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RE-EVALUATION OF OPTIMAL PARAMETERS FOR POST-FISSION BETA DECAY

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LOS ALAMOS SCIENTIFIC LABORATORY
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RE-EVALUATION OF OPTIMAL PARAMETERS FOR POST-FISSION BETA DECAY

by

James J. Griffin

LOS ALAMOS NATIONAL LABORATORY
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ABSTRACT

A calculational error in a previous calculation is corrected. The two adjustable parameters in the description are re-evaluated to optimize the fit with the corrected version of the theory. Previous qualitative conclusions about the encouraging consistency between the model and the available data remain valid. Quantitative modifications occur in certain extrapolations of the calculation reported earlier. The corrected extrapolations, which supersede earlier results, are presented.

I. INTRODUCTION

In a previous report[1] a model was constructed to describe the average properties of beta decay and subsequent emission of gamma radiation from the fragments of a given fission process. Two free parameters in the model were chosen to optimize the fit to relevant data available from the neutron-induced[2,3] fission of U^{235}. Then calculations were made for other fissioning nuclides and the results compared with experiment.[2]

The results indicated that the model provided an adequate description of the phenomena in question and suggested the use of the model with the optimized parameters for predictive extrapolation to nuclei not yet studied experimentally.

In reference 1 it was assumed that for fixed A, the nuclear ground state energies of odd A nuclei depend parabolically on displacement from the charge at the line of stability, and that even-even (odd-odd) nuclei are more (less) stable by an amount Δ:

$$M(A,Z) = M(A,Z^s) + c_1(Z^s - Z)^2 \pm \Delta \tag{1}$$

Then the maximum energy of a beta decay, $(A,Z) \rightarrow (A,Z + 1)$, is given by

$$E_\beta^{max} \pm 2\Delta = \left.\frac{\partial M(A,Z)}{\partial Z}\right|_{Z=Z+1/2} = M(A,Z) - M(A,Z+1) \qquad (2)$$

This equation differs from the corresponding equation of reference 1 in that the derivative is evaluated at Z + 1/2, rather than at Z. Equation 2 of reference 1 is erroneous in this respect.[4]

The error cited had the effect of assigning too high an energy to each beta decay. The consequences of this mistake were attenuated somewhat by the fact that in the previous calculation the average beta decay matrix element was chosen to optimize the fit to the data on U^{235}. The data then forced one to choose an erroneous value for the average matrix element in just such a way as to minimize the impact of the above error, at least as regards the U^{235} data.

II. RE-EVALUATION OF FREE PARAMETERS

It turns out that the recalculation of the optimal parameters with the expression (2) above is easily fit into the calculational framework of reference 1. This is a result of the fact that only a simple shift $Z \rightarrow Z + 1/2$ is involved in obtaining the correct equation (2), above. The corresponding change in displacement, $z = Z^s - Z$, is $z \rightarrow z - 1/2$.

Thus by repeating the calculation exactly as it was done in reference 1, except that each displacement from stability is reduced by one-half unit, one obtains the correct result. But such a replacement is very simply achieved by substituting the value $\bar{z}' = \bar{z} - 1/2$ for the parameter $\bar{z}$, which defines the initial situation through the distribution

$$P(Z,0) = (\pi)^{-1/2} \exp - (z - \bar{z})^2 \tag{3}$$

There remains only the question of checking the end effects, that is, the consequences of such a shift for beta decays near the line of stability. In particular, equation (2) above implies that any odd mass nuclide with $Z < 1/2$ will be assigned a maximum beta-decay energy less than zero. This is, in fact, a physically inevitable implication of the assumed parabolic mass dependence and the fact that beta decays must modify Z in unit steps. Indeed, in assigning a finite beta-decay energy to such nuclides, the previous calculation introduced a physical absurdity. It should be noted that as a matter of course in the numerical calculation any beta decay assigned a negative decay energy is also assigned an infinite decay lifetime.

Therefore, in the present work, one has simply deceived the computing machine by replacing the true value of $\bar{z}$ for each nuclide by $\bar{z}' = \bar{z} - 1/2$. This is equivalent to shifting each of the eight discrete groups inward by one-half unit. The groups can therefore be considered as centered on half-integer values of z in this calculation (instead of integral values as before) and to have maximum beta energies given by the present form of equation (2).

In this way the calculations of reference 1 were repeated to obtain new values of the two variable parameters, c_2 and E_γ^0, which optimize the fit to the U^{235} data. Then these values were used to calculate the rate of post-fission gamma radiation for U^{233}, Th^{232}, U^{238} and Pu^{239}, and the results compared to the data of Fisher and Engle.[2] Finally the predictive extrapolations made in reference 1, are re-estimated (Table I).

III. RESULTS

The new values of c_2 and E_γ^o were chosen to minimize χ^2 in exactly the manner described in reference 1. Moreover, all other constants remain unchanged from reference 1. Then one finds (Figure 1) that E_γ^o = 1.05 ± 0.1 MeV and $c_2 = 6.0 \pm 1.0 \times 10^{-6}$/sec provide the best fit for U^{235}. The former is remarkably close to the earlier result, E_γ^o = 1.03 MeV, whereas c_2 has increased by a factor of about 1.8 from its earlier value of 3.25×10^{-6}/sec. The minimal value of χ^2 has decreased to 2.69 from its earlier value of 3.13. Both are, of course, quite acceptable fits for a system with 8 - 2 degrees of freedom. The uncertainties noted in the parameters are the changes which would cause χ^2 to increase in magnitude by about one.

The new calculations for $\bar{\lambda}(t)$, $\dot{E}_d(t)$, and $\dot{E}_\gamma(t)$ are exhibited in Figures 2 through 16, together with the available data.[2,3] These figures correspond to Figures 1 through 15 of LA-2811 Addendum. Finally, summaries of the dependences of these various quantities on $\bar{z}$ at various times are presented in Figures 17 through 19 (analogous to Figures 16 through 18 of LA-2811 Addendum). It should be emphasized that all of these results are to supersede directly the earlier results. In particular, all definitions used here are identical to those furnished

TABLE I*

Extrapolated Estimates

Target Nuclide	E_n = 2 MeV $\bar{z}$	E_n = 2 MeV $\dot{E}^{o}_{\gamma}$ (MeV/Fiss.-Sec)	E_n = 14 MeV $\bar{z}$	E_n = 14 MeV $\dot{E}^{o}_{\gamma}(t=0)$	Godiva $\bar{z}$	Godiva $\dot{E}^{o}_{\gamma}(t=0)$
U^{233}	3.03	0.312	2.65	0.148	3.05	0.324
U^{234}	3.25	0.463	2.87	0.230	--	--
U^{235}	3.47**	0.669	3.09	0.349	3.52	0.726
U^{236}	3.69	1.952	3.31	0.513	--	--
U^{237}	3.91	1.34	3.53	0.738	--	--
U^{238}	4.13	1.85	3.75	1.046	4.12	1.820
U^{239}	4.35	2.496	3.97	1.463	--	--
U^{240}	4.57	3.320	4.19	2.008	--	--
Pu^{239}	3.29**	0.496	2.91	0.249	3.29	0.496
Pu^{240}	3.51	0.714	3.13	0.375	--	--
Pu^{241}	3.73	1.014	3.35	0.549	--	--
Pu^{242}	3.95	1.420	3.57	0.787	--	--
Pu^{243}	4.17	1.953	3.79	1.113	--	--
Th^{232}	--	--	--	--	3.97	1.463

*This table supersedes Table IV of reference 1.

**Values obtained as described in reference 1, caption to Table IV.

earlier. Only equation 2 has been changed in the manner described in Section II.

IV. DISCUSSION OF ILLUSTRATIONS

In Figure 1 are plotted contours of constant χ^2 for various values of c_2 and E_γ^o near the final optimal values of $6.0 \pm 1.0 \times 10^{-6}$/sec and 1.05 ± 0.10 MeV.

Figures 2 through 16 present the calculated time dependence of the three quantities, $\bar{\lambda}$, $\dot{E}_d$, and $\dot{E}_\gamma$, for each of the targets listed in the last two columns of Table I and studied in reference 2.

Figures 17 through 19 present the same calculated results as a function of $\bar{z}$, with time as a parameter. These figures are designed to facilitate estimations for target nuclides and experimental circumstances other than those calculated explicitly. They are simply smooth-curve interpolations in $\bar{z}$ of the numerical results in Figures 2 through 16.

