operator, in which case 1/a is the multiplication factor. For another example, L" might be in multigroup notation a square matrix whose elements are zero except on the main diagonal where the elements are inverse speeds, V_g^{-1}. In this case, a represents the Rossi-α parameter. For any particular breakup of the transport operator, there is a

set of eigenvalues and a corresponding set of eigenfunctions of which ordinarily only one, the fundamental, is real and non-negative. The fundamental is usually the only eigenfunction computed by conventional transport codes.

After nuclear data change, L becomes $L + \delta L$, and the eigenvalues and eigenfunctions change accordingly,

$$(L' + \delta L')(\psi_a + \delta\psi_a)$$

$$= (a + \delta a)(L'' + \delta L'')(\psi_a + \delta\psi_a) \quad . \quad (52)$$

Subtracting Eq. (51) from Eq. (52) and linearizing, we leave

$$\delta a L'' \psi_a = (L' - a L'')\delta\psi_a + (\delta L' - a\delta L'')\psi_a \quad . \quad (53)$$

Adjoint to Eq. (51) is the relation

$$L'^{+}\psi_a^{+} = a L''^{+}\psi_a^{+} \quad , \quad (54)$$

with the adjoint boundary conditions described earlier. Multiply Eq. (53) by ψ_a^{+} and integrate over phase space. The first terms on the right-hand side of Eq. (53) contribute nothing in view of the adjoint property expressed in Eq. (31), with the result that

$$\delta a = \frac{(\psi_a^{+}, [\delta L' - a\delta L'']\psi_a)}{(\psi_a^{+}, L''\psi_a)} \quad . \quad (55)$$

The fundamental eigenvalue is an integral quantity of considerable interest and according to Eq. (55) its change can be computed using only the fundamental eigenfunction and adjoint. A much more difficult situation arises when the integral quantity of interest is a ratio of reaction rates

$$R_{rq} = \frac{(S_r^{+}, \psi_a)}{(S_q^{+}, \psi_a)} \quad (56)$$

both measured in the fundamental flux. In this case

$$\frac{\delta R_{rq}}{R_{rq}} = \frac{(S_r^{+}/R_{rq} - S_q^{+}, \delta\psi_a)}{(S_q^{+}, \psi_a)} \quad , \quad (57)$$

and an inner product with $\delta\psi_a$ is required. Often in mathematical physics when an eigenfunction change is required it is expanded in a complete set of eigenfunctions. In the transport problem, however, we are unlikely to have available any eigenfunctions other than the fundamental.

Usachev[44] has developed an iterative algorithm for computation of Eq. (57) requiring repeated solution of the inhomogeneous transport equation. We develop an alternative algorithm and suggest its use in an appropriate way that only requires knowledge of the unperturbed flux and adjoint. Let ψ_r^{+} represent the solution to the inhomogeneous equation,

$$L'^{+}\psi_r^{+} = S_r^{+} \quad . \quad (58)$$

Insert this expression into Eq. (57) and apply the adjoint property, Eq. (54). Then

$$\delta R_{rq} = \frac{(\psi_r^{+}/R_{rq} - \psi_q^{+}, L'\delta\psi_a)}{(S_q^{+}, \psi_a)} \quad . \quad (59)$$

The linearized Eq. (52) can be rearranged to provide an iterative calculation of the nth approximation, $\delta\psi_{a,n}$, to the desired $\delta\psi_a$,

$$L'\delta\psi_{a,n} = (a\delta L'' + \delta a L'' - \delta L')\psi_a$$

$$+ a L''\delta\psi_{a,n-1} \quad , \quad (60)$$

with starting condition $\delta\psi_{a,0} = 0$. This constitutes an algorithm for computation of the required $L'\delta\psi_{a,n}$ under proper conditions of convergence. Here we terminate at $n = 1$, whereupon, approximately,

$$\delta R_{rq} = \frac{(\psi_r^{+}/R_{rq} - \psi_q^{+}, [a\delta L'' + \delta a L'' - \delta L']\psi_a)}{(S_q^{+}, \psi_a)} \quad . \quad (61)$$

The above approximation is not coded into this version of ALVIN.

Returning to the eigenvalue perturbations, we now apply the previously described techniques to develop the inner products in Eq. (55) for Rossi-α and for reactivity perturbations. For Rossi-α,

$$\frac{\partial \alpha}{\partial \Sigma_g} = -\frac{1}{D_\alpha} \int d\underline{r} \sum_j^\infty \frac{2j+1}{4\pi} \psi^+_{\alpha g j} \psi_{\alpha g j} \qquad (62)$$

and

$$\frac{\partial \alpha}{\partial \Sigma^j_{g'\to g}} = \frac{1}{D_\alpha} \int d\underline{r} \frac{2j+1}{4\pi} \psi^+_{\alpha g j} \psi_{\alpha g' j} \quad , \qquad (63)$$

where

$$D_\alpha = \int d\underline{r} \sum_g \sum_j^\infty \frac{2j+1}{4} \psi^+_{\alpha g j} \frac{1}{v_g} \psi_{\alpha g j} \qquad (64)$$

For reactivity perturbations, it is convenient to break up $\Sigma^j_{g'\to g}$ into a scattering portion and a fission portion

$$\Sigma^j_{g'\to g} = \Sigma^{sj}_{g'\to g} + \nu_g \Sigma_{fg'} \chi_{g'\to g} \delta_{jo} \quad . \qquad (65)$$

Then,

$$\frac{\partial \rho}{\partial \Sigma_g} = -\frac{1}{D_\rho} \int d\underline{r} \sum_j^\infty \frac{2j+1}{4\pi} \psi^+_{\rho g j} \psi_{\rho g j} \quad , \qquad (66)$$

and

$$\frac{\partial \rho}{\partial [\Sigma^{sj}_{g'\to g} + (1-\rho)\nu_g \Sigma_{g'} \chi_{g'\to g} \delta_{jo}]}$$

$$= \frac{1}{D_\rho} \int d\underline{r} \sum_g \frac{2j+1}{4\pi} \psi^+_{\rho g j} \psi_{\rho g' j} \quad , \qquad (67)$$

where

$$D_\rho = \int d\underline{r} \sum_g \sum_{g'} \psi^+_{\rho g 0} \frac{\nu_{g'}}{4\pi} \Sigma_{fg'} \chi_{g'\to g} \psi_{\rho g' 0} \cdot \quad (68)$$

These expressions also are in their simplest form.

Equations for sensitivities thus far have been expressed as derivatives with respect to macroscopic cross sections. Ordinarily a sensitivity of interest will be for a single material or nuclide, but this may occur with different number densities DENS(IR) in different spatial regions IR=1,2,...,NR. A derivative with respect to a microscopic cross section σ is obtained from the corresponding derivative with respect to a macroscopic cross section Σ as in Eqs. (46), (47), (62), (63), (66), and (67) by

$$\frac{\partial}{\partial \sigma} = \sum_{IR=1}^{NR} \int_{\text{Region IR}} d\underline{r} \; \text{DENS(IR)* remainder of} \atop \text{expression} \quad (69)$$

When sensitivity integrals are coded in SENSI, the number density of the material of interest is included as in Eq. (69) with one exception: The reactivity denominator in Eq. (68) requires the actual macroscopic fission cross sections for the assembly, so these are read in for each group and region and used as macroscopic quantities.

C. Sensitivity Profile

Expressions were developed in the previous sections for derivatives $\partial y/\partial \Sigma_g$ and $\partial y/\partial \Sigma_{g'\to g}$, where y is an integral parameter. These constitute the basic building blocks for compounding derivatives with respect to any differential data, e.g., the inelastic scattering cross sections for [238]U. We illustrate the compounding process not for a particular differential cross section, but for the "sensitivity profile," an interesting parameter characterizing a class of cross-section changes.[43,45]

Suppose that for a particular nuclide, say [238]U, we change the total cross section in group g by the amount $\delta\Sigma^x_g$, corresponding to a change in reaction x, say an inelastic cross section. The group-to-group transfer cross sections for this reaction, $\Sigma^{jx}_{g'\to g}$, change accordingly, and we make the particular assumption that

$$\frac{\delta\Sigma^{jx}_{g\to g'}}{\Sigma^j_{g\to g'}} = \frac{\delta\Sigma^x_g}{\Sigma^x_g} \quad , \qquad (70)$$

for all exit groups g' and for all Legendre orders j.

Cross sections for particle transfer into group g are unchanged. Then, compounding changes by the usual ($\delta_y = \sum_i \frac{\partial y}{\partial x_i} \delta x_i$) method we have, from Eqs. (46) and (47),

$$\frac{\delta(s^+,\psi)}{(s^+,\psi)} = P_g^x \frac{\delta\Sigma_g^x}{\Sigma_g^x} \quad , \tag{71}$$

where the sensitivity profile P_g^x is

$$P_g^x = \frac{1}{(s^+,\psi)} \int d\underline{r} \sum_{j=0}^{\infty} \frac{2j+1}{4\pi}$$

$$\times [-\Sigma_g^x \psi_{gj}^+ + \sum_{g'} \Sigma_{g \to g'}^{jx} \psi_{g'j}^+]\psi_{gj} \quad . \tag{72}$$

This inhomogeneous sensitivity profile characterizes certain interesting cross-section changes and is computed by subroutine PROFIL in ALVIN.

Similar profiles can be defined and computed for Rossi-α and for reactivity and in general under other assumptions than Eq. (70), but we do not carry these out in ALVIN.

Bartine et al.[45] use the definition Eq. (71) for inhomogeneous sensitivity profile, but they appear to state, on the basis of equations like Eqs. (34) and (39), that another definition can be used,

$$P_g^{x'} = \frac{1}{(s^+,\psi)} \int d\underline{r} \sum_{j=0}^{\infty} \frac{2j+1}{4\pi} \psi_{gj}^+$$

$$\times [-\Sigma_g^x \psi_{gj} + \sum_{g'} \Sigma_{g' \to g}^{jx} \psi_{g'j}] \quad . \tag{73}$$

In paraphrase, because $(\psi^+,\delta L\psi)$ equals $(\delta L^+\psi^+,\psi)$, then $\psi^+\delta L\psi$ equals $\psi\delta L^+\psi^+$ over some more limited region of phase space. There is no a priori reason to expect this to be generally true, although it is true for the fully absorptive case. We have devel-oped a complete analytic solution to the case of hydrogenous slowing down in an infinite homogeneous medium[46] to be used in clarifying a variety of problems. For this case the two definitions of P_g^x are

not equal and, because our definition, Eq. (71), was arrived at by the orderly process of compounding, we believe it it correct.

IV. ALVIN PROGRAMMING

ALVIN is programmed in FORTRAN-IV to be machine-independent except for large storage requirements which, in the distributed version, are specific to the CDC-7600. Five large arrays are stored in LCM and only a few statement lines would need to be changed for other machines. Otherwise, specific CDC features are avoided, e.g., Hollerith is used for formatting rather than asterisks. A code abstract is included as Appendix A.

The code consists of the main ALVIN routine and eight principal subroutines with about 1100 statement lines, approximately 20% of which are comment lines. The routines, the subroutines they call, and their tape requirements are listed in Table I. A subroutine SENRD2, which generates sensitivities specific to a sample problem, is provided as well. Sensitivity and variance-covariance matrices are so large, and so frequently have integral regularities, that it may be useful to create subroutines like SENRD2 to generate them for specific problems.

Program variables have the same significance in all parts of the code, and their values, with a few exceptions, are passed through labeled commons. All variables are defined in Appendix B, and these definitions hold in all parts of ALVIN. Certain systematics have been followed in variable naming.

TABLE I

ROUTINE CALLS AND TAPE REQUIREMENTS IN ALVIN

Routine	Calls or Requires
ALVIN	SENSRD (SENRD2), SENSI, DAFT2, DAFT3
SENSRD	
SENSI	REDFLX, CROSEF, CROSEC, PROFIL
REDFLX	Flux tape, adjoint flux tape
PROFIL	
CROSEF	
CROSEC	Cross section tape
DAFT2	INFO, MATINV
DAFT3	INFO, MATINV
INFO	
MATINV	

For example, indices have a fixed meaning, and temporaries are formed by suffix T. Variable names usually are similar to notation used in Secs. II and III describing the calculations.

The structure of ALVIN is shown in Fig. 1. Pairs of control parameters KSENS and KADJST define various modes of code operation. For example, KSENS=3 and KADJST=2 result in sensitivities being read in by SENSRD, some sensitivities being calculated and added by SENSI, and consistency tests and data adjustment being carried out for full correlation by DAFT2. A simple case is KSENS=1 and KADJST=1 where sensitivities are calculated by SENSI, and no data adjustment or consistency calculations are carried out.

