C. 3

LAMS-2415

LOS ALAMOS SCIENTIFIC LABORATORY

OF THE UNIVERSITY OF CALIFORNIA ○ LOS ALAMOS NEW MEXICO

CRITICAL DATA FOR NUCLEAR SAFETY GUIDANCE

LEGAL NOTICE

Printed in USA. Price $2.00. Available from the

Office of Technical Services
U. S. Department of Commerce
Washington 25, D. C.

LAMS-2415
CRITICALITY STUDIES
TID-4500, 15th Ed.

LOS ALAMOS SCIENTIFIC LABORATORY
OF THE UNIVERSITY OF CALIFORNIA LOS ALAMOS NEW MEXICO

REPORT COMPILED: February 1960

REPORT DISTRIBUTED: May 16, 1960

CRITICAL DATA FOR NUCLEAR SAFETY GUIDANCE

Compiled by

H. C. Paxton

Contract W-7405-ENG. 36 with the U. S. Atomic Energy Commission

PREFACE

This collection of critical data is intended for the convenience of those who wish to evaluate nuclear safety problems. It is made available in the present form pending consideration as a supplement to TID 7016, THE NUCLEAR SAFETY GUIDE. Major sources of information outside of the Los Alamos Scientific Laboratory are:

The Argonne National Laboratory

The Dow Chemical Company, Rocky Flats Plant

General Electric Company, Hanford Atomic Products Operation

Lawrence Radiation Laboratory, Livermore

Union Carbide Corporation, Oak Ridge National Laboratory

U K Atomic Energy Research Establishment, Harwell

U K Atomic Weapons Research Establishment, Aldermaston.

ILLUSTRATIONS

ILLUSTRATIONS (Continued)

ILLUSTRATIONS (Continued)

ILLUSTRATIONS (Continued)

INDIVIDUAL UNITS

Homogeneous, water-moderated systems

Figures 1 and 2 represent critical masses and critical volumes of homogeneous, water-moderated spheres of Oy(93.2), both bare (except for thin-wall container) and water-reflected.[1-6,31] Estimates of corresponding diameters of infinite critical cylinders appear in Figure 3, and thicknesses of infinite critical slabs in Figure 4.[4,6,11,12,32] Effective extrapolation lengths of Figures 27 and 28 are used for the shape conversions that are involved. Similar data for water-moderated Pu^{239} appear in Figures 5-8,[4,6-8,32,33] and for U^{233} in Figures 9-12.[4,6,8-10,34] The idealized metal-water mixtures of Figures 1-12 ($>$ 2 kg/liter) are denser, hence more limiting, than usually encountered.

Inhomogeneous water-moderated Oy

Figure 13 shows how the minimum critical mass of a water-moderated, water-reflected lattice of Oy(93.5) pieces (optimum spacing) depends upon size of piece.[26,35] Though measurements were on 1" cubes, 1/2" cubes, and 1/8" diameter rods, data appear in terms of approximate diameters of equivalent spheres. Surface-to-surface spacings that correspond to minima in critical mass vary from 0.7" for the 1" cubes to 0.6" for the 1/8" rods.

Oy at reduced U^{235} content

Minimum critical masses of homogeneous, water-moderated, water-reflected Oy are given as functions of U^{235} content of the Oy in Figure 14a.[24,36-38] Also shown are minimum critical masses of water-moderated lattices in the enrichment range through which these critical masses are less than those for homogeneous systems.[29,39] Similarly, Figure 14b displays minimum critical volumes. Critical masses of unmoderated Oy(93.5) metal vs. U^{235} concentration appear in Figure 15.[6]

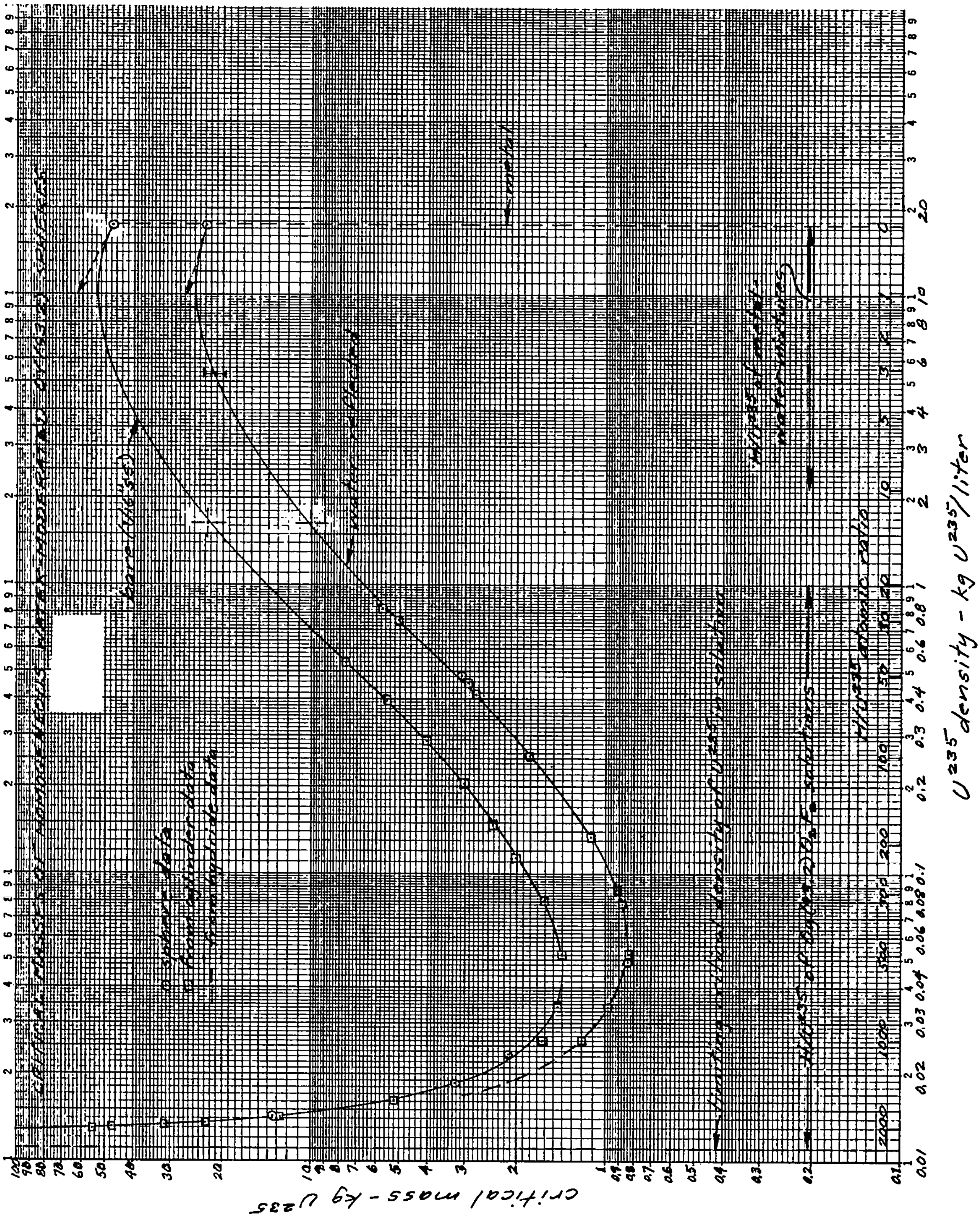

critical mass - kg U235
U235 density - kg U235/liter
bare ("the 5's")
water reflected
metal
H/U235 atomic ratio

Fig. 1.

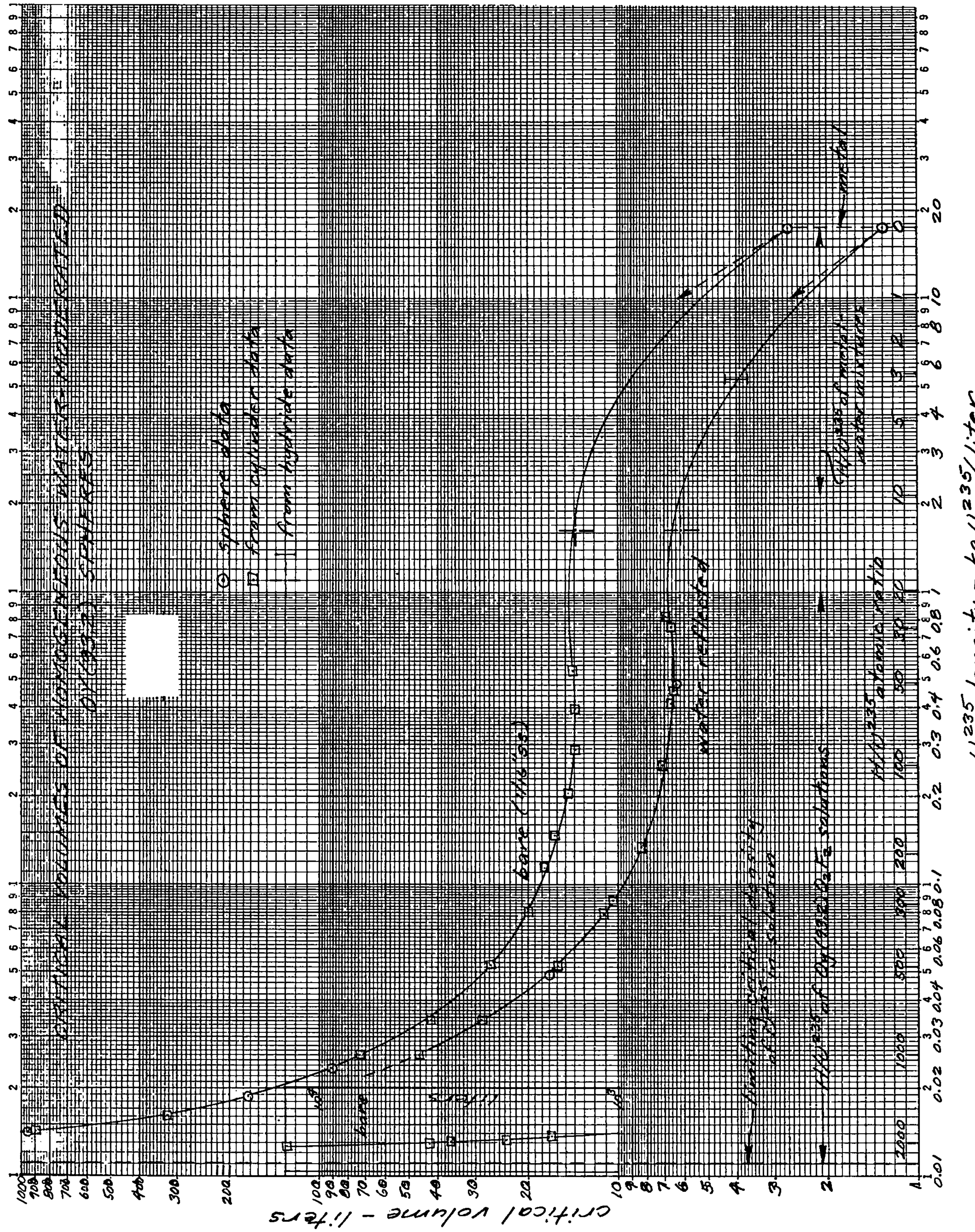

SPHERES
sphere data
from cylinder data
from hydride data
water reflected
H/U235 atomic ratio
critical volume - liters
U235 density - kg U235/liter
metal

Fig. 2.

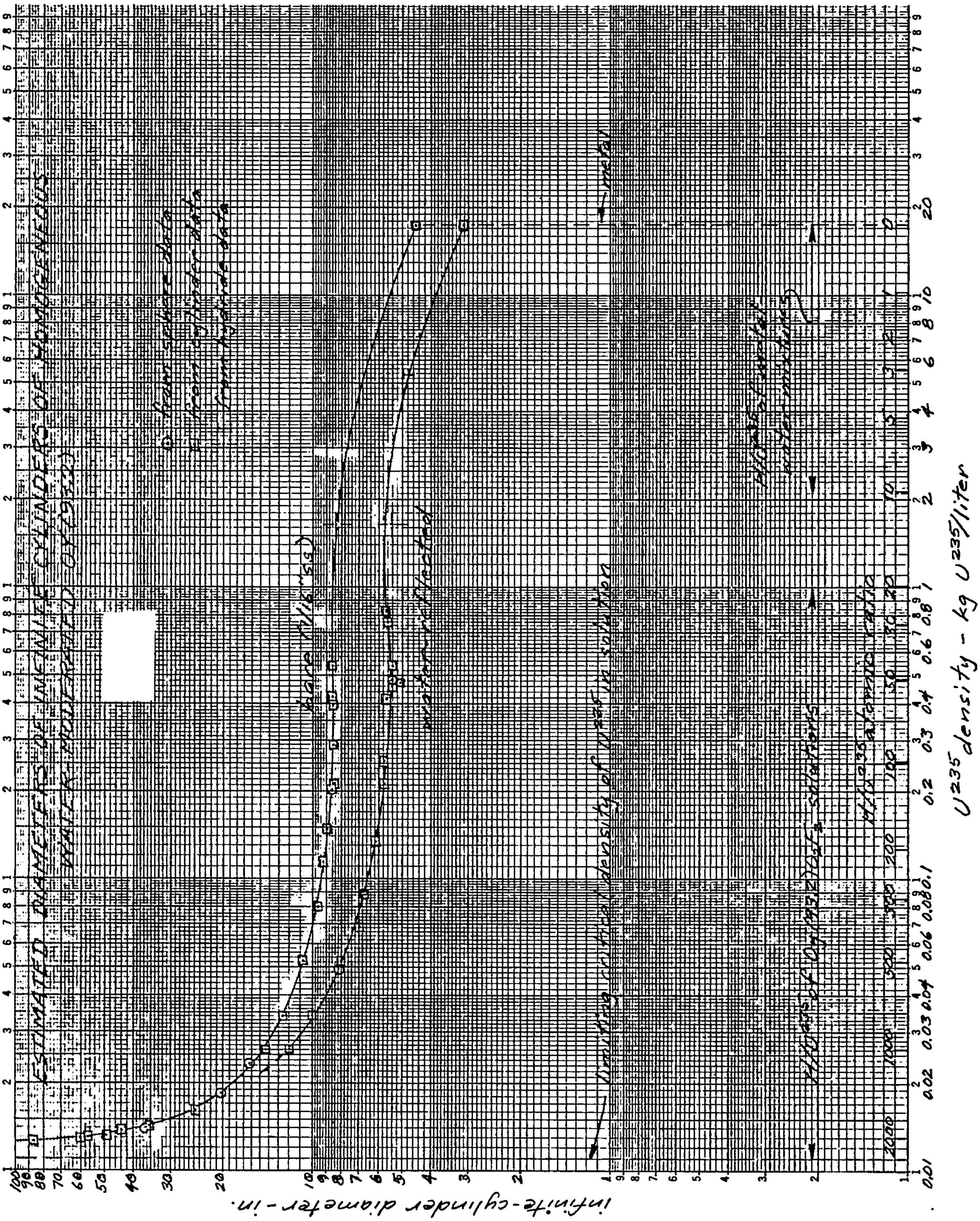

ESTIMATED DIAMETERS OF INFINITE CYLINDERS OF HOMOGENEOUS
WATER-MODERATED Oy(93.2)
from sphere data
from cylinder data
from hydride data
bare (1/16" ss)
water-reflected
limiting critical density of U^{235} in solution
metal
H/U^{235} of $Oy(93.2)O_2F_2$ solutions
H/U^{235} of metal-water mixtures
H/U^{235} atomic ratio
infinite-cylinder diameter - in.
U^{235} density - kg U^{235}/liter

Fig. 3.

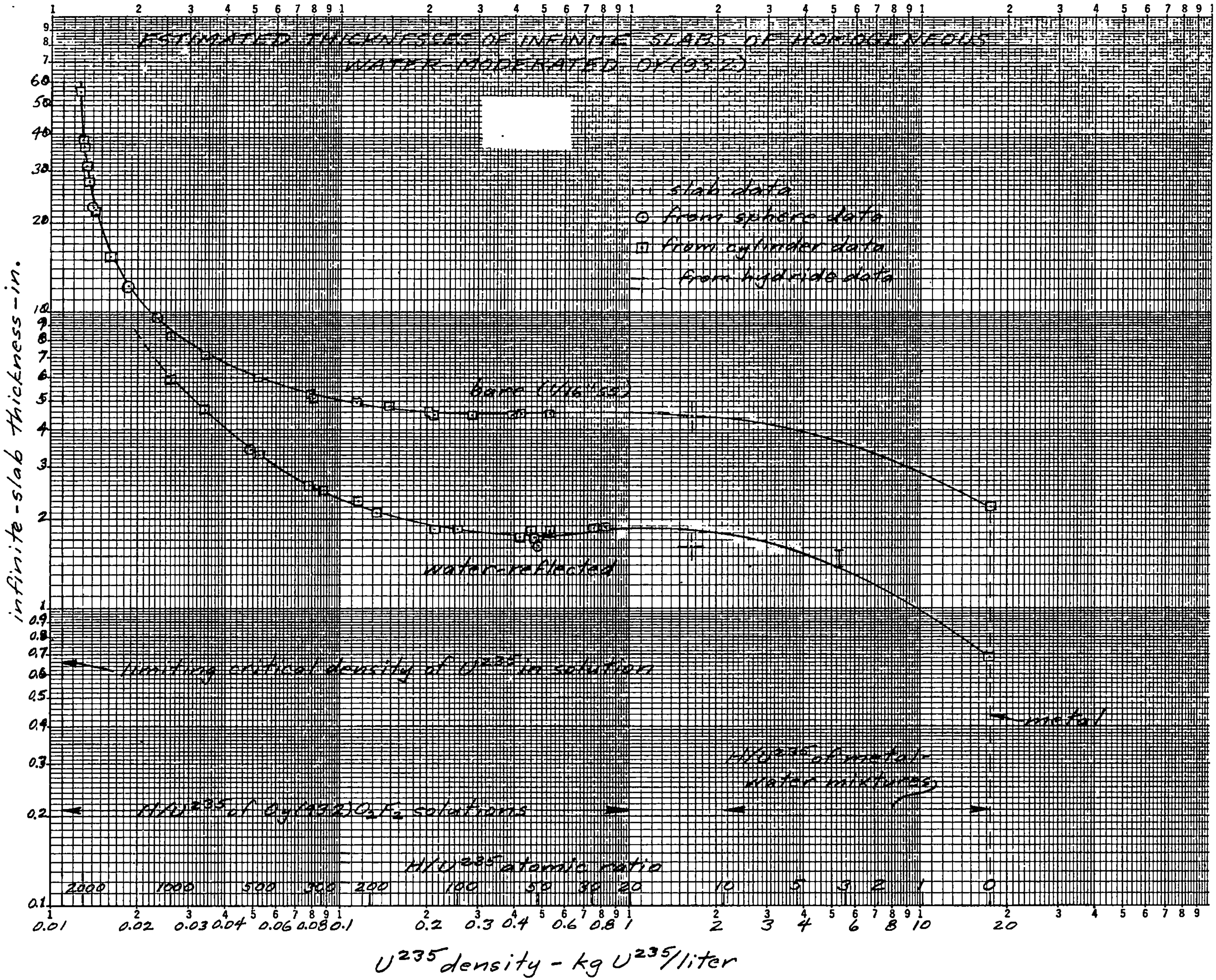

ESTIMATED THICKNESSES OF INFINITE SLABS OF HOMOGENEOUS
WATER-MODERATED OY(93.2)
slab data
⊙ from sphere data
□ from cylinder data
from hydride data
bare (U6'55)
water-reflected
limiting critical density of U^{235} in solution
metal
H/U^{235} of metal-water mixtures
H/U^{235} of Oy(93.2)O_2F_2 solutions
H/U^{235} atomic ratio
infinite-slab thickness-in.
U^{235} density - kg U^{235}/liter

Fig. 4.

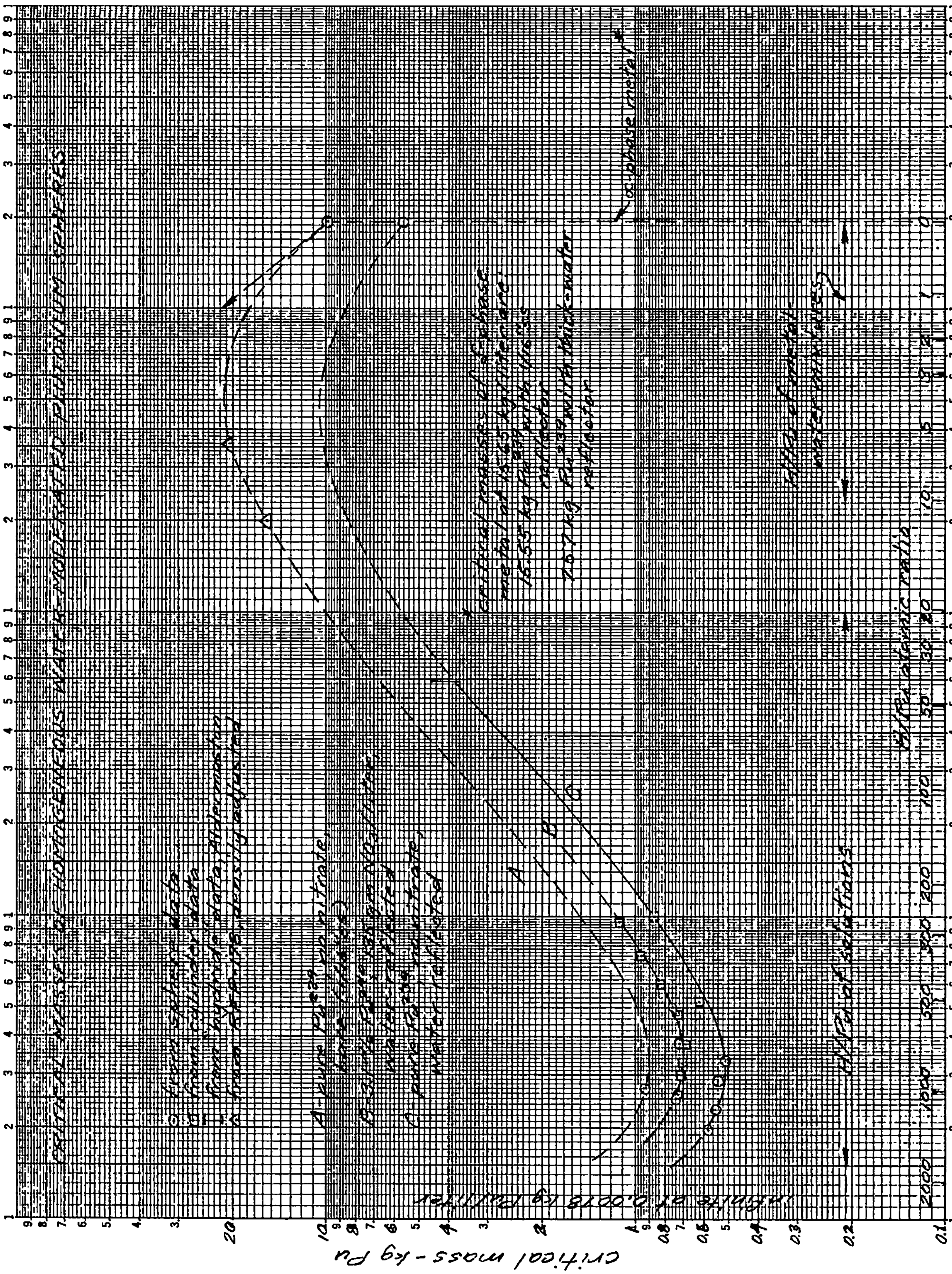
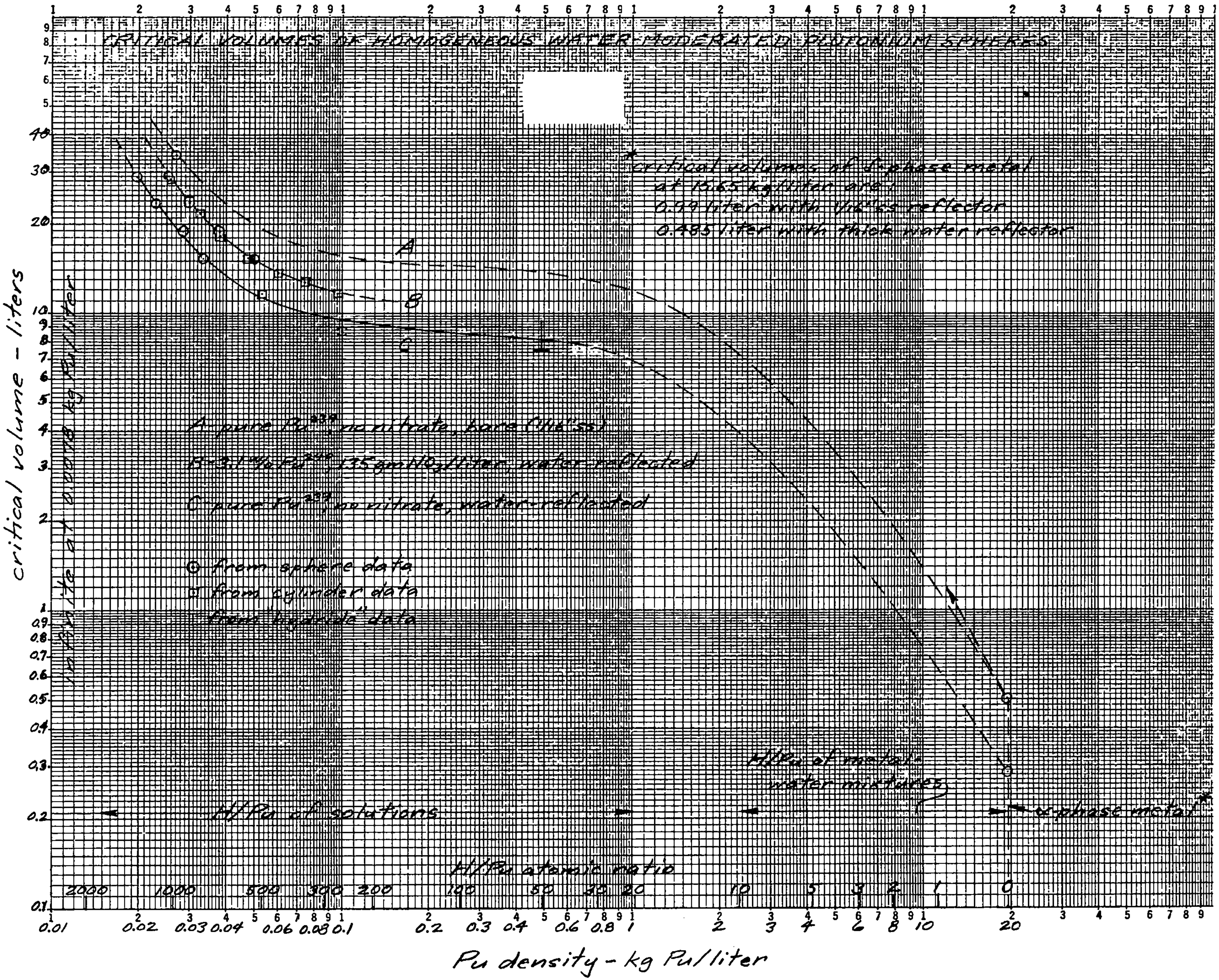

CRITICAL MASSES OF HOMOGENEOUS WATER-MODERATED PLUTONIUM SPHERES
○ from sphere data
□ from cylinder data
from "hydride" data, Aldermaston
△ from RFP-175, density adjusted
A-pure Pu239, no nitrate, bare
C-pure Pu239, no nitrate, water reflected
critical masses of α-phase metal
15.55 kg Pu239
reflector
7.67 kg Pu239 with thick-water reflector
α-phase metal
H/Pu of metal-water mixtures
H/Pu of solutions
H/Pu atomic ratio
critical mass - kg Pu
Pu density - kg Pu/liter

Fig. 5.