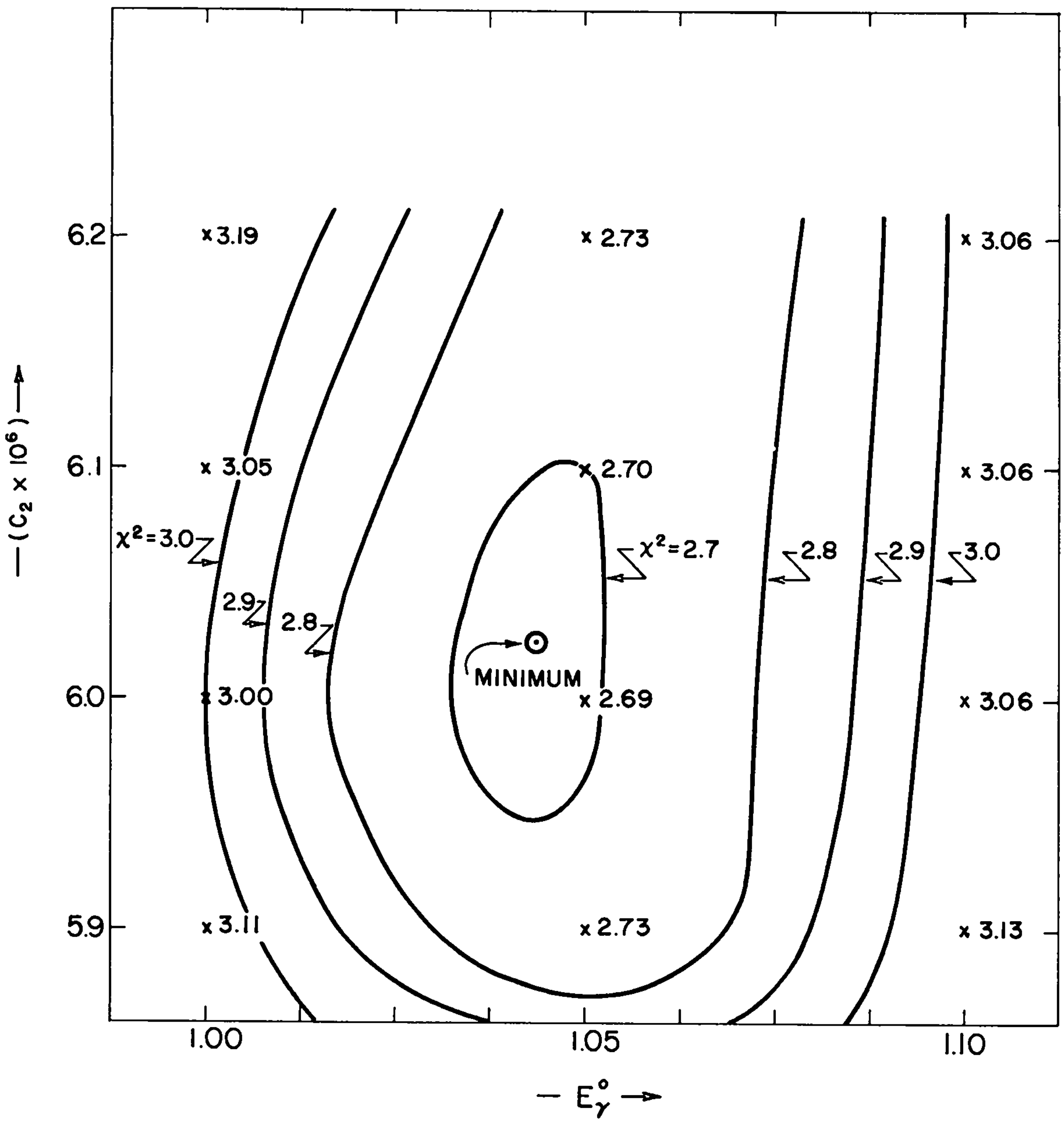

Fig. 1. Rough contours of constant χ^2 vs C_2 and E^o_γ.
(x indicates computed value of χ^2)

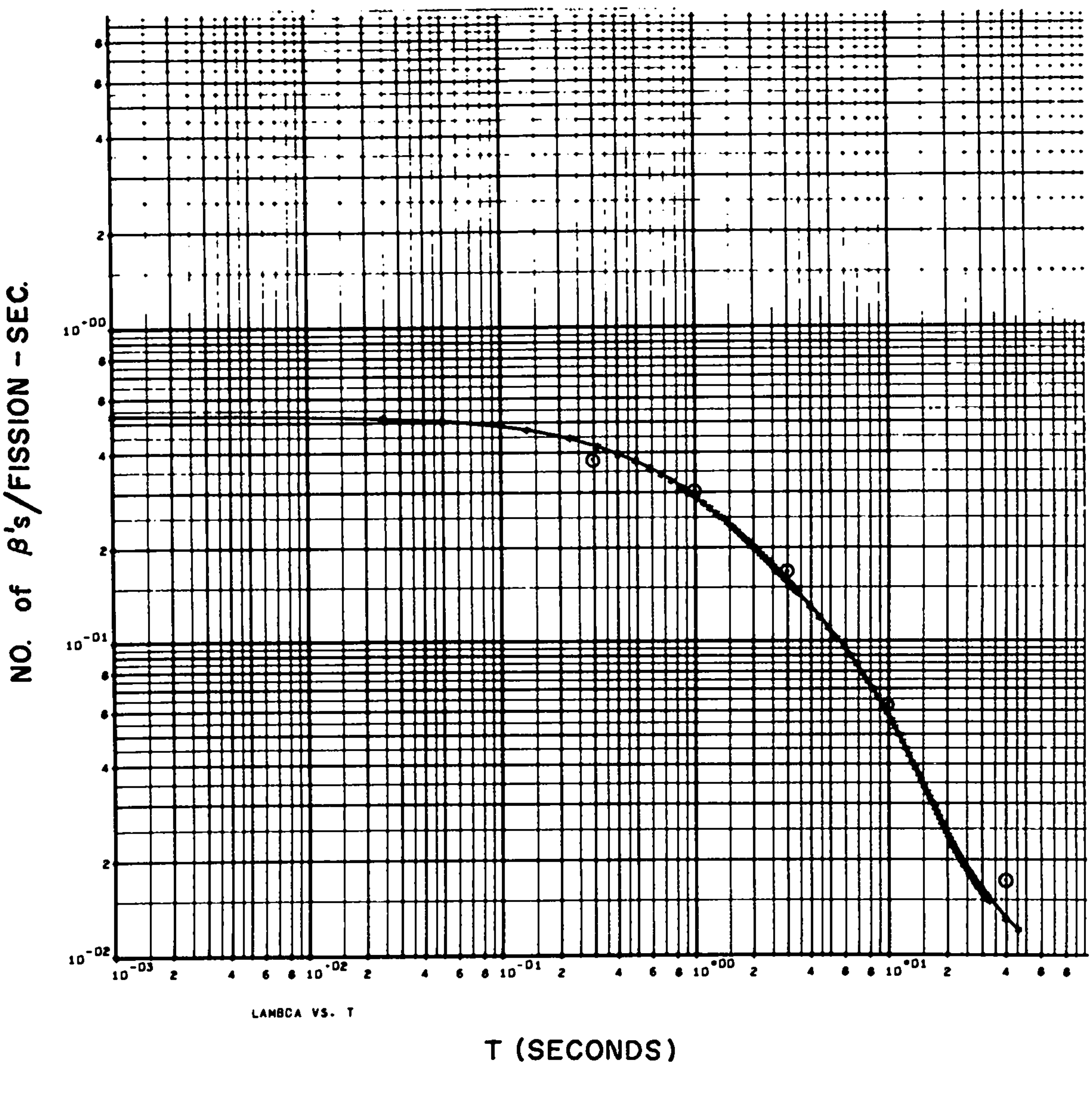

Fig. 2. $U^{235} + n$.

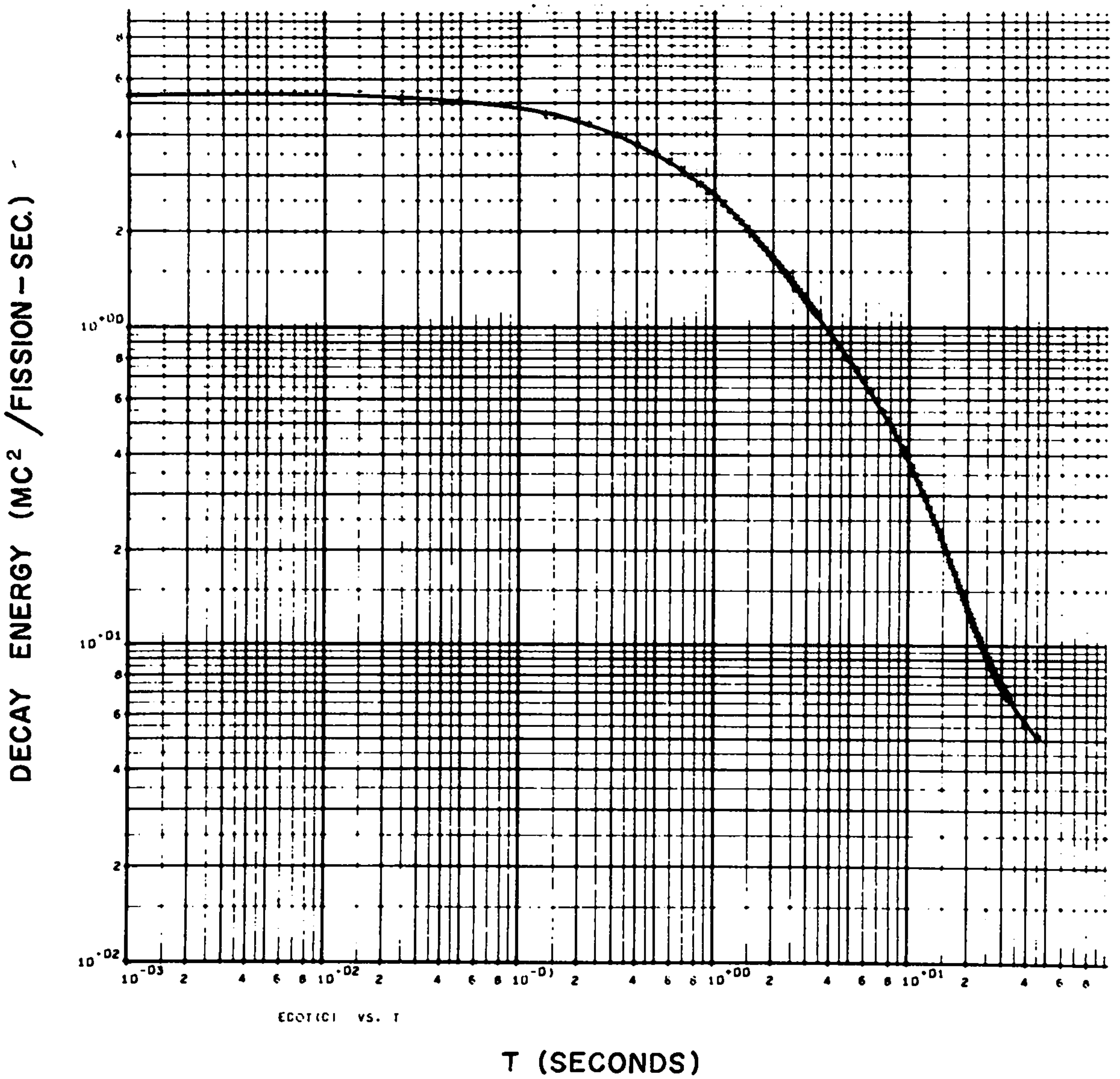

Fig. 3. U^{235} + n.