V. SAMPLE PROBLEMS AND CODE VALIDATION

Two sample problems are provided, one stressing the data consistency and adjustment parts of ALVIN, and one stressing the sensitivity parts of the code. The sample problems are used to illustrate capability, input, and output. In addition, however, the sample problems are used to validate the code by carrying out the same calculation in different ways.
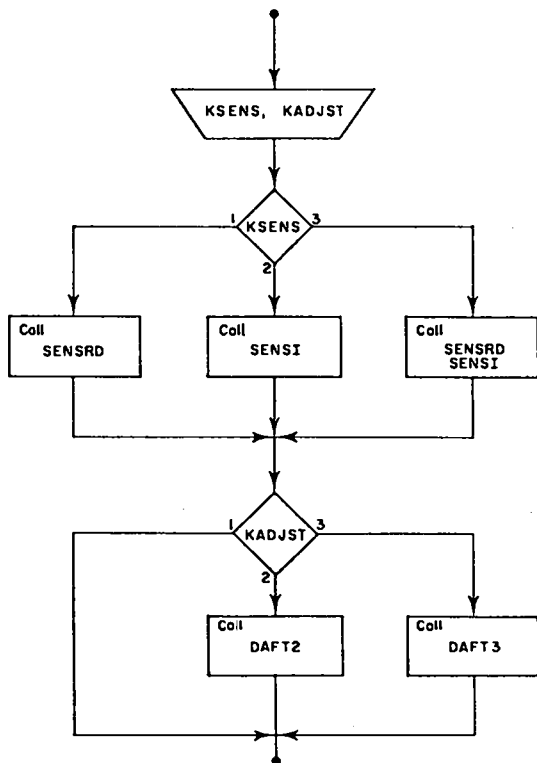
Fig. 1. ALVIN structure.

A. Data Consistency and Adjustment

Integral observations made on LMFBR-like criticals have frequently failed to agree with calculations. Reactivity worths in particular have been discrepant, thus presenting the nuclear designer with serious problems in view of the importance of reactivity worths in the design process. Data for three important criticals, ZPR-6-6A, ZPR-6-7, and ZPR-3-48, prominently display the reactivity worth discrepancy and have been compactly presented by Bohn.[47] Table II identifies 24 integral parameters y_i, i=1,2...,24, for these assemblies, and Table III identifies 19 differential nuclear quantities x_j, j=1,2...,19, of interest. It is convenient to allow y_i to represent the ratio of the computed value C_i of an integral parameter to its experimental value E_i, and to let x_j represent the ratio of the nuclear datum σ_j to its evaluated value σ_j^e. Then the evaluated quantities y_i^e and x_j^e are unity, and

$$\left.\frac{\partial y_i}{\partial x_j}\right|_{x^e} = \left.\left(\frac{\sigma_j}{C_i}\frac{\partial C_i}{\partial \sigma_j}\right)\right|_{x^e} - \left.\left(\frac{\sigma_j}{E_i}\frac{\partial E_i}{\partial \sigma_j}\right)\right|_{x^e} \quad . \quad (74)$$

This normalization is essentially that described in Sec. II-B-1. The effects of cross-section changes on inference of E_i from experiment are made explicit in Eq. (66). For i=1,2,3 the integral parameters are the C/E values for multiplication factors of ZPR-6-6A, SPR-6-7, and ZPR-3-48, indicated in the second column of Table II by subscripts A, 7, and 8, respectively. For i=4,5...,15 the integral parameters are the C/E values for central worths of ^{239}Pu, ^{235}U, ^{238}U, and ^{10}B, indicated by 49, 25, 28, and B, respectively, as superscripts on W; for example, the C/E value for the central worth of ^{239}Pu in the ZPR-6-7 assembly is indicated by W_7^{49} in Table II. Finally, for i greater than 15, the integral parameters y_i are C/E values of ratios of reaction rates, e.g., $_7R_{49f}^{28c}$ for y_{20} represents the C/E value of the ^{238}U capture rate relative to the ^{239}Pu fission rate measured in ZPR-6-7. If y_i is $(\sigma_n/\sigma_m)/(\sigma_n/\sigma)_E$, then to first order (unchanged flux spectrum),

$$\left.\frac{\partial y_i}{\partial x_j}\right|_{x^e} = \delta_{nj} - \delta_{mj} \quad . \quad (75)$$

TABLE II

INTEGRAL PARAMETERS y_i AND VALUES $y_i(x^e)$ COMPUTED USING EVALUATED NUCLEAR DATA PARAMETERS x_j^e

[If no adjustment were necessary $y_i(x^e)$ would = unity]

i	y_i	$y_i(x^e)$	y_i Adjusted by ALVIN	y_i Selected by Bohn
1	k_A	0.9920 ± 0.004	0.993	
2	k_7	1.0024 ± 0.004	1.001	
3	k_8	0.9927 ± 0.004	1.002	
4	w_A^{49}	1.10 ± 0.025	0.990	1.06
5	w_A^{25}	1.15 ± 0.025	1.014	1.05
6	w_A^{28}	1.24 ± 0.035	1.090	1.09
7	w_A^B	0.92 ± 0.075	0.854	0.96
8	w_7^{49}	1.25 ± 0.035	1.052	1.14
9	w_7^{25}	1.24 ± 0.035	1.036	1.08
10	w_7^{28}	1.16 ± 0.025	0.936	0.95
11	w_7^B	1.18 ± 0.035	1.027	1.17
12	w_8^{49}	1.25 ± 0.035	1.042	1.12
13	w_8^{25}	1.26 ± 0.035	1.046	1.08
14	w_8^{28}	1.27 ± 0.035	1.024	0.99
15	w_8^B	1.09 ± 0.035	0.960	1.06
16	$_A R^{28f}_{25f}$	0.90 ± 0.03	0.942	
17	$_A R^{28c}$	1.03 ± 0.03	1.060	
18	$_7 R^{28f}_{49f}$	0.99 ± 0.02	0.975	
19	$_7 R^{25f}_{49f}$	1.05 ± 0.02	1.032	
20	$_7 R^{28c}_{49f}$	1.09 ± 0.02	1.053	
21	$_7 R^{28f}_{25f}$	0.94 ± 0.02	0.924	
22	$_7 R^{28c}_{25f}$	1.04 ± 0.02	1.023	
23	$_8 R^{28f}_{25f}$	0.96 ± 0.05	0.961	
24	$_8 R^{28c}_{25f}$	0.94 ± 0.05	0.924	

TABLE III

PRIMARY NUCLEAR DATA PARAMETERS x_j, THEIR UNCERTAINTIES, AND THEIR ADJUSTED VALUES

j	x_j	x_j^e	x_j Adjusted by Alvin	x_j Selected by Bohn
1	σ_f^{25}	1 ± 0.1	0.978	0.93
2	σ_c^{25}	1 ± 0.1	0.932	
3	σ_f^{49}	1 ± 0.1	0.995	0.97
4	σ_c^{49}	1 ± 0.1	0.989	
5	σ_c^{28}	1 ± 0.15	0.961	0.88
6	σ_f^{28}	1 ± 0.1	0.979	0.97
7	σ_{inel}^{28}	1 ± 0.15	0.840	0.88
8	σ_{el}^{Na}	1 ± 0.1	1.059	
9	σ_{el}^{O}	1 ± 0.1	1.063	
10	σ_c^{fe}	1 ± 0.1	0.979	
11	σ_c^{Ni}	1 ± 0.1	1.001	
12	σ_c^{Cr}	1 ± 0.1	0.995	
13	$\bar{\nu}_d^{28}$	1 ± 0.06	1.138	1.10
14	$\bar{\nu}_d^{25}$	1 ± 0.04	1.106	1.012
15	$\bar{\nu}_d^{49}$	1 ± 0.06	1.166	1.024
16	$\bar{\nu}_d^{40}$	1 ± 0.1	1.009	1.016
17	$\bar{\nu}_d^{41}$	1 ± 0.1	1.009	
18	σ_c^{Na}	1 ± 0.1	0.995	
19	σ_{el}^{C}	1 ± 0.1	1.085	

The third columns of Tables II and III list the values of the parameters determined at the evaluated point together with standard errors, most of them assigned by Bohn, for the quantities. Most of the computed reactivity worths, y_4 through y_{15}, are high and differ from unity by many standard errors; this is the reactivity worth discrepancy. Uncertainties in reactivity worths due to uncertainties in delayed neutron yields are not included, because the delayed data ($j=13,17$) are assigned uncertainties and

TABLE IV

SENSITIVITY COEFFICIENTS $(x_j/y_i)\, \partial y_i/\partial x_j$ FOR ZPR-6-6A, ZPR-6-7, AND ZPR-3-48 (REF. 47)

```
   I  DYDX(I.J).J=1.MJ
 1  6.1000E-01 -1.0000E-01 -0.         -0.          8.0000E-02 -2.7000E-01 -3.0000E-02 -0.         -0.         -0.
    -0.         -0.         -0.         -0.         -0.         -0.         -0.         -0.         -0.
 2  -0.         -0.          6.6000E-01 -7.0000E-02  8.0000E-02 -2.6000E-01 -6.0000E-02 -0.         -0.         -0.
    -0.         -0.         -0.         -0.         -0.         -0.         -0.         -0.         -0.
 3  -0.         -0.          6.6000E-01 -6.0000E-02  9.0000E-02 -2.1000E-01 -7.0000E-02 -0.         -0.         -0.
    -0.         -0.         -0.         -0.         -0.         -0.         -0.         -0.         -0.
 4  -9.9100E-01  4.9000E-02  1.1600E+00 -1.6100E-01 -2.3900E-01 -2.5500E-01  4.7000E-02 -4.5000E-02 -1.1500E-01  2.0000E-03
    -6.0000E-03 -3.0000E-03 -1.4400E-01 -8.5600E-01 -0.         -0.         -0.         -5.0000E-03 -0.
 5   2.6400E-01 -1.7400E-01 -0.         -0.         -2.0600E-01  2.1000E-01  8.8000E-02 -2.4000E-02 -2.7000E-02  1.0000E-02
     2.0000E-03  5.0000E-03 -1.4400E-01 -8.5600E-01 -0.         -0.         -0.          4.0000E-03 -0.
 6  -1.3450E+00 -9.4000E-02 -0.         -0.         -5.8300E-01  1.2250E+00  3.4600E-01  4.3000E-02  1.4800E-01 -1.4000E-02
     2.0000E-03 -2.0000E-03 -1.4400E-01 -8.5600E-01 -0.         -0.         -0.          2.0000E-03 -0.
 7  -1.2520E+00 -1.4300E-01 -0.         -0.         -3.2900E-01 -9.2000E-02  2.0800E-01  9.8000E-02  2.1300E-01 -2.0000E-02
     4.0000E-03 -3.0000E-03 -1.4400E-01 -8.5600E-01 -0.         -0.         -0.          2.0000E-03 -0.
 8  -0.         -0.          2.1300E-01 -8.3000E-02 -4.1700E-01  1.9900E-01  1.1100E-01 -2.7000E-02 -5.2000E-02  3.0000E-03
    -3.0000E-03 -2.0000E-03 -3.2600E-01 -2.5200E-02 -5.9700E-01 -2.0600E-02 -3.1200E-02 -1.0000E-03 -0.
 9   1.1790E+00 -1.9000E-01 -1.1010E+00  5.0000E-02 -3.9100E-01  1.3700E-01  1.5000E-01 -4.0000E-03  3.5000E-02  9.0000E-03
     3.0000E-03  4.0000E-03 -3.2600E-01 -2.5200E-02 -5.9700E-01 -2.0600E-02 -3.1200E-02  6.0000E-03 -0.
10  -0.         -0.         -1.2070E+00 -1.2100E-01 -7.7800E-01  1.0450E+00  4.9000E-01  6.4000E-02  1.9200E-01 -2.0000E-02
     3.0000E-03 -3.0000E-03 -3.2600E-01 -2.5200E-02 -5.9700E-01 -2.0600E-02 -3.1200E-02  1.0000E-03 -0.
11  -0.         -0.         -1.1290E+00 -2.1600E-01 -5.0900E-01 -2.6000E-01  2.8300E-01  1.4900E-01  3.1600E-01 -3.3000E-02
     4.0000E-03 -4.0000E-03 -3.2600E-01 -2.5200E-02 -5.9700E-01 -2.0600E-02 -3.1200E-02  2.0000E-03 -0.
12  -0.         -0.          2.4700E-01 -6.9000E-02 -4.6500E-01  2.1100E-01  1.4400E-01 -9.0000E-03 -0.          1.0000E-03
    -3.0000E-03 -2.0000E-03 -3.5600E-01 -2.1000E-02 -5.9900E-01 -1.2100E-02 -1.2100E-02 -2.0000E-03 -6.0000E-02
13   1.1990E+00 -2.1400E-01 -1.0740E+00  6.8000E-02 -4.1700E-01  1.6200E-01  1.8300E-01 -2.0000E-03 -0.          6.0000E-03
     2.0000E-03  3.0000E-03 -3.5600E-01 -2.1000E-02 -5.9900E-01 -1.2100E-02 -1.2100E-02  3.0000E-03  5.7000E-02
14  -0.         -0.         -1.4460E+00 -1.6700E-01 -1.0260E+00  1.2730E+00  6.4000E-01  4.7000E-02 -0.         -1.2000E-02
    -0.          4.0000E-03 -3.5600E-01 -2.1000E-02 -5.9900E-01 -1.2100E-02 -1.2100E-02 -1.0000E-03  3.9500E-01
15  -0.         -0.         -1.1990E+00 -2.7300E-01 -5.6100E-01 -2.2600E-01  3.3800E-01  8.5000E-02 -0.         -1.8000E-02
     1.0000E-03 -4.0000E-03 -3.5600E-01 -2.1000E-02 -5.9900E-01 -1.2100E-02 -1.2100E-02 -0.          5.1700E-01
16   0.         -1.0000E+00  0.          0.          0.          1.0000E+00  0.          0.          0.          0.
     0.          0.          0.          0.          0.          0.          0.          0.          0.
17   0.         -1.0000E+00  0.          0.          1.0000E+00  0.          0.          0.          0.          0.
     0.          0.          0.          0.          0.          0.          0.          0.          0.
18   0.          0.         -1.0000E+00  0.          0.          1.0000E+00  0.          0.          0.          0.
     0.          0.          0.          0.          0.          0.         -0.          0.
19   1.0000E+00  0.         -1.0000E+00  0.          0.          0.          0.          0.          0.          0.
     0.          0.          0.          0.          0.          0.          0.          0.          0.
20   0.          0.         -1.0000E+00  0.          1.0000E+00  0.          0.          0.          0.          0.
     0.          0.          0.          0.          0.          0.          0.          0.          0.
21  -1.0000E+00  0.          0.          0.          1.0000E+00  0.          0.          0.          0.          0.
     0.          0.          0.          0.          0.          0.          0.          0.          0.
22  -1.0000E+00  0.          0.          0.          1.0000E+00  0.          0.          0.          0.          0.
     0.          0.          0.          0.          0.          0.          0.          0.          0.
23  -1.0000E+00  0.          0.          0.          0.          1.0000E+00  0.          0.          0.          0.
     0.          0.          0.          0.          0.          0.          0.          0.          0.
24  -1.0000E+00  0.          0.          0.          1.0000E+00  0.          0.          0.          0.          0.
     0.          0.          0.          0.          0.          0.          0.          0.
```