CRITICAL VOLUMES OF HOMOGENEOUS WATER MODERATED PLUTONIUM SPHERES
*critical volumes of δ-phase metal
at 15.65 kg/liter are:
0.99 liter with 1/16" ss reflector
0.485 liter with thick water reflector
A
B
C
A-pure Pu239 no nitrate, bare (1/16" ss)
B-3.1% Pu240, 135 gm NO3/liter, water-reflected
C pure Pu239, no nitrate, water-reflected
from sphere data
from cylinder data
from hydride data
H/Pu of solutions
H/Pu of metal-water mixtures
α-phase metal*
H/Pu atomic ratio
critical volume - liters
Infinite at 0.0078 kg Pu/liter
Pu density - kg Pu/liter

Fig. 6.

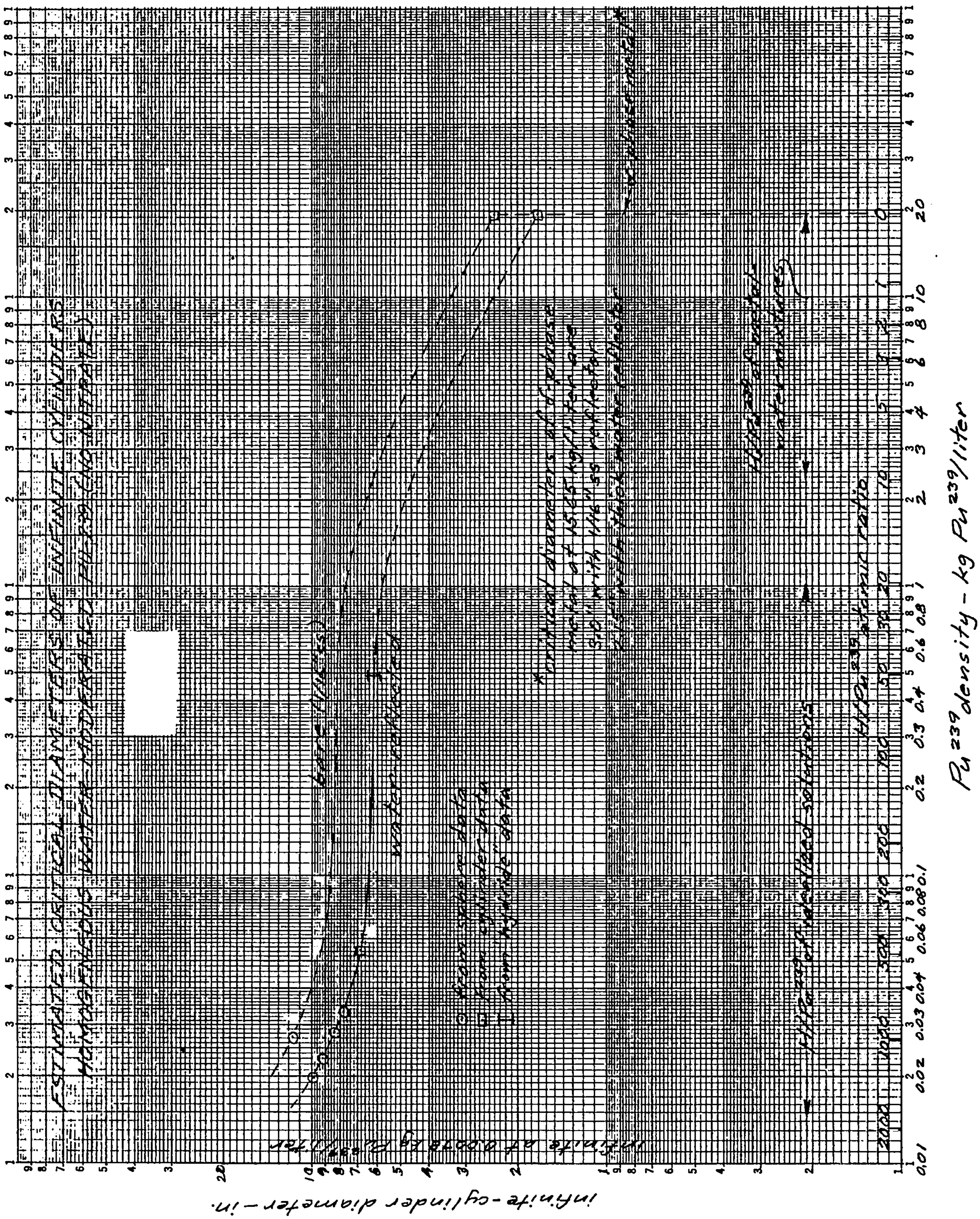

Pu239 density - kg Pu239/liter
infinite-cylinder diameter - in.
infinite at 0.0078 kg Pu239/liter
bare (1/16" SS)
water-reflected
from sphere data
from cylinder data
from hydride data
H/Pu239 atomic ratio

Fig. 7.

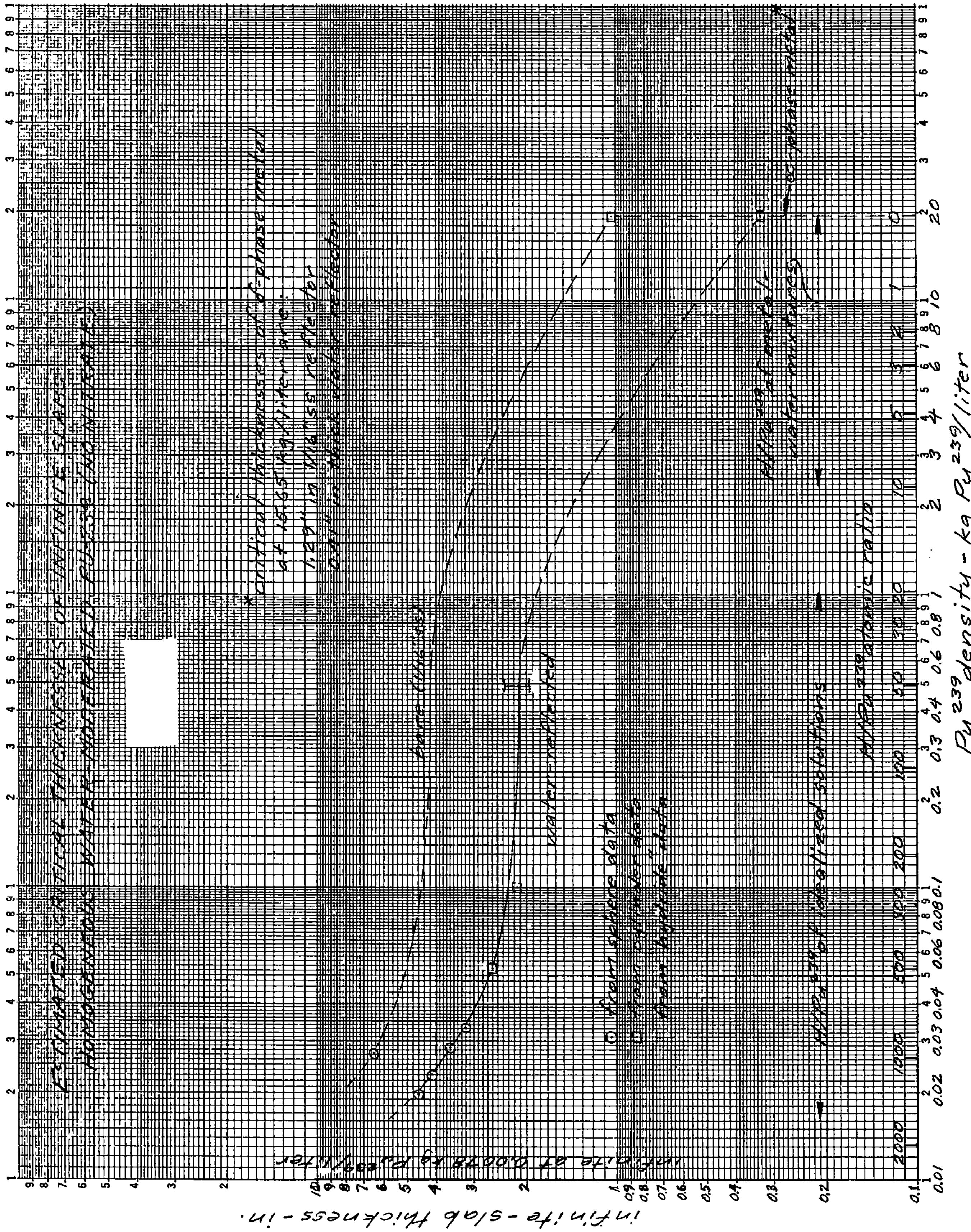

ESTIMATED CRITICAL THICKNESSES OF INFINITE SLABS
HOMOGENEOUS WATER MODERATED PU-239 (NO NITRATE)
* critical thicknesses of δ-phase metal
at 15.65 kg/liter are:
1.23" in 1/16" SS reflector
0.81" in thick water reflector
bare (unreflected)
water-reflected
○ from sphere data
□ from cylinder data
from hydride data
infinite at 0.0078 kg Pu239/liter
H/Pu239 of idealized solutions
H/Pu239 atomic ratio
H/Pu239 of metal-water mixtures
α-phase metal
Pu239 density - kg Pu239/liter
infinite-slab thickness - in.

Fig. 8.

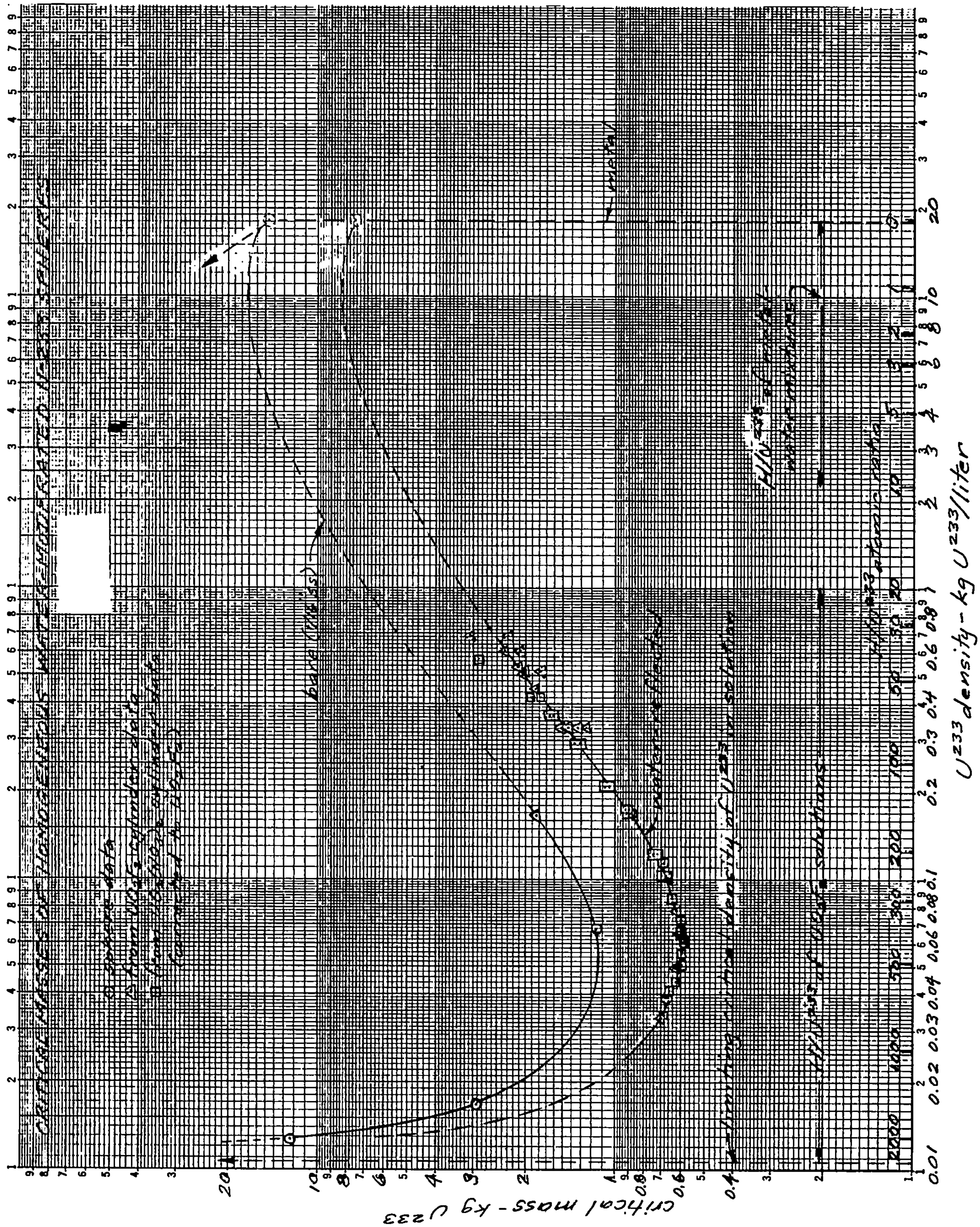

CRITICAL MASSES OF HOMOGENEOUS WATER-MODERATED U-233 SPHERES
○ sphere data
△ from UO2F2 cylinder data
□ from U(NO3)4 cylinder data
bare (Unless)
water-reflected
metal
limiting critical density of U233 in solution
H/U233 atomic ratio
U233 density - kg U233/liter
critical mass - kg U233

Fig. 9.

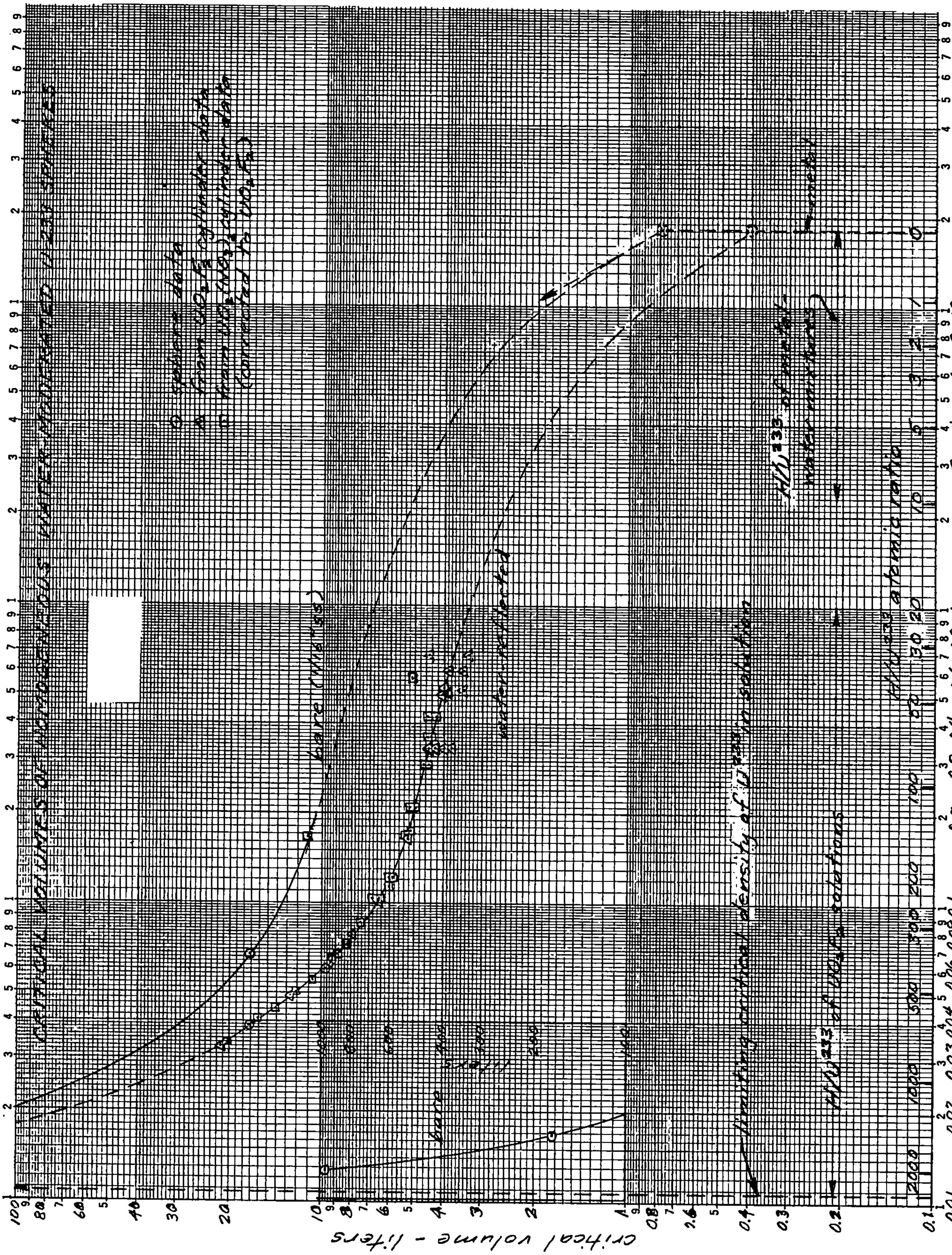
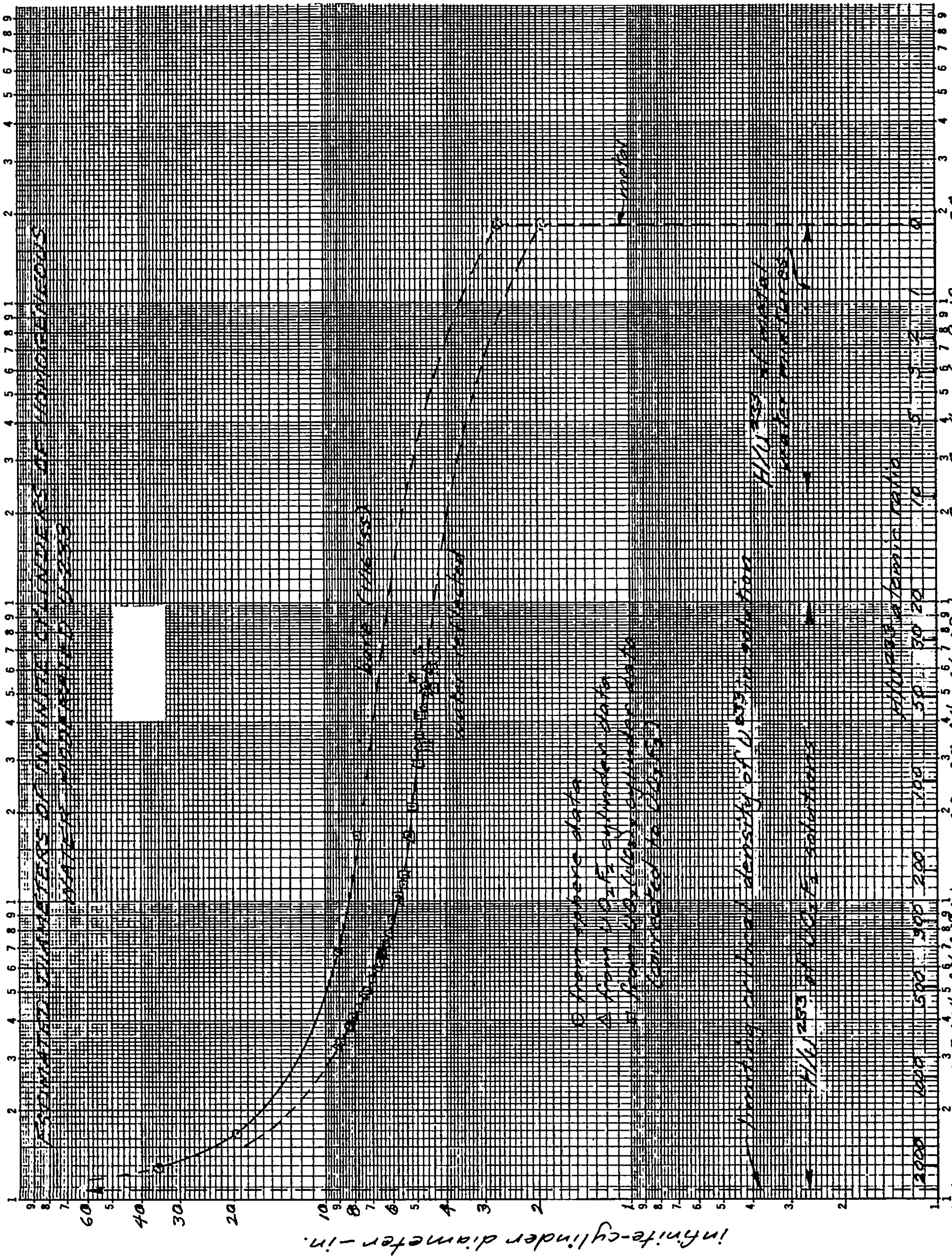
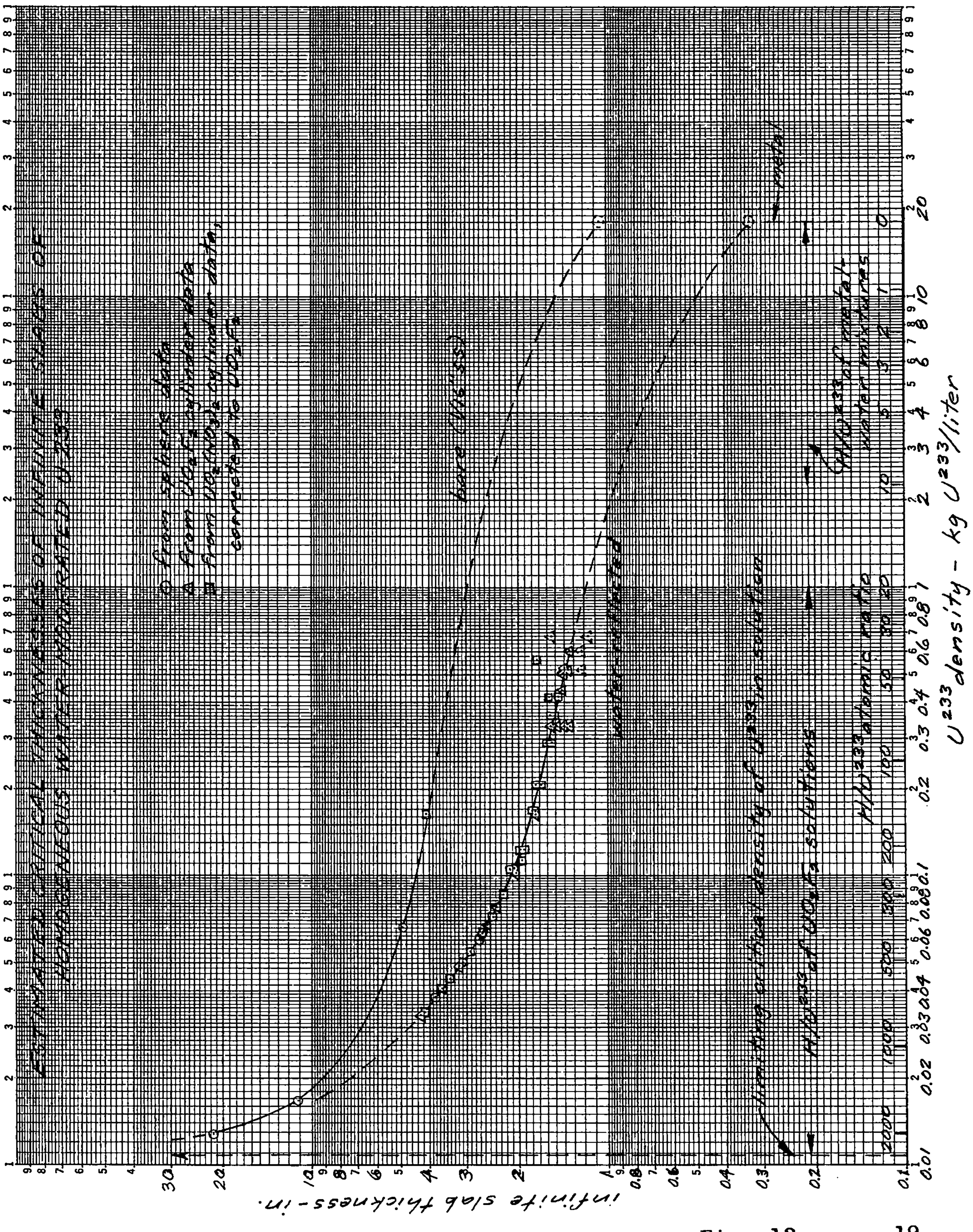

CRITICAL VOLUMES OF HOMOGENEOUS WATER-MODERATED U-233 SPHERES
○ sphere data
△ from UO2F2 cylinder data
□ from UO2(NO3)2 cylinder data
(corrected to UO2F2)
bare
water reflected
bare
limiting critical density of U233 in solution
H/U233 of metal-water mixtures
metal
H/U233 of UO2F2 solutions
H/U233 atomic ratio
U233 density - kg U233/liter
critical volume - liters

Fig. 10.