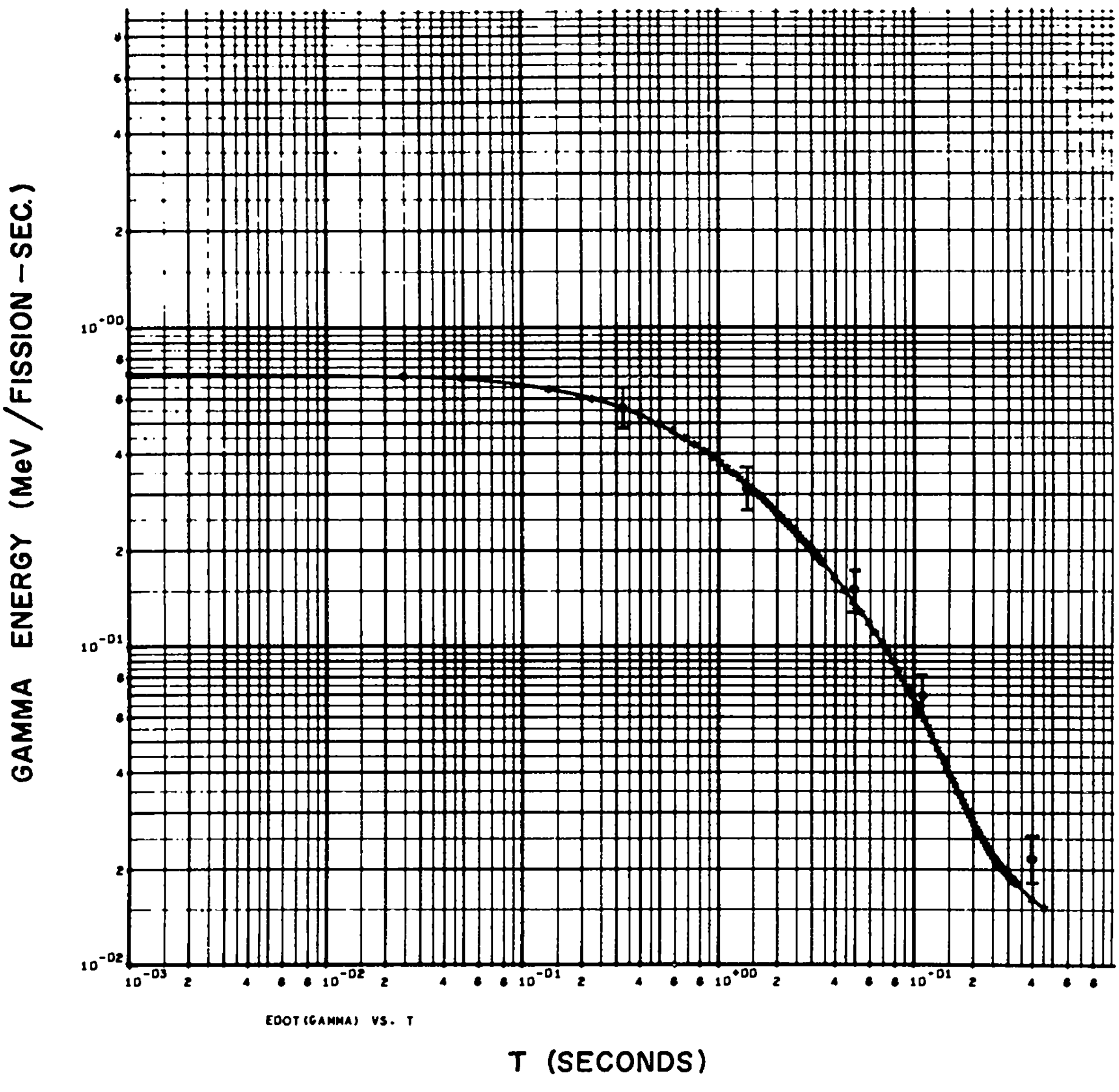

Fig. 4. U^{235} + n.

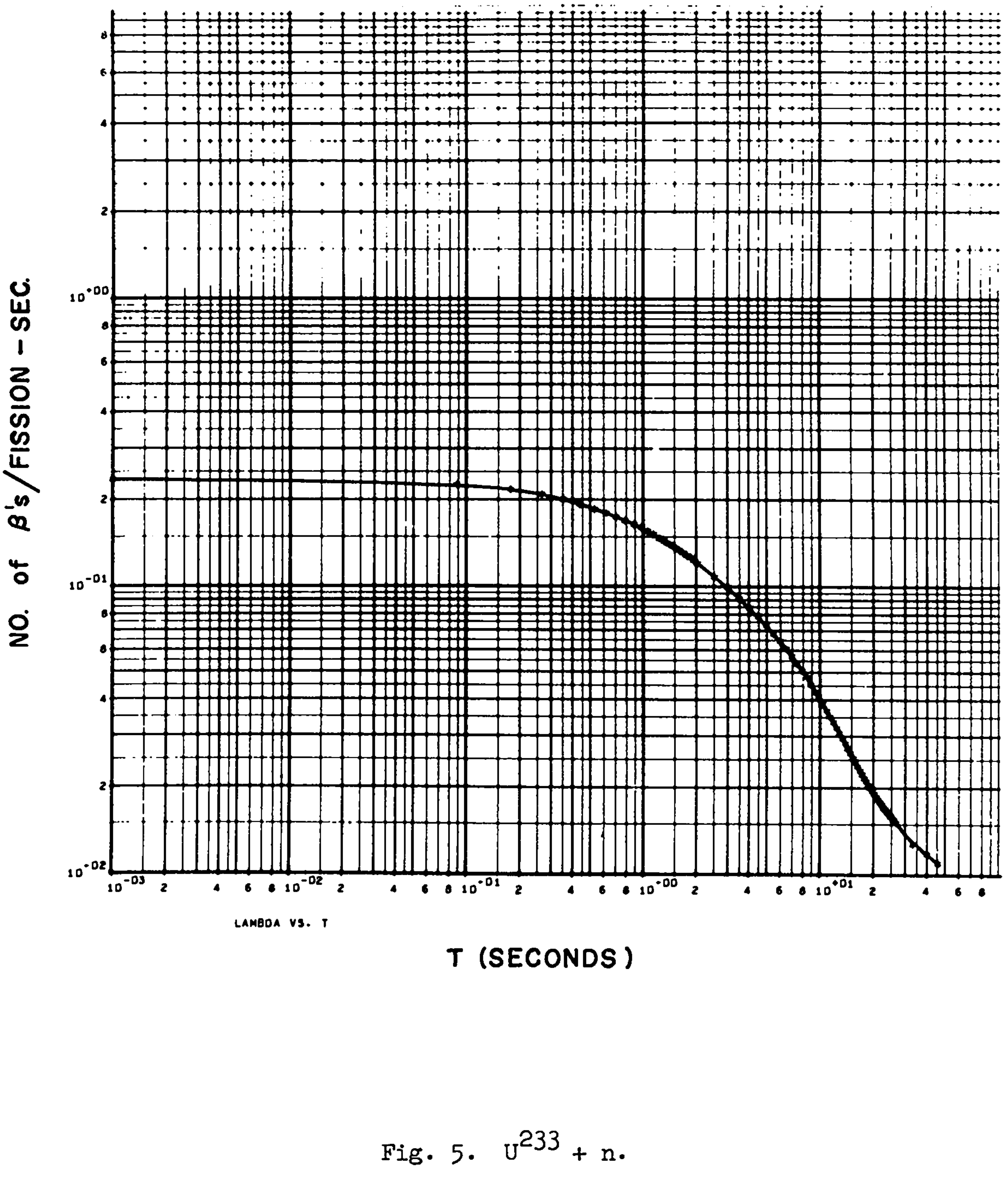
NO. of β's/FISSION - SEC.
10+00
10-01
10-02
8
6
4
2
10-03
10-02
10-01
10+00
10+01
LAMBDA VS. T
T (SECONDS)

Fig. 5. $U^{233} + n$.

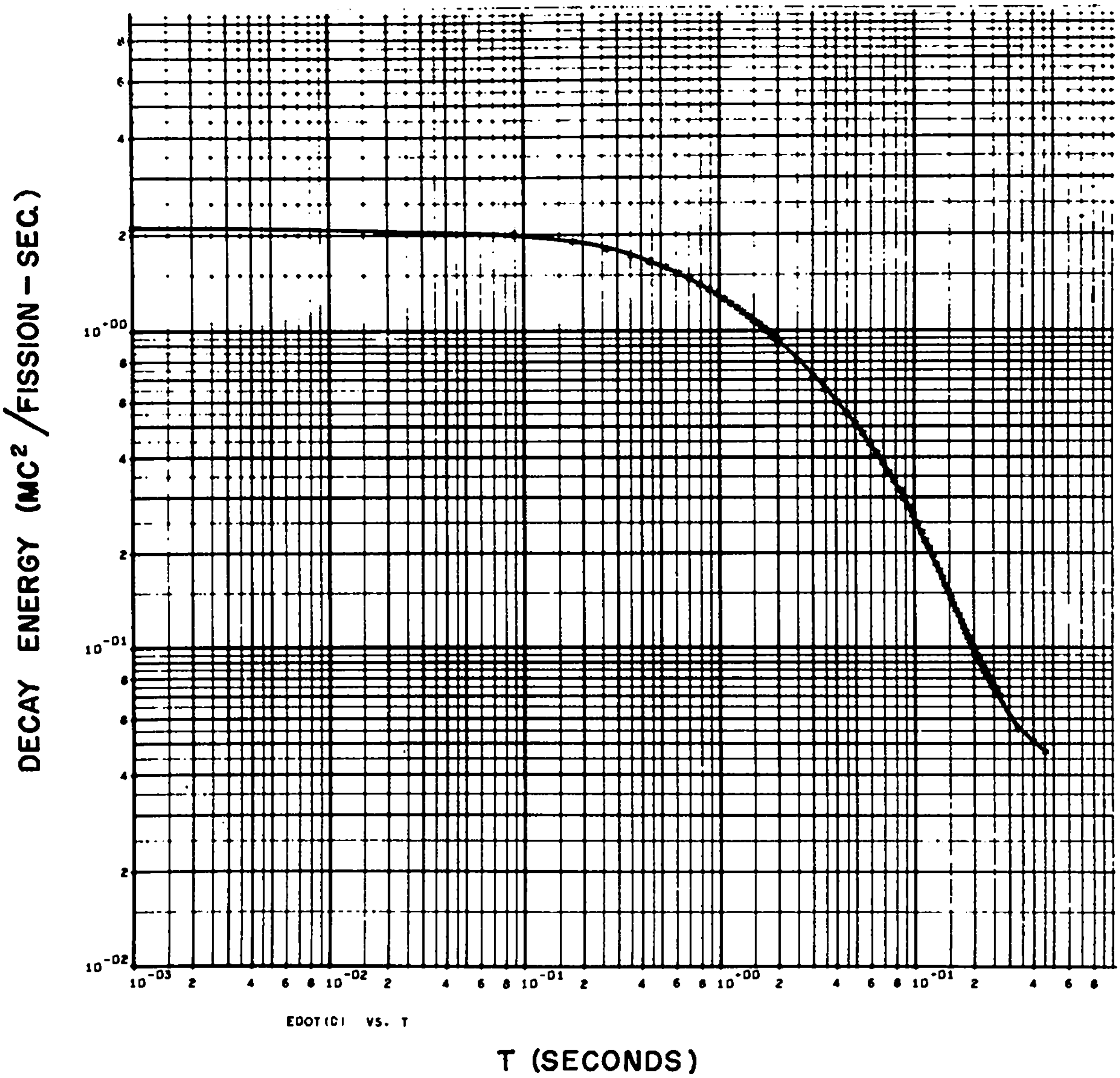

Fig. 6. $U^{233} + n$.

