# LOS ALAMOS SCIENTIFIC LABORATORY OF THE UNIVERSITY OF CALIFORNIA O LOS ALAMOS NEW MEXICO 

FAST REACTOR ROCKET ENGINES--CRITICALITY


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FAST REACTOR ROCKET ENGINES--CRITICALITY
by
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## ABSTRACT

Critical sizes are determined for a variety of fast assemblies appropriate for nuclear rocket reactors. These are based on cores of $\mathrm{UO}_{2}-\mathrm{W}$ cermets or UC-metal carbide solid solutions reflected by beryllium. Rocket reactors weighing as little as 200 lb . are possible, and, in larger sizes, either high power density or high exit gas temperature can be achieved. The fast spectrum allows the use of the most refractory materials (such as HfC) in bulk to obtain high performance. Use of $U^{233}$ in place of $U^{235}$ can lead to substantial improvements in reactor weight, specific power, and/or temperature capability at the cost of the radiation associated with $U^{233}$. The core sizes are generally quite small, which is valuable where shielding may be significant. Nuclear aspects, including control, uranium investment, power distribution, and reflector materials, are briefly discussed.

## INIRODUCIION

Although the fact that fast spectra could lead to the development of small reactors for nuclear rocket engines was observed as early as 1949 (I), no comprehensive and quantitative study of this possibility has appeared. In this paper we present some results of such a study, augmenting by calculation details the criticality aspects which were reported in Ref. 2. Subsequent work will deal with heat transfer, materials, application, etc.

The application of direct cycle nuclear heat exchangers to space propulsion puts a premium on low reactor weight, high power density, and very high temperature $\left(>1500^{\circ} \mathrm{C}\right)$ capability. We shall show that fast reactors can meet these requirements over a wide range of interest, particularly for lightweight engines with thrusts of the order of 10,000 to $100,000 \mathrm{lb}$ and for heavier engines with very high melting fuel elements. In addition, fast reactors with small dimensions would require mach less shielding where it is necessary.

## Criticality

Of the various problems associated with fast reactors for rocket propulsion, the nuclear criticality results are the least questioned, as there are considerable experimental data and theoretical techniques in this area. Furthermore, those features that cause major uncertainties in criticality surveys (heterogeneity, resonance effects) have a minimum effect on this type of reactor (homogeneous and fully enriched). Void space and fuel element thicknesses will be of the order of 0.1
inches, which is one-twentieth of the neutron mean free path. Since nuclear criticality results have been obtained with rather sophisticated methods and are checked against experimental data where possible, they should be useful as check points for other methods of calculation of small fast systems, particularly those with thermalizing reflectors, such as are considered for compact power sources. Note that while our basic criticality results for fully reflected spheres are computed as precisely as possible, the results for the rocket reactors will be more approximately and conservatively computed to allow for control, departure from the idealized geometry and composition, etc.

Because of the effective homogeneity of the core, critical masses Whll be relatively independent of core details such as fuel element thickness, and will depend primarily upon the composition and average void fraction. Thus, results will be presented as masses vs total cross-sectional flow area. The achievable power level is contingent on core details, such as heat transfer area, fuel element thickness and conductivity, operating pressure, and exit gas Mach number. For orientation purposes, with the above items considered, up to 2000 Mw (or $100,000 \mathrm{lb}$ thrust) per square foot of cross-sectional flow area seems feasible. This corresponds to an exit Mach number of 0.25 at an operating pressure of 1000 psi and is based on a detailed heat transfer study which will be reported when complete. While this is very high power density on an absolute basis ( $\sim 30 \mathrm{kw} / \mathrm{cc}$ ) the relatively short lifetime requirements (minutes to hours) allow greater leeway in this direction and
theoretical values of this order and higher have been reported (3).

## Basis of Calculations

Most of the criticality computations were made with a one-dimensional $S_{4}$ transport code (4) employing 20 to 40 space points and 13 energy groups. Uranium and zirconium cross sections were transcribed from a Hansen-Roach 16 group (5) and rechecked against experimental assemblies. Various theoretical and empirical correlations were used where data were insufficient. "Cloudy crystal ball model" results (6) were used to obtain high energy transport cross sections and the statistical model for inelastically scattered neutron distributions. (7) Where appropriate (e.g., for $U, W, M O$ ), resonance effects were accounted for by using the work of G. Bell. (8) Energy-dependent cross sections were, in general, weighted by appropriate flux spectra within the group energy interval to obtain group values. The 13 groups covered energies from 10 Mev to thermal, and are listed in Table I along with the spectra and fission rates for a typical fast reactor. By comparison with an Oy sphere reflected with 2 in. of $B e$, we see that the propulsion reactior cores are truly fast, although coverage of the complete energy range is necessary due to the thermalizing effect of the reflector.