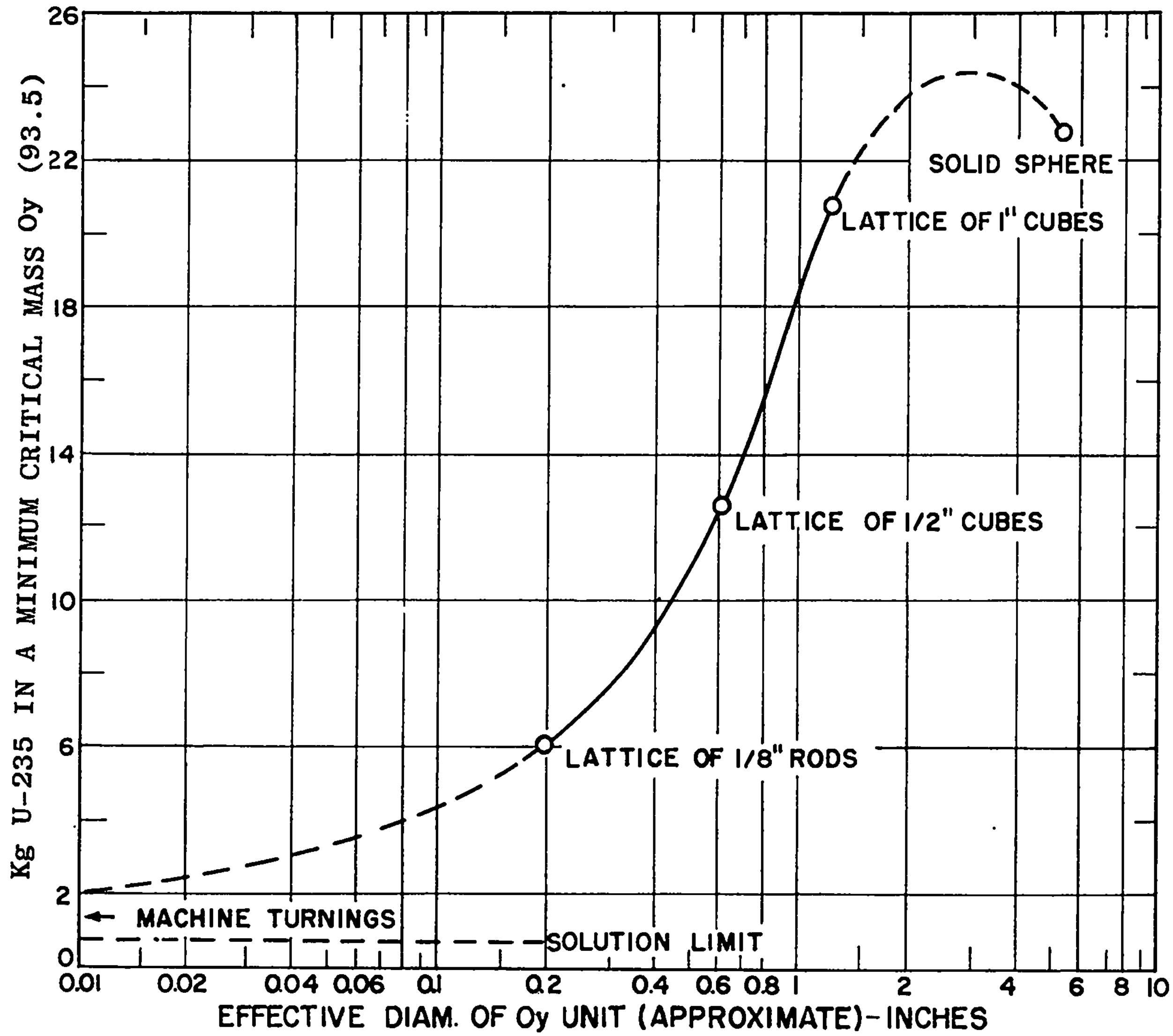
infinite-cylinder diameter-in.
U^{233} density-kg U^{233}/liter
H/U233 atomic ratio
limiting critical density of U233 in solution
H/U233 of UO2F2 solutions
H/U233 of metal-water mixtures
bare
water reflected
metal
O from sphere data
Δ from UO2F2 cylinder data

Fig. 11.



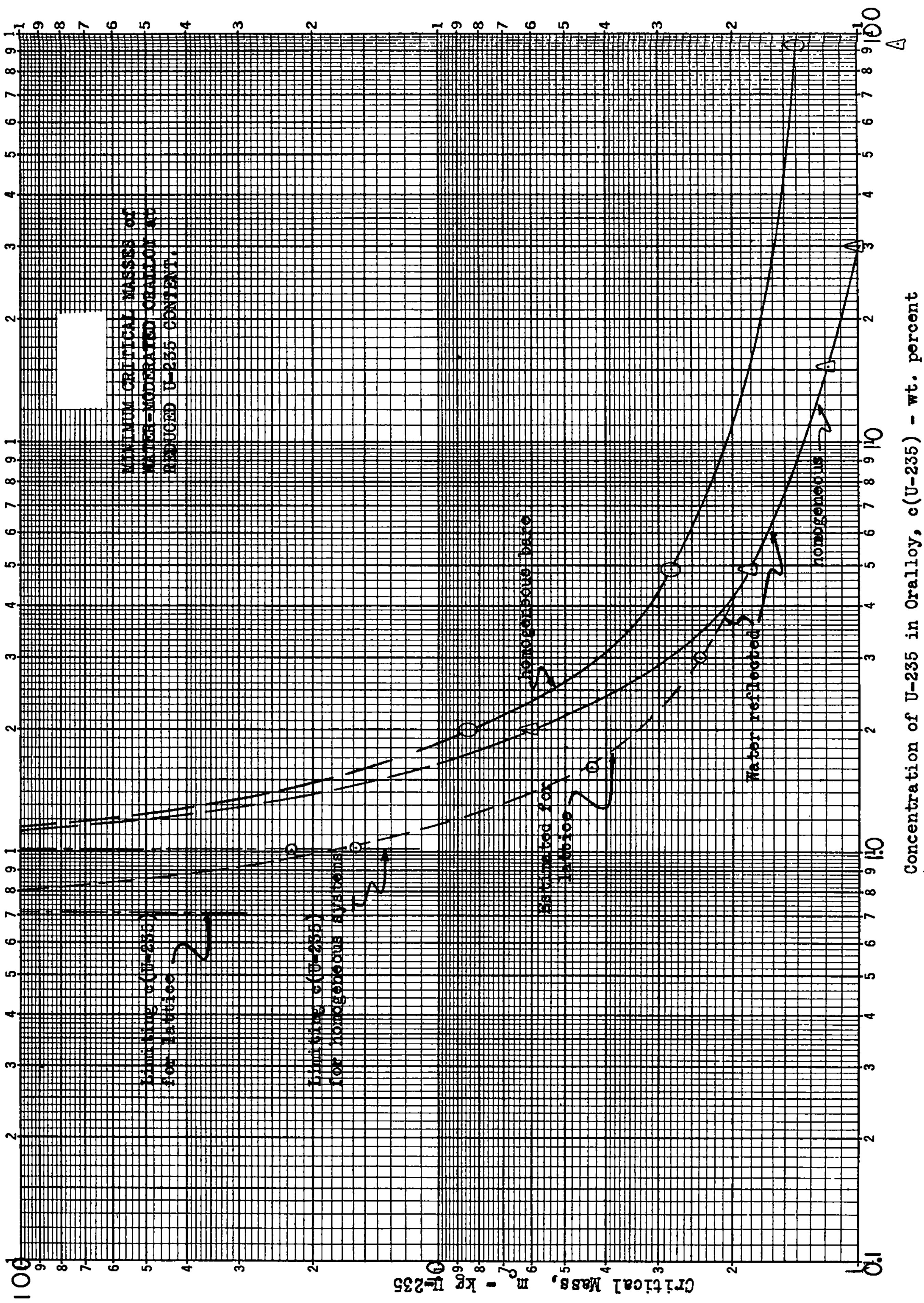
ESTIMATED CRITICAL THICKNESSES OF INFINITE SLABS OF HOMOGENEOUS WATER MODERATED U233
○ from sphere data
△ from UO2F2 cylinder data
□ from UO2(NO3)2 cylinder data, corrected to UO2F2
bare (1/16" SS)
water reflected
metal
limiting critical density of U233 in solution
H/U233 of UO2F2 solutions
H/U233 of metal water mixtures
H/U233 atomic ratio
U233 density - kg U233/liter
infinite slab thickness - in.

Fig. 12.



Minimum critical mass of flooded Oy (93.5) metal lattices as a function of oralloy unit size.

Fig. 13.



MINIMUM CRITICAL MASSES of WATER-MODERATED ORALLOY at REDUCED U-235 CONTENT.
homogeneous bare
Water reflected
homogeneous
Estimated for lattice
Limiting c(U-235) for lattice
Limiting c(U-235) for homogeneous systems
Concentration of U-235 in Oralloy, c(U-235) - wt. percent
Critical Mass, m_c - kg U-235

Fig. 14a.

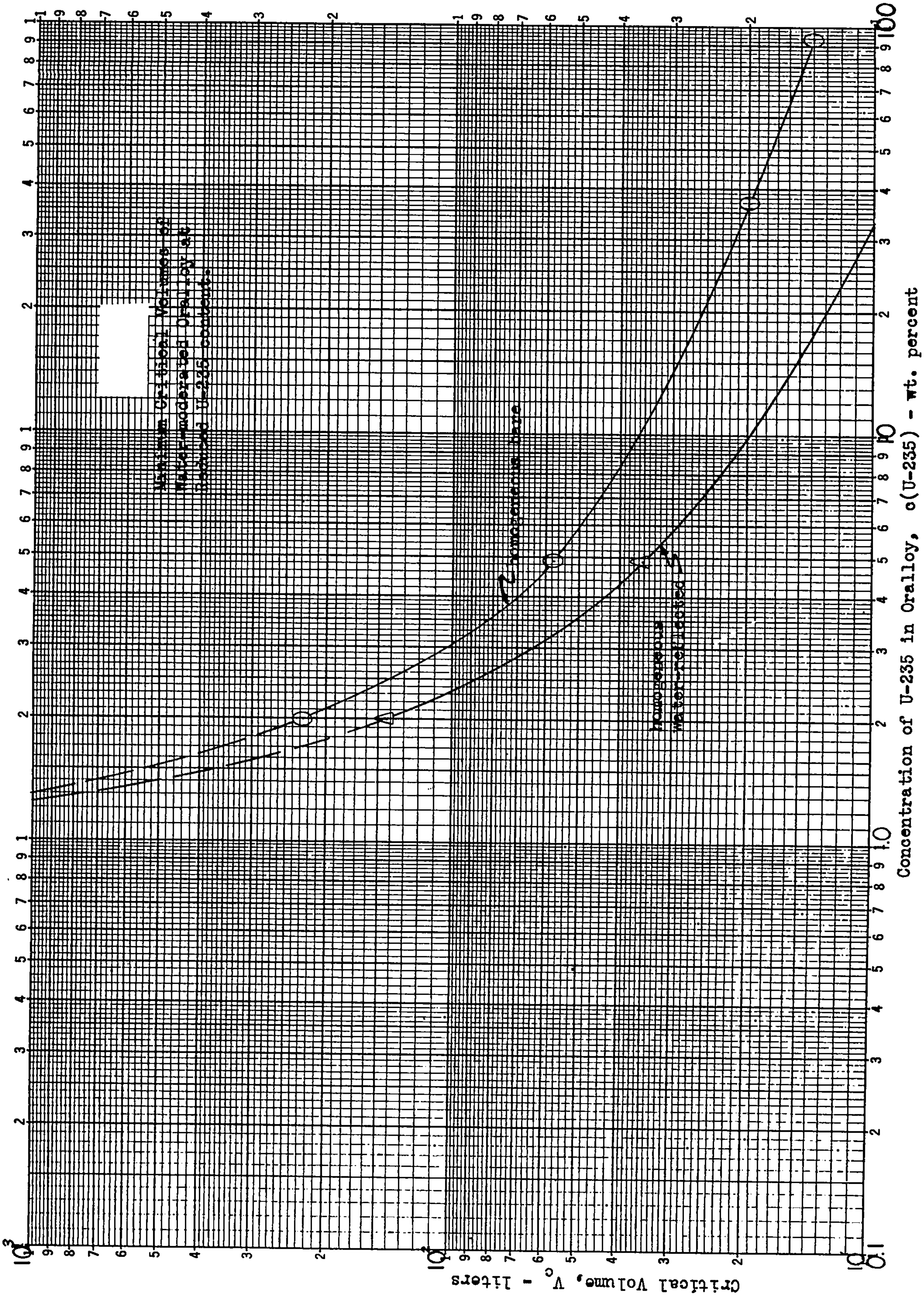
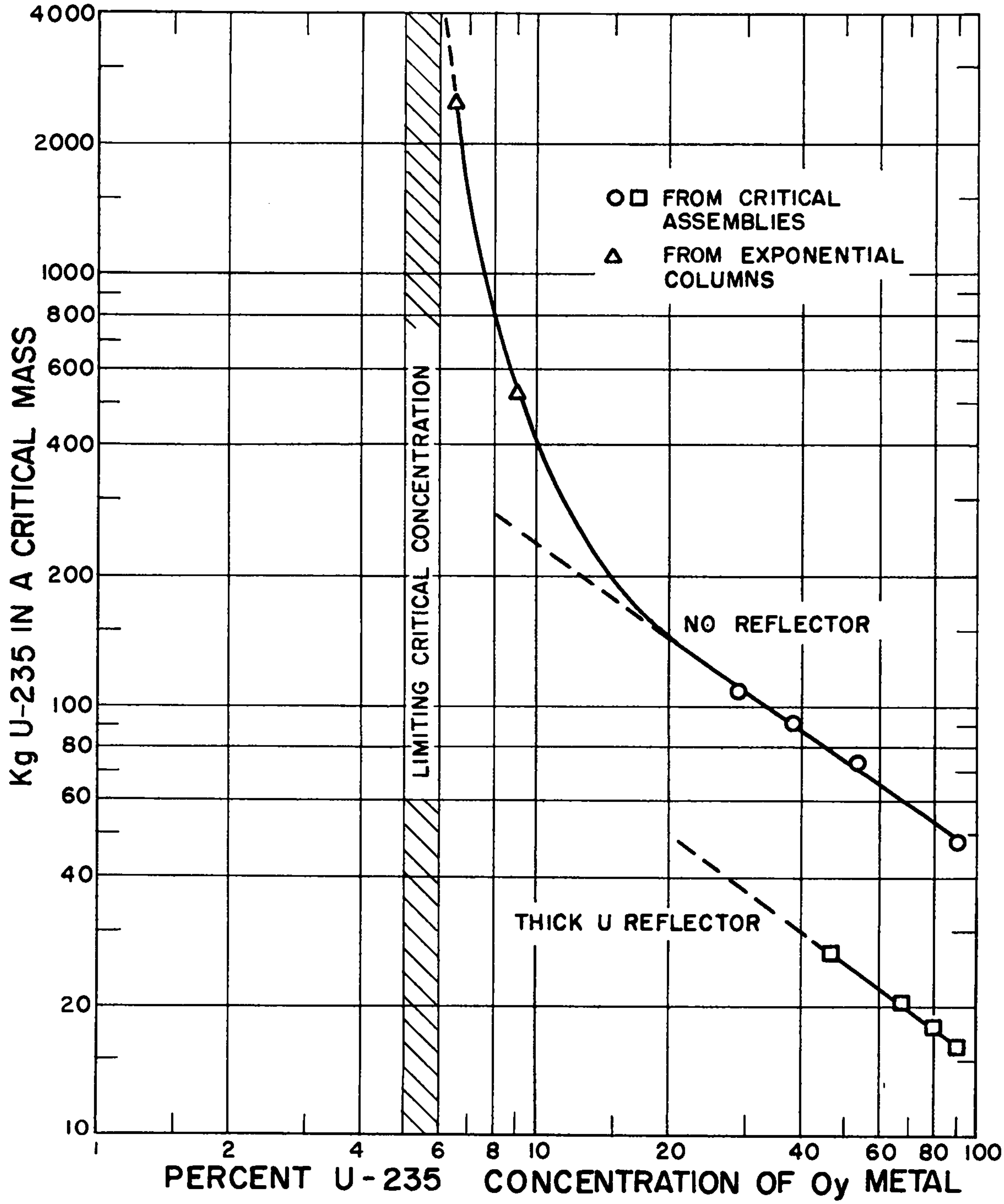

Minimum Critical Volumes of
Water-moderated Oralloy at
Reduced U-235 content
Homogeneous bare
Homogeneous
water-reflected
Concentration of U-235 in Oralloy, c(U-235) - wt. percent
Critical Volume, V_c - liters
0.1
1.0
10
100

Fig. 14b.



Critical mass vs. U-235 concentration of oralloy metal. (The shaded strip represents the range of uncertainty in the value of U-235 concentration below which oralloy metal cannot be made critical.)

Fig. 15.

Poisoned solutions

The influence of excess nitrate on critical mass of water-reflected Pu^{239} solutions is presented in Figure 16.[7] Observations on effects of heterogeneous poisons in U^{235} solutions are summarized in Table I[40-43] and Figure 17. The figure shows the influence of various degrees of Pyrex poisoning on the critical height of 20"-diam. aqueous solutions of U^{235}, both bare and water-reflected. The Raschig rings with which one point was obtained were 2.375" OD x 2" ID x 2.375" long and were packed randomly throughout the solution volume.

Data from the Physical Constants Testing Reactor[36] establish the quantity of uniformly-distributed boron that is required to reduce to unity the k_∞ of a fissionable mixture. For Oy(3.04% U^{235})O_3 - polyethylene mixtures, 0.37 atom B per atom of U^{235} (17 gm B/kg U^{235}) protects against criticality for the entire range of H/U^{235}; at $H/U^{235} = 1430$, $k_\infty = 1$ without boron. In the case of an Oy(2% U^{235})F_4 - paraffin mixture at $H/U^{235} = 195$, 0.25 atom B per atom of U^{235} gives $k_\infty = 1$, from which it is estimated that 0.26 atom B per atom of U^{235}($\sim$ 12 gm B/kg U^{235}) protects for all H/U^{235}.

Systems with nonhydrogenous diluents

Some effective cross sections from reactivity coefficient data and resulting dilution exponents for bare Oy(94) (Godiva), Oy(94) in an 8-1/2"-thick U reflector (Topsy), and bare Pu (Jezebel) are listed in Table II.[44] In terms of the dilution exponent n(x) for the material x, the critical mass of fissionable material diluted homogeneously with the volume fraction F of the material x is

$$m_c = m_{co}(1-F)^{-n}, \; F \ll 1,$$

where m_{co} is the critical mass of the undiluted system. In the cases of D_2O, graphite ($\rho_o = 1.67$ gm/cm^3) and BeO ($\rho_o = 2.86$ gm/cm^3) diluting unreflected Oy($\sim$ 93),[45-47] data exist over an extended range

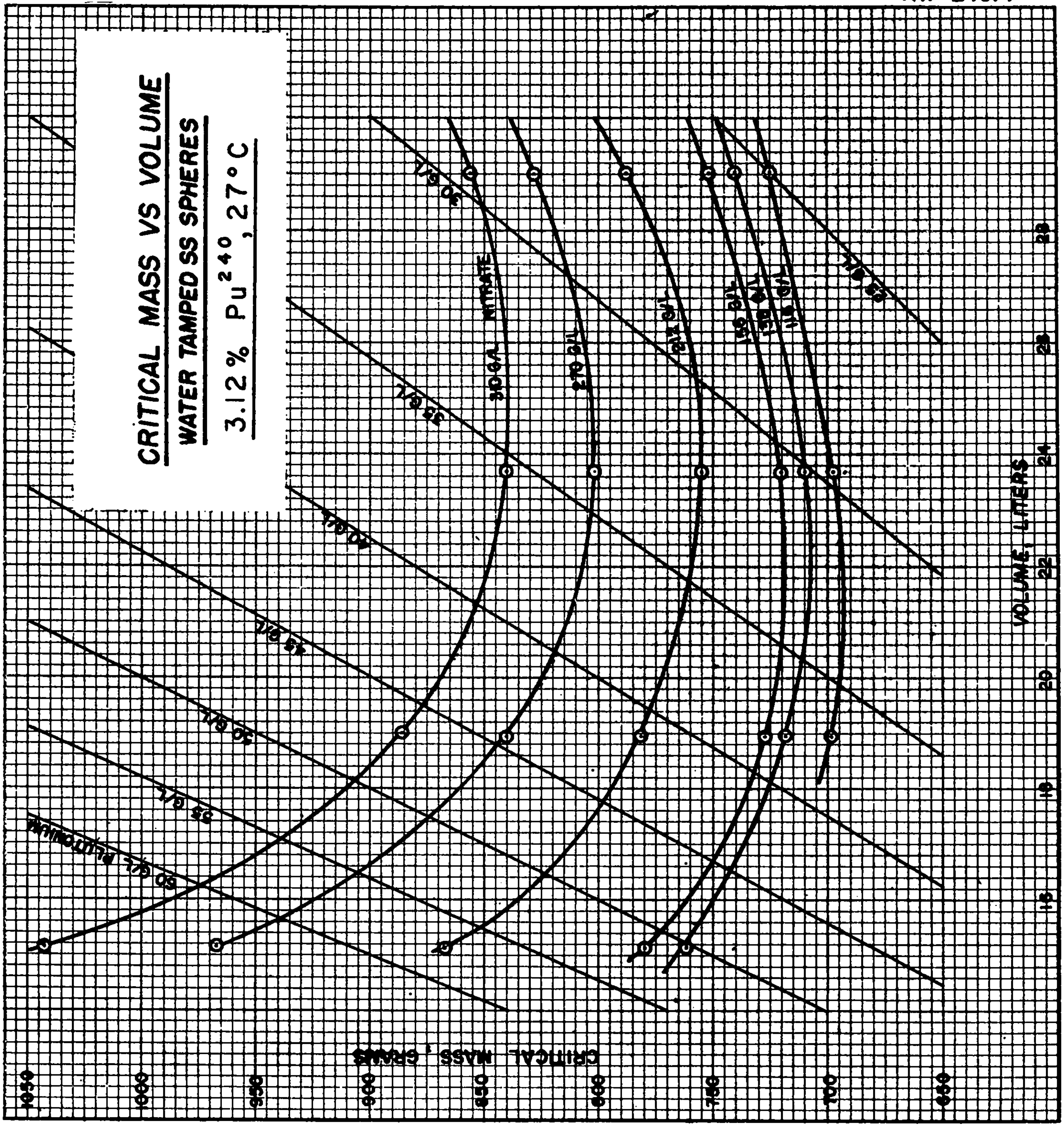
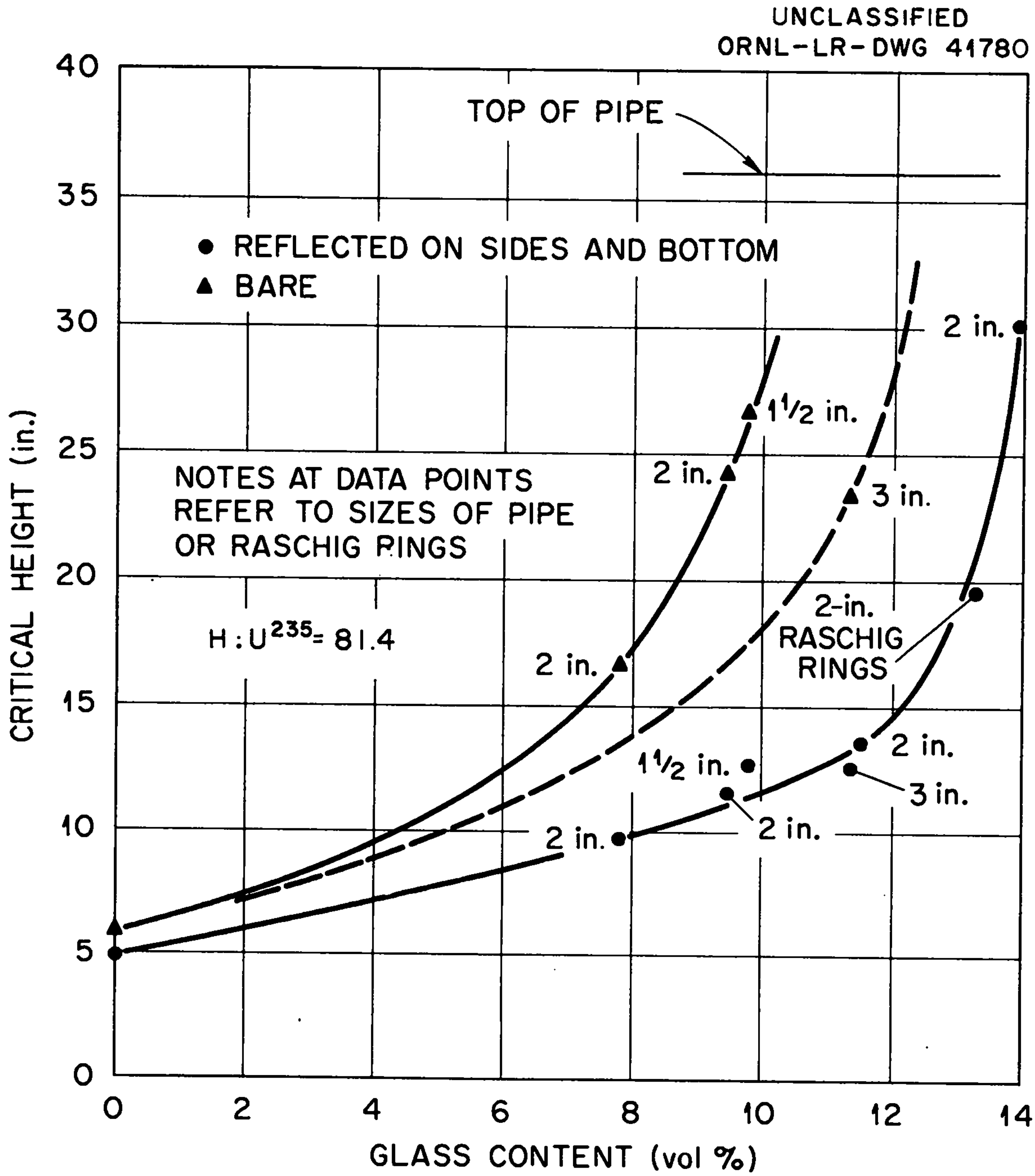

CRITICAL MASS VS VOLUME
WATER TAMPED SS SPHERES
3.12% Pu240, 27°C
VOLUME, LITERS
CRITICAL MASS, GRAMS

Fig. 16.

TABLE I.

U^{235} SOLUTIONS WITH HETEROGENEOUS POISONS

Container	Solution	Reflector	Poison	Critical Height
Oy (~ 93):				
15" diam. ss cylinder	OyO_2F_2 $H/U^{235} = 73.0$	water	136 steel rods, 7/8"diam.(49.2v/o of core)	37.5"
30" x 60" Al tank	OyO_2F_2 $H/U^{235} = 78.7$	water (half-reflected)	10 boral partitions 2.3" wide (3/8" boral, ~ 0.3 gmB/cm^3)	6.9"
10" diam. Al cylinder in 1/4"-thick Cu	OyO_2F_2 $H/U^{235} = 52.6$	water outside 1/4" Cu	33.7v/o Cu~0.15" thick, min. spacing ~ 3/4"	60"
42" diam. ss tank	$OyO_2(NO_3)_2$ $\leq$ 360 gm U^{235}/liter	concrete (on sides)	random-packed. Pyrex raschig rings, 1.5"ODx1.5"highx7/64"wall (17.8 v/o Pyrex containing 12-1/2 w/o B_2O_3)	subcritical at 460 liters solution
Oy (~ 87):				
20" diam. Al cylinder	$OyO_2(NO_3)_2$ $H/U^{235} = 81.4$	water on sides, bottom	Pyrex tubing, or rings $\leq$ 2" ID (~ 4 w/o B):	
			7.8 v/o glass	9.75"
			9.45 v/o glass	11.6"
			11.5 v/o glass	13.6"
			13.3 v/o glass	19.6"
			13.95 v/o glass	30.1"
			16.7 v/o glass	subcritical at 36" depth
"	same except $H/U^{235} = 141$	"	same, 7.8 v/o glass	12.5"
"	same except $H/U^{235} = 276$	"	same, 7.8 v/o glass	subcritical at 36" depth



CRITICAL HEIGHTS of 20-in-diam STAINLESS STEEL CYLINDERS

CONTAINING PYREX-POISONED SOLUTIONS of Oy(87.4)O_2 $(NO_3)_2$

Fig. 17.