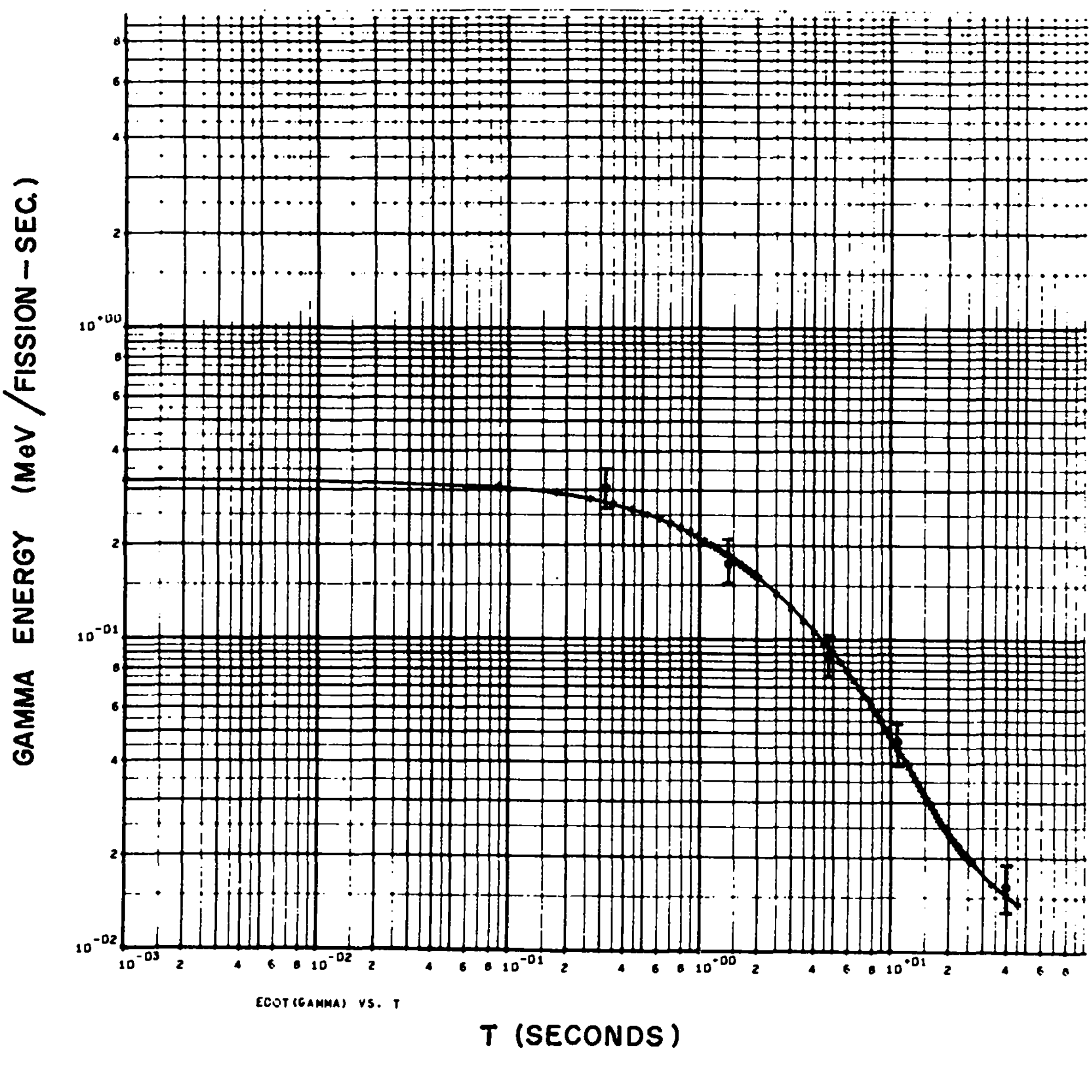
GAMMA ENERGY (MeV /FISSION - SEC.)
T (SECONDS)
ECOT(GAMMA) VS. T
10^-02
10^-01
10^+00
10^-03
10^+01

Fig. 7. U^{233} + n.

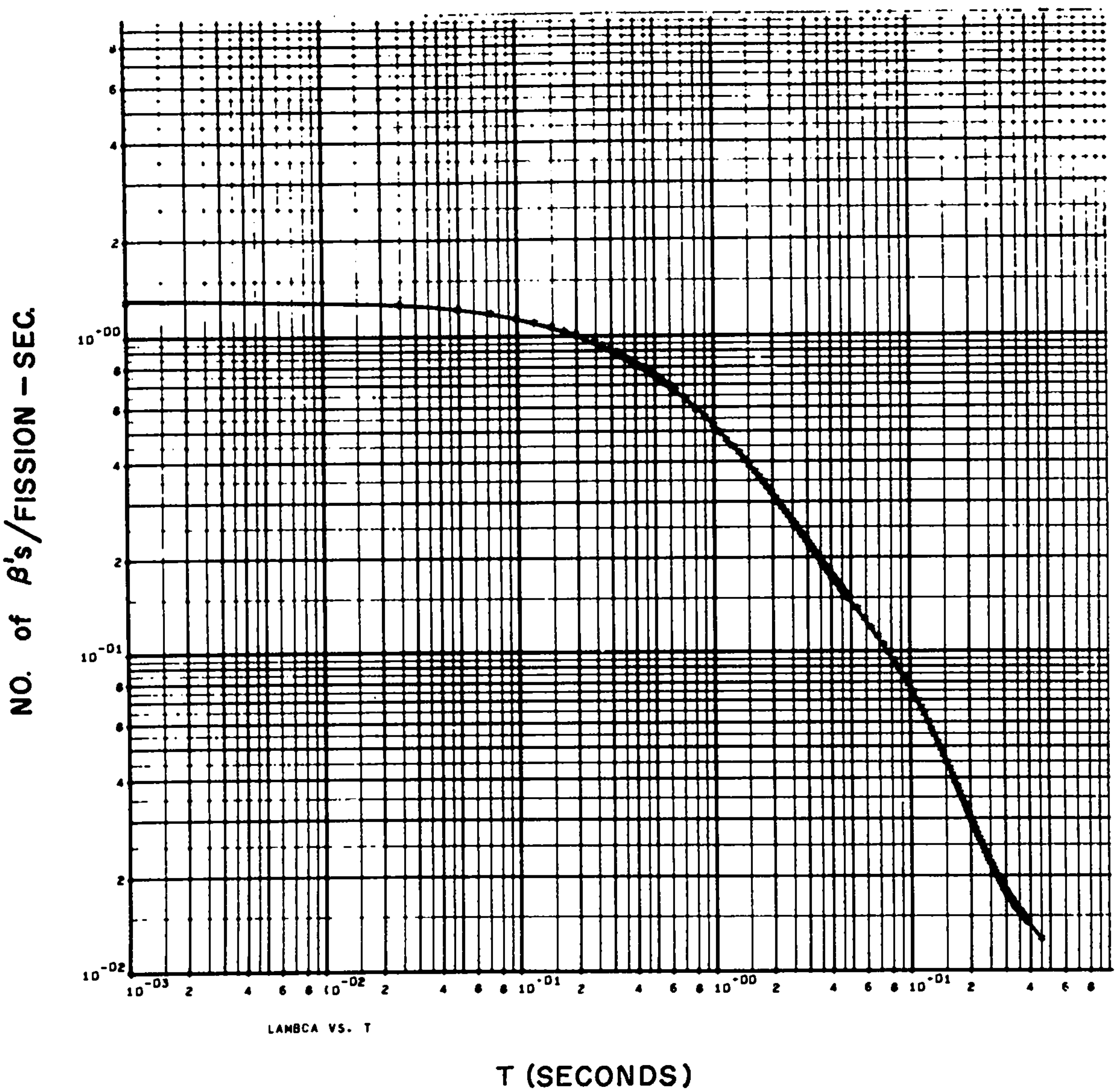

Fig. 8. $U^{238} + n$.