The microscopic cross sections were checked against a wide variety of critical assemblies (2) and replacement in fast spectra (10). The uranium cross sections gave $k$ within $1 \%$ for spheres of various enrichments (also see Ref. 5 for more comparisons). Some of the non-fissile materials ( $\mathrm{C}, \mathrm{Be}, \mathrm{W}, \mathrm{Ni}$ ) were checked against experiments where Oy
spheres were reflected by several inches of the other material. These gave $k$ to within about $2 \%$ and in some cases (e.g.,Be, Ni) the high energy cross sections were altered to match the experiments to within $1 \%$ in k . After the cross sections were developed they were compared with unreflected, diluted uranium fast critical experiments conducted by G. Jarvis at Los Alamos. Based on preliminary results the material cross sections which had been checked against reflected assembles ( $W, C$ ) gave critical radii differing by an average of $1 \%$ from experiment, whereas the relatively unchecked Ta cross sections gave critical radii about $3 \%$ larger than experiment. A further check of the calculations was made with an independent set of 18 group cross sections, (11) and gave good agreement for the few cases examined.

The one-dimensional transport code was used for straightforward calculations, such as variation of core composition, reflector thickness and density, fission distribution, and reactivity worth of $\mathrm{H}_{2}$. We ran a series of two-dimensional $S_{4}$ transport theory calculations (4) to check a few of the more difficult points: the control worth of a movable inlet end reflector, the extra core length required in the absence of an exit end reflector, and the conversion for spheres to cylinders. In order to conserve computing machine time, the 13 group cross sections were collapsed to four groups based on the neutron spectra obtained from the one-dimensional calculations. Group collapsing was done with the ZOT code (12) and was based on equations by G. Bell. The four group constants gave satisfactory results for the reactivity
( $k=1.006$ in contrast to 1.000 for 13 groups) for the one-dimensional case(Fig. la).

For the first two dimensional calculation (Fig. lb) the core was assumed to be a circular cylinder of the same radius as the spherical core, and the core height was determined by equating the bucklings of the cores assuming no reflector savings or extrapolation distance. This is deliberately conservative (the reflector saving is $\sim 12 \mathrm{~cm}$ for this case) and results in an excess reactivity of 0.032 , giving some leeway for control, elimination of power peaking, uncertainty in cross sections, etc. Burnup and fission product poisoning will be negligible over the short life of these reactors. Fig. lc shows two-dimensional check of the reflector savings for the exit reflector which was computed to be 12 cm from spherical calculations. This is the geometry assumed for the rocket reactors. One method of control is motion of the inlet reflector away from the core. The calculation (Fig. ld) indicated adequate reactivity change ( $\sim \$ 1 / \mathrm{cm}$ ) for control purposes.

RESULIS
Uranium Dioxide-Tungsten
One of the most obvious classes of fast reactor types for use as nuclear rocket engines is $\mathrm{UO}_{2}$ dispersed in a refractory metal such as Mo or W. A comparison of their properties (Table II) shows that their nuclear properties are somewhat similar, but that $W$ is about twice as dense and, most significantly, has a much higher melting point. The larger $W$ absorption, particularly in the resonance region, tends to
offset its larger scattering cross section. This leads to larger critical radii for a fixed $\mathrm{OU}_{2}$ concentration ( $14 \%$ larger core for 30 vol $\% \mathrm{UO}_{2}$ and 30 vol $\%$ refractory metal) and, because of its much higher density, to heavier reactors for the same $\mathrm{UO}_{2}$ investment. Owing to tungsten's higher melting point, we will emphasize the $W$ systems.