TABLE II.

SELECTED MATERIAL REPLACEMENT RESULTS FOR TOPSY, GODIVA, AND JEZEBEL

		Topsy (Oy 94% in U)			Godiva (bare Oy 94%)			Jezebel (bare Pu)		
Element (x)	Density gm-atom/cm^3	$\bar{\sigma}_a(x)$[a] barn	$\bar{\sigma}_{tr}(x)$ barn	Dilution exponent[b] n(x)	$\bar{\sigma}_a(x)$[a] barn	$\bar{\sigma}_{tr}(x)$ barn	Dilution exponent[b] n(x)	$\bar{\sigma}_a(x)$[a] barn	$\bar{\sigma}_{tr}(x)$ barn	Dilution exponent[b] n(x)
C	0.185	-0.022	2.13	0.86	-0.028	2.17	1.02	0.016	2.15	1.30
O		-0.013	2.20					0.023	2.22	
Al	0.100	-0.006	2.12	1.04	-0.006	2.14	1.51	0.033	2.30	1.61
Cr	0.138	0.015	2.41	0.98						
Mn	0.135	0.009	2.70	0.95						
Fe	0.137	0.020	2.29	1.01	0.006	2.29	1.28	0.050	2.44	1.45
Ni	0.152	0.066	2.77	1.02	0.056	2.65	1.22	0.111	2.77	1.39
Cu	0.141	0.035	2.68	0.99	0.022	2.73	1.18	0.074	2.83	1.37
Zr	0.071	0.022	3.87	1.02				0.070	4.10	1.51
Nb	0.092	0.068	3.99	1.01						
Mo	0.106	0.032	4.58	0.89				0.105	3.99	1.33
Ta	0.092	0.155	3.91	1.12				0.232	4.34	1.48
W	0.105	0.097	4.40	0.99				0.182	4.60	1.30
Th	0.049_5	0.069	4.48	1.08	0.017	4.92	1.46	0.141	5.00	1.66
U^{233}	0.080	-3.22_0								
U^{235}	0.080	-1.89_3[c]			-1.86_0[c]			-1.82_8	5.3	
U^{238}	0.080	-0.228	5.10[c]		-0.299	5.0[c]		-0.238	5.1[a]	
Pu^{239}		-3.63_6			-3.56_1			-3.60_0[a]	5.3	
Pu^{240}		-2.58						-2.34		
Void				1.20			2.00			2.00

(Footnotes on next page)

TABLE II Footnotes

[a] $\bar{\sigma}_a(x) = \bar{\sigma}_c(x) - \bar{\sigma}_f(x) - \Delta\gamma\bar{\sigma}_s(x)$, where $\bar{\sigma}_c$ and $\bar{\sigma}_f$ are capture and fission cross-sections (suitably averaged), $\Delta\gamma$ is the increase in neutron effectiveness per central scattering and $\bar{\sigma}_s$ is scattering cross section.

[b] The critical mass of a system diluted by the volume fraction $F(x)$ of element x, $m_c(x)$, is related to the critical mass of the undiluted system $m_c(o)$, according to $m_c(x)/m_c(o) = [1-F(x)]^{-n}$; if $F(x) \ll 1$. Where $\rho_o(x)$ is the normal density of x in gm-atom/cm^3,

$$n(x) = 1.20 - \rho_o(x)\left[0.735\ \bar{\sigma}_{tr}(x) - 12.82\ \bar{\sigma}_a(x)\right], \text{ for Topsy;}$$

$$n(x) = 2.00 - \rho_o(x)\left[2.25\ \bar{\sigma}_{tr}(x) - 14.27\ \bar{\sigma}_a(x)\right], \text{ for Godiva;}$$

$$n(x) = 2.00 - \rho_o(x)\left[1.846\ \bar{\sigma}_{tr}(x) - 9.964\ \bar{\sigma}_a(x)\right], \text{ for Jezebel.}$$

[c] These values are used for normalization.

(Figure 18). Figure 19 gives critical masses of bare and U-reflected cylinders of Pu diluted by Al, Fe, U, and Th.[48]

Systems at reduced density

The dependence of critical mass (m_c) upon core density (ρ) has been determined for several spheres or nearly equilateral cylinders.[6,13] Values of n in the relation m_c = const $(\rho/\rho_o)^{-n}$ are

1.20 for Oy(94) metal in 8-1/2" U reflector

1.57 for Oy(93) H_3C in 8" thick U reflector

1.88 for Oy(93) O_2F_2 solution at H/U^{235} = 230 in thick water reflector (possibly influenced by void geometry)

~ 1.1 for Pu^{239} metal in a reflector corresponding to thick U (from Figure 22)

Where density of both core and reflector of a spherical system are changed by the ratio ρ/ρ_o, and the ratio of reflector thickness to core radius is maintained, then n = 2 (the value for an unreflected spherical core).

In the case of an infinite slab, the critical mass per unit area is necessarily independent of ρ.

Spherical systems with various reflectors

Critical masses of unmoderated Oy (93.5) metal spheres are given for various reflectors as functions of reflector thickness in Figures 20 and 21, with supplementary data in Table III.[6] Figure 22 gives critical masses of U^{233} metal, δ-phase Pu^{239} and α-phase Pu^{239} in terms of the critical mass of Oy (93.5) metal in a reflector of the same composition and thickness.[34,49-51] As the existing data show no distinction between nonmoderating and moderating reflectors (of limited thickness), these curves provide a basis for estimating critical masses of the other materials from the abundant data for Oy.

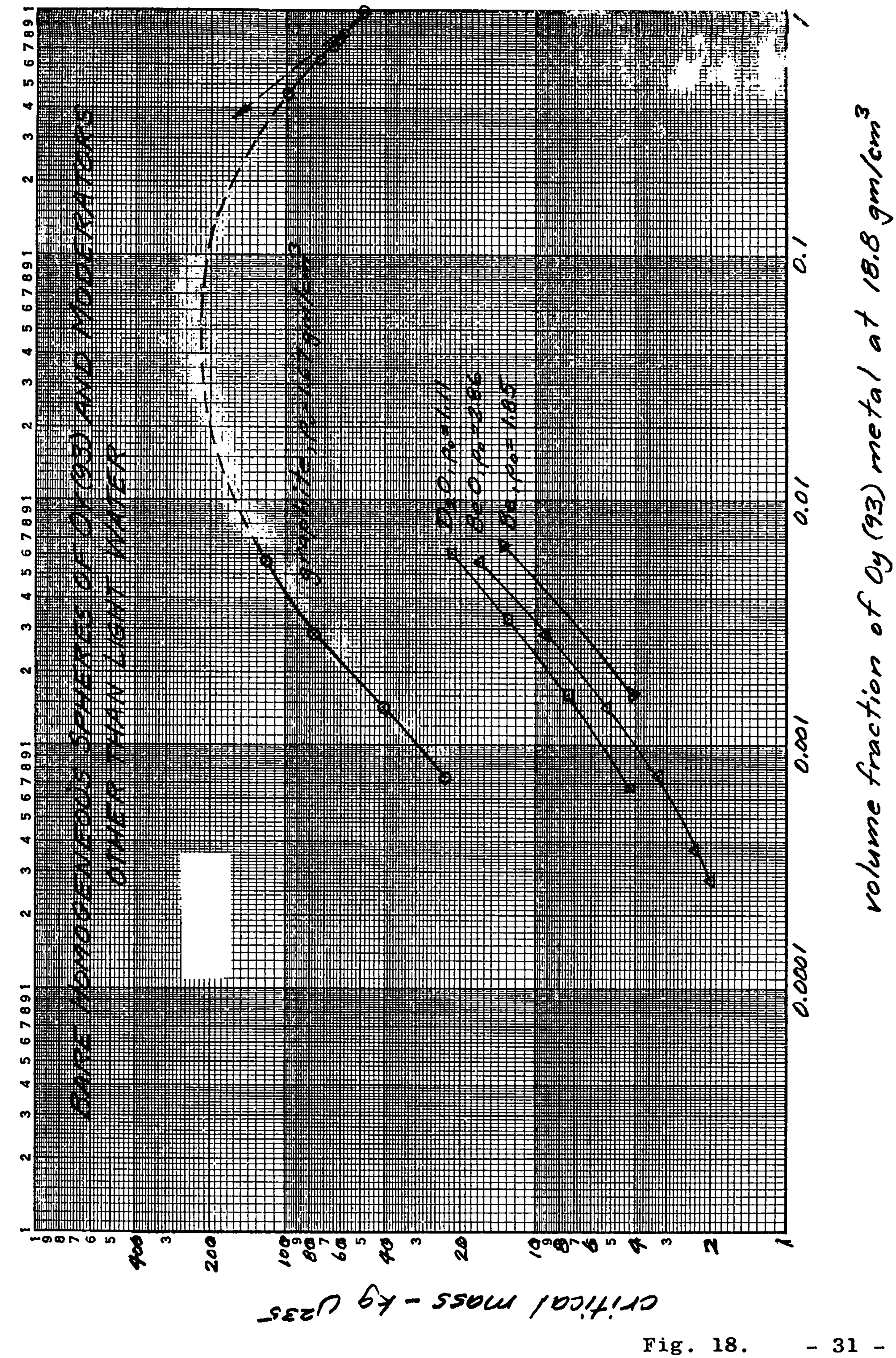

BARE HOMOGENEOUS SPHERES OF Oy(93) AND MODERATORS
OTHER THAN LIGHT WATER
graphite, ρ_0=1.67 gm/cm³
D_2O, ρ_0=1.1
BeO, ρ_0=2.86
Be, ρ_0=1.85
volume fraction of Oy (93) metal at 18.8 gm/cm³
0.0001
0.001
0.01
0.1
1
critical mass - kg U^{235}
2
4
10
20
40
60
80
100
200
400

Fig. 18.

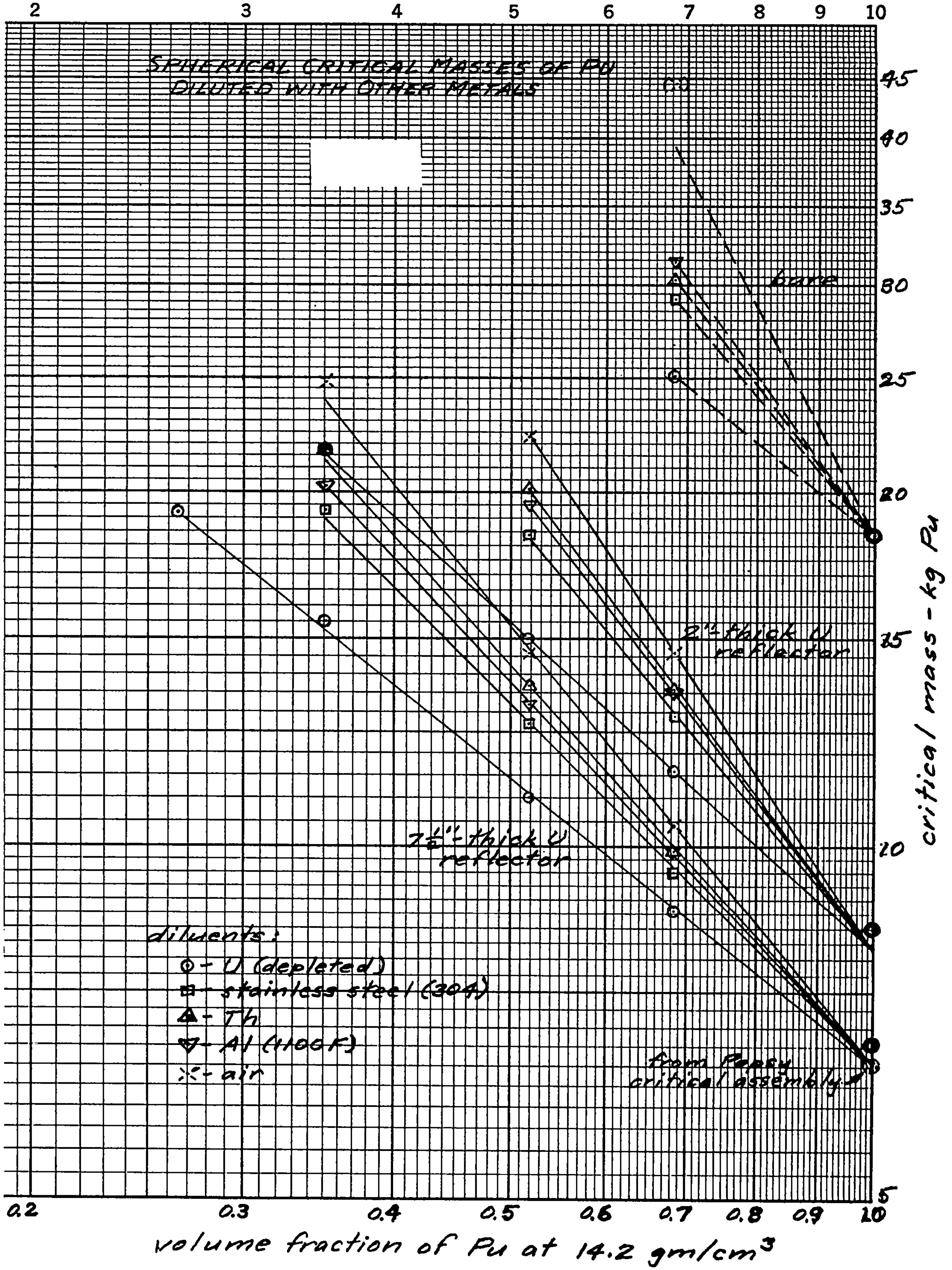

SPHERICAL CRITICAL MASSES OF Pu
DILUTED WITH OTHER METALS
bare
2"-thick U
reflector
7½"-thick U
reflector
diluents:
⊙ - U (depleted)
⊡ - stainless steel (304)
▲ - Th
▽ - Al (1100F)
✕ - air
from Popsy
critical assembly
critical mass - kg Pu
volume fraction of Pu at 14.2 gm/cm³
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
5
10
15
20
25
30
35
40
45

Fig. 19.

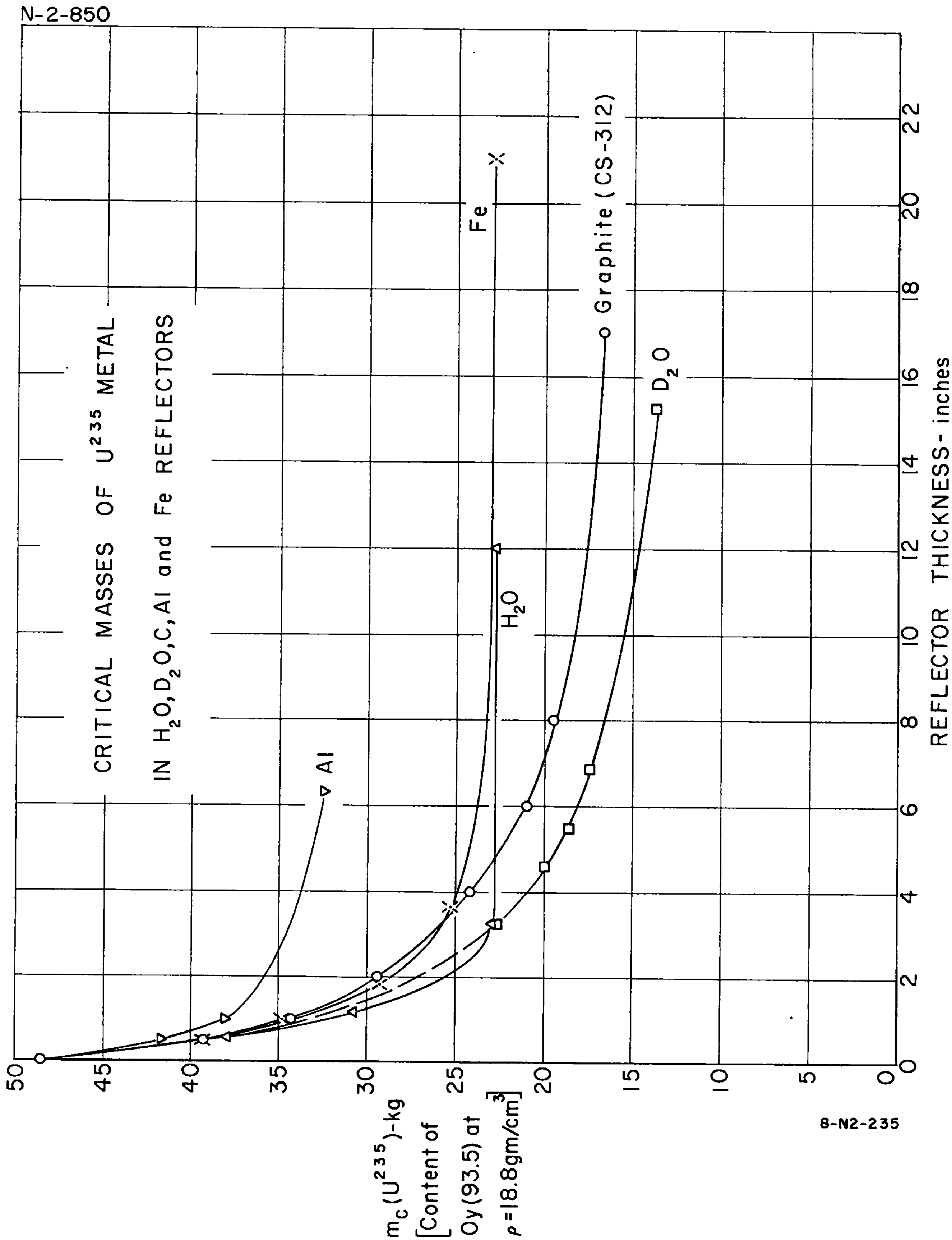

N-2-850
CRITICAL MASSES OF U^{235} METAL
IN H_2O, D_2O, C, Al and Fe REFLECTORS
Fe
Graphite (CS-312)
D_2O
H_2O
Al
REFLECTOR THICKNESS - inches
$m_c(U^{235})$-kg
[Content of
Oy(93.5) at
ρ=18.8gm/cm^3]
0
2
4
6
8
10
12
14
16
18
20
22
0
5
10
15
20
25
30
35
40
45
50
8-N2-235

Fig. 20.

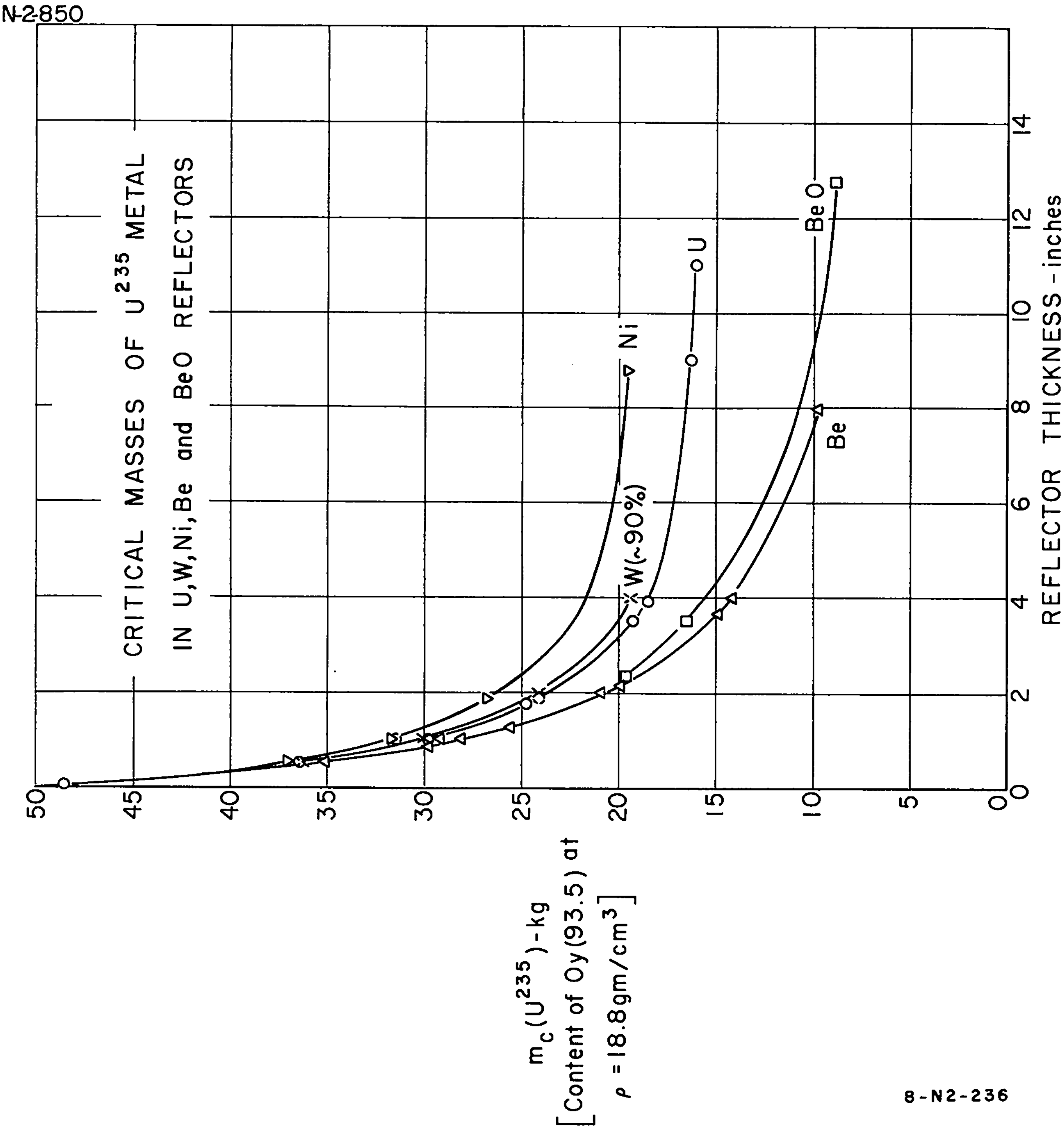
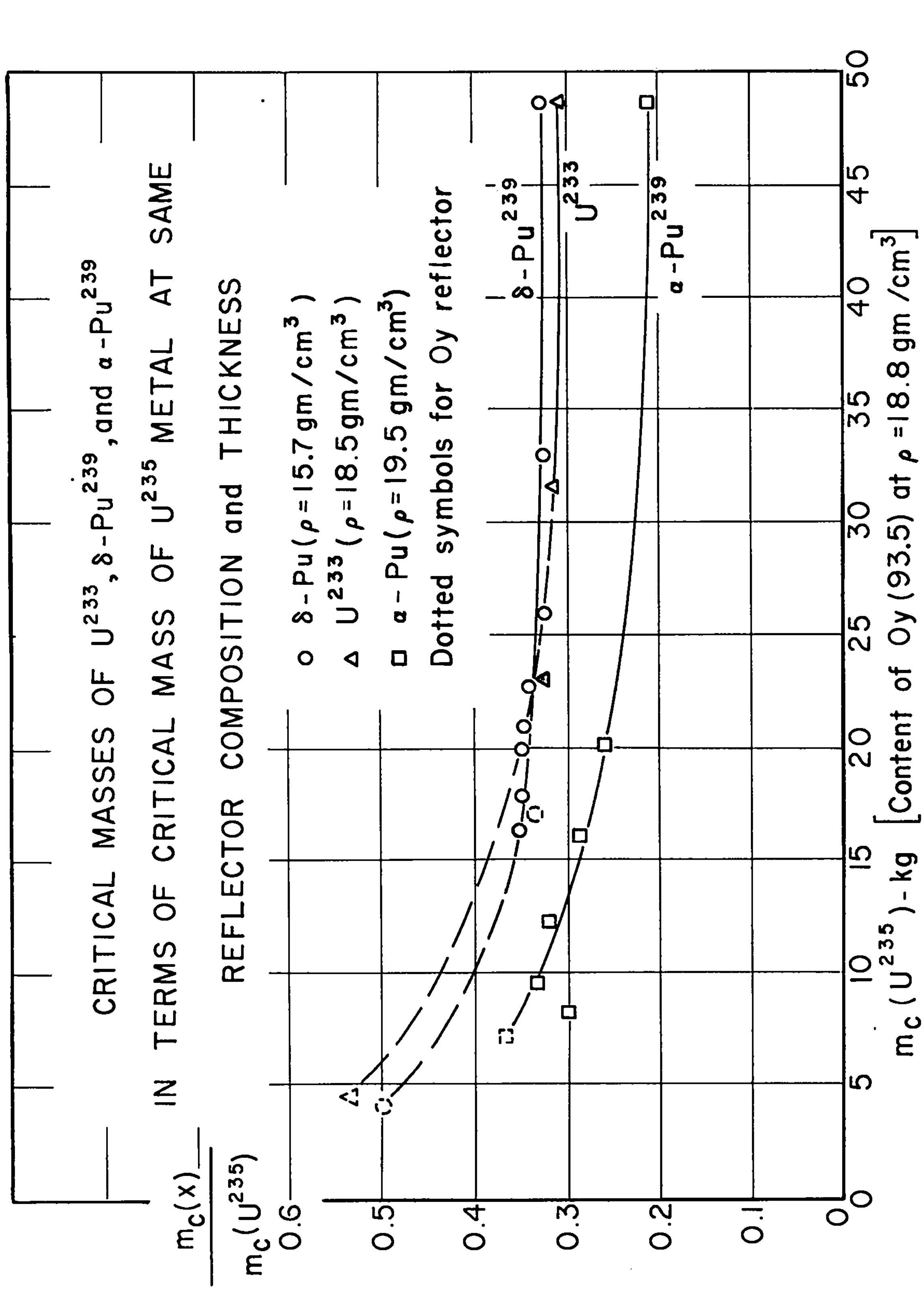

N-2850
CRITICAL MASSES OF U^{235} METAL
IN U,W,Ni,Be and BeO REFLECTORS
Ni
U
BeO
W(~90%)
Be
REFLECTOR THICKNESS - inches
$m_c(U^{235})$-kg
[Content of Oy(93.5) at
ρ = 18.8gm/cm^3]
8-N2-236

TABLE III.