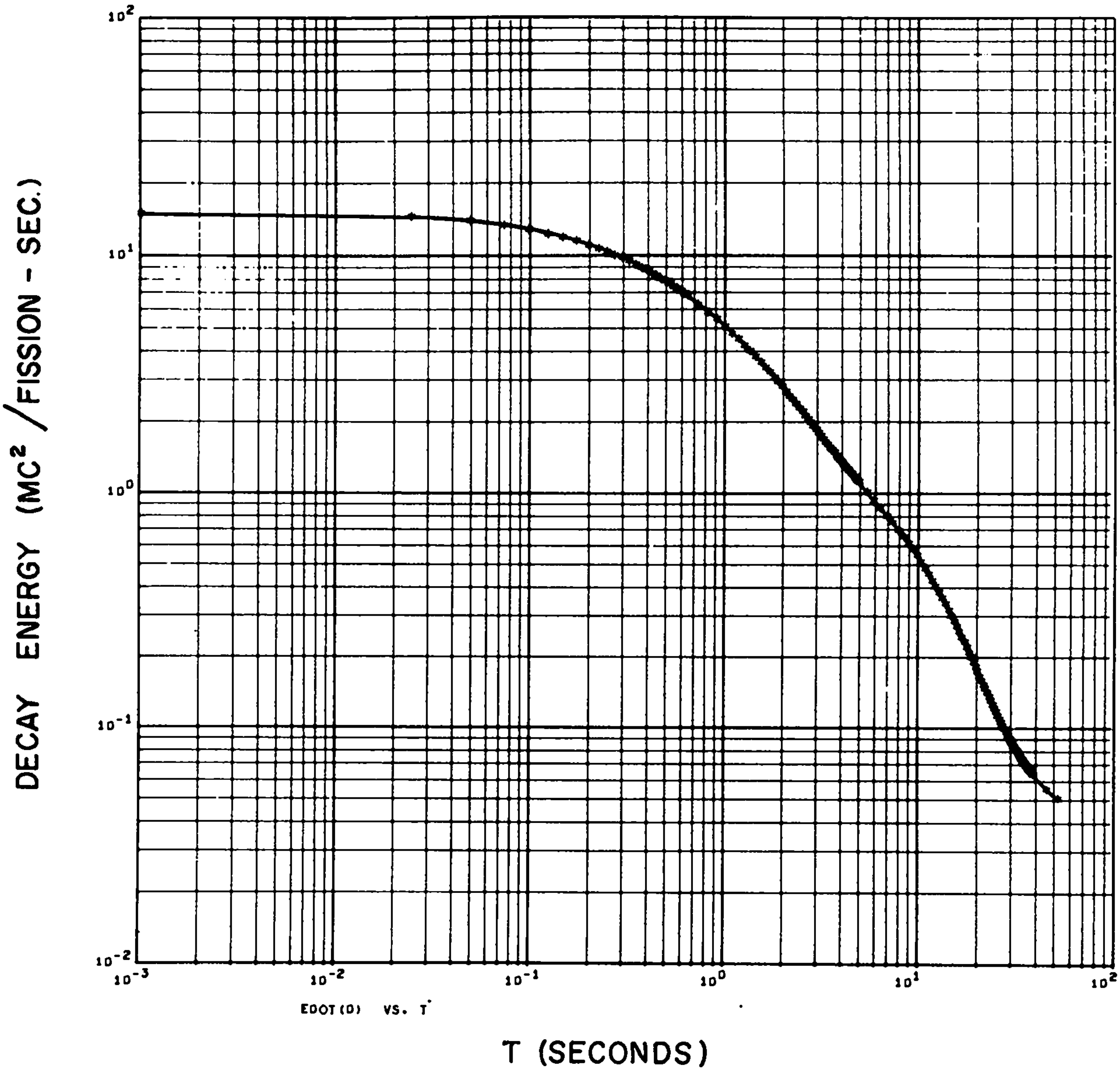

Fig. 9. U^{238} + n.

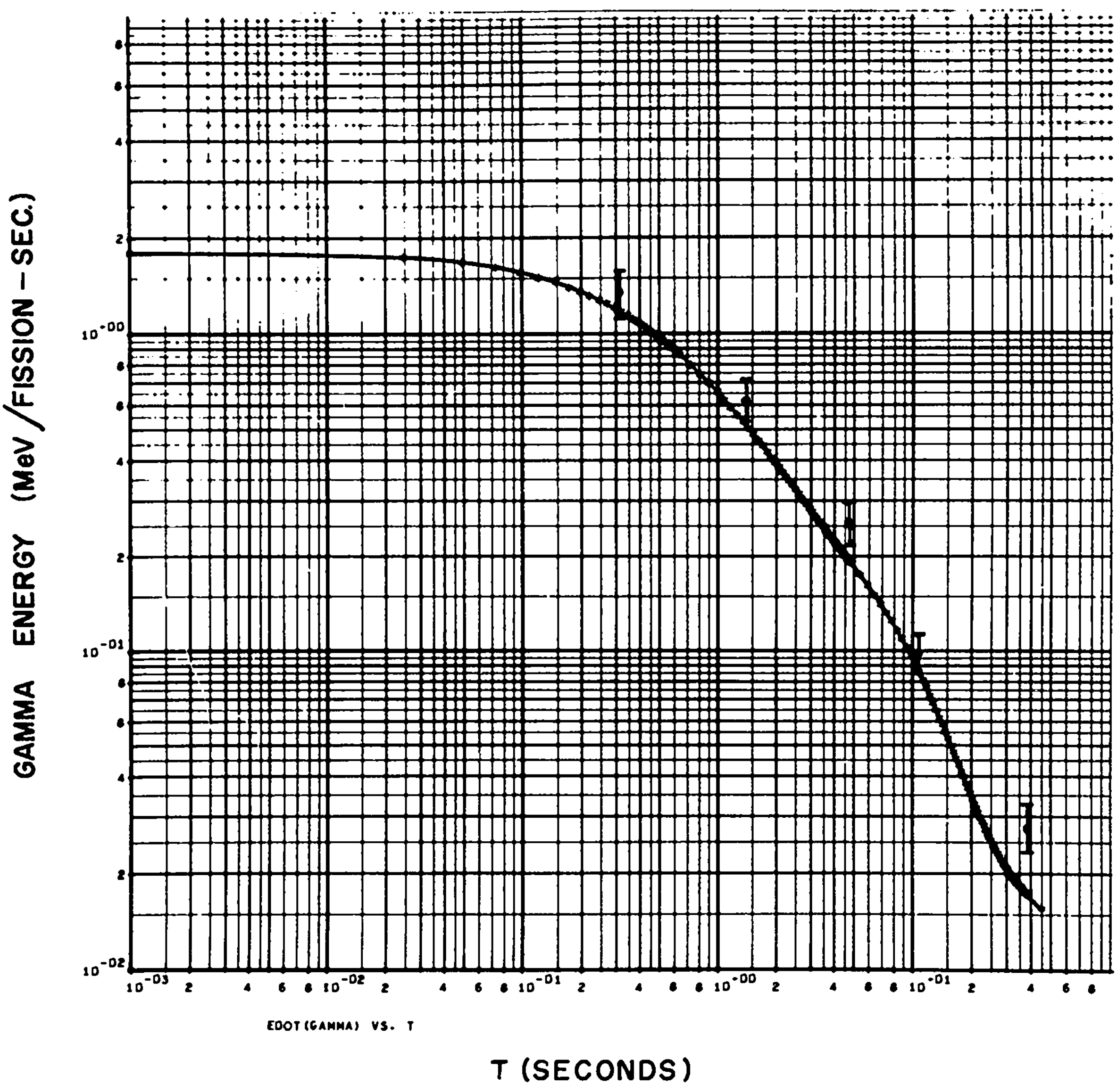

Fig. 10. U^{238} + n.

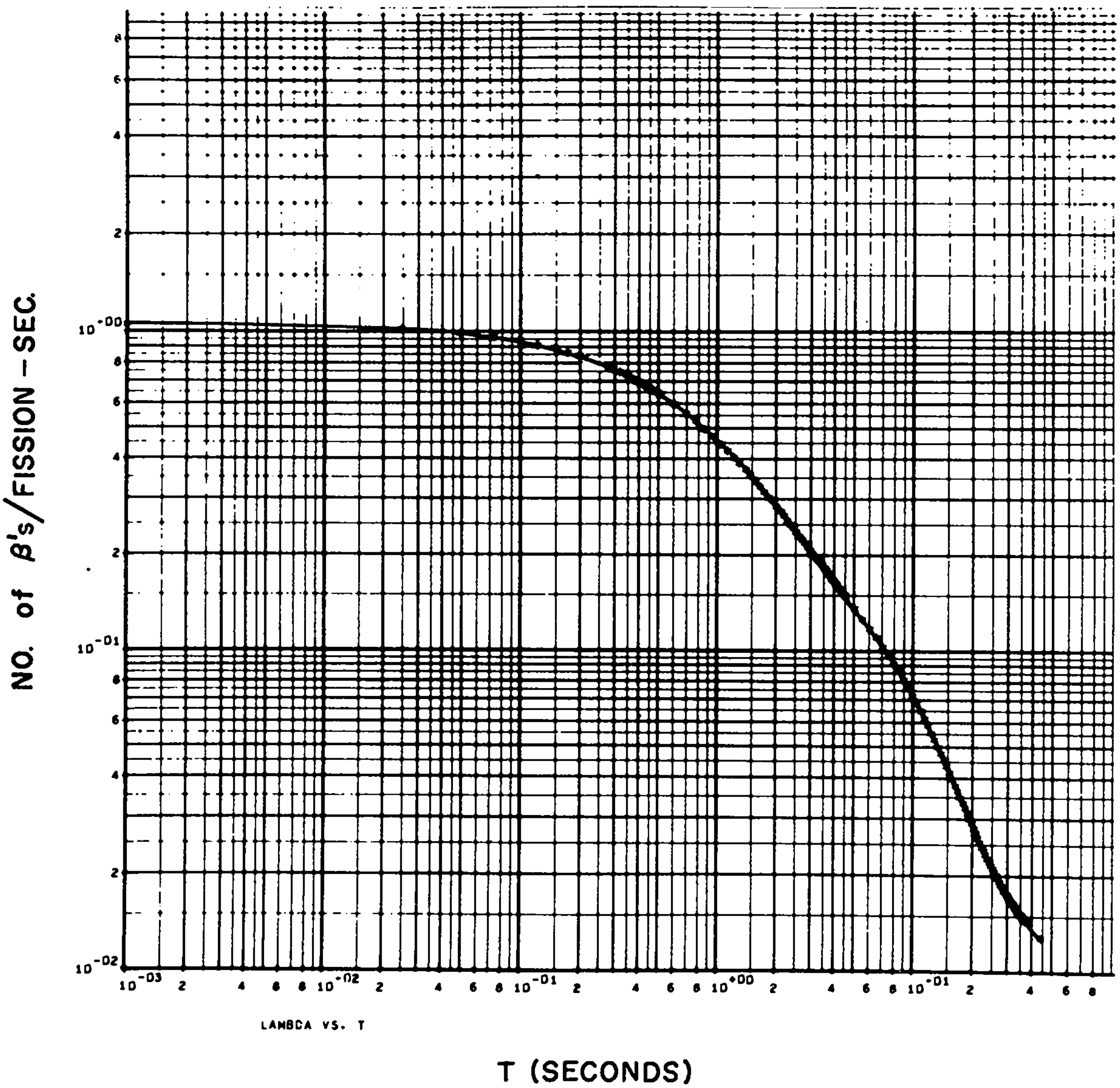

Fig. 11. $Th^{232} + n$.