examined separately.[48] The uncertainties ascribed to differential data in Table III assume that cross-section errors are correlated at all energies. Sensitivities presented by Bohn are listed in Table IV. Calculated results shown in Tables I through III used ENDF/B-III as the evaluated data base as processed into multigroup form by SDX. Values of differential and integral parameters adjusted by ALVIN also are listed in Tables II and III and show physically plausible trends. Values selected by Bohn from six trial sets of differential quantities also are listed. Our data adjustments are intended to be illustrative only of the techniques involved and, particularly, illustrative of consistency inferences. More detailed study of data uncertainties and sensitivities would be required to justify an adjusted data set for nuclear design application.

Input for this sample problem is shown in Appendix C. The output, shown in Appendix D, provides considerably more information, particularly consistency information, than do Tables II through IV. Values of chi-squared before and after adjustment both are improbably large, 503 and 91, respectively, for 24 degrees of freedom. The dispersion multiplier DISPR is 3.8 for this problem, thus accounting for the fact that errors in the adjusted data in Tables II and III are larger than input evaluated errors in many cases. Contributions of the 43 differential and integral parameters to chi-squared are listed before and after adjustment.

This sample problem is not fully illustrative of the capabilities of DAFT3 in that many more than 19 differential data could be treated by DAFT3. However, the sample problem is useful for code validation in that, with only 24 + 19 parameters, data adjustment can be carried out by DAFT2 as well as by DAFT3. To facilitate this comparison, inputs to DAFT2 and DAFT3 are made similar when only standard error information is provided. DAFT2 and DAFT3 results agree for this problem, thus validating the consistency and data adjustment part of the code.

B. Sensitivity Calculation

Illustrative of sensitivity calculations in ALVIN is a spherical representation of a thick iron shield-collimator.[49] The shield consists of a 70-cm-radius sphere of iron with a 4-cm-radius void at the center. An isotropic neutron source is uniformly distributed in a central 1-cm-radius sphere with the 0° spectrum of neutrons produced by 50-MeV deuterons on beryllium.[50] The response quantity consists of the product of the neutron flux and neutron fluence-to-dose equivalent conversion factor summed over all neutron energy groups and averaged over the volume of the 1-cm-thick air shell at the outer surface of the sphere.

The S_N transport code DTF[51] was used to calculate the neutron fluxes and adjoint fluxes throughout the shield, using 41-group, P-5 cross sections and S-16 quadrature. The source for the adjoint calculation, located in the 1-cm-thick shell surrounding the sphere, was the vector of neutron fluence-to-dose equivalent conversion factor.

Directed fluxes and adjoint fluxes were read in to ALVIN through REDFLX and converted to Legendre components. Nuclear data were read in through CROSEC and CROSEF. Fluxes and nuclear data were used by SENSI to compute detailed sensitivities as described in Sec. III-A. These were then used by PROFIL to compute sensitivity profiles as described in Sec. III-C. The computed sensitivity profile as a function of energy group is illustrated in Appendix E and output is listed in Appendix F.

All routines in ALVIN that are concerned with sensitivity are used in calculation of the sensitivity profile, and can be validated by a direct calculation. The sensitivity profile P(IG) represents the change in neutron dose-equivalent rate at the shield surface resulting from a change in cross section in group IG combined with proportionate changes in transfer cross sections from group IG to other groups. The ALVIN calculation uses first-order perturbation theory; this approximation, as well as code accuracy, can be validated by comparison with direct calculations.

The direct approach to determine the change in the result due to a change in the cross-section data involves the creation of an altered cross-section set, performing a neutron transport calculation using the altered cross sections, and converting the fluxes in the outer shell to the neutron dose equivalent rate. The fractional change in the dose equivalent rate divided by the fractional change in the cross sections of group IG thus yields the sensitivity P(IG) of the result to cross sections in group IG.

Sixteen separate, altered cross-section sets were formed with $\Sigma_T(IG)$ and $\Sigma^j(IG \rightarrow IG')$, $IG' = IG,NG$ for all j increased by 0.1, 0.5, 1.0 or 10.0% for
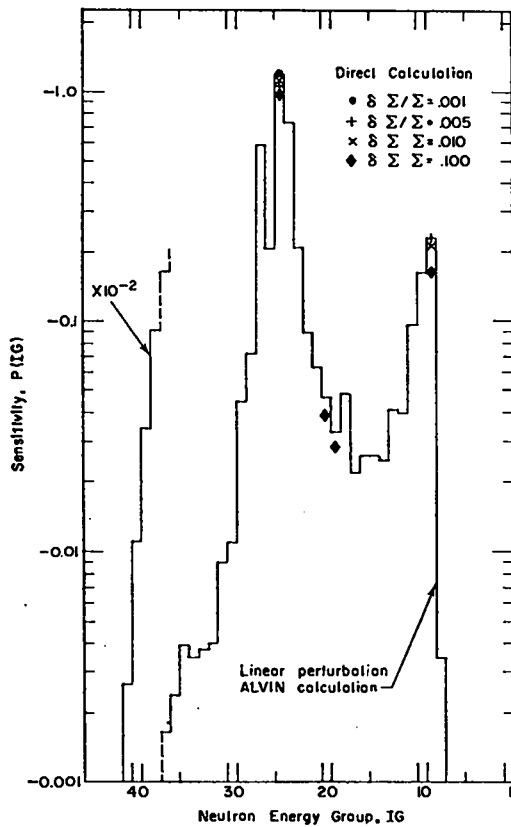
Fig. 2. Sensitivity of shield surface neutron dose-equivalent rate to neutron cross-section data computed by direct change in cross sections and by linear perturbation -- ALVIN calculation.

group IG = 9, 19, 20, or 25. The sensitivities resulting from the neutron transport calculation are superimposed on the sensitivity profile histogram of Fig. 2. The separate DTF calculations were converged to 10^{-4}, so the only directly calculated sensitivities shown are those for which the relative change in dose equivalent exceeded 10^{-4}. For example, the dose equivalent changed from 0.7189×10^{-13} rem/s to 0.7188×10^{-13}, 0.7187×10^{-13}, and 0.7186×10^{-13} rem/s, when the cross sections of group 19 were increased by 0.1, 0.5, and 1.0%, respectively. Therefore, these cases were disregarded.

In addition to accuracy problems in the direct calculation, there are nonlinearity problems in the perturbation calculation. Consider monoenergetic neutron penetration through a slab of thickness X with cross section Σ. Then,

$$P_{direct} = -[1- \frac{1}{2!} \, X\Sigma \, \frac{\delta\Sigma}{\Sigma} + \frac{1}{3!} \, (X\Sigma \, \frac{\delta\Sigma}{\Sigma})^2 - ...]X\Sigma \ . \quad (76)$$

If $\delta\Sigma/\Sigma$ is chosen to be 0.1 to insure accuracy in the dose equivalent change, then for the shield thickness studied here (about 12 mean free paths thick) the direct calculation underpredicts the result for very small $\delta\Sigma/\Sigma$ by about one-third according to Eq. (75). An underprediction of this order of magnitude can be observed in Fig. 2 for the direct calculation with $\delta\Sigma/\Sigma$ equal to 0.1. Conversely one can conclude from these results that linear perturbation theory will overpredict the change in dose equivalent by about one-third when fractional changes of 10% in cross section are considered.

Useful direct calculations for the purpose of validating ALVIN's linear perturbation calculations should have $\delta\Sigma/\Sigma$ sufficiently small to avoid the nonlinear effects discussed above and sufficiently large to avoid inaccuracy problems. The results shown in Fig. 2 are in good agreement and are believed to validate the inhomogeneous sensitivity parts of ALVIN. Calculations of sensitivities for Rossi-α and reactivity utilize inhomogeneous sensitivities and have not been validated separately.

VI. INPUT AND OUTPUT

Input requirements are shown in Table V. Formats and precise variable descriptions are given in terms of the variable definitions listed in Appendix B.

Outputs are labeled also by variable name and follow the examples shown in Appendixes D and F.

VII. SUMMARY OF ALVIN CAPABILITIES AND LIMITATIONS

ALVIN carries out sensitivity calculations for steady-state or time-dependent inhomogeneous transport. For eigenproblems, ALVIN computes sensitivities of eigenvalues to nuclear data changes (specifically reactivity and Rossi-α), but does not compute sensitivities of eigenfunction properties such as reaction rate ratios. Sensitivities are computed with respect to total cross section, with respect to individual Legendre components of group-to-group transfer cross sections, and with respect to fission parameters. Sensitivity profiles are computed.

ALVIN carries out data consistency and adjustment calculations for arbitrary variances and covariances among the differential and integral data. DAFT2 is used for these calculations and the number of parameters treated is limited by the necessity for in-

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TABLE V

ALVIN PROBLEM INPUT

(KSENS=1, KADJST=3, KVAR=2)

Order	Format	Contents
1	16A5	Title
2	12I6	KSENS,KADJST,MI,MJ
3	16A5	Title
4	6E12.6	DYDX(I,J),J=1,MJ A card set for each I=1,MI.
5	16A5	Title
6	6E12.6	YC(I),I=1,MI
7	16A5	Title
8	12I6	KVAR
9	6E12.6	VE(K1,K2),K2=1,MK A card set for each K1=1,MK.

version of large matrices. The DAFT3 subroutine carries out data consistency calculations and adjusts an arbitrarily large and correlated differential base uncorrelated with integral data. Least squares techniques are employed throughout. Limits on data adjustments are not used in ALVIN.

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The authors are grateful to R. J. LaBauve, D. W. Muir, M. Becker, and M. G. Stamatelatos for many pleasant and fruitful discussions on various aspects of this work. We also are grateful to E. M. Bohn for advice on data for the ZPR sample problem and to J. Y. Barré, J. B. Smathers, and W. E. Davey for useful discussions on the methodology.