CRITICAL MASSES OF SPHERICAL ORALLOY (93.5 w/o U-235) WITH VARIOUS REFLECTORS

Reflector (ρ-gm/cm^3)	Data Adjusted to the Following Standard Reflector Thicknesses. Critical mass - kg U-235 at ρ(Oy)=18.8 gm/cm^3: 1 in.	2 in.	4 in.	infinite	Effective σ_{tr}-cm^{-1}
Be (QMV, ρ = 1.84)	29.2	20.8	14.1		~0.25
BeO (ρ = 2.69)		21.3	15.5	~ 8.9	
WC (ρ = 14.7)		21.3	16.5	~16.0	
U (ρ = 19.0)	30.8	23.5	18.4	16.1	0.25
W-alloy (~92% W, ρ=17.4)	31.2	24.1	19.4		~0.25
Paraffin	(32.6)			21.8	
H_2O	(33.5)	~24.0	22.9	22.8	
D_2O		(27)	21.0	~13.6	
Cu (ρ = 8.88)	32.4	25.4	20.7		0.23
Ni (ρ = 8.88)	33.0	25.7	(21.5)	19.6	0.23
Al_2O_3 (ρ = 2.76)	35.1				
Graphite (CS-312, ρ=1.69)	35.5	29.5	24.2	~16.7	0.18
Fe (ρ = 7.87)	36.0	29.3	25.3	23.2	0.19
Zn (ρ = 7.04)		29.8	25.0		0.18
Th (ρ = 11.48)		33.3			~0.14
Al (2S, ρ = 2.70)	39.3	(35.5)	(32)	$<$30.0	0.13
Ti (ρ = 4.50)	39.7				0.12
Mg (ρ = 1.77)	41.0				0.10



CRITICAL MASSES OF U^{233}, δ-Pu^{239}, and α-Pu^{239}
IN TERMS OF CRITICAL MASS OF U^{235} METAL AT SAME
REFLECTOR COMPOSITION and THICKNESS
o δ-Pu (ρ=15.7 gm/cm³)
Δ U^{233} (ρ=18.5 gm/cm³)
□ α-Pu (ρ=19.5 gm/cm³)
Dotted symbols for Oy reflector
δ-Pu^{239}
U^{233}
α-Pu^{239}
$\frac{m_c(x)}{m_c(U^{235})}$
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
45
50
$m_c(U^{235})$-kg [Content of Oy (93.5) at ρ=18.8 gm/cm³]

Fig. 22.

No such range of reflector data exists for solutions. It has been observed that the same thickness of iron is essentially equivalent to the inner two inches (or less) of a thick water reflector about U^{235} solution.[13] Similar replacements show that plexiglas is a slightly more effective reflector than water. Figure 23 shows critical height of a 10"-diameter U^{235} solution (0.337 kg U^{235}/liter) as a function of thickness of lateral water reflector and of lateral furfural reflector.[5] The critical height of a slab of U^{235} solution (0.483 kg U^{235}/liter), 4' wide x 6" thick, vs. thickness of Al reflector on each face is given by Figure 24.[11]

Cylinders of various height/diameter ratios

Ratios of critical masses of cylinders (height h, diameter d) to those for spheres appear vs. h/d in Figure 25 for U^{235} solutions[2] and in Figure 26 for Oy(93.5) metal.[6,32] For extrapolation to broad slabs and long cylinders, the following alternative representation is more convenient. The interrelationships between critical cylinders of various height/diameter may be given in terms of effective extrapolation lengths, δ_c, which satisfy

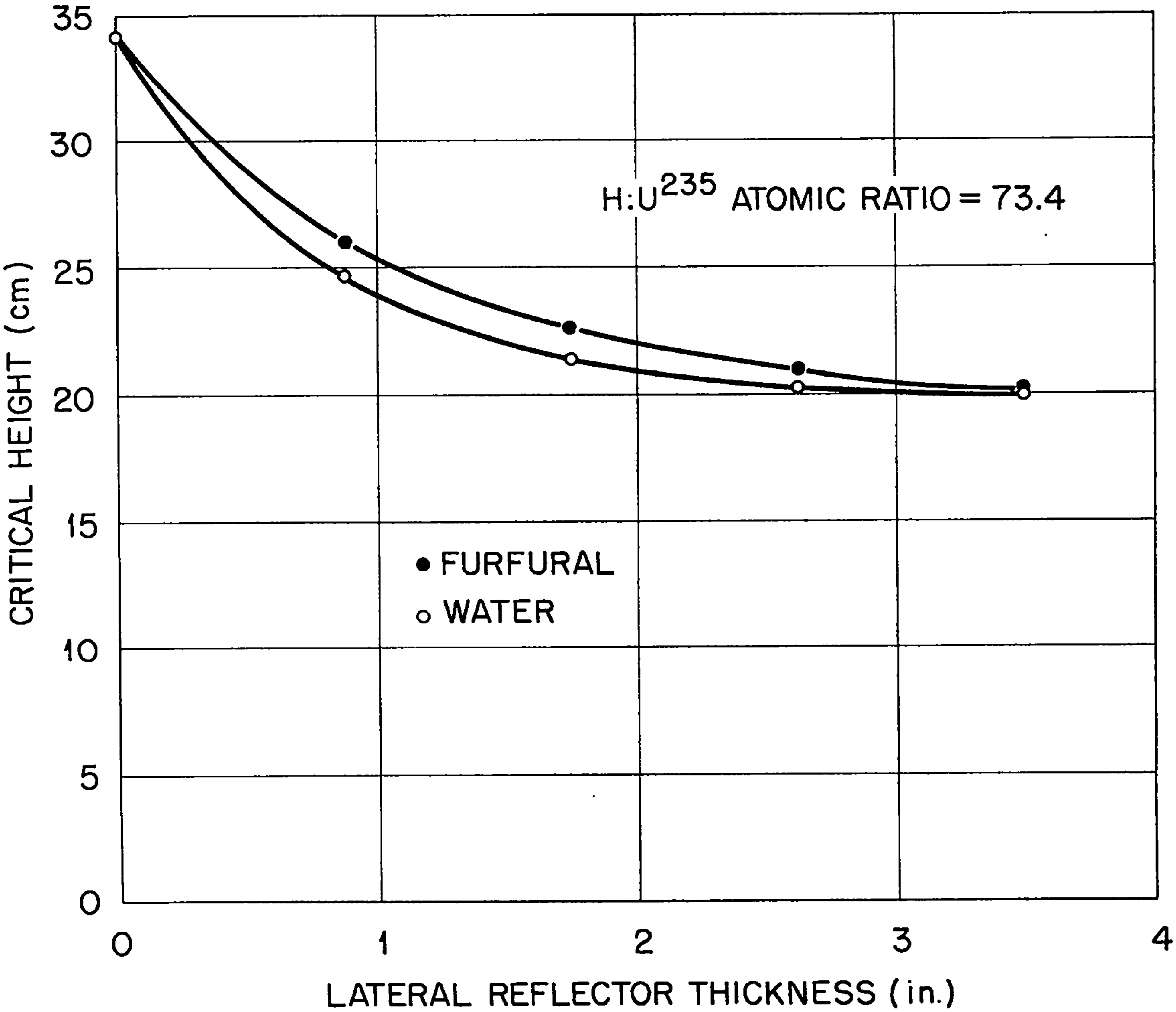
$$\left(\frac{2.405}{\frac{d}{2} + \delta_c}\right)^2 + \left(\frac{\pi}{h + 2\delta_c}\right)^2 = B_s^{\,2}$$

where $B_s^{\,2}$ is an assumed constant buckling (e.g., that of the corresponding sphere). Such extrapolation lengths are shown by Figure 27 for families of solution cylinders that are either bare or water-reflected, and similarly by Figure 28 for metals.[52]

Other shapes

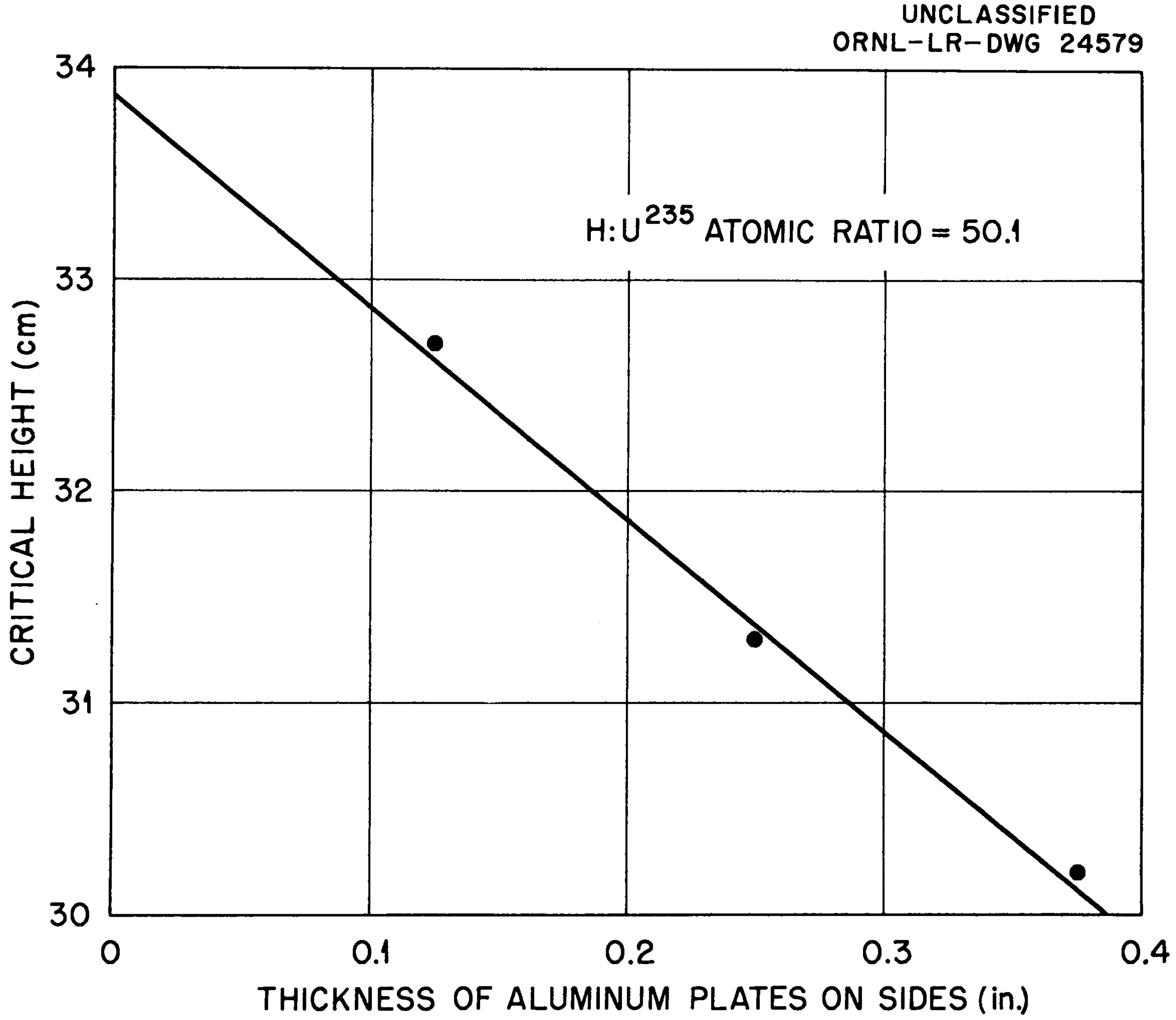
Investigations of the possibility of large-volume solution storage in annular cylinders led to the data of Figures 29 and 30, which apply to critical annuli with inner cylinder Cd-lined and water-filled.[5,53] Similar data exist for solution annuli with internal water but no Cd, and without either water or Cd.

UNCLASSIFIED
ORNL–LR–DWG 16556



Critical Height as a Function of the Thickness of a Water or Furfural Reflector on the Lateral Surface of a 10-in.-dia Aluminum Cylinder Containing an Enriched U^{235} Solution.

Fig. 23.



CRITICAL HEIGHT of a 6-in-thick SLAB of U^{235} SOLUTION as a FUNCTION of THICKNESS of Al on EACH FACE of the SLAB

Fig. 24.

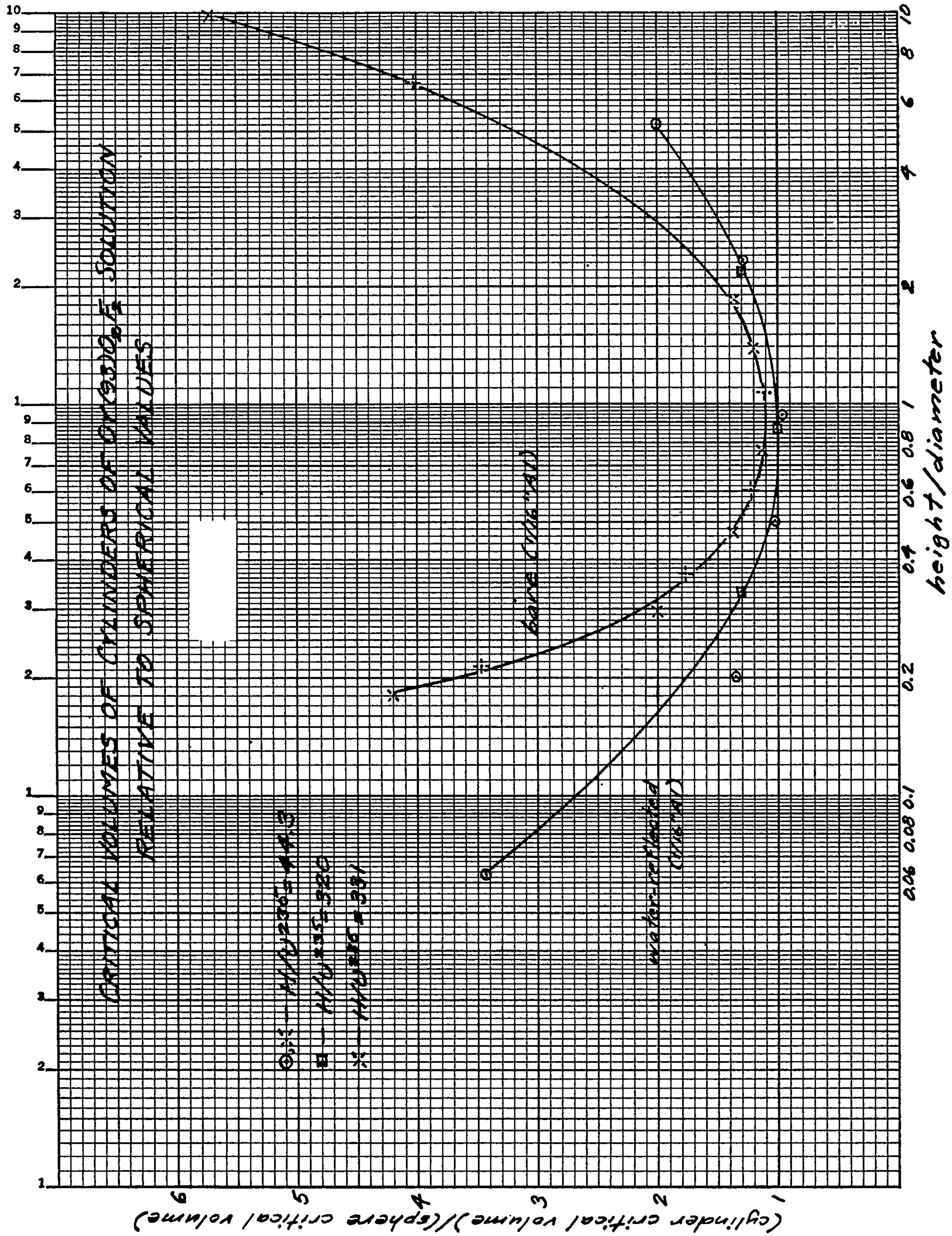
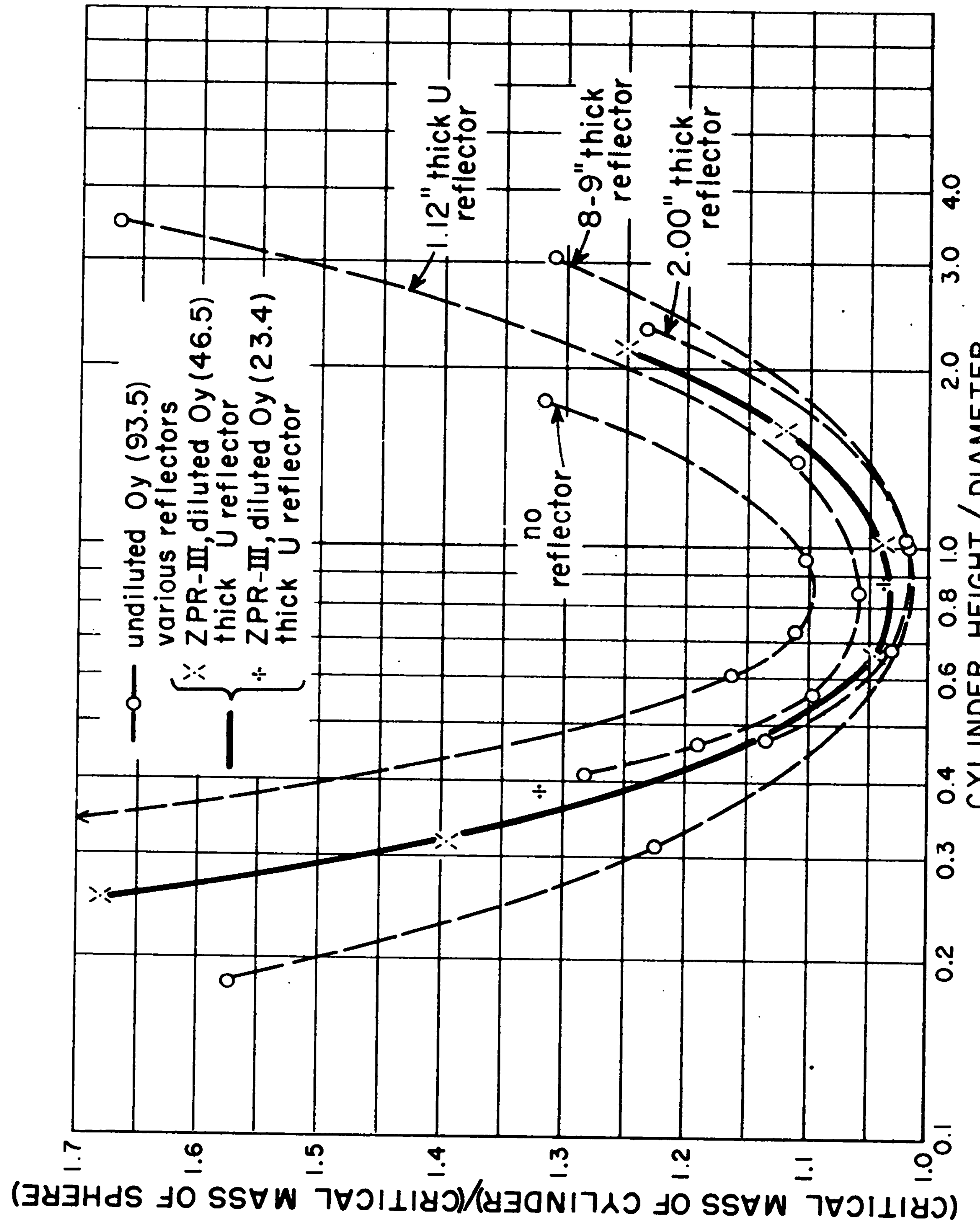

CRITICAL VOLUMES OF CYLINDERS OF U(93)O2F2 SOLUTION
RELATIVE TO SPHERICAL VALUES
⊙ — H/U235 = 44.3
■ — H/U235 = 320
✳ — H/U235 = 331
bare
water-reflected
height/diameter
(cylinder critical volume)/(sphere critical volume)

Fig. 25.



undiluted Oy (93.5) various reflectors
ZPR-III, diluted Oy (46.5) thick U reflector
ZPR-III, diluted Oy (23.4) thick U reflector
1.12" thick U reflector
8-9" thick reflector
2.00" thick reflector
no reflector
CYLINDER HEIGHT / DIAMETER
(CRITICAL MASS OF CYLINDER)/(CRITICAL MASS OF SPHERE)
0.1
0.2
0.3
0.4
0.6
0.8
1.0
2.0
3.0
4.0
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7

Fig. 26.

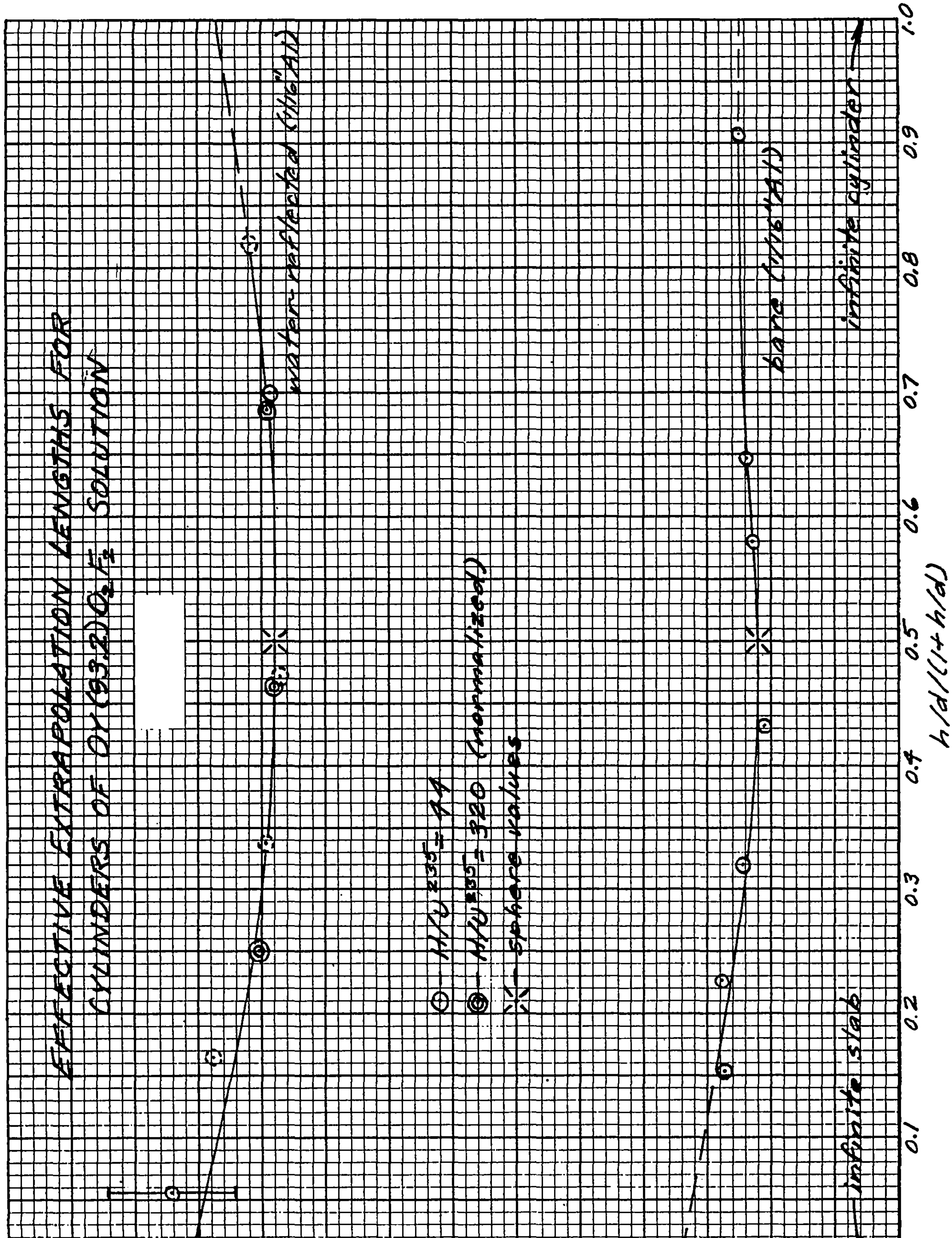
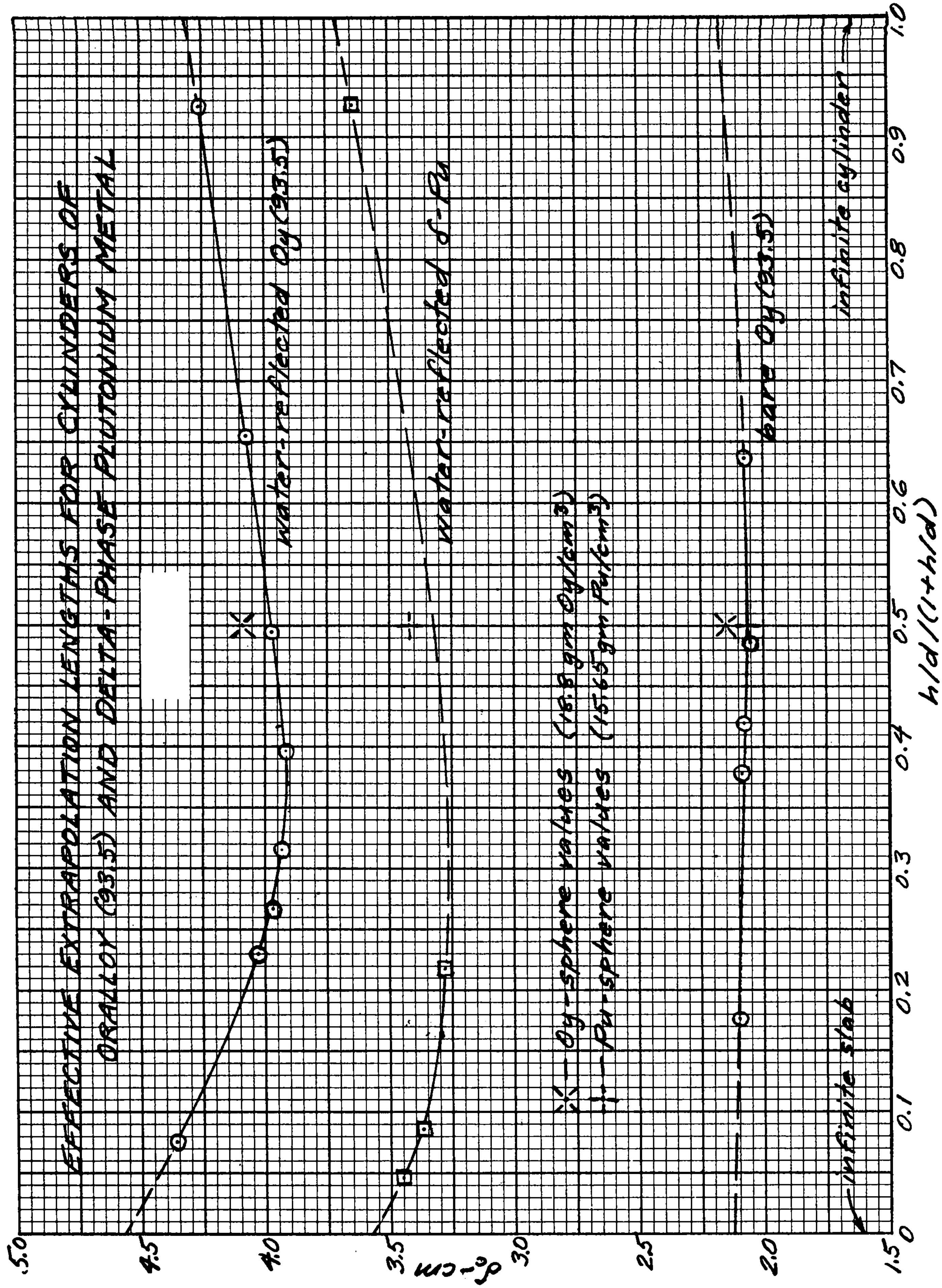
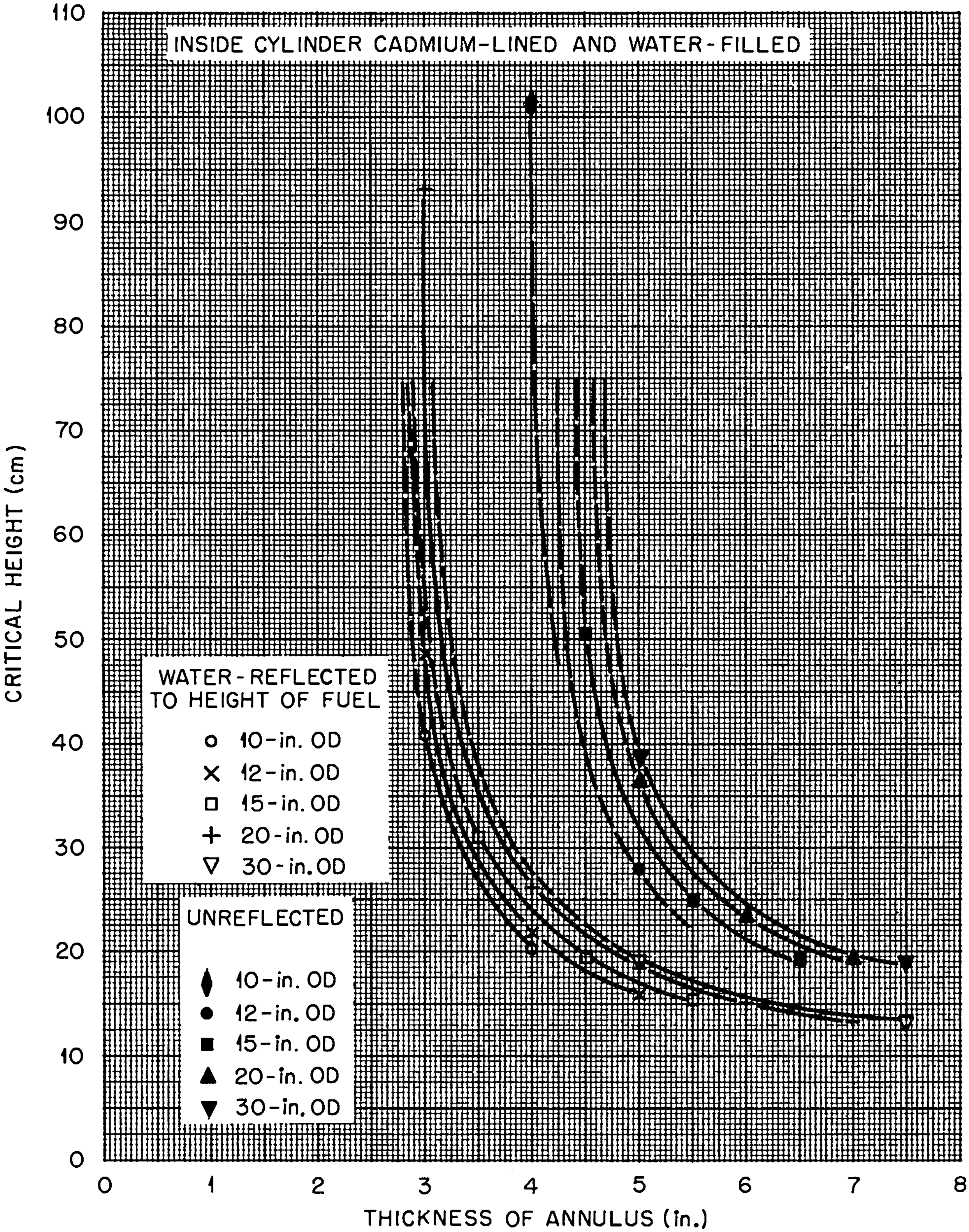