Though results vary $\mathrm{wi}^{\mathrm{th}} \mathrm{UO}_{2}$ and metal concentration the reactivities of Mo and $W$ are approximately zero in reactors of practical interest (Be reflected, 10 to 30 vol $\% \mathrm{UO}_{2}$ and 20 to $60 \%$ refractory metal in the core). Molybdenum has a positive worth; e.g. in a core with $\sim 15 \%$ vol $\mathrm{UO}_{2}$, doubling the Mo from $30 \%$ to $60 \%$ reduced the core radius $4.6 \%$, giving an $0.15 \%$ decrease in radius per vol $\%$ increase of Mo in the core. Tungsten has a much smaller worth, usually negative. The core radii for three cases with $30 \mathrm{vol} \% \mathrm{UO}_{2}$ and 0,30 and 50 vol \% W were 26.21 , 26.08 and 26.30 cm ,respectively, less than $1 \%$ variation over the range. In a case with 15 vol \% $\mathrm{U}^{233} \mathrm{O}_{2}$ in the core, the radius increased $1.7 \%$ in doubling the W content from $22.5 \%$ to $45 \%$. W. Kirk, who first surveyed the $\mathrm{UO}_{2}-\mathrm{W}$ system, reported (13) the core radius constant within 2 or $3 \%$ for flyed amount of $\mathrm{UO}_{2}$ in the core ( 15 to $40 \mathrm{vol} \%$ ) over a range of $W$ concentrations (about 25 to $60 \mathrm{vol} \%$ ). Thus the critical core radius ( $R_{s}$ ) for a Be-reflected sphere is a function only of the volume fraction of $\mathrm{UO}_{2}$ in the core, which we shall call $u$. This relation,

$$
\begin{equation*}
R=f(u) \tag{1}
\end{equation*}
$$

together with an assumption of the volume fraction $(l)$ of $\mathrm{UO}_{2}$ in the fuel (fuel loading) define the reactor critical weight and (with a specification of the core $L / D$ ratio) the cross-sectional flow area.

Although a variety of compositions and thickness were examined, for this survey we fix the reflector to be 12 cm of Be at $75 \%$ of theoretical density ( $\rho=1.38 \mathrm{gm} / \mathrm{cc}$ ) to allow for coolant flow channels. This is a reasonable reflector for the cores of interest in terms of reducing reactor weight and flattening the power distribution, but is not optimized for any given reactor. These considerations are discussed later. The fuel materials are assumed to be $\sim 95 \%$ of their theoretical densities, i.e., $10.4 \mathrm{~g} / \mathrm{cc}$ for $\mathrm{UO}_{2}$ and $18.4 \mathrm{~g} / \mathrm{cc}$ for W . In converting from spheres to idealized reactors (cylinders with one end bare), we have made the conservative assumption of equal core buckling (zero reflector savings) and replacement of one end reflector by an equivalent amount of core. Examination of these results and the two-dimensional calculations has led to a simple generalization that is adequate for preliminary criticality estimates. This is that a right circular cylinder of radius $R$ and height $2 R(L / D=1)$, reflected on the sides and one end, is neutronically equivalent to a fully reflected sphere of equal radius. Defining the core void fraction $V$, we have

$$
\begin{equation*}
V=1-\frac{u}{l} \tag{2}
\end{equation*}
$$

and the core cross-sectional flow area $A_{f}$ becomes

$$
\begin{equation*}
A_{f}=\pi R^{2}\left(1-\frac{u}{l}\right) \tag{3}
\end{equation*}
$$

The component masses are given by

$$
\begin{align*}
& M\left(U O_{2}\right)=2 \pi R^{3} u p\left(U O_{2}\right)  \tag{4a}\\
& M(W)=2 \pi R^{3} u \frac{(1-l)}{l} \rho(W)  \tag{4b}\\
& M(B e)=\pi t\left(5 R^{2}+4 R t+t^{2}\right) \rho(B e) \tag{4c}
\end{align*}
$$

where $t$ is the reflector thickness and $\rho$ the assumed material densities. Thus, the reactor mass is given by

$$
\begin{equation*}
M_{r}=M\left(U_{2}\right)+M(W)+M(B e) \tag{5a}
\end{equation*}
$$

which specifically excludes pump, pressure shell, nozzle, piping and valves, and ignores such items as structural supports in core and reflector, and controls and instrumentation in and about the reactor. For our assumptions

$$
\begin{equation*}
M_{r}=65 \cdot 3 R^{3} u+115.5 u \frac{(1-\ell)}{\ell}+260\left(R^{2}+9.6 R+28.8\right) \tag{5b}
\end{equation*}
$$