REFERENCES

1. B. M. O. Ballance et al., "The Optimization of Neutron Cross-Section Data Adjustments to Give Agreement with Spectrum Measurements at the Center of Critical Systems," Physics of Fast Reactor Operation and Design, Intern. Conf. at the Inst. of Civil Engineers, London, 1969, Proc. BNES, London (1969), p.149.

2. K. O. Ott, R. B. Pond, J. M. Kallfelz, "A Study of Methods of Cross-Section Error Identification Utilizing Integral Data from Fast Critical Assemblies," Argonne National Laboratory report ZPR-TM-87 (1972).

3. J. Barré, M. Heindler, T. Lacapelle, and J. Ravier, "Lessons Drawn from Integral Experiments on a Set of Multigroup Cross Sections," Proc. BNES Conf., London (1969), p. 165.

4. G. G. Campbell and J. L. Rowlands, "The Relationship of Microscopic and Integral Data," Nuclear Data for Reactors 2, Proc. Helsinki Conf., 1970, Vol. 2, IAEA, Vienna (1970), p. 391.

5. J. P. Chaudat, J. Y. Barré, and A. Khairallah, "Improvements of the Predicted Characteristics for Fast Power Reactors from Integral Experiments, Cadarache Version III Multigroup Cross Section Set," Proc. Symp. Physics of Fast Reactors, Tokyo (1973).

6. A. Gandini, M. Petilli, M. Salvatores, "Nuclear Data and Integral Measurements Correlation for Fast Reactors. Statistical Formulation and Analysis of Method. The Consistent Approach," Proc. Symp. Physics of Fast Reactors, Tokyo (1973).

7. G. Cecchini, A. Gandini, I. Dal Bono, and B. Faleschine, "Comparison Between Experimental and Theoretical Integral Data on Fast Critical Facilities, CALI, A Program for Generating Effective Nuclear Group Constants by a Correlation Method," Argonne National Laboratory report ANL-7320 (1966), p. 107.

8. J. B. Dragt, "Statistical Considerations on Techniques for Adjustment of Differential Cross Sections with Measured Integral Parameters," Reactor Centrum Nederland, Petten, RCN-122 (1970), pp. 85-105.

9. E. T. Y. Chow, "An Investigation of Methods for Neutron Cross-Section Error Identification Utilizing Integral Data," Ph.D. Dissertation, Georgia Institute of Technology (1974).

10. H. Häggblom, "Adjustment of Neutron Cross Section Data by a Least Square Fit of Calculated Quantities to Experimental Results. Part 1, Theory," AE-422, Aktiebolaget Atomenergi, Studsvik, Sweden (1971).

11. H. Häggblom, "Adjustment of Neutron Cross Section Data by a Least Square Fit of Calculated Quantities to Experimental Results. Part 2, Theory," AE-439, Aktiebolaget Atomenergi, Studsvik, Sweden (1971).

12. H. Häggblom, "Adjustments of Neutron Data from ENDF/B-III by Using Integral Data in a Least-Square Fit," Aktiebolaget Atomenergi, Studsvik, Sweden, AE-RF-74-3037 (1974).

13. P. C. E. Hemment and E. D. Pendlebury, "The Optimization of Neutron Cross Section Adjustments to Give Agreement with Experiment," Argonne National Laboratory report ANL-7320 (1966), p. 88.

14. H. Kuroi et al., "Correlation of Integral Data and Cross Sections in Intermediate Energy Cores," Physics of Fast Reactor Operation and Design, Intern. Conf. at the Inst. of Civil Engineers, London, 1969, Proc. BNES, London (1969), p.82.

15. H. Mitani and H. Kuroi, "Adjustment of Group Cross Sections by Means of Integral Data, (I) Theoretical Study," J. Nucl. Sci. Technol. 9, 383 (1972).

16. H. Mitani and H. Kuroi, "Adjustment of Group Cross Sections by Means of Integral Data, (II) Numerical Study," J. Nucl. Sci. Technol. 9, 642 (1972).

17. T. Moorhead, "The Effects of Errors in Cross-Section Data on the Calculations for a Large Dilute Fast Reactor," International Atomic Energy Seminar on the Physics of Fast and Intermediate Reactors, Vienna (1961).

18. A Pazy, G. Rakavy, and Y. Reiss, "Use of Integral Measurements as Supplementary Data in Neutron Cross Section Evaluation," Argonne National Laboratory report ANL-7320 (1966), p. 270.

19. A. Pazy, G. Rakavy, Y. Reiss, J. J. Wagschal, A. Ya'ari, and T. Yeivin, "The Role of Integral Data in Neutron Cross Section Evaluation," Nuc. Sci. Eng. 55, 280 (1974).

20. E. D. Pendlebury, "On the Optimization of Neutron Cross Section Data Adjustments to Give Agreement with the Results of Integral Experiments," Atomic Weapons Research Establishment, Aldermaston, report AWREO-92/64 (1964).

21. J. L. Rowlands and J. D. MacDougall, "The Use of Integral Measurements to Adjust Cross-Sections and Predict Reactor Properties," Physics of Fast Reactor Operation and Design, Intern. Conf. at the Inst. of Civil Engineers, London, 1969, Proc. BNES, London (1969), p. 180.

22. M. Salvatores, "Adjustment of Multigroup Neutron Cross Sections by A Correlation Method," Nuc. Sci. Eng. 50, 345 (1973).

23. L. N. Usachev and Yu. G. Bobkov, "Planning an Optimum Set of Microscopic Experiments and Evaluations to Obtain a Given Accuracy in Reactor Parameter Calculations," International Atomic Energy Agency, Vienna, INDC(CCP)-19/U (1972).

24. L. N. Usachev and Yi. G. Bobkov, "Determination of Required Accuracy of Nuclear Data," International Atomic Energy Agency, Vienna, INDC(CCP)-33/L, (1973).

25. P. H. West, "The Application of Linear Programming to the Optimum Adjustment of Group Cross-Section Data," private communication (1972).

26. P. H. West, "PENICUIK3HS, A Program to Calculate Optimum Adjustments to Neutron Group Cross Section Data Using Results of Integral Experiments," Atomic Weapons Research Establishment, Aldermaston report AWREA 34/73 (1973).

27. D. R. Harris, "Consistency Among Differential Nuclear Data and Integral Observations." Trans. Am. Nuc. Soc. 18, 340 (1974).

28. D. R. Harris, "DAFT1--A FORTRAN Program for Least Square Fitting of 0.0254 eV Neutron Data for Fissile Nuclides," Westinghouse Electric Corporation report WAPD-TM-761 (1968).

29. W. C. Hamilton, Statistics in Physical Science, Estimation, Hypothesis, Testing, and Least Squares (Ronald Press Company, New York, 1964).

30. D. R. Harris, P. G. Young, and G. M. Hale, "Characterization of Uncertainties in Evaluated Cross Section Sets," Trans. Am. Nuc. Soc. 16, 323 (1973).

31. R. N. Hwang, "Characteristics of Uncertainties in Sensitivity Studies," Argonne National Laboratory report FRA-TM-63 (1974).

32. H. O. Hartley, "The Modified Gauss-Newton Method for the Fitting of Non-Linear Regression Functions by Least Squares," Technometrics 3, 269 (1961).

33. Yu. V. Linnik, Method of Least Squares and Principles of the Theory of Observations (Pergamon Press, New York 1961).

34. G. E. Hansen and H. A. Sandmeier, "The Effect of Basic Neutron Reaction Cross Sections of Nitrogen (n,n'), (n,2n), (n,γ), (n,p), and (n,α) on High Energy Neutron Transport in Air," Los Alamos Scientific Laboratory report LA-3810 (1967).

35. P. C. E. Hemment, E. D. Pendlebury, M. J. Adams, B. A. Brett, and D. Sams, "The Multigroup Neutron Transport Perturbation Program DUNDEE," Atomic Weapons Research Establishment, Aldermaston report AWRE O 040/66 (1966).

36. P. C. E. Hemment, E. D. Pendlebury, P. H. West, and W. H. G. Wilson, "The Multigroup Neutron Diffusion Theory Perturbation Program SPECTRE," Atomic Weapons Research Establishment, Aldermaston report AWRE O 074/70 (1970).

37. E. M. Oblow, "General Sensitivity Theory for Radiation Transport," Oak Ridge National Laboratory report ORNL-TM-4110 (1973).

38. D. L. Prezbindowski and H. A. Sandmeier, "The Effects of Cross Section Inaccuracies on Integral Data," Trans. Am. Nuc. Soc. 11, 193 (1968).

39. W. E. Preeg, "Flux Sensitivity to Cross-Section Data for Iron and Oxygen," Doctoral Thesis, Columbia University (1970).

40. E. A. Straker, "Sensitivity of Neutron Transport in Oxygen to Various Cross-Section Sets," Oak Ridge National Laboratory report ORNL-TM-2552 (1968).

41. P. H. West, P. C. E. Hemment, and W. H. G. Wilson, "The Multigroup Neutron Diffusion Theory Perturbation Program JINX," Weapons Research Establishment, Aldermaston, report AWRE O 040/71 (1971).

42. P. H. West, "A Description of New Capabilities of the Multigroup Neutron Transport Perturbation Theory Programs DUNDEE, SPECTRE, and JINX," Atomic Weapons Research Establishment, Aldermaston, report AWRE O 36/73 (1973).

43. D. E. Bartine, E. Oblow, and F. B. Mynatt, "Neutron Cross-Section Sensitivity Analysis: A General Approach Illustrated for a Na-Fe System," Oak Ridge National Laboratory report ORNL-TM-3944 (1972).

44. L. N. Usachev, "Perturbation Theory for the Breeding Ratio and for Other Number Ratios Pertaining to Various Reactor Processes," J. Nuc. Energy A/B 18, 571 (1964).

45. D. E. Bartine, E. R. Mynatt, and E. M. Oblow, "SWANLAKE, A Computer Code Utilizing ANISN Radiation Transport Calculations for Cross Section Sensitivity Analysis," Oak Ridge National Laboratory report ORNL-TM-3809 (1973).

46. G. M. Hale, D. R. Harris, and R. E. MacFarlane, Eds., "Applied Nuclear Data Research and Development Quarterly Progress Report, October 1 through December 31, 1974," Los Alamos Laboratory report LA-5944-PR (1975).

47. E. M. Bohn, "The Central Worth Discrepancy in Three Fast Reactor Benchmark Critical Assemblies," Argonne National Laboratory report ZPR-TM-185 (1974).

48. S. Ramchandran, "New LASL Delayed Neutron Parameters and the Impact on FTR Nuclear Design," FRP-771, private communication (1972), Internal FFTF Project Correspondence, March 14, 1972.

49. G. M. Hale, D. R. Harris, and R. E. MacFarlane, Eds., "Applied Nuclear Data Research and Development Quarterly Progress Report, January 1 through March 31, 1974," Los Alamos Scientific Laboratory report LA-5655-PR (1974).

50. J. P. Meulders, University of Louvain, private communication (1974).

51. K. D. Lathrop, "DTF-IV, A FORTRAN-IV Program for Solving the Multigroup Transport Equation with Anisotropic Scattering," Los Alamos Scientific Laboratory report LA-3373 (1965).

APPENDIX A

PROGRAM ABSTRACT

1. <u>Name of program</u>: ALVIN.

2. <u>Name of computer for which program is designed</u>: CDC-7600.

   <u>Programming language</u>: FORTRAN-IV.

3. <u>Nature of the problem solved</u>: ALVIN analyzes the consistency of a set of differential and integral nuclear data, adjusts the differential nuclear data to improve agreement with integral observations, and identifies inconsistent data. ALVIN also computes required sensitivities and related quantities such as sensitivity profiles.

4. <u>Method of solution</u>: Linear perturbation theory is used for sensitivity calculations. Data consistency and adjustment computations use least squares techniques.

5. <u>Restrictions on the complexity of the problem</u>: The DAFT2 consistency and adjustment subroutine treats fully or partially correlated differential and integral parameters, but only as many as the order of the largest matrix that can be inverted. The DAFT3 consistency and adjustment subroutine treats arbitrarily large differential data sets, but only if they are uncorrelated with the integral data.

6. <u>Related and auxiliary programs</u>: None.

7. <u>Typical running times</u>: About 1 min, depending on size of problem.

8. <u>Description</u>: Equations, calculational methods, input and output are described in Ref. 1.

9. <u>Unusual features</u>: Data set consistency is analyzed. A special algorithm is used in DAFT3 to treat arbitrarily large data sets.

10. <u>Status</u>: The program is currently in use and can be obtained from the Argonne Code Center.

11. <u>Machine requirements</u>: Distributed version uses LCM to store five large arrays. Otherwise machine-independent.

12. <u>Operating system</u>: Independent.

13. <u>Other programming information</u>: None.