EFFECTIVE EXTRAPOLATION LENGTHS FOR
CYLINDERS OF Oy(93.2)O2F2 SOLUTION
water reflected (1/16" Al)
bare (1/16" Al)
H/U235 = 44
H/U235 = 320 (normalized)
sphere values
infinite slab
infinite cylinder
h/d/(1+h/d)
dc-cm
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1
2
3
4
5
6
7

Fig. 27.



EFFECTIVE EXTRAPOLATION LENGTHS FOR CYLINDERS OF
ORALLOY (93.5) AND DELTA-PHASE PLUTONIUM METAL
water-reflected Oy (93.5)
water-reflected δ-Pu
bare Oy (93.5)
infinite cylinder
infinite slab
Oy-sphere values (18.8 gm Oy/cm³)
Pu-sphere values (15.65 gm Pu/cm³)
h/d/(1+h/d)
δ-cm
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

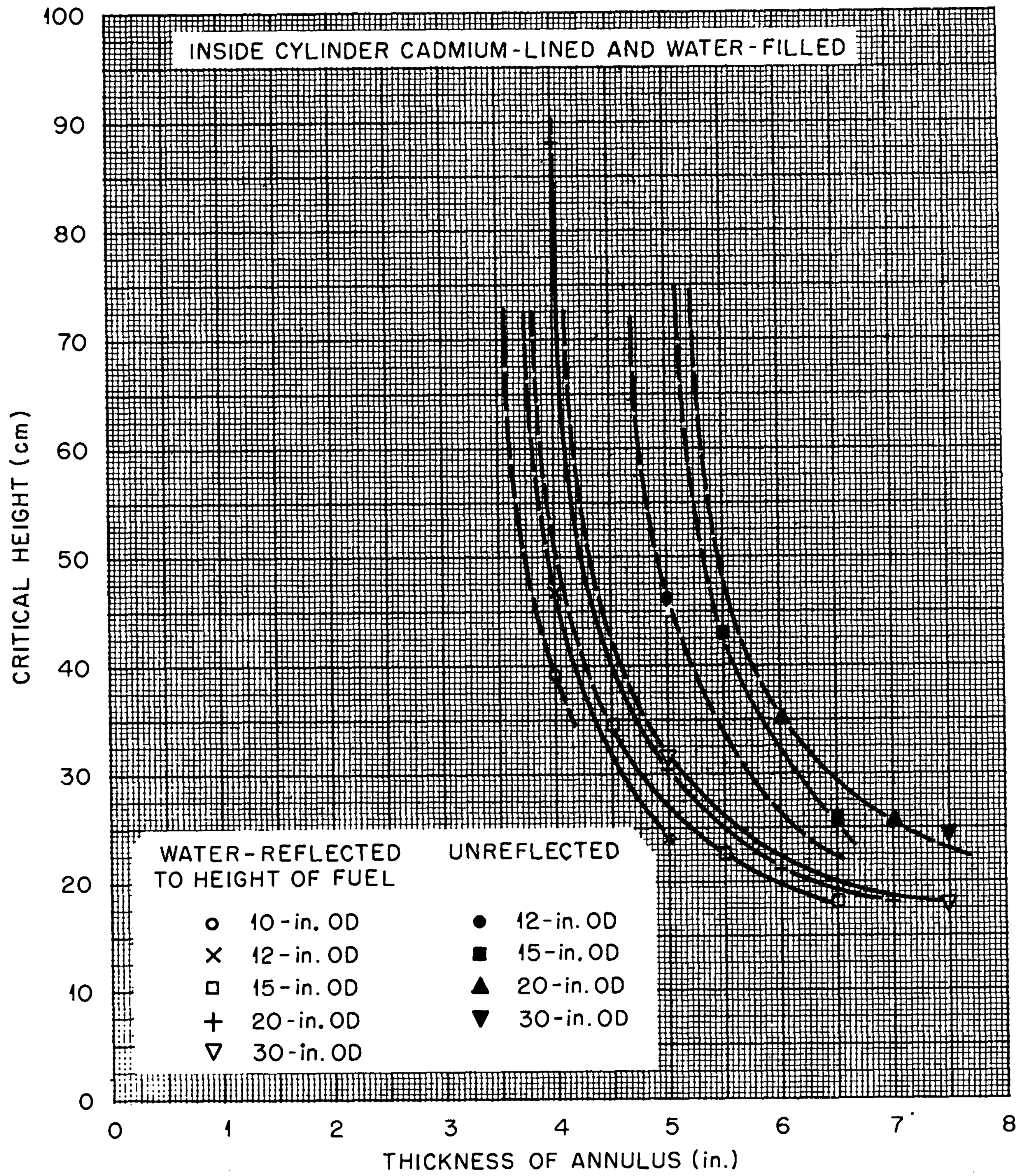
Fig. 28.



Critical Heights of Cylindrical Annuli Containing Aqueous Solutions of 93.2% U^{235}-Enriched Uranyl Fluoride as a Function of the Thicknesses of the Annuli: $H:U^{235}$ Atomic Ratio = 50.4

Fig. 29.

UNCLASSIFIED
ORNL-LR-DWG 31744



Critical Heights of Cylindrical Annuli Containing Aqueous Solutions of 93.2% U^{235}-Enriched Uranyl Fluoride as a Function of the Thicknesses of the Annuli: $H:U^{235}$ Atomic Ratio = 309.

Fig. 30.

Table IV gives results of a few observations on critical Oy(~ 93) metal annuli in various reflectors.

Critical and subcritical data on several solution-filled crosses and diagonal pipe intersections appear in Table V.(5)

<u>Metal-solution systems</u>

Figure 31 shows the relation between critical thickness of a 10" x 16" slab of Oy(~ 90) metal vs. U^{235} concentration of a uranyl nitrate solution in which the slab is immersed.(54) The solution is a cylinder 30" diam. x 28" high. From these data and measurements on 16" x 20" slabs, the curves of Figure 32 for slabs in infinitely-thick solution have been deduced. With the 10" x 16" slab, 1 gm Cd per liter of solution (as cadmium nitrate) compensates for 7 gm U^{235} per liter.

The critical thickness of a 5" x 8" slab of Oy(~ 90) metal on the axis of a 9.45" diam. x 16" cylinder of uranyl nitrate solution appears in Figure 33 vs. U^{235} concentration in the solution.(55)

<u>Some subcritical observations</u>

Numerous multiplication measurements, while not establishing actual critical configurations, have been sufficient to show that certain systems are subcritical. A few conservatively subcritical systems that help fill gaps in critical data follow (others appear in Tables I and V).

1a) Close-packed array of 4 polyethylene containers (7-1/4" ID x 1/4" wall) containing 7"-deep Oy(~ 93)O_2F_2 solution at $H/U^{235} = 260$ (480 gm U^{235} per container), standing on stainless-steel floor of a hood.(56)

1b) Close-packed array of 6 of the units of 1a) after precipitation of the uranium as a 2-1/4"-deep peroxide layer at $H/U^{235} \sim 75$; 5"-thick uranium-free solution above the peroxide. Apparently less reactive than 1a).

TABLE IV.

CRITICAL MASSES OF 12-1/4" OD x 6" ID ANNULI

OF OY (~ 93) METAL

reflector (material, thickness)	critical mass (kg Oy)	critical height (in. Oy)
1" normal U, complete (some excess)	82.7 ± 0.3	3.01
3" normal U, complete	55.9 ± 0.3	2.03
3" polyethylene, complete	60.6 ± 0.3	2.20
2" CS-312 graphite (inner cyl. completely filled)	78.5 ± 0.3	2.86
2" graphite crucible, same as last except without top reflector (wall extends 5" above base of Oy)	97 ± 2	3.54
1" normal U in 2" polyethylene, complete	54.5 ± 0.3	1.98
1" normal U in 2" polyethylene, no reflector in inner 6" cyl.	60.8 ± 0.3	2.21

TABLE V.

CRITICAL PARAMETERS OF ENRICHED U^{235} SOLUTIONS IN CYLINDRICAL 60° "Y" AND 90° "CROSS" GEOMETRIES

diameter of cylinders (in.)	geometry	H/U^{235} atomic ratio	kg U^{235} per liter	critical height (in.)[a]
effectively infinite water reflector except at top:				
4	cross	44.3	0.538	b
5	cross	44.3	0.538	5.75
5	cross	73.4	0.337	7.8
5	Y	73.4	0.337	15.6
no reflector:				
5	Y	73.4	0.337	b
5	cross	73.4	0.337	b
7.5	cross	44.3	0.538	b
7.5	cross	72.4	0.342	b

[a] Above the intersection of the center lines.

[b] Extrapolation of the reciprocal source-neutron multiplication curve from an observation taken at least 36 cm above the intersection of the center lines indicates that this vessel will not be critical at any height.

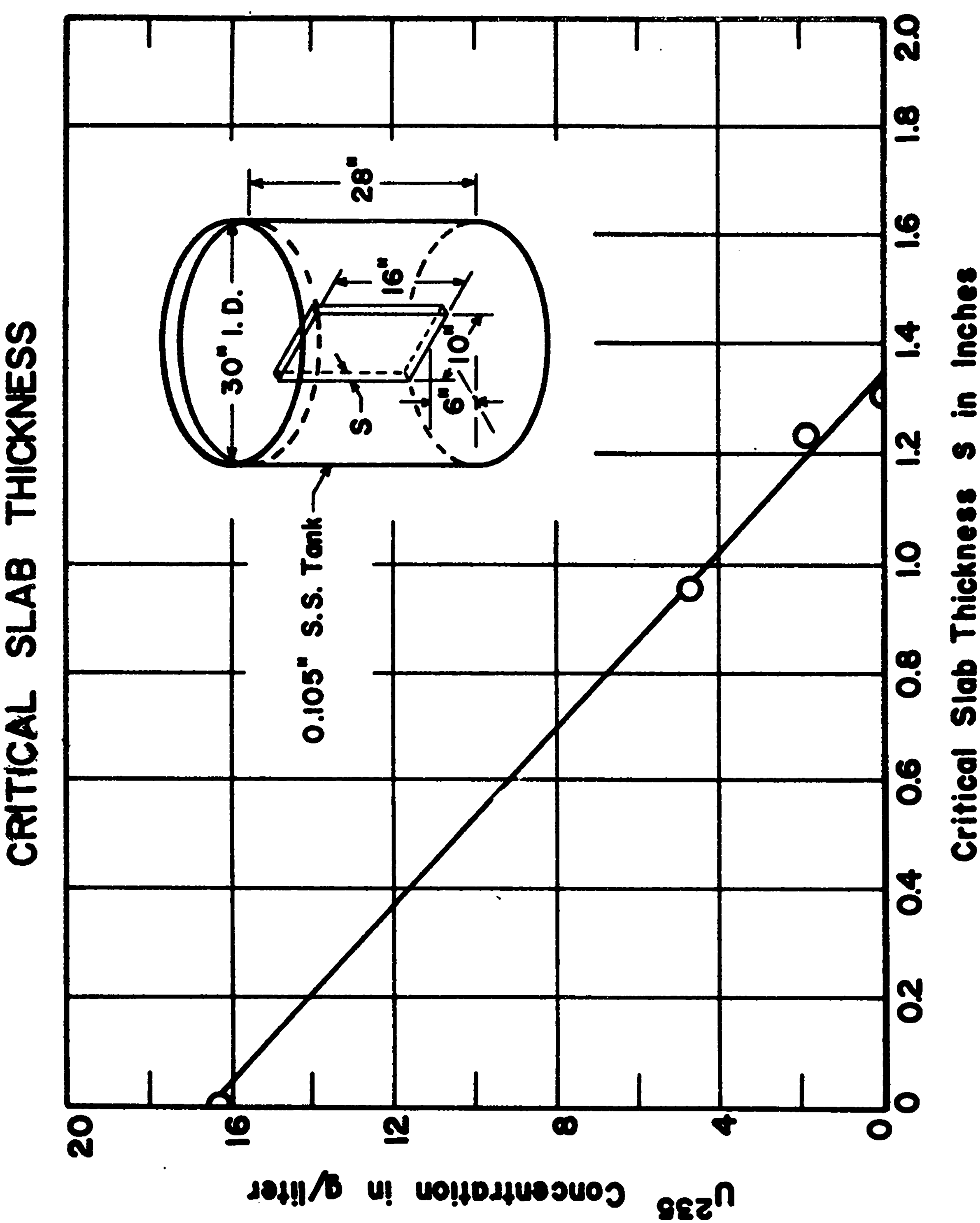

CRITICAL SLAB THICKNESS
30" I.D.
28"
16"
10"
6"
S
0.105" S.S. Tank
Critical Slab Thickness S in Inches
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
U^{235} Concentration in g/liter
0
4
8
12
16
20

Fig. 31.

CRITICAL SLAB THICKNESS
IN AN INFINTE SYSTEM

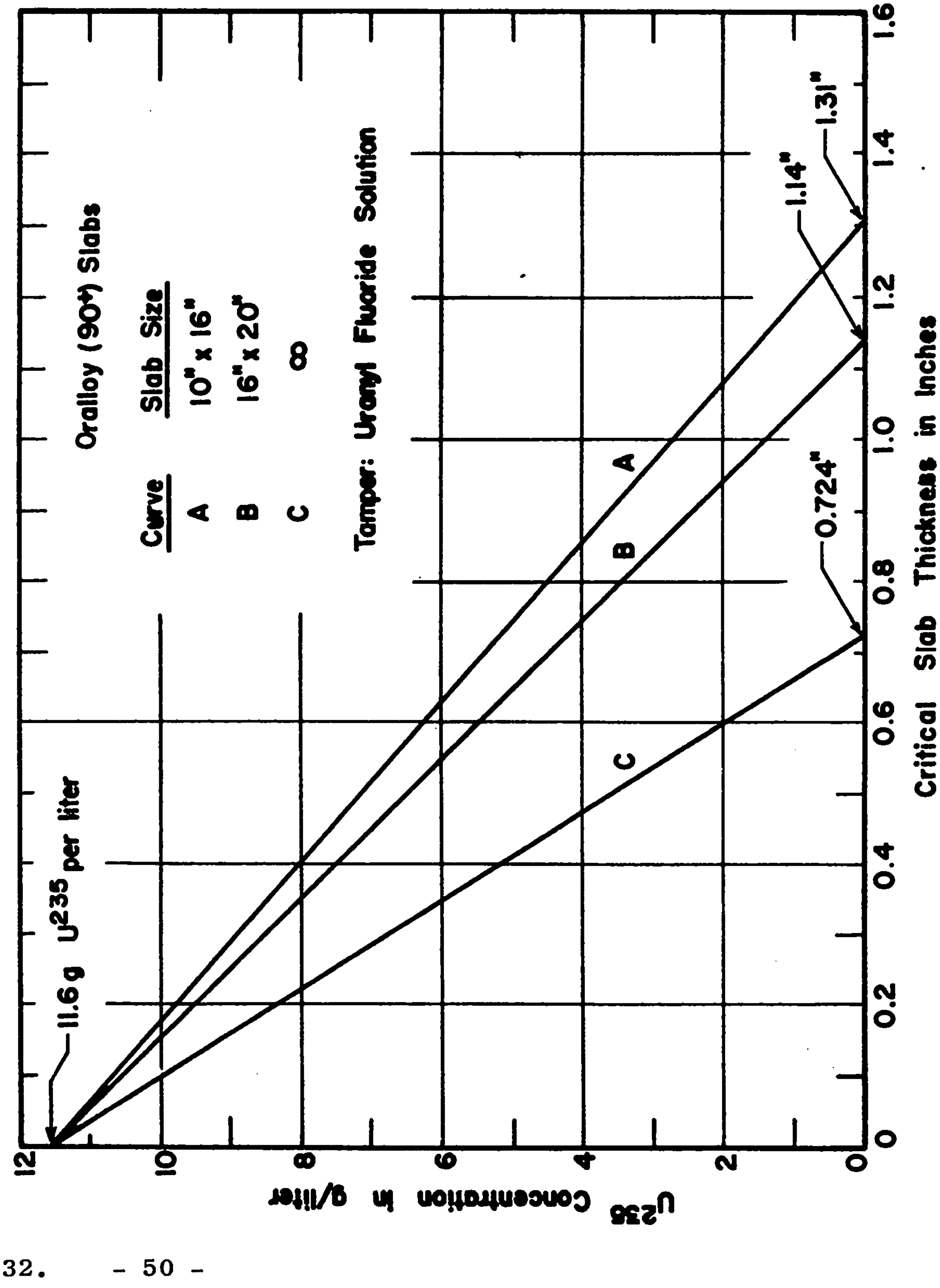


Fig. 32.

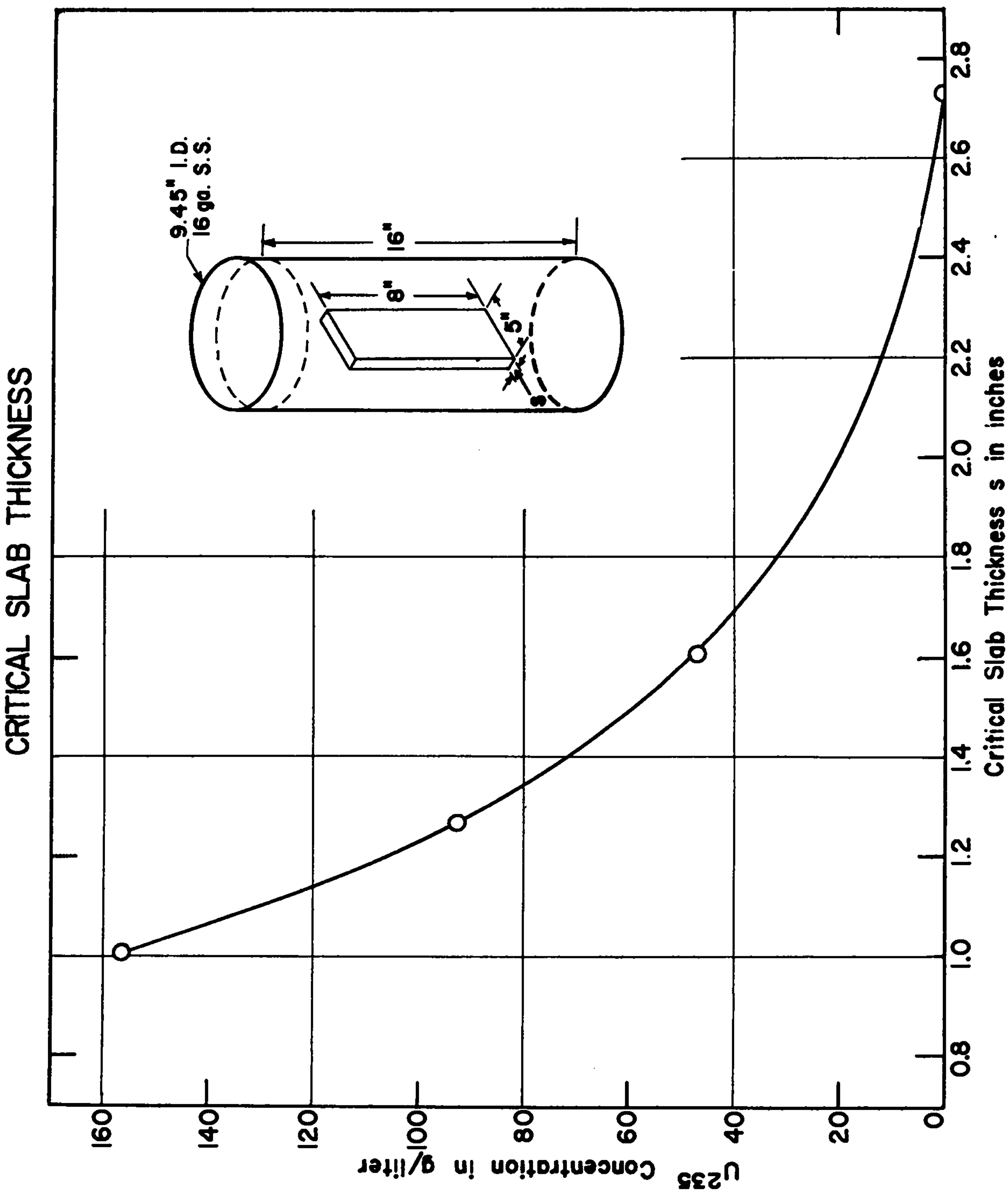

CRITICAL SLAB THICKNESS
9.45" I.D.
16 ga. S.S.
16"
8"
5"
s
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
Critical Slab Thickness s in inches
0
20
40
60
80
100
120
140
160
U235 Concentration in g/liter

Fig. 33.