with $M$ in grams and $R$ in centimeters.
The critical spherical core radii for $\mathrm{UO}_{2}-\mathrm{W}$ reactors (for both $\mathrm{U}^{235}$ and $\mathrm{U}^{233}$ ) are given in Fig. 2 as functions of the $\mathrm{UO}_{2}$ volume fraction, u. The calculations were performed with $\ell=0.5$, but apply within $\sim 3 \%$ over a range of values for $\ell$ from 0.3 to 0.6 . Using these results for $R(u)$ and the above assumptions, one can compute $M_{r}$ and $A_{f}$ for various fuel loadings. Results (Fig. 3) are given as $M_{r}$ vs $A_{f}$ (in English units), which can also be interpreted as $M_{r}$ vs reactor power or engine thrust ( $1 \mathrm{ft}^{2}=2000 \mathrm{Mw}$ or $100,000 \mathrm{lb}$ thrust). Masses are given for fixed fuel loadings ( $\ell$ ) and for fixed $u$ (or fixed core radius). The latter is strictly linear, which follows simply from our assumptions. The former (fixed l) are approximately linear, although somewhat fortuitously so due to the dependence of $R$ upon $u$. One can expect linearity for small void fractions ( $A_{f} \ll \pi R^{2}$ ). Since criticality studies for Bereflected graphite reactors (14) indicate weights of 4000 to 6000 lb in the range of 0 to $3 \mathrm{ft}^{2}$ of flow area, we see that fuel loadings of 40 to $50 \%$ are necessary for the $\mathrm{UO}_{2}-\mathrm{W}$ reactors to surpass them on a power per weight basis. Other factors, such as physical size and recycling ability, must also enter into comparison of the two systems.

Table III presents details of $\mathrm{UO}_{2}$ reactors with void fractions in the range of 0.2 to 0.5 . The enriched uranium requirement is relatively high, 500 to $1000 \mathrm{lb} \mathrm{UO}_{2}\left(93 \% \mathrm{U}^{235}\right.$ ) being necessary. The use of Mo in place of $W$ generally reduces reactor weights by a factor of 2 for the same flow area at the cost of lower fuel element melting point.

## Uranium Carbide-Metal Carbide Reactors

Uranium carbide-metal carbide reactors present a more complex picture for several reasons. First, UC forms solid solutions with many metal carbides, generally with a continuous range of properties (e.g., melting point) dependent upon the metal carbide and the composition. Thus, one must examine and select from a great many possible systems. Second, the metal carbide usually has a nonzero (positive or negative) reactivity worth, which prevents the simplifying approximation for critical core radius made for $\mathrm{UO}_{2}-\mathrm{W}$ reactors and therefore requires a greater number of criticality calculations. Finally, the mechanical properties of the carbides are such that some unknown amount of metallic support structure might be required, which would lead to uncertainties in the reactor mass.

Selected properties of some carbides are listed in Table IV. One could conceive of a reactor core composed of unalloyed UC (which melts at $2450^{\circ} \mathrm{C}$ ). Although the gas temperature would be limited, useful exhaust velocities ( $\geq 18,000 \mathrm{ft} / \mathrm{sec}$ ) could be achieved with very small reactors ( 200 to 300 lb ). The desire for higher performance leads to consideration of solid solutions with the higher melting carbides. We choose ZrC for our examples, as many of its chemical and nuclear properties are known, and seem suitable for this application. Furthermore, UC-ZrC solid solutions have been studied (15) and used in connection with the plasma thermocouple program at Los Alamos, including in-pile tests in the Omega West reactor. (16)

From the UC-ZrC phase diagram (Fig. 4), we see that a continuous series of solid solutions with varying melting points exists. We must compromise between the desire for high melting point (low UC concentration) and low reactor weight (high UC concentration). Three compositions will be examined--100\% UC, 50 vol $\%$ UC, and 30 vol $\%$ UC -- with emphasis on the second value. The core radius vs vol $\%$ UC in the core is shown in Fig. 5 for these three fuel compositions. Since the reactivity worth of Mo and $W$ is small in these cores also, we can ignore the structure in estimating criticality and exchange structure for void when necessary. Actually, most of the calculations included $10 \%$ by volume of the core of Mo or $W$, which was assumed as a conservative value for the structure. The value in practice will depend upon detailed core design, but limiting conditions (no structure and $10 \% \mathrm{~W}$ ) are examined for the 50 vol $\%$ UC solid solution core. If we assume zero reactivity worth, the effect of structure is to reduce the flow area and increase the core mass, leaving the core radius fixed.