14. <u>References</u>:

    (1) D. R. Harris, W. A. Reupke, and W. B. Wilson, "Consistency Among Differential Nuclear Data and Integral Observations -- the ALVIN Code for Data Adjustment, for Sensitivity Calculations, and for Identification of Inconsistent Data," Los Alamos Scientific Laboratory report LA-5987 (1975).

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APPENDIX B

DEFINITIONS OF VARIABLES USED IN ALVIN

DEFINITIONS
    A(I1,I2)=TEMPORARY MATRIX USED WITH DIFFERENT DEFINITIONS IN DIFFERENT
        SUBROUTINES
    B(J)=VECTOR SOURCE TO REARRANGED NORMAL EQUATIONS
    C(J1,J2)=MATRIX WHOSE INVERSE IS THE ADJUSTED VARIANCE-COVARIANCE MATRIX
        OF INDEPENDENT QUANTITIES. LATER THE INVERSE IS STORED IN C
    CHI2F=CHI-SQUARED AFTER ADJUSTMENT USING EVALUATED SECOND MOMENTS
    CHI2I=CHI-SQUARED INITIAL
    CHIDA=DIAGONAL PART OF CHI2A
    CHIDF=DIAGONAL PART OF CHI2F
    CHIDI=DIAGONAL PART OF CHI2I
    DENOMA=DENOMINATOR OF D(ALPHA)/D(CROSS SECTION) RELATION
    DENOMR=DENOMINATOR OF D(REACTIVITY)/D(CROSS SECTION) RELATION
    DENS(IR)=NUMBER DENSITY IN REGION IR
    DETERM=DETERMINANT OF A MATRIX
    DISPR=DISPERSION MULTIPLIER
    DYDG(IG)=DERIVATIVE OF INTEGRAL QUANTITY WITH RESPECT TO TOTAL CROSS
        SECTION IN MULTIGROUP IG
    DYDL(IL,IG1,IG)=DERIVATIVE OF INTEGRAL QUANTITY WITH RESPECT TO IL-1 TH
        LEGENDRE ORDER CROSS SECTION FOR TRANSFER FROM MULTIGROUP IG1 TO
        MULTIGROUP IG
    DYDX(I,J)=DERIVATIVE OF INTEGRAL QUANTITY I WITH RESPECT TO DIFFERENTIAL
        QUANTITY J COMPUTED AT THE EVALUATED DATA POINT
    FLAXL(IM,IL,IG)=IL-1 TH LEGENDRE COMPONENT OF ADJOINT FLUX AVERAGED OVER
        SPACE MESH INTERVAL IM, IN ENERGY GROUP IG
    FLUXL(IM,IL,IG)=IL-1 TH LEGENDRE COMPONENT OF FLUX AVERAGED OVER SPACE
        MESH INTERVAL IM, IN ENERGY GROUP IG
    IA=QUADRATURE ANGLE INDEX
    IDG(ISAVE)=IG INDEX OF SAVED SENSITIVITY CASE
    IDG1(ISAVE)=IG1 INDEX OF SAVED SENSITIVITY CASE
    IDL(ISAVE)=IL INDEX OF SAVED SENSITIVITY CASE
    IDX(ISAVE)=IX INDEX OF SAVED SENSITIVITY CASE
    IDY(ISAVE)=IY INDEX OF SAVED SENSITIVITY CASE
    IG=MULTIGROUP INDEX
    IL=LEGENDRE ORDER INDEX
    IM=SPATIAL MESH INDEX
    IMMAX(IR)=MESH POINT WITH HIGHEST INDEX IN REGION IR
    IMMIN(IR)=MESH POINT WITH LOWEST INDEX IN REGION IR
    INDEX(I,J)=TEMPORARY ARRAY USED IN MATRIX INVERSION
    IPIVOT(I)=TEMPORARY ARRAY USED IN MATRIX INVERSION
    IPOS=POSITION OF CROSS SECTION IN OTF CROSS SECTION FORMAT
    IR=REGION INDEX
    IT=TIME INTERVAL INDEX
    ITAPE= FILE SET NUMBER (1 OR 2)
    ITYPE=TYPE OF SAVED SENSITIVITY CASE, =1 IF INHOMOGENEOUS
        SENSITIVITY, =2 IF D(ALPHA)/D(CROSS SECTION) SENSITIVITY,
        =3 IF D(REACTIVITY)/D(CROSS SECTION) SENSITIVITY
    IYR=INDEX OF CASE OF SENSITIVITIES CALCULATED FROM FLUXES AND
        ADJOINT FLUXES READ IN
    JTAPE=ASSIGNED TAPE NUMBER
    KADJST=2 IF DIFFERENTIAL DATA ARE TO BE ADJUSTED USING DAFT2, =3 IF
        DIFFERENTIAL DATA ARE TO BE ADJUSTED USING DAFT3, =1 OTHERWISE
    KFLUX=1 IF LEGENDRE FLUXES ARE READ IN, =2 IF DIRECTED FLUXES ARE READ IN
        FROM WHICH LEGENDRE FLUXES ARE COMPUTED FOR USE IN SENSITIVITY
        CALCULATIONS
    KTYPE=TYPE OF SENSITIVITY CALCULATION, =0 IF INHOMOGENEOUS
        SENSITIVITY, =1 IF D(EIGENVALUE)/D(CROSS SECTION)
    KGEOM=1 IF GEOMETRY IN SENSITIVITY COMPUTATIONS IS SLAB, =2 IF SPHERICAL,
        =3 IF CYLINDRICAL
    KRFLX=CONTROL SET IN CODE, =1 IF FLUXES ARE BEING PROCESSED,=2 IF ADJOINT
        FLUXES ARE BEING PROCESSED
    KSENS=1 IF SENSITIVITIES ARE ONLY READ IN, =2 IF SENSITIVITIES ARE ONLY
        COMPUTED FROM INPUT FLUXES AND ADJOINT FLUXES, =3 IF SENSITIVITIES
        ARE BOTH READ IN AND COMPUTED
    KTIMS=1 IF TIME DEPENDENT FLUXES ARE USED IN SENSITIVITY COMPUTATION
    KVAR=1 IF EVALUATED STANDARD ERRORS ONLY ARE READ IN FOR DIFFERENTIAL AND
        INTEGRAL PARAMETERS, =2 IF FULL EVALUATED VARIANCE-COVARIANCE ARRAY
        IS READ IN FOR DIFFERENTIAL -INTEGRAL PARAMETERS

MI=NUMBER OF INTEGRAL PARAMETERS
MJ=NUMBER OF DIFFERENTIAL PARAMETERS
MK=MI+MJ=NUMBER OF INTEGRAL-DIFFERENTIAL PARAMETERS
NA=NUMBER OF QUADRATURE ANGLES FOR DIRECTED FLUX
NG=NUMBER OF MULTIGROUPS
NL=NUMBER OF LEGENDRE FLUX ORDERS USED IN SENSITIVITY COMPUTATION
NLR=NUMBER OF LEGENDRE FLUX ORDERS READ IN
NLCX=NUMBER OF LEGENDRE CROSS SECTION ORDERS READ IN
NM=NUMBER OF MESH INTERVALS IN SENSITIVITY CALCULATION
NR=NUMBER OF REGIONS IN SENSITIVITY CALCULATIONS
NSAVE=NUMBER OF CASES OF COMPUTED SENSITIVITIES SAVED INTO DYDX
    (IY,IX) MATRIX FOR CASE IYR
NT=NUMBER OF TIME INTERVALS IN SENSITIVITY COMPUTATION
NXR=NUMBER OF IYR CASES FOR WHICH NEW SET OF UN-NORMALIZED
    FVALUATED DIFFERENTIAL PARAMETERS ARE TO BE READ IN AND USED
    FOR SENSITIVITY NORMALIZATION OR PROFILE CALCULATION
NYR=NUMBER OF CASES FOR WHICH FLUXES AND ADJOINT FLUXES ARE READ IN AND
    SENSITIVITIES COMPUTED
P(IG)=SENSITIVITY PROFILE FOR GROUP IG
PIVOT(I)=TEMPORARY ARRAY USED IN MATRIX INVERSION
QCOSA(IA)=COSINE OF QUADRATURE ANGLE NUMBER IA FOR DIRECTED FLUX
QLEGP(IL,IA)=LEGENDRE POLYNOMIAL OF ORDER IL-1 OF QCOSA(IA)
QWTSA(IA)=WEIGHT FOR QUADRATURE ANGLE NUMBER IA
R=RESPONSE, INTEGRATED OVER APPROPRIATE VOLUME OF PHASE SPACE,
    USED IN INHOMOGENEOUS SENSITIVITY NORMALIZATION AND PROFILE
    CALCULATION
RELXE=(XE(J)-XE(J))/SX(J)
RELXF=(XA(J)-XE(J))/SX(J)
RELYE=(YC(I)-YE(I))/SY(I)
RELYF=(YA(I)-YE(I))/SY(I)
RM(IM)=LEFT-HAND COORDINATE OF MESH INTERVAL IM
SIGFN(IG,IR)=MACROSCOPIC FISSION CROSS SECTION IN GROUP IG IN REGION IR
    TIMES FISSION NEUTRON YIELD
SIGTO(IG)=MICROSCOPIC TOTAL CROSS SECTION IN GROUP IG
SIGTR(IG,IG2,IL)=MICROSCOPIC IL-1 TH LEGENDRE COMPONENT CROSS SECTION FOR
    TRANSFER FORM GROUP IG TO GROUP IG2
SORF(IG2)=NORMALIZED FISSION NEUTRON SPECTRUM INTO GROUP IG2
SPDS(IG)=PARTICLE SPEEDS IN GROUP IG
SX(J)=EVALUATED STANDARD ERROR FOR DIFFERENTIAL PARAMETER J
SXA(J)=STANDARD ERROR OF DIFFERENTIAL PARAMETER J AFTER ADJUSTMENT
SY(I)=EVALUATED STANDARD ERROR FOR INTEGRAL PARAMETER I
SYA(I)=STANDARD ERROR OF INTEGRAL PARAMETER I AFTER ADJUSTMENT
TEMSTO(IG,IPOS)=TEMSTO(IM,IA)=TEMPORARY STORAGE FOR CROSS SECTIONS AND FOR
    DIRECTED FLUXES AND ADJOINT FLUXES
TITLE=DESCRIPTIVE TITLE FOR JOB AND FOR SUBSECTIONS OF INPUT
TM(IT)=LOWER BOUND OF TIME INTERVAL IT
VE(K1,K2)=EVALUATED VARIANCE-COVARIANCE VALUE FOR INTEGRAL-DIFFERENTIAL
    PARAMETERS K1 AND K2. LATER THE INVERSE, IE, WEIGHT MATRIX, IS
    STORED IN VE
VZ(K1,K2)=COMPUTED VARIANCE-COVARIANCE VALUE FOR INTEGRAL-DIFFERENTIAL
    PARAMETERS K1 AND K2 AFTER ADJUSTMENT. INCLUDES DISPERSION FACTOR
    LATER THE ADJUSTED CORRELATION COEFFICIENT MATRIX IS STORED
    IN VZ.
XA(J)=ADJUSTED VALUE OF DIFFERENTIAL PARAMETER J
XE(J)=EVALUATED VALUE OF DIFFERENTIAL PARAMETER J
YA(I)=ADJUSTED VALUE OF INTEGRAL PARAMETER I
YC(I)=VALUE OF INTEGRAL PARAMETER I COMPUTED FOR EVALUATED VALUES OF
    DIFFERENTIAL PARAMETERS
YCU(ISAVE)=VALUE OF INTEGRAL PARAMETER, UN-NORMALIZED, COMPUTED
    FROM EVALUATED DIFFERENTIAL PARAMETERS, USED TO NORMALIZE
    SAVED SENSITIVITY CASE ISAVE. YCU=R IF ITYPE=1,
    =EIGENVALUE IF ITYPE=2,3
YE(I)=EVALUATED VALUE OF INTEGRAL PARAMETER I
ZA(K)=ADJUSTED VALUE OF INTEGRAL-DIFFERENTIAL PARAMETER
ZC(K)=VALUE OF INTEGRAL-DIFFERENTIAL PARAMETER K COMPUTED FOR EVALUATED
    VALUES OF DIFFERENTIAL PARAMETERS
ZE(K)=EVALUATED VALUE OF INTEGRAL-DIFFERENTIAL PARAMETER K