2) Close-packed array of 17 porcelain filter boats (4" diam.) containing 3-1/2"-deep Oy ($\sim$ 93) peroxide at $H/U^{235} \sim 18$; reflected on 3 sides by thick water and concrete.[56]

3) Slab on concrete floor, made up of 23 - 2-3/4" x 2-3/4" x 3-3/4"-deep units of $(Oy \sim 93)_3O_8$ containing water such that $H/U^{235} = 12$ (705 gm U^{235} per unit in milk carton).[56]

4a) Oy ($\sim$ 93)-metal slab 8" x 8-1/2" x 1-3/32"-thick, reflected by 6"-thick salt eutectic consisting of 55 w/o K_2CO_3 and 45 w/o Li_2CO_3. (16" x 17" x 1-3/32" slab also subcritical but at high multiplication).[56]

4b) Stack of four 8" x 8-1/2" x 1-3/2"-thick Oy-metal slabs separated by 2"-thick layers of the salt of 4a), essentially unreflected. Data also exist for Oy ($\sim$ 93) sheet distributed in 65 w/o K_2CO_3, 30 w/o Li_2CO_3 and 5 w/o Na_2CO_3.[57]

5) Four 30" x 6'-high cylinders of condensed Oy $(2\%)F_6$ at $H/U^{235} \sim 4$, in contact, water reflected.[58]

INTERACTING UNITS

Three-dimensional arrays

Critical data for cubic lattices of fissionable metal units are summarized in Figure 34, where the ordinate is critical capacity of the array in terms of number of bare, spherical critical masses of the material, and the abscissa is volume-fraction F of the lattice that is occupied by the unit.[59] (Consistent densities of units are used for determining coordinates.) Though data do not exist for cubic lattices of nearly equilateral solution units, information about clusters of solution cylinders or slabs can be forced into the form of Figure 34 by confining attention to roughly equilateral lattices ($1/2 \leq h/d \leq 2$). The data of Figure 35 represent this sort

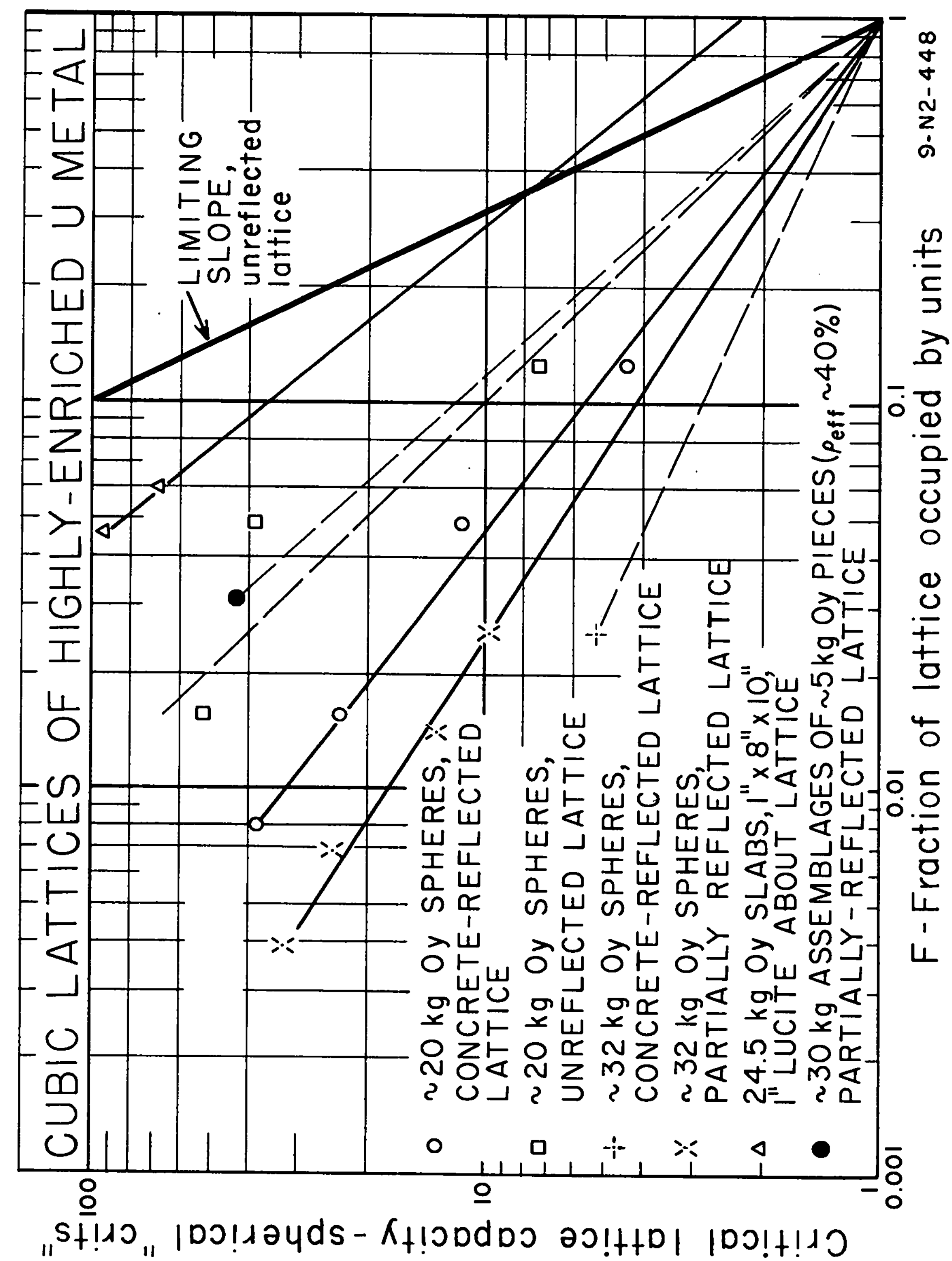

CUBIC LATTICES OF HIGHLY-ENRICHED U METAL
LIMITING SLOPE, unreflected lattice
○ ~20 kg Oy SPHERES, CONCRETE-REFLECTED LATTICE
□ ~20 kg Oy SPHERES, UNREFLECTED LATTICE
~32 kg Oy SPHERES, CONCRETE-REFLECTED LATTICE
~32 kg Oy SPHERES, PARTIALLY REFLECTED LATTICE
△ 24.5 kg Oy SLABS, 1" x 8" x 10", 1" LUCITE ABOUT LATTICE
● ~30 kg ASSEMBLAGES OF ~5kg Oy PIECES (ρ_{eff} ~40%) PARTIALLY-REFLECTED LATTICE
Critical lattice capacity - spherical "crits"
100
10
1
0.001
0.01
0.1
1
F-Fraction of lattice occupied by units
9-N2-448

Fig. 34.

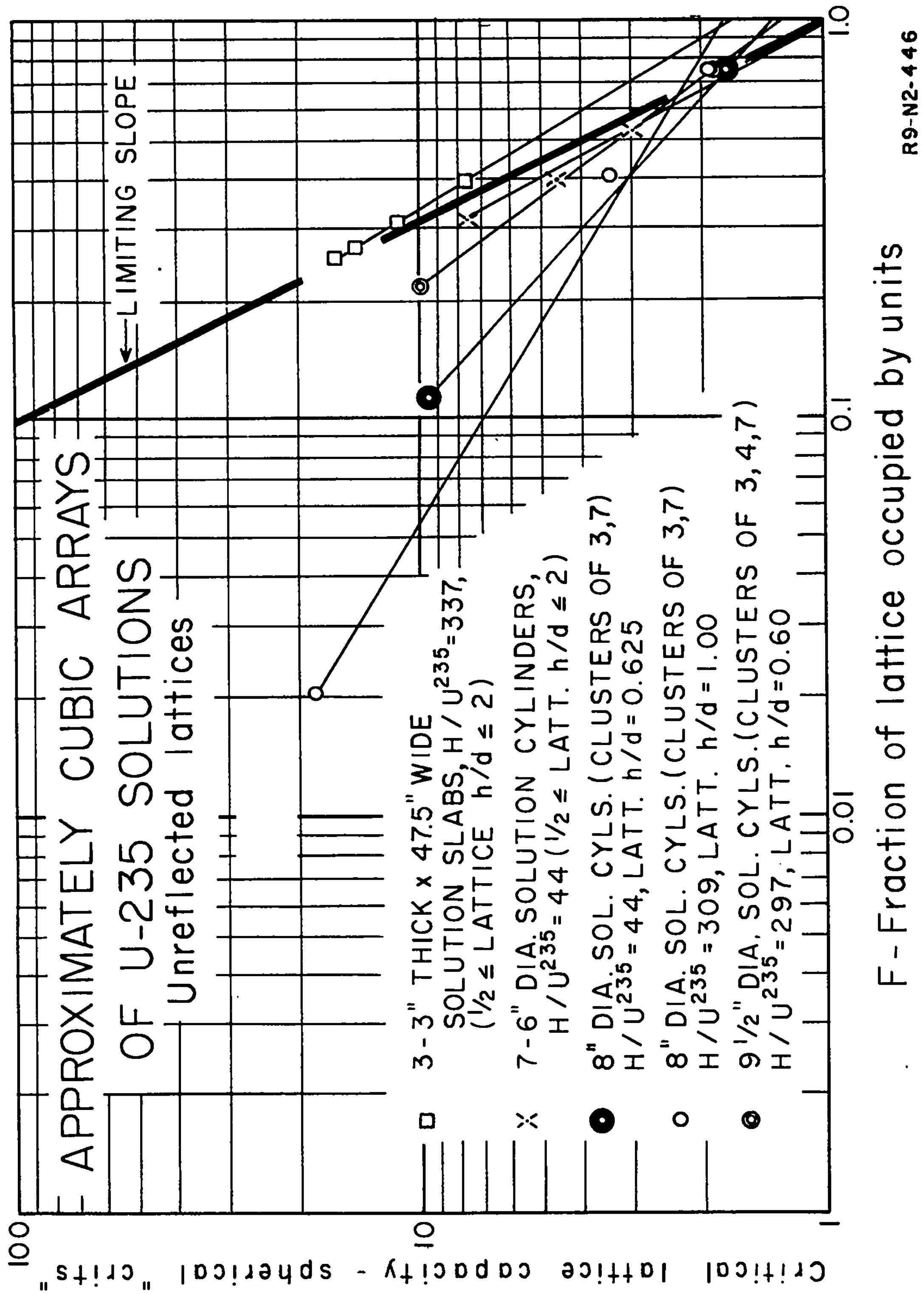

APPROXIMATELY CUBIC ARRAYS
OF U-235 SOLUTIONS
Unreflected lattices
LIMITING SLOPE
□ 3-3" THICK x 47.5" WIDE SOLUTION SLABS, H/U^{235}=337, ($1/2 \leq$ LATTICE $h/d \leq 2$)
× 7-6" DIA. SOLUTION CYLINDERS, H/U^{235}= 44 ($1/2 \leq$ LATT. $h/d \leq 2$)
● 8" DIA. SOL. CYLS. (CLUSTERS OF 3,7) H/U^{235} = 44, LATT. h/d = 0.625
○ 8" DIA. SOL. CYLS. (CLUSTERS OF 3,7) H/U^{235} = 309, LATT. h/d = 1.00
◎ 9 1/2" DIA. SOL. CYLS. (CLUSTERS OF 3,4,7) H/U^{235}=297, LATT. h/d=0.60
100
10
1
0.01
0.1
1.0
Critical lattice capacity - spherical "crits"
F - Fraction of lattice occupied by units
R9-N2-446

Fig. 35.

of compromise for 3-3" slabs and 7-6" diam. cylinders.[5,20,21,59] In the cases of 8"-diam. and 9-1/2" diam. cylinders, where data exist for clusters of different numbers, shape is preserved (assuming lattice extrapolation lengths equal to one-half of the surface-to-surface separation of units).

Each slope, -s, of Figures 34 and 35 corresponds to a density exponent if the lattice is thought of as a single low-density unit. Figure 36 is a correlation of s with quantity of reflector about the lattice and reactivity (fraction critical) of an individual unit.

It has been observed that 1"-thick plexiglas between all pairs of 1" x 8" x 10" Oy (~ 93)-metal units decreases the critical number in a cubic lattice by the factor ~ 5.[60]

Linear and planar arrays

Figure 37 gives cross-multiplication data for linear and two-dimensional arrays of Oy (~ 93)-metal units. It suggests that interactions for large linear or planar arrays can be predicted from measurements on a few units, provided $1-1/M_x$ is an undistorted measure of reactivity.[59]

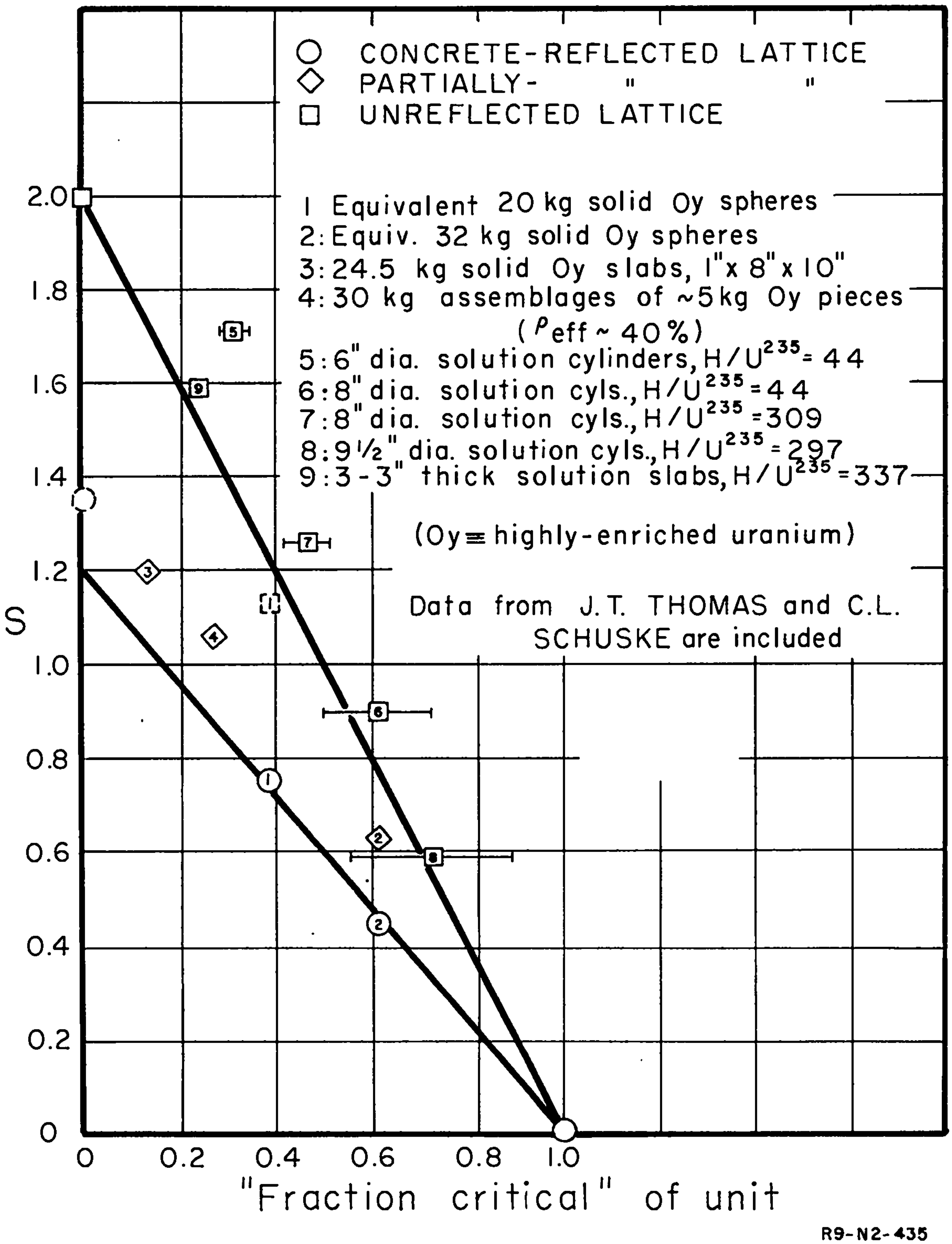
The influence of spacing on interaction between various numbers of bare in-plane solution cylinders is shown in Figure 38.

Pairs of water-immersed units

Figure 39 gives a measure of interaction between pairs of units immersed in water vs. separation of units. Whereas 4"-thick water effectively isolates small spheres, about 8" is required for long cylinders and large slabs that are face to face.[5,6,20,21,61]

Effects of incidental reflectors

Figure 40 shows the critical height of a 9"-diameter U^{235} solution as a function of distance from a concrete slab.[5] Effects of carbon



CONCRETE-REFLECTED LATTICE
PARTIALLY- " "
UNREFLECTED LATTICE
1 Equivalent 20 kg solid Oy spheres
2: Equiv. 32 kg solid Oy spheres
3: 24.5 kg solid Oy slabs, 1"x 8"x 10"
4: 30 kg assemblages of ~5 kg Oy pieces (ρ_{eff} ~ 40%)
5: 6" dia. solution cylinders, H/U^{235} = 44
6: 8" dia. solution cyls., H/U^{235} = 44
7: 8" dia. solution cyls., H/U^{235} = 309
8: 9 1/2" dia. solution cyls., H/U^{235} = 297
9: 3-3" thick solution slabs, H/U^{235} = 337
(Oy ≡ highly-enriched uranium)
Data from J.T. THOMAS and C.L. SCHUSKE are included
S
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
"Fraction critical" of unit
R9-N2-435

Fig. 36.

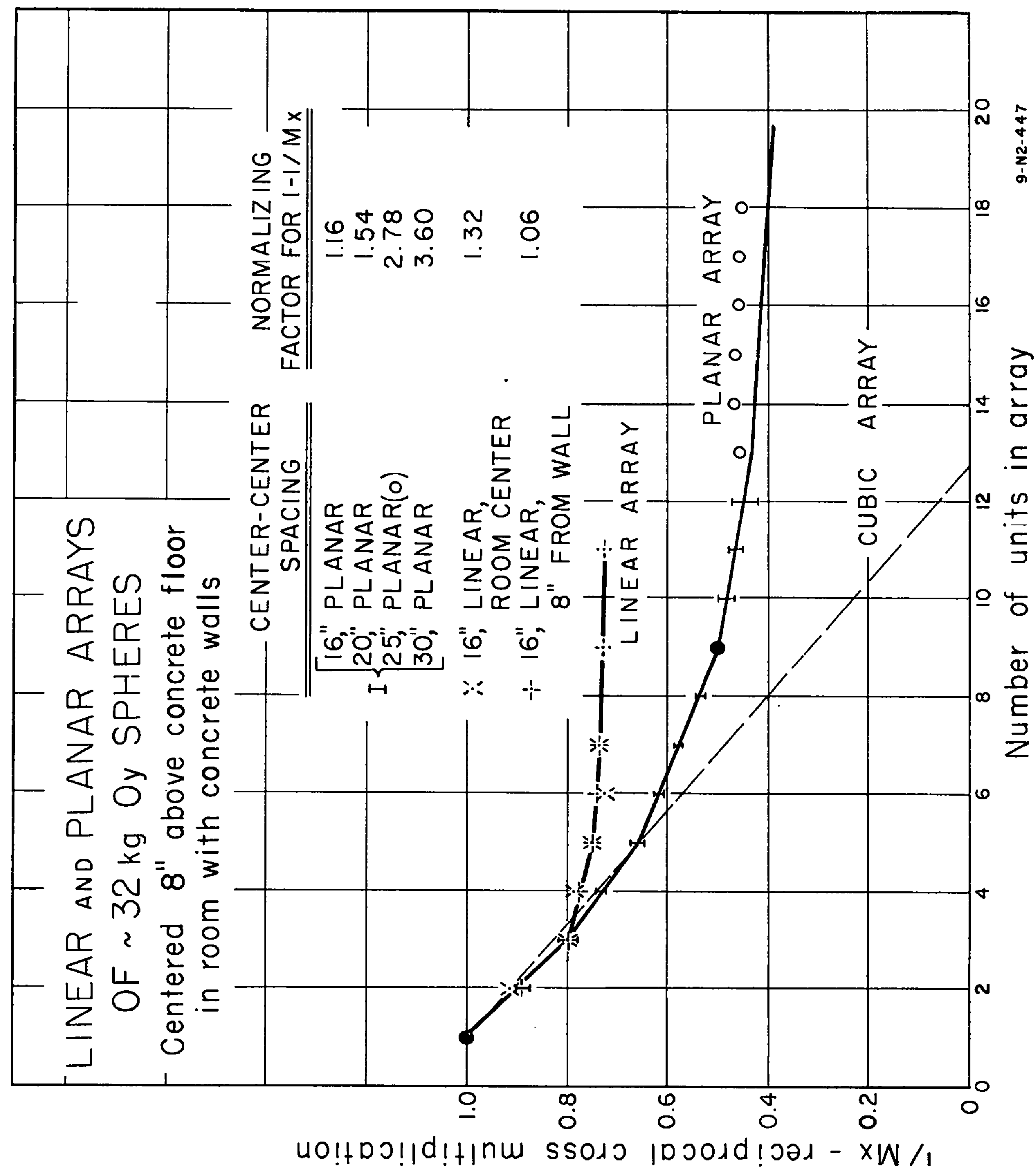

LINEAR AND PLANAR ARRAYS
OF ~ 32 kg Oy SPHERES
Centered 8" above concrete floor
in room with concrete walls
CENTER-CENTER SPACING
NORMALIZING FACTOR FOR 1-1/Mx
16", PLANAR 1.16
20", PLANAR 1.54
25", PLANAR(o) 2.78
30", PLANAR 3.60
16", LINEAR, ROOM CENTER 1.32
16", LINEAR, 8" FROM WALL 1.06
LINEAR ARRAY
PLANAR ARRAY
CUBIC ARRAY
1/Mx - reciprocal cross multiplication
0 0.2 0.4 0.6 0.8 1.0
Number of units in array
0 2 4 6 8 10 12 14 16 18 20
9-N2-447

Fig. 37.

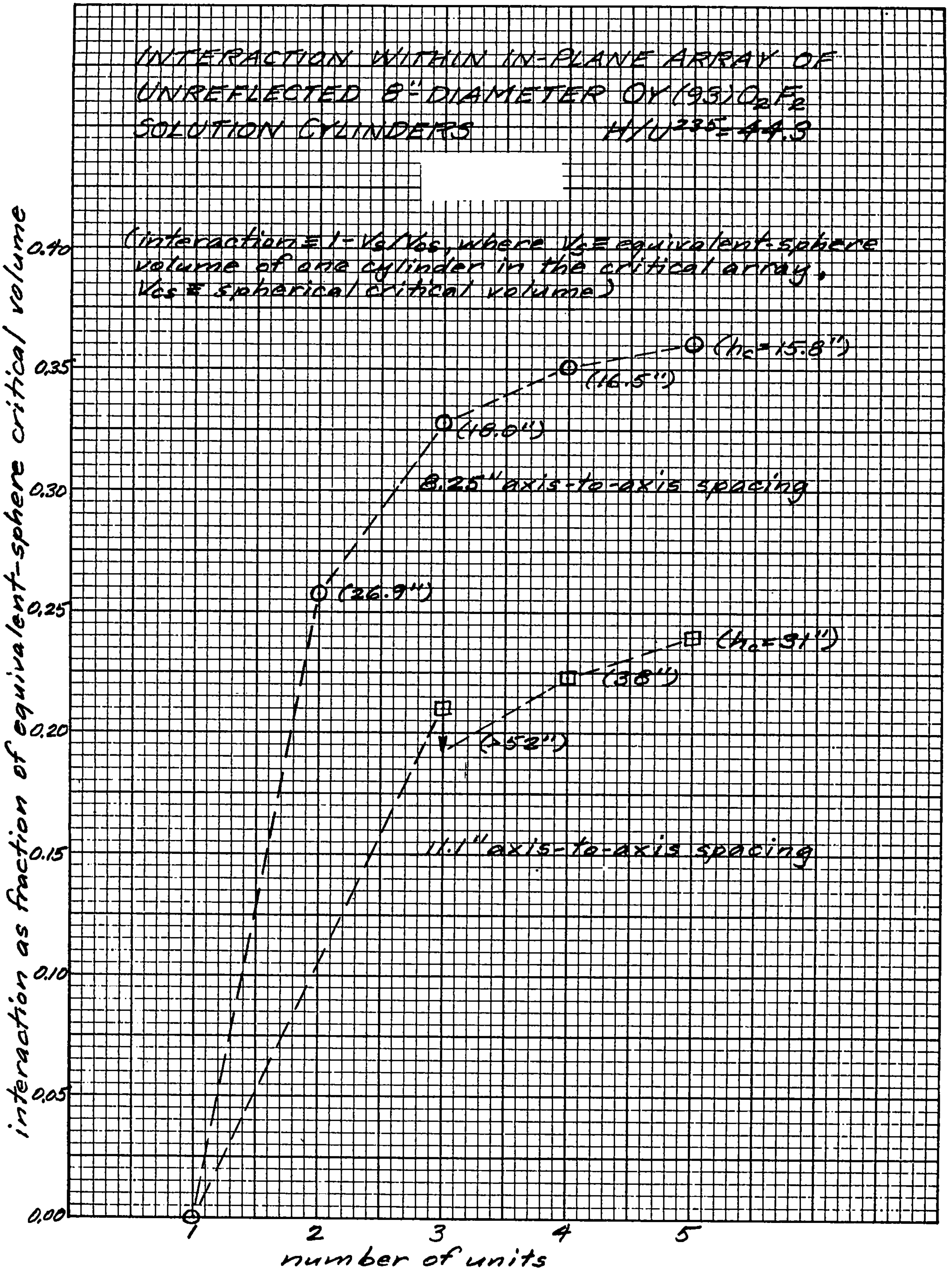

INTERACTION WITHIN IN-PLANE ARRAY OF
UNREFLECTED 8" DIAMETER OY(93)O2F2
SOLUTION CYLINDERS
H/U235=44.3
(interaction ≡ 1-Vs/Vcs, where Vs ≡ equivalent-sphere
volume of one cylinder in the critical array,
Vcs ≡ spherical critical volume)
(hc=15.8")
(16.5")
(18.0")
8.25" axis-to-axis spacing
(26.9")
(hc=31")
(38")
(>52")
11.1" axis-to-axis spacing
interaction as fraction of equivalent-sphere critical volume
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
1
2
3
4
5
number of units

Fig. 38.