Reactor weights vs flow area are presented in Fig. 6 with an ordinate scale different from that for the $\mathrm{UO}_{2}-\mathrm{W}$ reactors, since the UC-ZrC reactor weights are much smaller, generally by a factor of about 2 for the same flow area. This shows the variation in reactor weight which can occur for changes in the amount of structure required (curves $b$ and c) or changes in fuel composition (curves cand d). $100 \%$ UC fueled reactors (curve a) are very low in weight ( 200 to 600 lbs ) but are restricted to low flow areas for moderate void fractions ( $<60 \%$ ).

Quantities of interest for typical reactors are shown in Table $V$. The 50 vol \% UC-50 vol \% ZrC reactors require 200 to 400 lb enriched uranium and weigh 600 to 2000 lb . Their diameters are similar to the $\mathrm{UO}_{2}-\mathrm{W}$ reactors ( 12 to 24 in. ), which are half those of graphite reactors. Pure UC core reactors can be made with only 100 lb of $U^{235}$ and with total weights of less than 300 lb , again at the expense of lower melting point. To go in the direction of higher exit gas temperature requires lower concentrations of UC in the solid solution, higher melting diluents ( $\mathrm{HfC}, \mathrm{TaC}$ ), and/or varying the UC concentration through the core. (The use of $\mathrm{U}^{233}$ in this connection will be discussed later.) For example, raising the fuel element melting point $200^{\circ} \mathrm{C}$ by changing the composition from $50 \% \mathrm{UC}$ to $30 \% \mathrm{UC}$ in ZrC adds about 400 lb for a uniformly loaded core with the same flow area. Zirconium has very good neutronic properties, and UC composition can be lowered to the point where the system is intermediate or thermal ( $<1 \% \mathrm{UC}$ ). For very high exit gas temperatures, one might use TaC or $\mathrm{HfC}\left(\mathrm{MP} \sim 3800^{\circ} \mathrm{C}\right.$ ) with low concentrations of UC. Although Ta and Hf are strong poisons in intermediate and thermal spectra, their use in bulk is permissible in fast reactors even where the $C / U$ ratio is $\sim 5$. Based on the best available cross-section data, Hf appears neutronically superior, as can be seen from Table VI and Fig. 7, which give critical radii for several such reactors. No metallic support structure has been included, since metal-carbide eutectics might form at lower temperatures, defeating the goal of high exit gas temperature.

Thus, such reactors might have to use the carbides structurally as well as for fuel elements. Hafnium carbide has a negative coefficient of reactivity that tends to limit lasings to a minimum of about $25 \%$ Oyc in the fuel if the reactor core is to be less than 3 ft in diameter. Further dilution of the OyC leads to greater neutron leakage, a degraded spectrum, and a greater capture in the Hf , which causes the critical radius and reactor mass to increase very rapidly.

Uranium-233
The nuclear properties of $\mathrm{U}^{233}$ are much superior to those of oralloy ( $93 \%$ enriched $\mathrm{U}^{235}$ ) for fast spectra, as can be seen from comparison of their bare sphere critical masses of 16 and 52 kg , respectively (2). One can take advantage of this superiority in a number of different ways, e.g., reduce the minimum reactor size by a factor of about 2, reduce the reactor weight for a given flow area (power level), increase the void space and therefore the flow area for a flxed core size, alter the power distribution with fixed uranium loadings, or reduce the loadings while maintaining the same core size and void fraction. The last is probably the most promising, particularly with regard to achieving high gas temperatures (and consequently obtaining hydrogen dissociation). This nuclear superiority extends to the intermediate spectra of small graphite reactors $(C / U \sim 150)$ of the type described in reference (14) which could be reduced to 2000 to 3000 1b with $\mathrm{U}^{233}$.