SAMPLE INPUT FOR DATA CONSISTENCY AND ADJUSTMENT PROBLEM

```
SAMPLE PROBLEM--CONSISTENCY AND ADJUSTMENT OF ZPR DATA
          1      2     24     19
SENSITIVITIES FOR ZPR PROBLEM
.61              -.1                             .08        -.27
-.03


                        .66       -.07          .08        -.26
-.06


                        .66       -.06          .09        -.21
-.07


-.991        .049      1.160      -.161         -.239      -.255
.047         -.045     -.115       .002         -.006      -.003
-.144        -.056                                         -.005

.264         -.174                              -.206       .210
.088         -.024     -.027       .010          .002       .005
-.144        -.056                                          .004

-1.345       -.094                              -.583      1.225
.346          .043      .148      -.014          .002      -.002
-.144        -.056                                          .002

-1.252       -.143                              -.329      -.092
.208          .098      .213      -.020          .004      -.003
-.144        -.856                                          .002

                        .213      -.083         -.417       .199
.111         -.027     -.052       .003         -.003      -.002
-.326        -.0252    -.597      -.0206        -.0312      -.001

1.179        -.190     -1.101      .050         -.391       .137
.150         -.004      .035       .009          .003       .004
-.326        -.0252    -.597      -.0206        -.0312       .006

                       -1.207     -.121         -.778      1.045
.490          .064      .192      -.020          .003      -.003
-.326        -.0252    -.597      -.0206        -.0312       .001

                       -1.129     -.216         -.509      -.260
.283          .149      .316      -.033          .004      -.004
-.326        -.0252    -.597      -.0206        -.0312       .002

                        .247      -.069         -.465       .211
.144         -.009                 .001         -.003      -.002
-.356        -.0210    -.599      -.0121        -.0121      -.002
-.060
1.199        -.214     -1.074      .068         -.417       .162
.183         -.002                 .006          .002       .003
-.356        -.0210    -.599      -.0121        -.0121       .003
.057
                       -1.446     -.167         -1.026     1.273
.640          .047                -.012                    -.004
-.356        -.0210    -.599      -.0121        -.0121      -.001
.395
                       -1.199     -.273         -.561      -.226
.338          .005                -.018          .001      -.004
```

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```
-.356        -.0210       -.599        -.0121       -.0121
.517
INT PARAMETERS COMPUTED FROM EVALUATED DIFF PARAMETERS
.9920        .9924        .9927        1.10         1.15         1.24
.92          1.25         1.24         1.16         1.18         1.25
1.26         1.27         1.09         .90          1.03         .99
1.05         1.09         .94          1.04         .96          .94
EVALUATED STANDARD ERRORS
         1
.1           .1           .1           .1           .15          .1
.15          .1           .1           .1           .1           .1
.06          .04          .06          .1           .1           .1
.1
.004         .004         .004         .10          .10          .11
.15          .11          .11          .10          .11          .11
.11          .11          .11          .03          .03          .02
.02          .02          .02          .02          .05          .05
```

APPENDIX D

SAMPLE OUTPUT FOR DATA CONSISTENCY AND ADJUSTMENT PROBLEM

SAMPLE PROBLEM--CONSISTENCY AND ADJUSTMENT OF ZPR DATA

        KSENS   KADJST   MI   MJ
          1   2   24   19

          XE(J),J=1,MJ
 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01
 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01

          YE(I),I=1,MI
 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01
 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01 .100000E+01
 .100000E+01 .100000E+01 .100000E+01 .100000E+01

INT PARAMETERS COMPUTED FROM EVALUATED DIFF PARAMETERS

          YC(I),I=1,MI
 .992000E+00 .992400E+00 .992700E+00 .110000E+01 .115000E+01 .124000E+01 .920000E+00 .125000E+01 .124000E+01 .116000E+01
 .118000E+01 .125000E+01 .126000E+01 .127000E+01 .109000E+01 .900000E+00 .103000E+01 .990000E+00 .105000E+01 .109000E+01
 .940000E+00 .104000E+01 .960000E+00 .940000E+00

EVALUATED STANDARD ERRORS

         KVAR
          1

          SX(J),J=1,MJ
 .100000E+00 .100000E+00 .100000E+00 .100000E+00 .150000E+00 .100000E+00 .150000E+00 .100000E+00 .100000E+00 .100000E+00
 .100000E+00 .100000E+00 .600000E-01 .400000E-01 .600000E-01 .100000E+00 .100000E+00 .100000E+00 .100000E+00

          SY(I),I=1,MI
 .400000E-02 .400000E-02 .400000E-02 .250000E-01 .250000E-01 .350000E-01 .750000E-01 .350000E-01 .350000E-01 .250000E-01
 .350000E-01 .350000E-01 .350000E-01 .350000E-01 .350000E-01 .300000E-01 .300000E-01 .200000E-01 .200000E-01 .200000E-01
 .200000E-01 .200000E-01 .500000E-01 .500000E-01

    CHI2I   REDUCED CHI2I
 .502816E+03 .209507E+02

    CHI2F   REDUCED CHI2F   DISPR
 .909101E+02 .378792E+01 .378792E+01

J1 C(J1,J2),J2=1,M.1 ADJUSTED VARIANCE-COVARIANCE MATRIX FOR DIFFERENTIAL PARAMETERS

```
 1  .39300E-03  .62006E-03  .25074E-03  .23897E-03  .34804E-03  .41226E-03  .91392E-03  .36280E-03  .58895E-04 -.96503E-04
    -.29095E-04 -.35524E-04  .66727E-04 -.75741E-04  .15369E-03  .46379E-05 -.30904E-06 -.321A6E-04  .13262E-03

 2  .60000E-03  .23893E-02  .36940E-03  .94504E-03  .74244E-03  .62785E-03  .95461E-03  .62256E-03  .48116E-03 -.13553E-03
    -.15532E-04 -.32287E-04 -.18038E-03 -.35889E-03 -.92722E-04 -.132A3E-04 -.23313E-04 -.18213E-04  .56812E-03

 3  .25074E-03  .36940E-03  .40240E-03  .14041E-02  .30219E-03  .29220E-03  .14274E-02  .46053E-03  .19556E-03 -.87850E-04
    -.11042E-04 -.17145E-04  .53326E-04  .73756E-04  .61723E-05 -.17140E-04 -.30862E-04 -.14335E-04  .390A1E-03

 4  .23897E-03  .94504E-03  .14041E-02  .15014E-01  .66868E-03  .84932E-04  .14592E-02  .32224E-02  .37120E-02 -.11368E-02
    -.95899E-03 -.31761E-03 -.10744E-02 -.41659E-03 -.19206E-02 -.27923E-03 -.49203E-03 -.223A2E-03  .55436E-02

 5  .34804E-03  .74244E-03  .30219E-03  .66868E-03  .75934E-03  .40872E-03  .14645E-02  .38947E-03  .10306E-03 -.91098E-04
    -.23697E-04 -.29190E-04 -.48581E-04 -.10078E-03 -.39015E-04 -.12250E-04 -.24738E-04  .25140E-04  .25053E-03

 6  .41226E-03  .62785E-03  .29220E-03  .84932E-04  .40872E-03  .96087E-03 -.633A6E-03  .72330E-03  .35810E-03 -.17752E-03
    -.32056E-04 -.50774E-04 -.93753E-04 -.13063E-03 -.10175E-03 -.18212E-04 -.33731E-04 -.42966E-04  .68404E-03

 7  .91392E-03  .95461E-03  .14274E-02  .14592E-02  .14645E-02 -.633A6E-03  .17805E-01 -.12140E-02 -.416A6E-02  .79177E-03
    -.63610E-05  .24027E-03  .20775E-02  .14763E-02  .26456E-02  .17906E-03  .21705E-03  .12722E-03 -.46038E-02

 8  .36280E-03  .62256E-03  .46053E-03  .32224E-02  .38947E-03  .72330E-03 -.12140E-02  .34368E-01 -.74250E-02  .71742E-03
    -.17371E-03  .445A9E-04 -.40292E-03 -.64613E-03 -.26154E-03 -.11092E-04 -.66451E-05 -.95450E-04 -.13310E-02

 9  .58895E-04  .48116E-03  .19556E-03  .37120E-02  .10306E-03  .35810E-03 -.416A6E-02 -.74250E-02  .17801E-01  .14710E-02
    -.49185E-03 -.14881E-04 -.59056E-03 -.73158E-03 -.25421E-04  .23854E-03  .53637E-03 -.34969E-03  .5A5AAE-02

10 -.96503E-04 -.13553E-03 -.87850E-04 -.11368E-02 -.91098E-04 -.17752E-03  .79177E-03  .71742E-03  .14710E-02  .37640E-01
    -.19516E-05 -.60749E-04  .19130E-03  .16344E-03  .20452E-03  .41692E-05 -.48996E-05 -.31191E-04 -.53671E-04

11 -.29095E-04 -.15532E-04 -.11042E-04 -.95899E-04 -.23697E-04 -.32056E-04 -.63610E-05 -.17371E-03 -.49185E-03 -.19516E-05
     .37848E-01 -.23942E-04 -.85828E-05 -.34616E-05  .32114E-06  .60124E-05  .13452E-04 -.31805E-04 -.99540E-04

12 -.35524E-04 -.32287E-04 -.17145E-04 -.31761E-03 -.29190E-04 -.50774E-04  .24027E-03  .445A9E-04 -.14881E-04 -.60749E-04
    -.23942E-04  .37845E-01  .49446E-04  .68367E-04  .44081E-04  .43771E-05  .67399E-05 -.31668E-04 -.42454E-04

13  .66727E-04 -.18038E-03  .53326E-04 -.10744E-02 -.48581E-04 -.93753E-04  .20775E-02 -.40292E-03 -.59056E-03  .19130E-03
    -.85828E-05  .49446E-04  .10575E-01 -.12033E-02 -.45413E-02 -.30780E-03 -.37355E-03  .48250E-04 -.239A9E-03

14 -.75741E-04 -.35898E-03  .73756E-04 -.41659E-03 -.10078E-03 -.13063E-03  .14763E-02 -.64613E-03 -.73158E-03  .16344E-03
    -.34416E-05  .68367E-04 -.12033E-02  .13396E-02  .10203E-02  .70696E-04  .87386E-04  .5AA49E-04 -.82559E-03

15  .15369E-03 -.92722E-04  .61723E-05 -.19206E-02 -.39015E-04 -.10175E-03  .26456E-02 -.26154E-03 -.25421E-04  .20452E-03
     .32114E-06  .44081E-04 -.45413E-02  .10203E-02  .48199E-02 -.70393E-03 -.96359E-03  .70374E-04 -.71723E-03

16  .46379E-05 -.132A3E-04 -.17140E-04 -.27923E-03 -.12250E-04 -.18212E-04  .17906E-03 -.11092E-04  .23854E-03  .41692E-05
     .60124E-05  .43771E-05 -.30780E-03  .70696E-04 -.70393E-03  .37776E-01 -.18290E-03  .13916E-04 -.43763E-03

17 -.30904E-06 -.23313E-04 -.30862E-04 -.49203E-03 -.24738E-04 -.33731E-04  .21705E-03 -.66451E-05  .53637E-03 -.48996E-05
     .13452E-04  .67399E-05 -.37355E-03  .873A6E-04 -.96359E-03 -.18290E-03  .37536E-01  .262A8E-04 -.93A04E-03

18 -.321A6E-04 -.18213E-04 -.14335E-04 -.223A2E-03  .25140E-04 -.42966E-04  .12722E-03 -.95450E-04 -.34969E-03 -.31191E-04
    -.31805E-04 -.31668E-04  .48250E-04  .5AA49E-04  .70374E-04  .13916E-04  .262A8E-04  .37841E-01 -.93775E-04

19  .13262E-03  .56812E-03  .390A1E-03  .55436E-02  .25053E-03  .68404E-03 -.46038E-02 -.13310E-02  .5A5AAE-02 -.53671E-04
    -.99540E-04 -.42454E-04 -.239A9E-03 -.82559E-03 -.71723E-03 -.43763E-03 -.930A4E-03 -.93775E-04  .12590E-01
```

K1 VZ(K1,K2),K2=1,M1 ADJUSTED VARIANCE-COVARIANCE MATRIX FOR INTEGRAL PARAMETERS

```
1   .40650E-04   .49838E-05   .27325E-05 -.55321E-04   .51449E-04 -.48885E-04 -.20766E-04 -.41715E-04   .53821E-04 -.24668E-05
   -.72616E-05 -.32650E-04   .63293E-04   .21654E-05 -.13811E-04 -.65803E-05   .57399E-04 -.36825E-04   .52471E-04   .28167E-04
   -.22183E-04 -.25096E-04 -.83682E-04 -.226A3E-04

2   .49838E-05   .28231E-04   .26509E-04   .30977E-04 -.12666E-04 -.16559E-04 -.21776E-04   .18360E-04 -.36012E-04   .78293E-05
    .16156E-04   .20491E-04 -.40486E-04   .23272E-05   .83385E-05 -.77859E-05   .27750E-05 -.21711E-04 -.15977E-04 -.11538E-04
    .43529E-05   .48159E-05 -.64459E-05   .43529E-05