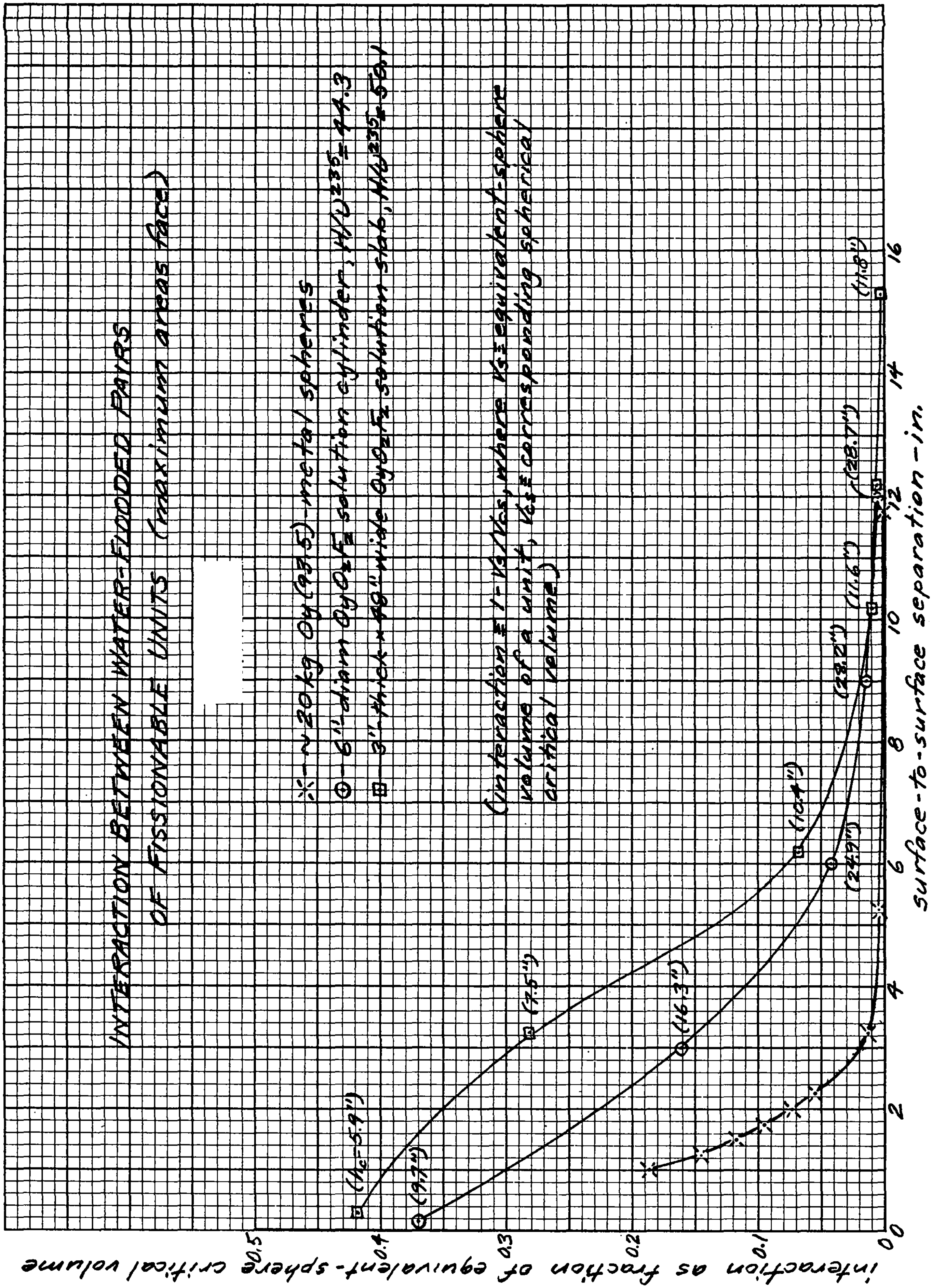
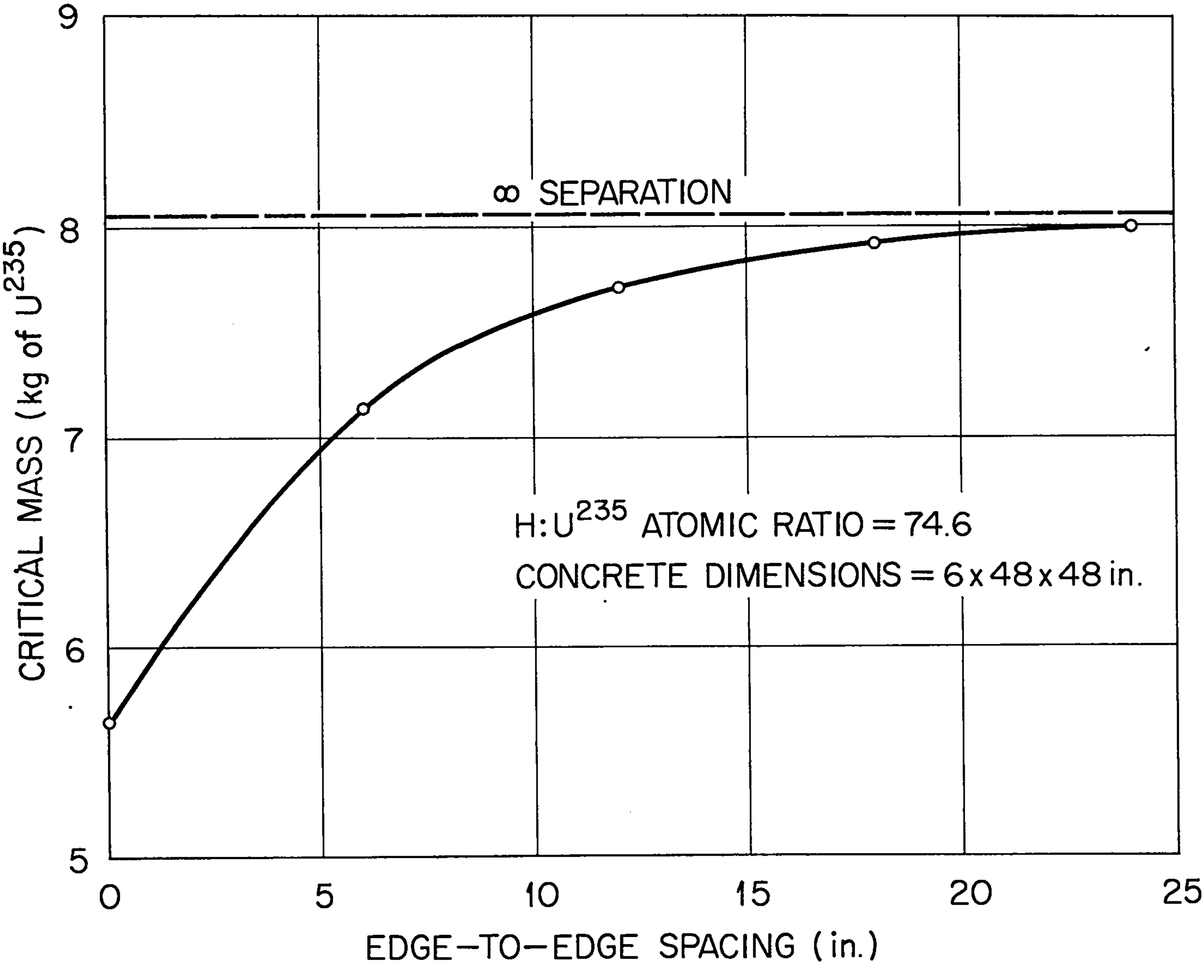

INTERACTION BETWEEN WATER-FLOODED PAIRS
OF FISSIONABLE UNITS (maximum areas face)
~20 kg Oy(93.5)-metal spheres
6" diam OyO2F2 solution cylinder, H/U235=44.3
3" thick × 48" wide OyO2F2 solution slab, H/U235=50.1
(interaction ≡ 1-Vs/Vcs, where Vs ≡ equivalent-sphere
volume of a unit, Vcs ≡ corresponding spherical
critical volume)
surface-to-surface separation - in.
interaction as fraction of equivalent-sphere critical volume

Fig. 39.

UNCLASSIFIED
ORNL-LR-DWG 16557



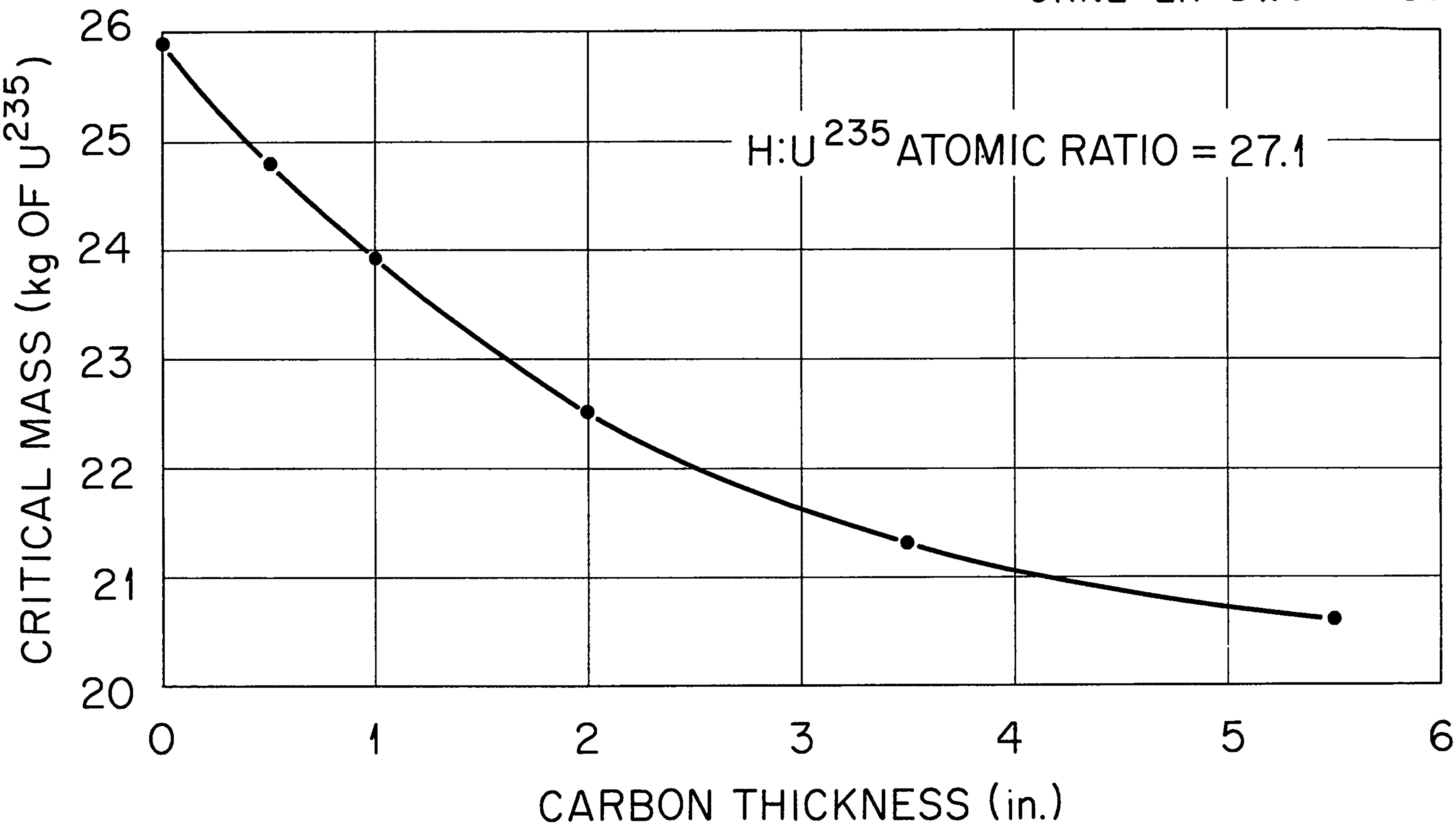
Critical Mass as a Function of the Distance Between a 9-in.-dia Stainless Steel Cylinder Containing an Enriched U^{235} Solution and a 6-in.-thick Concrete Wall.

Fig. 40.

and firebrick as reflectors on the base of a 20"-diameter U^{235} solution cylinder are given by Figures 41 and 42.

The influence of a concrete wall about 8-1/2" from a vertical plane array of Oy-metal units appears in Figure 43 as a function of concrete thickness.[22] Figure 44 shows the degree to which a concrete wall of various thicknesses isolates plane arrays of the Oy units 8-1/2" from each side of the wall. The ordinate is ratio of multiplication of the two arrays, with wall between, to that of a single array.

UNCLASSIFIED
ORNL-LR-DWG 16559



Critical Mass as a Function of the Thickness of Carbon on the Bottom of a 20-in.-dia Aluminum Cylinder Containing an Enriched U^{235} Solution.

Fig. 41.

UNCLASSIFIED
ORNL-LR-DWG 16558

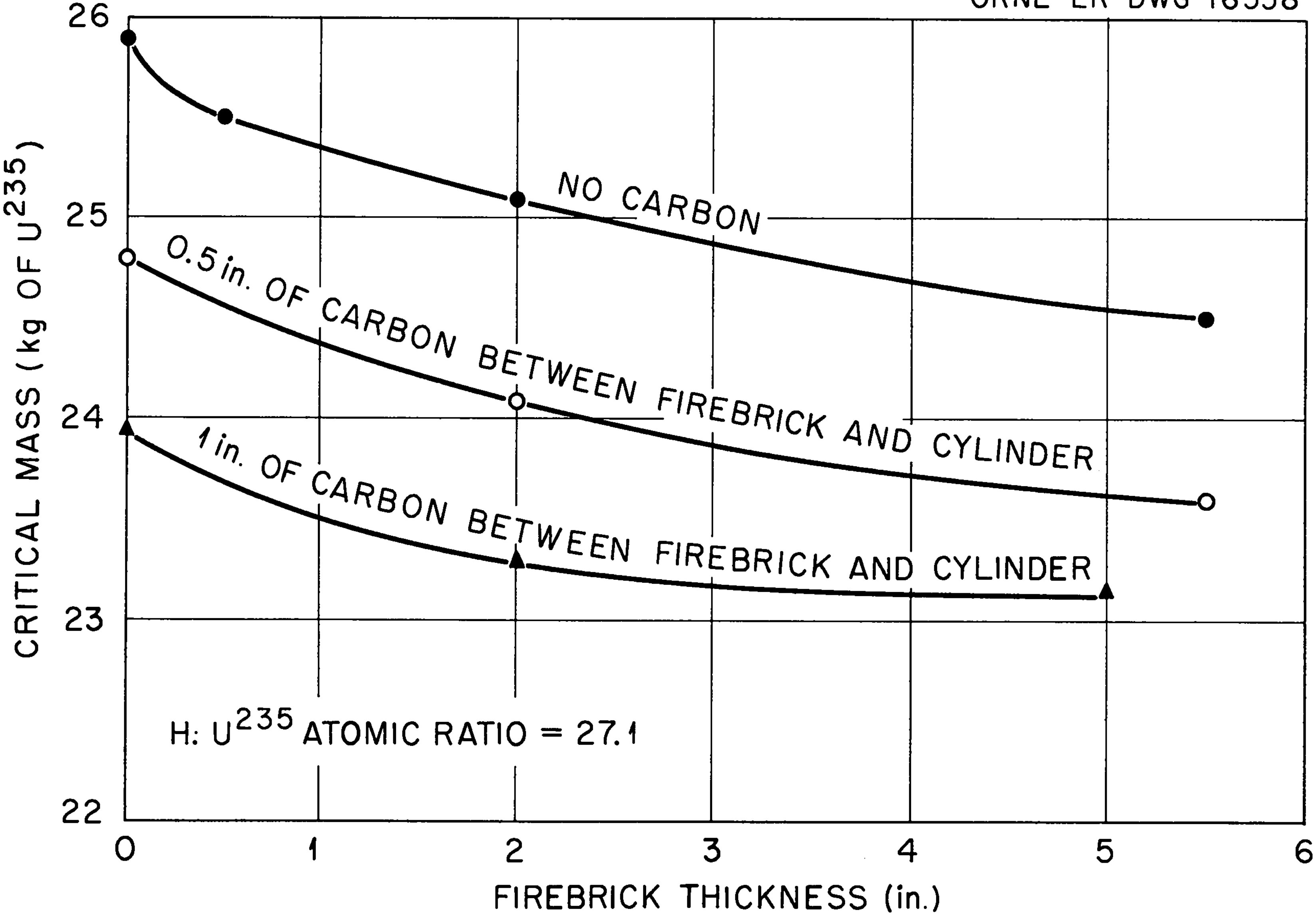


Fig. 42. Critical Mass as a Function of the Thickness of Carbon and Firebrick on the Bottom of a 20-in.-dia Aluminum Cylinder Containing an Enriched U^{235} Solution.

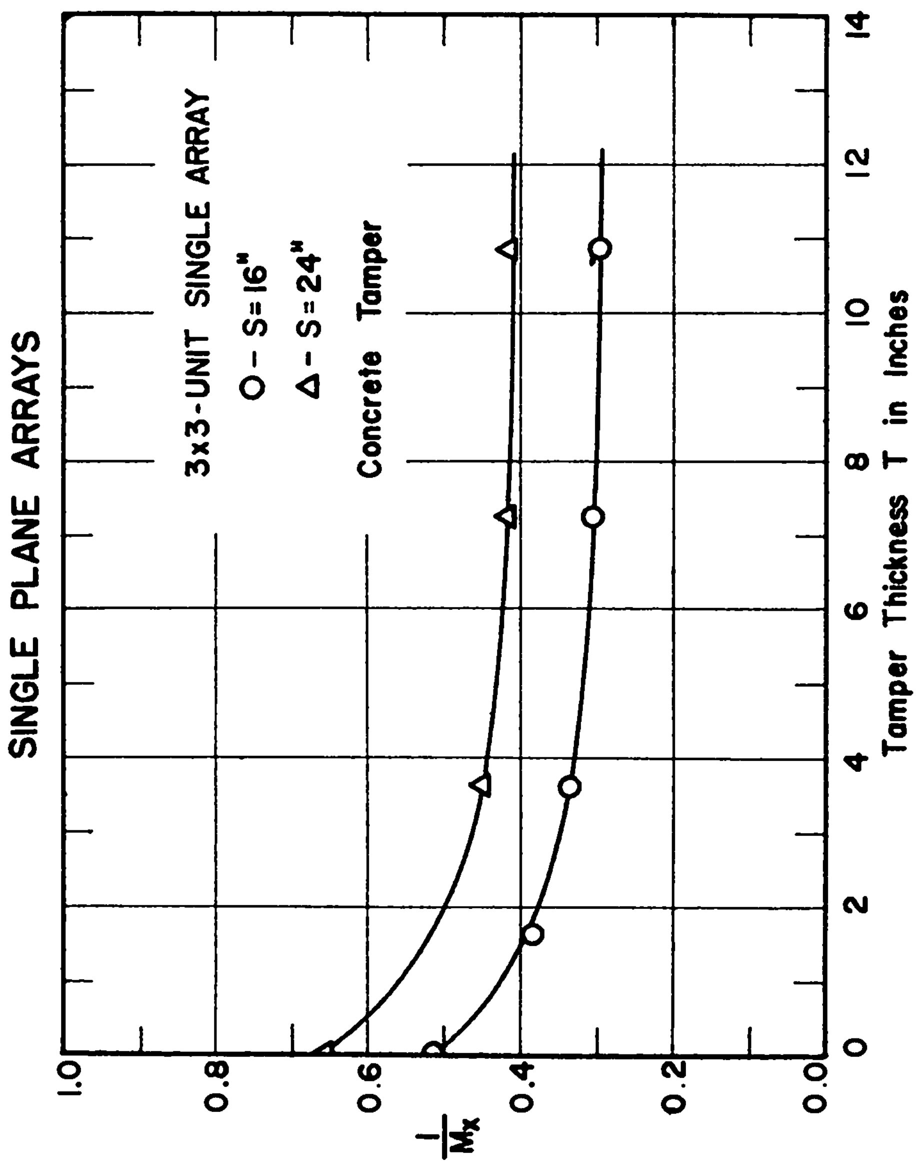

SINGLE PLANE ARRAYS
3x3-UNIT SINGLE ARRAY
O - S=16"
Δ - S=24"
Concrete Tamper
1/Mx
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
12
14
Tamper Thickness T in Inches

Fig. 43.

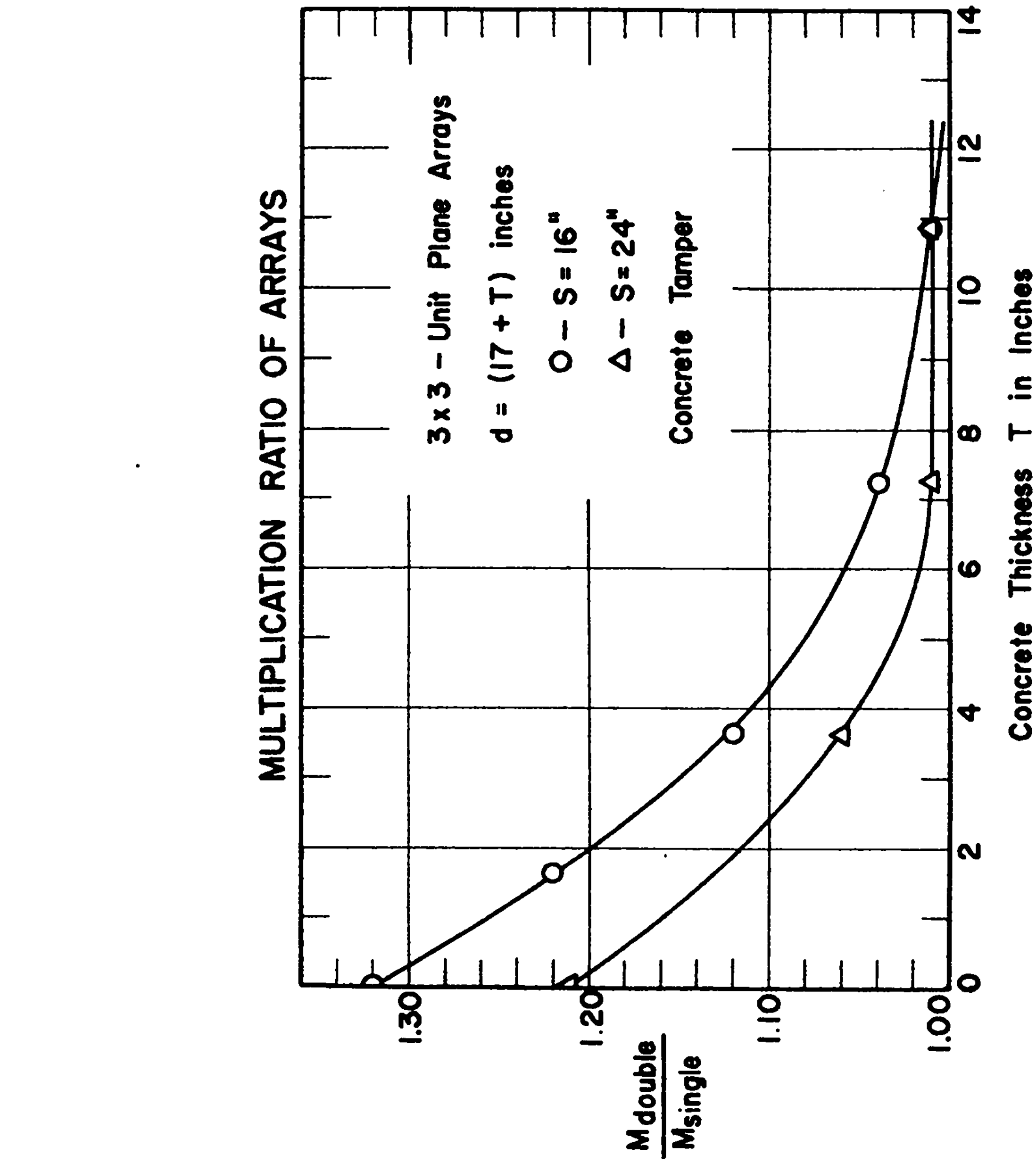

MULTIPLICATION RATIO OF ARRAYS
3 x 3 – Unit Plane Arrays
d = (17 + T) inches
O – S = 16"
Δ – S = 24"
Concrete Tamper
Concrete Thickness T in Inches
Mdouble / Msingle
1.30
1.20
1.10
1.00
0
2
4
6
8
10
12
14

Fig. 44.

REFERENCES

References 1-30 appear in TID-7016, Nuclear Safety Guide.

31. D. W. Magnuson, R. Gwin, Comparison of Critical Experiments for the Determination of Eta of U^{233}, Trans. of the Am. Nuclear Soc. 1, No. 1, p. 146 (June 1959).

32. G. E. Hansen, H. C. Paxton, D. P. Wood, Critical Plutonium and Oralloy Metal Cylinders of Extreme Shape, to appear in Nuclear Sci. and Eng.

33. Communication from John McEnhill, Aldermaston, England.

34. E. A. Plassmann, D. P. Wood, Critical Reflector Thicknesses for Spherical U^{233} and Pu^{239} Systems, to appear in Nuclear Sci. and Eng.

35. J. C. Hoogterp, Critical Masses of Oralloy Lattices Immersed in Water, LA-2026, March 1957.

36. R. E. Heineman, Experience in the Use of the Physical Constants Testing Reactor, Paper UN-1929, Second International Conf. on the Peaceful Uses of Atomic Energy, September 1958.

37. J. J. Lynn, J. T. Mihalczo, W. C. Connelly, Homogeneous Hydrogen Moderated Critical Assemblies with Two Percent U^{235} Enriched Uranium, ORNL-2609, 1958.

38. D. Callihan, D. F. Cronin, ORNL-CF-55-10-97, October 1955 (Classified).

39. H. Kouts, Lattices of Slightly Enriched Uranium in Ordinary Water, Paper UN-1841, Second International Conf. on the Peaceful Uses of Atomic Energy, September 1958.

40. D. Callihan, et al, Physics Division Semiannual Progress Report for Period Ending March 10, 1954, ORNL-1715, July 1954 (Classified).

41. L. W. Gilley, A. D. Callihan, Nuclear Safety Tests on a Proposed Ball Mill, ORNL-54-9-89, September 1954.

42. J. K. Fox, L. W. Gilley, Critical Parameters for 20-in.-dia. Stainless Steel Cylinders Containing Aqueous Solutions of U^{235} Poisoned with Pyrex Glass, Neutron Physics Division Annual Progress Report for Period Ending September 1, 1959, ORNL-2842, pp. 78-81.

43. F. H. Langell, et al, Rocky Flats Technical Quarterly Progress Report for January, February, March 1959, RFP-151, p. 44 (Classified).

44. L. B. Engle, G. E. Hansen, H. C. Paxton, Reactivity Contributions of Various Materials in Topsy, Godiva and Jezebel, to appear in Nuclear Sci. and Eng.

45. R. N. Olcott, Homogeneous Heavy Water Moderated Critical Assemblies. Part I. Experimental, Nuclear Sci. and Eng. 1, No. 4, p. 327, August 1956.

46. J. E. Schwager, F. A. Kloverstrom, W. S. Gilbert, Critical Measurements on Intermediate-Energy Graphite-U^{235} Systems, UCRL-5006, November 1957.

47. F. A. Kloverstrom, R.M.R. Deck, A. J. Reyenga, Critical Measurements on Near-Homogeneous BeO-Moderated, Oralloy-Fueled Systems, UCRL-5369 Pt. 1, July 1959.

48. D. P. Wood, C. C. Byers, L. C. Osborn, Critical Masses of Cylinders of Plutonium Diluted with Other Metals, to appear in Nuclear Sci. and Eng.

49. F. A. Kloverstrom, Spherical and Cylindrical Plutonium Critical Masses, UCRL-4957, September 1957.

50. H. R. Ralston, UCRL-4457, February 1955 (Classified).

51. H. R. Ralston, UCRL-5610, June 1959 (Classified).

52. G. E. Hansen, Properties of Elementary Fast-Neutron Critical Assemblies, Paper UN-562, International Conference on the Peaceful Uses of Atomic Energy, September 1958.

53. J. K. Fox, L. W. Gilley, Critical Parameters for Poisoned Annular Cylinders Containing Aqueous Solutions of U^{235}, ORNL-CF-58-8-5, August 1958.

54. C. L. Schuske, M. G. Arthur, D. F. Smith, RFP-66, August 1956 (Classified).

55. C. L. Schuske, M. G. Arthur, D. F. Smith, RFP-69, October 1956 (Classified).

56. C. L. Schuske, M. G. Arthur, D. F. Smith, RFP-58, January 1956 (Classified).

57. C. L. Schuske, M. G. Arthur, A. Goodwin, Jr., A. N. Nickel, D. F. Smith, RFP-89, November 1957 (Classified).

58. A. D. Callihan, A Test of Neutron Multiplication by Slightly Enriched Uranium, Part II, ORNL-1698, March 1954.

59. H. C. Paxton, Standards for Fissionable Materials Outside Reactors, STANDARDS WORK IN THE AMERICAN NUCLEAR SOCIETY, A Special Session of Invited Papers, pp. 55-63, June 1959.

60. J. T. Mihalczo, J. J. Lynn, Multiplication Measurements with Slabs of Enriched Uranium, Neutron Physics Division Annual Progress Report for Period Ending September 1, 1959, ORNL-2842, pp. 67-68.

61. C. L. Schuske, A. N. Nickel, Isolation Thickness of Water for UO_2F_2 Solution Systems, RFP-169, October 1959.