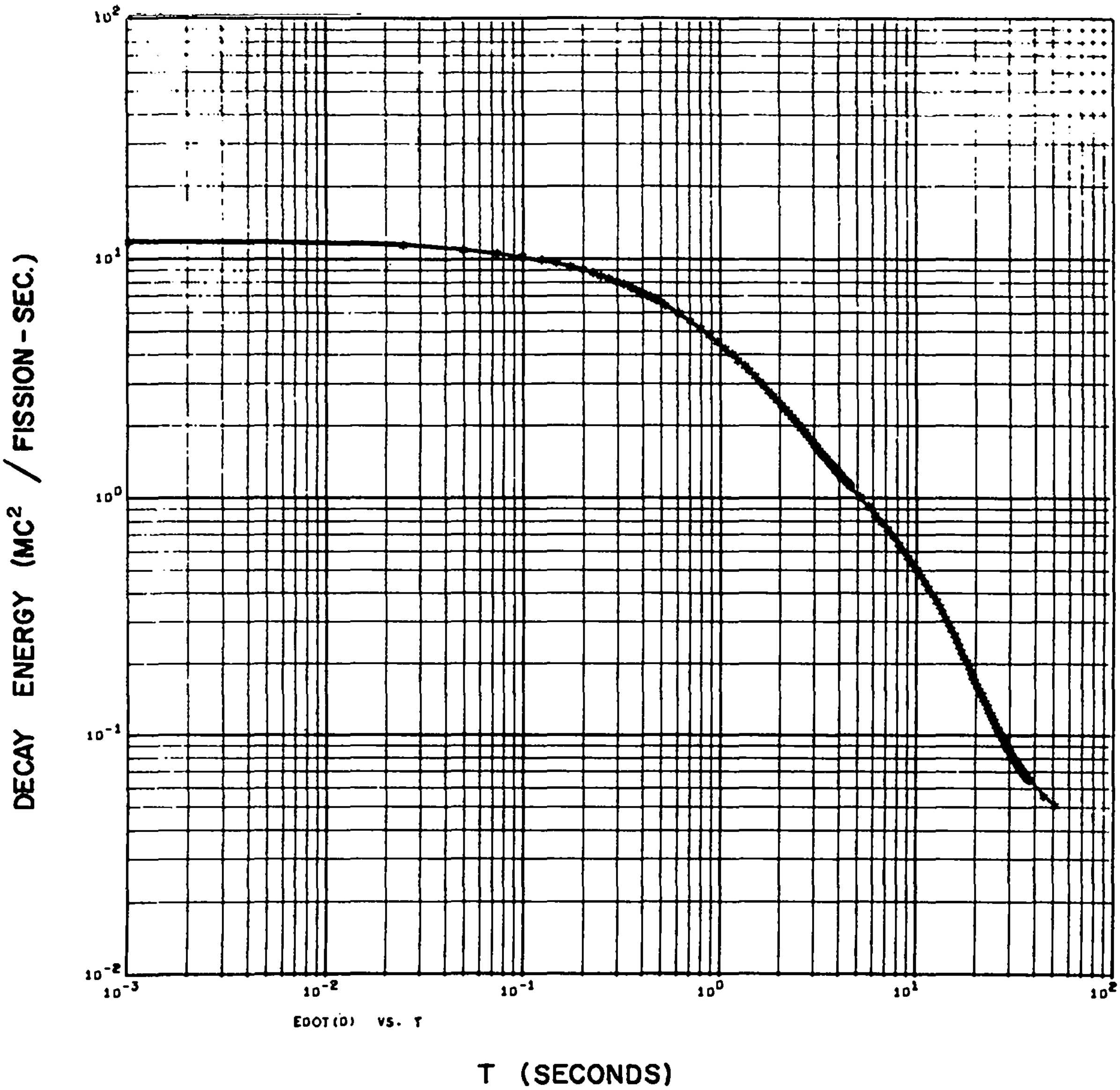

Fig. 12. $Th^{232} + n$.

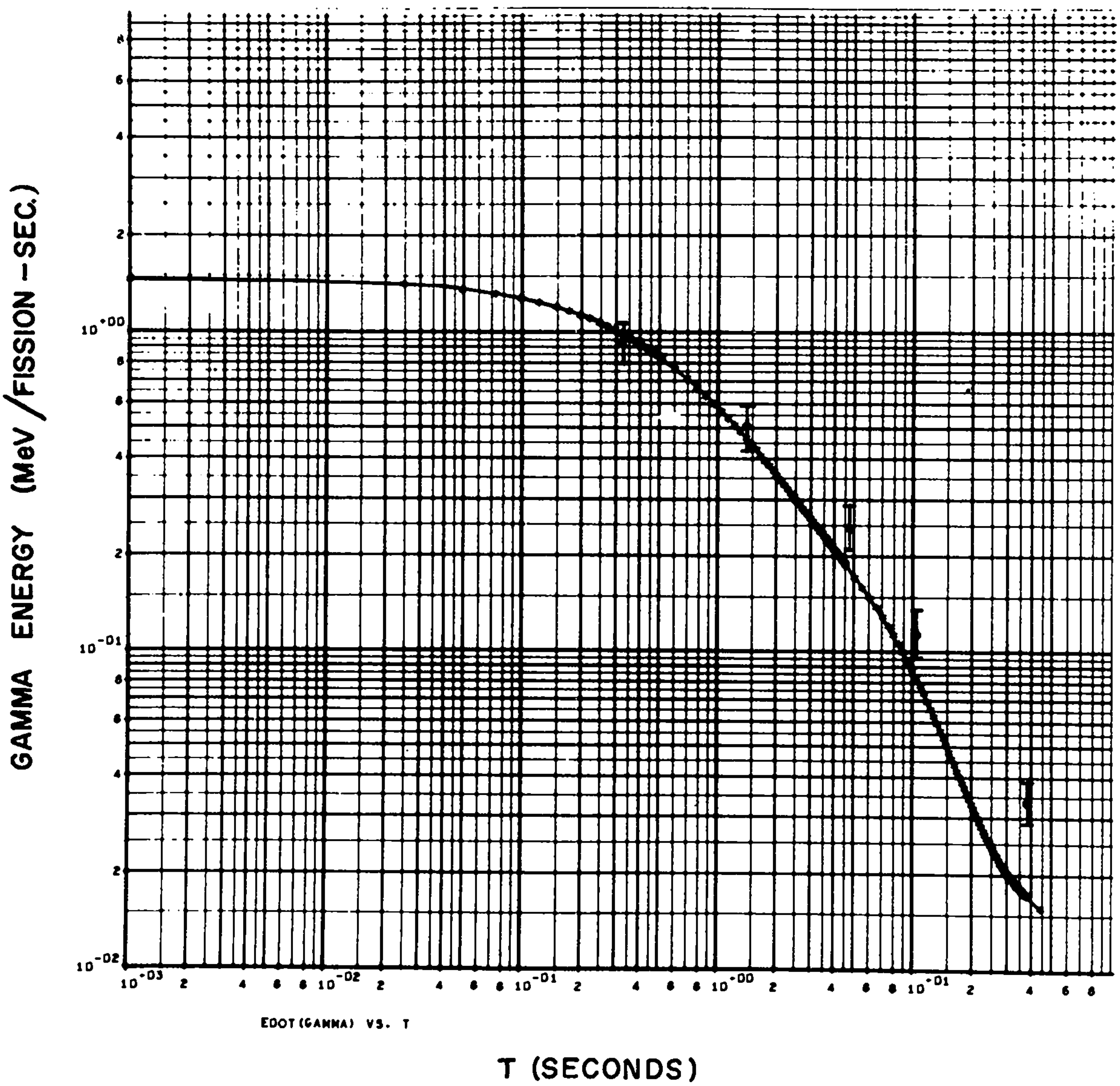

Fig. 13. $Th^{232} + n$.

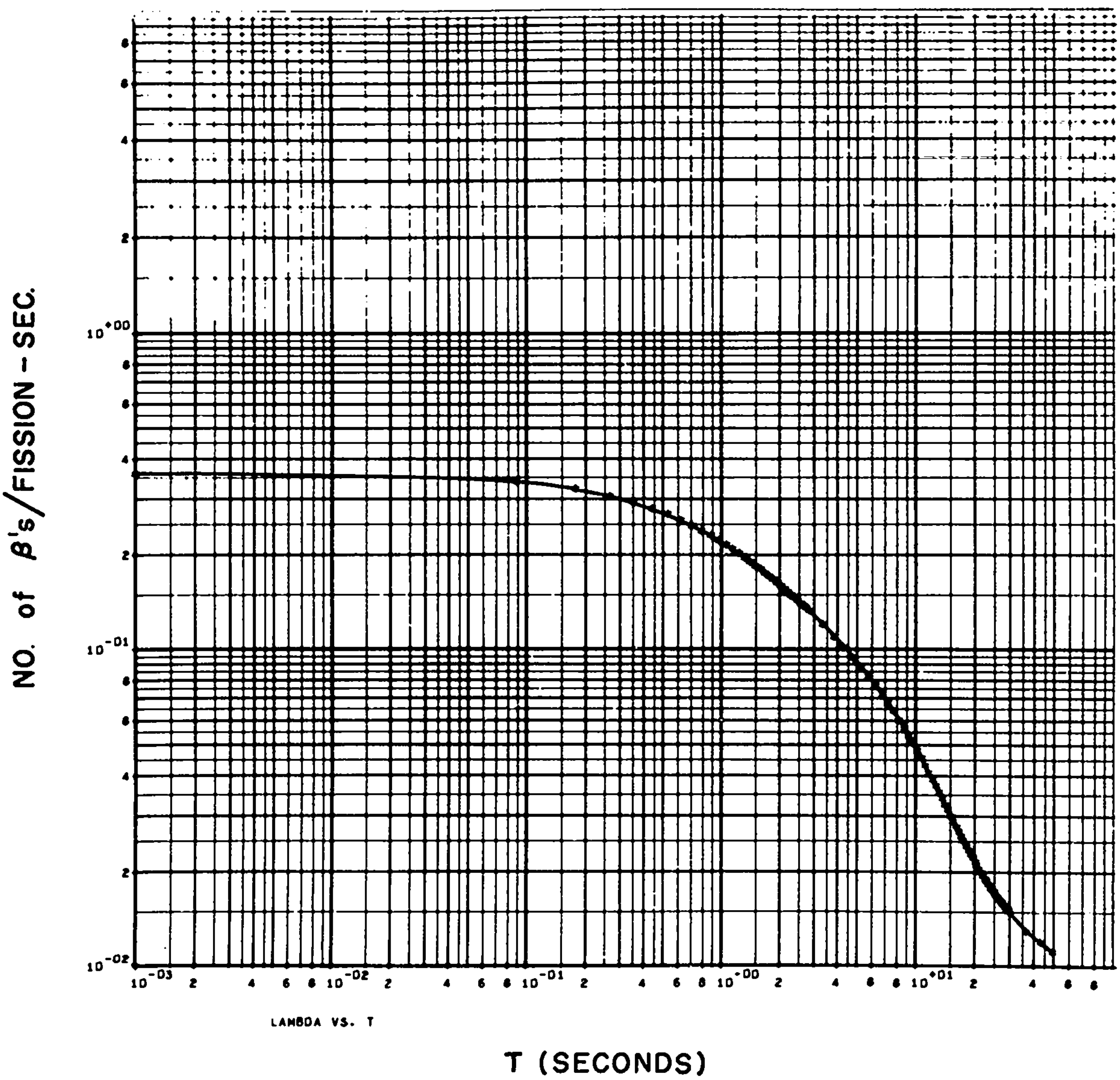

Fig. 14. Pu^{239} + n.