3   .27325E-05   .26509E-04   .31128E-04 -.63411E-05   .23907E-05   .55745E-05 -.39354E-04   .32121E-04 -.10466E-04 -.65423E-05
   -.30712E-04   .28895E-04 -.10600E-04 -.63212E-05 -.16471E-04   .10614E-04 -.16294E-04   .19480E-04 -.16039E-04 -.86505E-05
    .68991E-05   .76330E-05   .53554E-05   .68991E-05

4  -.35321E-04   .30977E-04 -.63411E-05   .12476E-02   .62436E-03   .31730E-03   .48955E-03   .25528E-03 -.11437E-03 -.20736E-03
    .64119E-04   .172A8E-03 -.13241E-03 -.96550E-04   .33267E-03 -.12792E-03   .28922E-04 -.20689E-03 -.43244E-04 -.42251E-04
    .22772E-05   .25195E-05 -.16108E-03   .22772E-05

5   .51449E-04 -.12666E-04   .23907E-05   .62436E-03   .93843E-03   .80184E-03   .438A3E-03   .2111AE-04   .18117E-03 -.37849E-04
   -.25117E-03 -.7647BE-06   .20474E-03   .30374E-04 -.11598E-03   .49822E-04 -.99172E-05   .13545E-03   .12050E-03   .78303E-04
   -.40351E-04 -.44643E-04   .21177E-04 -.40351E-04

6  -.48885E-04 -.16559E-04   .55745E-05   .31730E-03   .80184E-03   .19471E-02   .10457E-02 -.36279E-03 -.23616E-03   .59643E-03
    .73790E-04 -.15386E-03 -.20012E-03   .44722E-03 -.50596E-03   .27877E-03   .10893E-03   .42956E-04 -.21959E-03 -.17505E-03
    .45630E-04   .52484E-04   .24242E-03   .45630E-04

7  -.20766E-04 -.21776E-04 -.39354E-04   .48955E-03   .438A3E-03   .10457E-02   .17354E-02 -.40765E-03 -.64823E-04 -.38327E-04
    .12978E-02 -.24232E-03 -.12782E-03 -.62251E-03   .43637E-03   .10166E-04   .31023E-03 -.46107E-03 -.24783E-03 -.19165E-03
    .56592E-04   .62613E-04 -.22051E-03   .56592E-04

8  -.41713E-04   .18360E-04   .32121E-04   .25528E-03   .2111AE-04 -.36279E-03 -.40765E-03   .10809E-02   .75274E-03   .20635E-03
   -.52178E-04   .99725E-03   .75776E-03   .12969E-03   .72701E-04   .669A4E-04 -.52570E-04   .58874E-04 -.89684E-04 -.71936E-04
    .18252E-04   .20194E-04   .13909E-03   .18252E-04

9   .53821E-04 -.36012E-04 -.10466E-04 -.11437E-03   .18117E-03 -.23616E-03 -.64823E-04   .75274E-03   .10896E-02   .201A3E-03
    .31377E-03   .73209E-03   .98964E-03 -.12141E-03   .86230E-04   .66455E-04   .28551E-04   .71696E-04   .77082E-04   .28667E-04
   -.44285E-04 -.48996E-04 -.95170E-06 -.44285E-04

10 -.24668E-05   .78293E-05 -.65423E-05 -.20736E-03 -.37849E-04   .59643E-03 -.38327E-04   .20635E-03   .201A3E-03   .123A9E-02
    .64050E-03   .42515E-03   .123A9E-03   .810A5E-03 -.36010E-03 -.10794E-04 -.56948E-04   .75573E-04   .48321E-04   .45702E-04
   -.38456E-05 -.42547E-05   .29103E-04 -.38456E-05

11 -.72616E-05   .16156E-04 -.30712E-04   .64119E-04 -.25117E-03   .73790E-04   .12978E-02 -.52178E-04   .31377E-03   .64050E-03
    .27605E-02   .15006E-03 -.72125E-05 -.68563E-03   .89472E-03 -.15296E-03   .11062E-03 -.22325E-03   .93321E-04   .56513E-04
   -.34808E-04 -.38511E-04 -.30181E-03 -.34808E-04

12 -.32650E-04   .20491E-04   .28893E-04   .172A8E-03 -.76478E-06 -.15386E-03 -.24232E-03   .99725E-03   .73209E-03   .42515E-03
    .15006E-03   .10829E-02   .74499E-03   .90429E-05 -.27344E-03 -.92847E-06 -.53403E-04 -.49622E-04 -.132A9E-03 -.110A2E-03
    .24085E-04   .26647E-04   .7338AE-04   .24085E-04

13  .63293E-04 -.40486E-04 -.10600E-04 -.13241E-03   .20474E-03 -.20012E-03 -.12782E-03   .75776E-03   .98964E-03   .123A9E-03
   -.72125E-05   .74499E-03   .11078E-02   .21752E-03   .238A1E-03   .618A5E-04   .68163E-04 -.11909E-04   .77933E-05 -.1592AE-04
   -.20713E-04 -.22917E-04 -.18673E-04 -.20713E-04

14  .21654E-05   .23272E-05 -.63212E-05 -.96550E-04   .30374E-04   .44722E-03 -.62251E-03   .12969E-03 -.12141E-03   .810A5E-03
   -.68563E-03   .90429E-05   .21752E-03   .27809E-02   .11767E-02   .297A4E-04 -.62848E-04   .66816E-04 -.7909AE-05 -.29016E-04
   -.17942E-04   .19850E-04   .72923E-04 -.17942E-04

15 -.13811E-04   .83385E-05 -.16471E-04   .33267E-03 -.11598E-03 -.50596E-03   .43637E-03   .72701E-04   .86230E-04 -.36010E-03
    .89472E-03 -.27344E-03   .238A1E-03   .11767E-02   .27883E-02 -.57204E-04   .11126E-03 -.12767E-03   .47294E-04   .46455E-04
```

```
   -.22775E-05 -.25198E-05 -.16704E-03 -.22775E-05

16 -.65803E-05 -.77859E-05  .10614E-04 -.12792E-03  .49822E-04  .27877E-03  .10166E-04  .669A4E-04  .66455E-04 -.18794E-04
   -.15296E-03 -.92847E-06  .61885E-04  .29704E-04 -.57204E-04  .16965E-02  .13235E-02  .36550E-03 -.10452E-03 -.25164E-03
   -.12344E-03 -.13657E-03  .44999E-03 -.12344E-03

17  .57399E-04  .27750E-05 -.16294E-04  .28922E-04 -.99172E-05  .10893E-03  .31023E-03 -.52570E-04  .28551E-04 -.56948E-04
    .11062E-03 -.53493E-04  .68163E-04 -.62808E-04  .11126E-03  .13235E-02  .17651E-02 -.15491E-03 -.19987E-03  .94442E-04
    .26037E-03  .28807E-03  .32521E-04  .26037E-03

18 -.36825E-04 -.21711E-04  .19480E-04 -.20689E-03  .13545E-03  .42956E-04 -.46107E-03  .58874E-04  .71696E-04  .75573E-04
   -.22325E-03 -.49622E-04 -.11909E-04  .66816E-04 -.12767E-03  .36550E-03 -.15491E-03  .76344E-03  .28254E-03  .23396E-03
   -.51173E-04 -.56617E-04  .48199E-03 -.51173E-04

19  .52471E-04 -.15977E-04 -.16039E-04 -.43244E-04  .12050E-03 -.21959E-03 -.24783E-03 -.89684E-04  .77082E-04  .48321E-04
    .93321E-04 -.13209E-03  .77933E-05 -.79409E-05  .47294E-04 -.10452E-03 -.19987E-03  .28254E-03  .32415E-03  .22615E-03
   -.95164E-04 -.10529E-03 -.22393E-04 -.95164E-04

20  .28167E-04 -.11538E-04 -.86505E-05 -.42251E-04  .78303E-04 -.17505E-03 -.19165E-03 -.71936E-04  .28667E-04  .45702E-04
    .56513E-04 -.11002E-03 -.15928E-04 -.29016E-04  .46455E-04 -.25164E-03  .94442E-04  .23396E-03  .22615E-03  .66230E-03
    .36870E-03  .40793E-03  .20104E-04  .36870E-03

21 -.22683E-04  .43529E-05  .68991E-05  .22772E-05 -.40351E-04  .45630E-04  .56592E-04  .18252E-04 -.44285E-04 -.38456E-05
   -.34808E-04  .24085E-04 -.20713E-04 -.17942E-04 -.22775E-05 -.12344E-03  .26037E-03 -.51173E-04 -.95164E-04  .36870E-03
    .40316E-03  .44605E-03  .37384E-04  .40316E-03

22 -.25096E-04  .48159E-05  .76330E-05  .25195E-05 -.44643E-04  .50484E-04  .62613E-04  .20194E-04 -.48996E-04 -.42547E-05
   -.38511E-04  .26647E-04 -.22917E-04 -.19854E-04 -.25198E-05 -.13657E-03  .28807E-03 -.56617E-04 -.10529E-03  .40793E-03
    .44605E-03  .49350E-03  .41362E-04  .44605E-03

23 -.83682E-04 -.64459E-05  .33554E-04 -.16108E-03  .21177E-04  .24242E-03 -.22051E-03  .13909E-03 -.95170E-06  .29103E-04
   -.30181E-03  .73383E-04 -.18673E-04  .72023E-04 -.16704E-03  .44999E-03  .32521E-04  .48199E-03 -.22393E-04  .20104E-04
    .37384E-04  .41362E-04  .48786E-03  .37384E-04