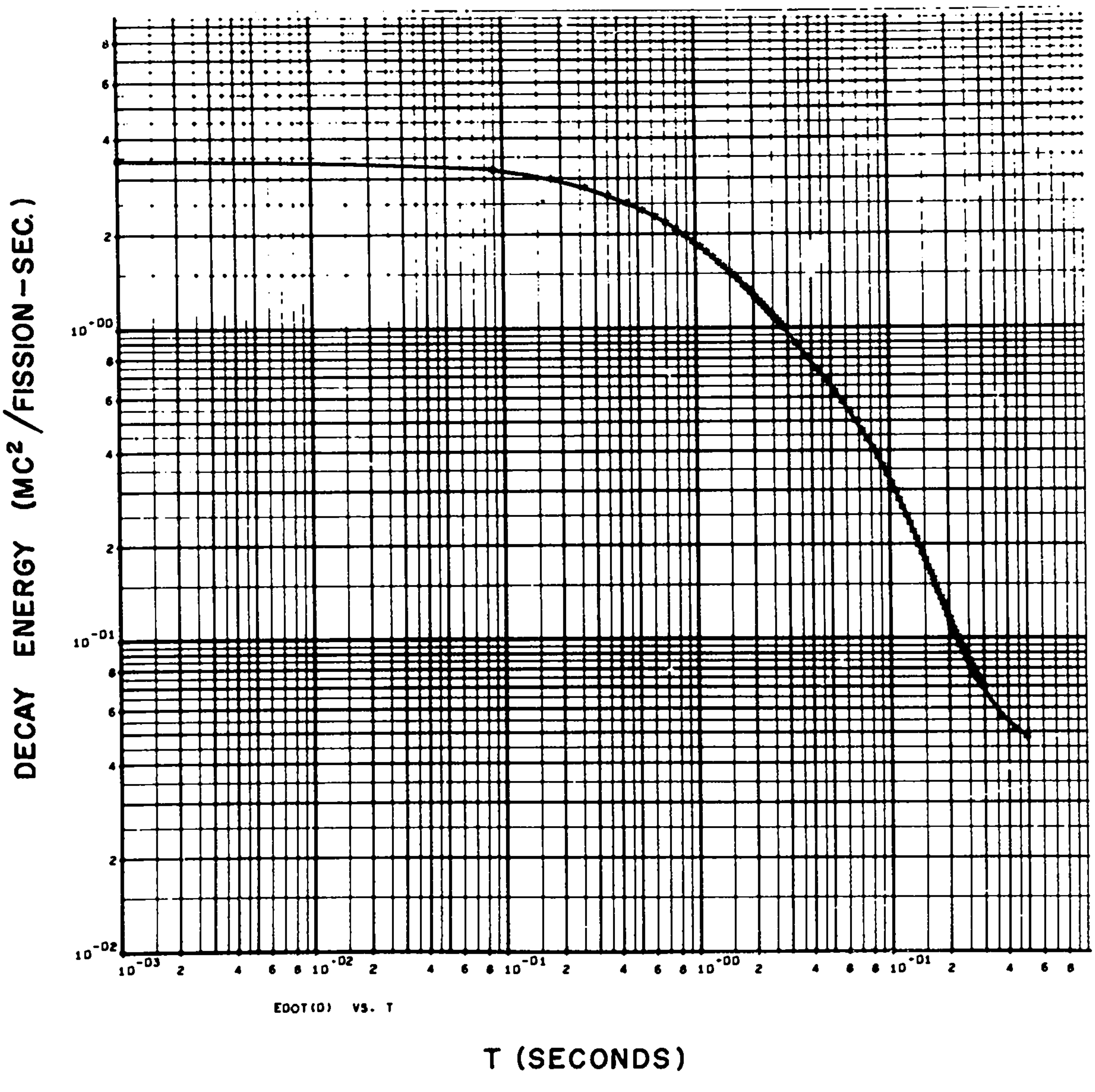

Fig. 15. $Pu^{239} + n$.

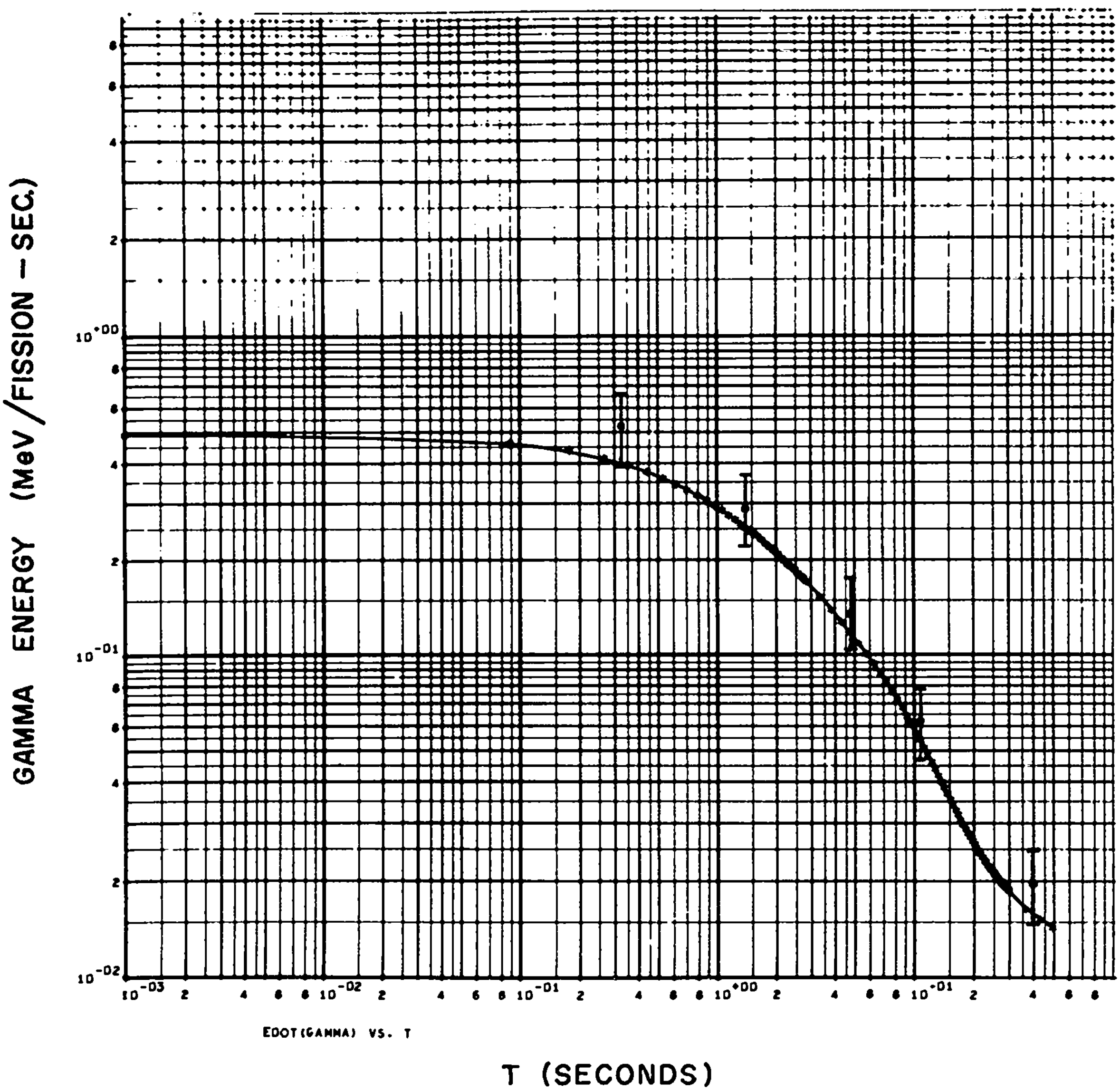

Fig. 16. $Pu^{239} + n$.