24 -.22683E-04  .43529E-05  .68991E-05  .22772E-05 -.40351E-04  .45630E-04  .56592E-04  .18252E-04 -.44285E-04 -.38456E-05
   -.34808E-04  .24085E-04 -.20713E-04 -.17942E-04 -.22775E-05 -.12344E-03  .26037E-03 -.51173E-04 -.95164E-04  .36870E-03
    .40316E-03  .44605E-03  .37384E-04  .40316E-03
```

SUMMARY FOR DIFFERENTIAL PARAMETERS

J	XE(I)	SX(I)	XC(T)	RELXE	RELXE**2	XA(I)	SXA(I)	RELXF	RELXF**2
1	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	9.78012E-01	1.98243E-02	-2.19885E-01	4.83494E-02
2	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	9.32236E-01	4.88807E-02	-6.77641E-01	4.59197E-01
3	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	9.94606E-01	2.00620E-02	-5.39420E-02	2.90974E-03
4	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	9.88789E-01	1.22533E-01	-1.12109E-01	1.25684E-02
5	1.00000E+00	1.50000E-01	1.00000E+00	0.	0.	9.61475E-01	2.75562E-02	-2.56835E-01	6.59644E-02
6	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	9.79102E-01	3.09979E-02	-2.08985E-01	4.36747E-02
7	1.00000E+00	1.50000E-01	1.00000E+00	0.	0.	8.39995E-01	1.33436E-01	-1.06670E+00	1.13785E+00
8	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	1.05883E+00	1.85363E-02	5.88324E-01	3.46125E-01
9	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	1.06331E+00	1.33422E-01	6.33065E-01	4.00771E-01
10	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	9.79160E-01	1.94011E-01	-2.08402E-01	4.34316E-02
11	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	1.00134E+00	1.94546E-01	1.34117E-02	1.79872E-04
12	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	9.95451E-01	1.94538E-01	-4.54924E-02	2.06956E-03
13	1.00000E+00	6.00000E-02	1.00000E+00	0.	0.	1.13790E+00	1.02834E-01	2.29827E+00	5.28204E+00
14	1.00000E+00	4.00000E-02	1.00000E+00	0.	0.	1.10604E+00	3.66005E-02	2.65103E+00	7.02798E+00
15	1.00000E+00	6.00000E-02	1.00000E+00	0.	0.	1.16569E+00	6.94256E-02	2.76147E+00	7.62574E+00
16	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	1.00909E+00	1.94360E-01	9.08541E-02	8.25446E-03
17	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	1.00882E+00	1.93743E-01	8.82264E-02	7.78390E-03
18	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	9.95512E-01	1.94528E-01	-4.48831E-02	2.01449E-03
19	1.00000E+00	1.00000E-01	1.00000E+00	0.	0.	1.08546E+00	1.12206E-01	8.54607E-01	7.30352E-01

MJ		CHIDI		CHIDF
19		0.		2.3247E+01

MK	CHI2I		CHI2F
43	5.0282E+02		9.0910E+01

SUMMARY FOR INTEGRAL PARAMETERS

I	YE(I)	SY(I)	YC(I)	RELYE	RELYE**2	YA(I)	SYA(I)	RELYF	RELYF**2
1	1.00000E+00	4.00000E-03	9.92000E-01	-2.00000E+00	4.00000E+00	9.92718E-01	6.68208E-03	-1.82041E+00	3.31390E+00
2	1.00000E+00	4.00000E-03	9.92400E-01	-1.90000E+00	3.61000E+00	1.00151E+00	5.31330E-03	3.76681E-01	1.41888E-01
3	1.00000E+00	4.00000E-03	9.92700E-01	-1.82500E+00	3.33063E+00	1.00187E+00	5.57924E-03	4.66705E-01	2.17813E-01
4	1.00000E+00	2.50000E-02	1.10000E+00	4.00000E+00	1.60000E+01	9.90511E-01	3.53207E-02	-3.79562E-01	1.44067E-01
5	1.00000E+00	2.50000E-02	1.15000E+00	6.00000E+00	3.60000E+01	1.01368E+00	3.06338E-02	5.47022E-01	2.99233E-01
6	1.00000E+00	3.50000E-02	1.24000E+00	6.85714E+00	4.70204E+01	1.08997E+00	4.41262E-02	2.57055E+00	6.60774E+00
7	1.00000E+00	7.50000E-02	9.20000E-01	-1.06667E+00	1.13778E+00	8.53378E-01	4.16583E-02	-1.95496E+00	3.82188E+00
8	1.00000E+00	3.50000E-02	1.25000E+00	7.14286E+00	5.10204E+01	1.05249E+00	3.28765E-02	1.49961E+00	2.24883E+00
9	1.00000E+00	3.50000E-02	1.24000E+00	6.85714E+00	4.70204E+01	1.03574E+00	3.17746E-02	1.02120E+00	1.04284E+00
10	1.00000E+00	2.50000E-02	1.16000E+00	6.40000E+00	4.09600E+01	9.36055E-01	3.51980E-02	-2.55780E+00	6.54233E+00
11	1.00000E+00	3.50000E-02	1.18000E+00	5.14286E+00	2.64490E+01	1.02748E+00	5.25404E-02	7.85003E-01	6.16230E-01
12	1.00000E+00	3.50000E-02	1.25000E+00	7.14286E+00	5.10204E+01	1.04182E+00	3.29078E-02	1.19498E+00	1.42797E+00
13	1.00000E+00	3.50000E-02	1.26000E+00	7.42857E+00	5.51837E+01	1.04629E+00	3.32832E-02	1.32258E+00	1.74922E+00
14	1.00000E+00	3.50000E-02	1.27000E+00	7.71429E+00	5.95102E+01	1.02388E+00	5.27346E-02	6.82303E-01	4.65537E-01
15	1.00000E+00	3.50000E-02	1.09000E+00	2.57143E+00	6.61224E+00	9.59831E-01	5.28047E-02	-1.14768E+00	1.31718E+00
16	1.00000E+00	3.00000E-02	9.00000E-01	-3.33333E+00	1.11111E+01	9.42179E-01	4.11891E-02	-1.92737E+00	3.71474E+00
17	1.00000E+00	3.00000E-02	1.03000E+00	1.00000E+00	1.00000E+00	1.06012E+00	4.20133E-02	2.00387E+00	4.01548E+00
18	1.00000E+00	2.00000E-02	9.90000E-01	-5.00000E-01	2.50000E-01	9.74651E-01	2.76305E-02	-1.26746E+00	1.60646E+00
19	1.00000E+00	2.00000E-02	1.05000E+00	2.50000E+00	6.25000E+00	1.03258E+00	1.80042E-02	1.62880E+00	2.65299E+00
20	1.00000E+00	2.00000E-02	1.09000E+00	4.50000E+00	2.02500E+01	1.05389E+00	2.57353E-02	2.69436E+00	7.25955E+00
21	1.00000E+00	2.00000E-02	9.40000E-01	-3.00000E+00	9.00000E+00	9.24455E-01	2.00788E-02	-3.77723E+00	1.42675E+01
22	1.00000E+00	2.00000E-02	1.04000E+00	2.00000E+00	4.00000E+00	1.02280E+00	2.22148E-02	1.14009E+00	1.29980E+00
23	1.00000E+00	5.00000E-02	9.60000E-01	-8.00000E-01	6.40000E-01	9.61046E-01	2.20875E-02	-7.79072E-01	6.06953E-01
24	1.00000E+00	5.00000E-02	9.40000E-01	-1.20000E+00	1.44000E+00	9.24455E-01	2.00788E-02	-1.51089E+00	2.28279E+00

MK	CHIDI	CHIDF
24	5.0282E+02	6.7663E+01

MK	CHI2I	CHI2F
43	5.0282E+02	9.0910E+01

SUMMARY FOR COMBINED PARAMETERS

MK	CHI2I	CHIDI	CHI2F	CHIDF	DISPR
43	5.0282E+02	5.0282E+02	9.0910E+01	9.0910E+01	3.7879E+00

APPENDIX E

SAMPLE INPUT FOR SENSITIVITY CALCULATION

```
SENSITIVITY ANALYSIS, 41-GP NEUTRON TRANSPORT IN IRON SPHERE      ALVIN
     2     1     0     0                                           ALVIN
     1     6                                                       SENSI
DATA FOR SENSITIVITY ANALYSIS  CASE ONE OF ONE                     SENSI
     2     1     0     0     1     0                               SENSI
     8    73                                                       SENSI
  .0848 E+00                                                       SENSI
     2    74    41     0    17     6                               REDFLX
     6                                                             CROSEC
8.01443E-09                                                        R
```

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APPENDIX F

SAMPLE OUTPUT FOR SENSITIVITY CALCULATION

SENSITIVITY ANALYSIS, 41-GP NEUTRON TRANSPORT IN IRON SPHERE

KSENS	KADJST	MI	MJ
2	1	0	0

DATA FOR SENSITIVITY ANALYSIS CASE ONE OF ONE

NYR	NXR	KGEOM	NR	NSAVE	KTIMS	NT	KTYPE
1	6	2	1	0	0	1	0

IMMIN(IR),IMMAX(IR),IR=1,NR
```
     8          73
```

DENS(IR),IR=1,NR
```
.848000E-01
```

KFLUX	NM	NG	NLR	NA	NL
2	74	41	0	17	6

(RM(IM),IM=1,NM+1)
INPUT RADTI(J),J=1,75
```
0.            .250000E+00 .500000E+00 .750000E+00 .100000E+01 .200000E+01
.300000E+01 .400000E+01 .500000E+01 .600000E+01 .700000E+01 .800000E+01
.900000E+01 .100000E+02 .110000E+02 .120000E+02 .130000E+02 .140000E+02
.150000E+02 .160000E+02 .170000E+02 .180000E+02 .190000E+02 .200000E+02
.210000E+02 .220000E+02 .230000E+02 .240000E+02 .250000E+02 .260000E+02
.270000E+02 .280000E+02 .290000E+02 .300000E+02 .310000E+02 .320000E+02
.330000E+02 .340000E+02 .350000E+02 .360000E+02 .370000E+02 .380000E+02
.390000E+02 .400000E+02 .410000E+02 .420000E+02 .430000E+02 .440000E+02
.450000E+02 .460000E+02 .470000E+02 .480000E+02 .490000E+02 .500000E+02
.510000E+02 .520000E+02 .530000E+02 .540000E+02 .550000E+02 .560000E+02
.570000E+02 .580000E+02 .590000E+02 .600000E+02 .610000E+02 .620000E+02
.630000E+02 .640000E+02 .650000E+02 .660000E+02 .670000E+02 .680000E+02
.690000E+02 .700000E+02 .710000E+02
```

(QCOSA(IA),IA=1,NA)
COSINE(I),I=1,17)
```
-.100000E+01-.982031E+00-.910582E+00-.833027E+00-.747468E+00-.650756E+00
-.536897E+00-.391194E+00-.133446E+00 .133446E+00 .391194E+00 .536897E+00
 .650756E+00 .747468E+00 .833027E+00 .910582E+00 .982031E+00
```

(QWTSA(IA),IA=1,NA)
WEIGHT(I),I=1,17)
```
0.            .270771E-01 .367965E-01 .402441E-01 .454261E-01 .523245E-01
 .628440E-01 .907613E-01 .144526E+00 .144526E+00 .907613E-01 .628440E-01
 .523245E-01 .454261E-01 .402441E-01 .367965E-01 .270771E-01
```

```
            (RM(IM),IM=1,NM+1)
    INPUT RADII(J),J=1,75
0.            .250000E+00 .500000E+00 .750000E+00 .100000E+01 .200000E+01
  .300000E+01 .400000E+01 .500000E+01 .600000E+01 .700000E+01 .800000E+01
  .900000E+01 .100000E+02 .110000E+02 .120000E+02 .130000E+02 .140000E+02
  .150000E+02 .160000E+02 .170000E+02 .180000E+02 .190000E+02 .200000E+02
  .210000E+02 .220000E+02 .230000E+02 .240000E+02 .250000E+02 .260000E+02
  .270000E+02 .280000E+02 .290000E+02 .300000E+02 .310000E+02 .320000E+02
  .330000E+02 .340000E+02 .350000E+02 .360000E+02 .370000E+02 .380000E+02
  .390000E+02 .400000E+02 .410000E+02 .420000E+02 .430000E+02 .440000E+02
  .450000E+02 .460000E+02 .470000E+02 .480000E+02 .490000E+02 .500000E+02
  .510000E+02 .520000E+02 .530000E+02 .540000E+02 .550000E+02 .560000E+02
  .570000E+02 .580000E+02 .590000E+02 .600000E+02 .610000E+02 .620000E+02

  .630000E+02 .640000E+02 .650000E+02 .660000E+02 .670000E+02 .680000E+02
  .690000E+02 .700000E+02 .710000E+02


            (QCOSA(IA),IA=1,NA)
COSINE(I),I=1,17)
 COSINES MULT BY -1. IN REDFLX  FOR ADJOINT FLUX EXPANSION

  .100000E+01 .982031E+00 .910582E+00 .833027E+00 .747468E+00 .650756E+00
  .536897E+00 .391194E+00 .133446E+00-.133446E+00-.391194E+00-.536897E+00
 -.650756E+00-.747468E+00-.833027E+00-.910582E+00-.982031E+00


            (QWTSA(IA),IA=1,NA)
WEIGHT(I),I=1,17
0.            .270771E-01 .367965E-01 .402441E-01 .454261E-01 .523245E-01
  .628440E-01 .907613E-01 .144526E+00 .144526E+00 .907613E-01 .628440E-01
  .523245E-01 .454261E-01 .402441E-01 .367965E-01 .270771E-01


        NLCX
          6
P 0 FE OF P0-P8,41GP CROSS SECTIONS ADJ TO EXP VAL    AFTER CHANGE IN CRANGE
P1 FE OF P0-P8,41GP CROSS SECTIONS ADJ TO EXP VAL    AFTER CHANGE IN CRANGE
P2 FE OF P0-P8,41GP CROSS SECTIONS ADJ TO EXP VAL    AFTER CHANGE IN CRANGE
P3 FE OF P0-P8,41GP CROSS SECTIONS ADJ TO EXP VAL    AFTER CHANGE IN CRANGE
P4 FE OF P0-P8,41GP CROSS SECTIONS ADJ TO EXP VAL    AFTER CHANGE IN CRANGE
P5 FE OF P0-P8,41GP CROSS SECTIONS ADJ TO EXP VAL    AFTER CHANGE IN CRANGE


        R
.801443E-08
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SENSITIVITY PROFILE

GROUP	SENSITIVITY
1	0.
2	0.
3	0.
4	0.
5	0.
6	0.
7	0.
8	-1.52696E-04
9	-1.02585E-02
10	-7.16549E-03
11	-4.26670E-03
12	-1.75711E-03
13	-1.83075E-03
14	-1.10495E-03
15	-1.15348E-03
16	-1.15511E-03
17	-9.72420E-04
18	-2.16578E-03
19	-1.46277E-03
20	-2.03188E-03
21	-2.80632E-03
22	-3.94025E-03
23	-9.26679E-03
24	-3.24802E-02
25	-5.23364E-02
26	-9.26052E-03
27	-2.59525E-02
28	-3.21015E-03
29	-1.99605E-03
30	-4.84289E-04
31	-3.99885E-04
32	-1.83508E-04
33	-1.68276E-04
34	-1.53271E-04
35	-1.35074E-04
36	-1.06397E-04
37	-7.35754E-05
38	-4.06675E-05
39	-1.55001E-05
40	-4.99287E-06
41	-1.19203E-06