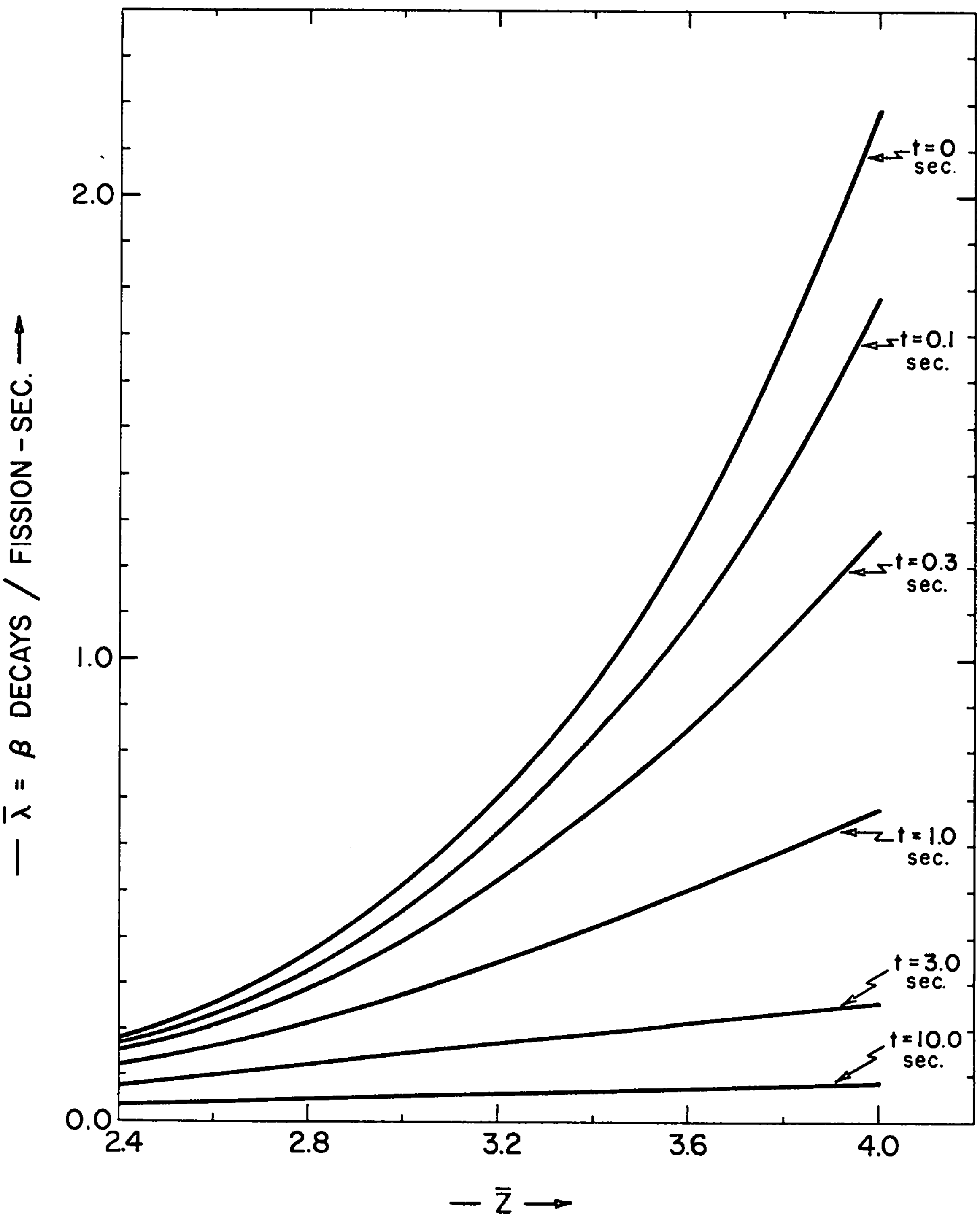

Fig. 17. $\bar{\lambda}$ vs $\bar{z}$ at various times.

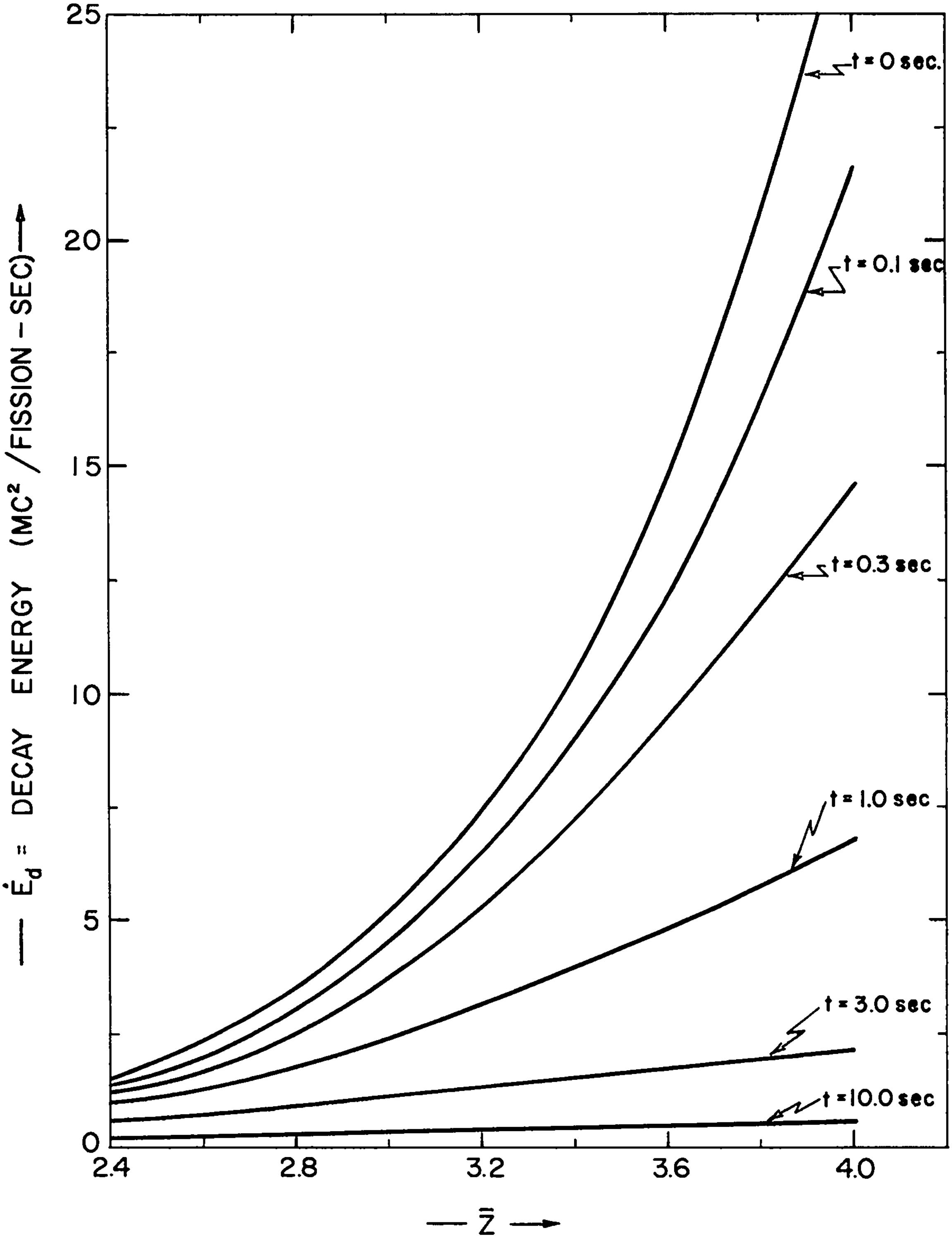

Fig. 18. $\dot{E}_d$ vs $\bar{z}$ at various times.

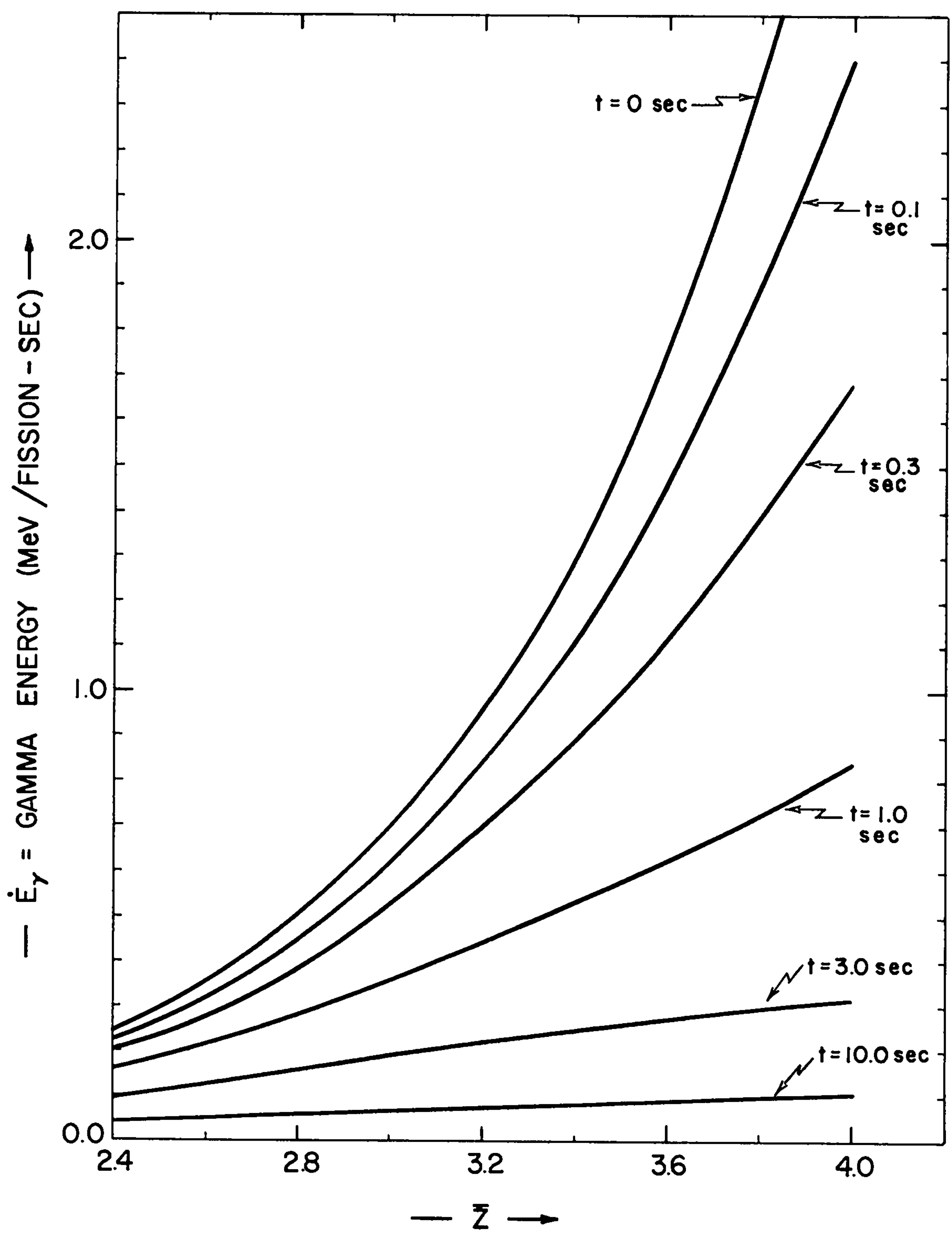

Fig. 19. $\dot{E}_\gamma$ vs $\bar{z}$ at various times.

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1. J. J. Griffin, Los Alamos Scientific Laboratory Report LA-2811, December 1962 (also LA-2811 Addendum, October 1963). Some of the present results have been submitted for publication in The Physical Review.

2. L. B. Engle and P. C. Fisher, Los Alamos Scientific Laboratory Report LAMS-2642, January 1962, and The Physical Review, to be published.

3. P. Armbruster and H. Meister, Z.f. Physik, 170, 274 (1962).

4. The author is grateful to Dr. R. B. Walton for pointing out this error in a private communication.