The major practical disadvantage of $\mathrm{U}^{233}$ is radioactivity, some of which is intrinsic, but most of which is due to a contaminant $\left(U^{232}\right)$. As does $P u^{239}, U^{233}$ emits $\alpha^{\prime}$ s followed by soft $\gamma^{\prime} \mathrm{s}$. A greater difficulty is due to the $2.6 \mathrm{Mev} \gamma$-ray from the decay products of the $U^{232}$ impurity formed with the $U^{233}$. Briefly this decay chain (4n series) is

$$
\mathrm{U}^{232} \underset{70 \mathrm{yr}}{\alpha} \mathrm{Th}^{228} \underset{1.9 \mathrm{yr}}{\alpha} \underset{\mathrm{fast}}{\alpha \beta} \mathrm{Bi}^{212} \frac{\alpha(34 \%)}{\text { fast }} \mathrm{mI}^{208} \frac{\beta}{\mathrm{fast}} \mathrm{~Pb}^{208 *}
$$

The $\mathrm{Pb}^{208}$ is formed only in its 2.6 Mev excited state and is the source of the trouble. The 1.9 yr half-life of $\mathrm{Th}-228$ controls the buildup of activity for short times ( $\leq 4$ years), which increases linearly from zero activity just after separation of the $U$ from the $T h$. (17) One can try to minimize the $U^{232}$ production or fabricate the reactor soon after the metal separation. In either case, complications are involved, and one would expect $U^{233}$ to be used only after much experience is gained with $U^{235}$ reactors. Much of the $U^{233}$ technology would follow directly (fuel element fabrication, etc.) because of the chemical and physical similarities, and even $U^{238}$ would be adequate for the early non-nuclear research (thermal conductivity, melting points, etc.).

A U ${ }^{233}$ fueled reactor would be somewhat radioactive before startup, necessitating additional ground support effort; but after one operation, the fission product decay would probably dominate the activity, and reuse would not be further complicated. Uranium-233 (and $\mathrm{Pu}{ }^{239}$ ) have smaller delayed neutron fractions, which may make control more
difficult but not unfeasible, as demonstrated by the existing Pu fast critical assemblies and reactors. Whereas $P u$ itself might also be considered as a fuel, in general, the physical properties (melting point, vapor pressure, etc.) of its compounds are not as desirable for this application.

The spherical core radius as a function of $U^{233} O_{2}$ fraction is given in Fig. 2, curve b. Only 50 to $60 \%$ as much $U^{233} O_{2}$ as $U^{235} O_{2}$ ( $93 \%$ enriched) is required for a fixed core radius; another way of interpreting this is that a 25 to $30 \%$ loading of $U^{233} O_{2}$ in the fuel is equivalent to a $50 \%$ loading of $U^{235} O_{2}$. Alternatively, one could keep the loading high and reduce the reactor weight by a factor of 2 to 3 for equal flow area, as illustrated by the examples of Table VII. The value of reducing the loading for $\mathrm{UO}_{2}-W$ fuel elements is difficult to assess quantitatively, since the metal retains its melting point for all compositions, whereas the maximum operating temperature would be some unknown function of composition of the $\mathrm{UO}_{2}-\mathrm{W}$ two-phase cermet. On the other hand, the UC-MC solid solutions are single phase and have definite relations between melting point and composition. Although other factors may limit temperature (structural members, uranium loss from the fuel, etc.), we shall assume the melting point as an index of the achievable exit gas temperature (which may be some fraction of the melting point or some number of degrees below it). Then the performance is simply related to the composition. A calculation has shown that 22 vol $\% U^{233}$ C-78 vol $\% \mathrm{ZrC}$ fuel is neutronically equivalent to
an equal volume of 50 vol $\% 0 y C-50$ vol $\% \mathrm{ZrC}$ fuel in a 15 in. diameter core. The former melts at $\sim 3090^{\circ} \mathrm{C}$, which is $315^{\circ}$ higher than the Oyc-ZrC fuel, and in addition weighs only $81 \%$ as much.

## DISCUSSION

There are a number of topicsincluding control, uranium investment, power distribution, reflector materials, and shielding that are associated with the nuclear aspects of these engines and warrant some consideration here.

Control of fast reactors was a problem that inhibited their consideration in the early period of nuclear development. Normal control of fast reactors depends upon the delayed neutrons, as is the case with thermal reactors, and thus the shorter prompt neutron lifetime does not strongly affect the normal control problem. Should the reactor become prompt critical, the shorter lifetime, smaller temperature coefficient, and smaller mass of nonfissile material to absorb the energy release would make fast reactors more likely to melt or vaporize. Nevertheless, fast critical assemblies (18) have been pulsed above prompt critical without damage. The high reactivity worth of hydrogen in fast cores necessitates care in preventing large amounts (such as in slugs of liquid $H_{2}$ ) from suddenly entering the core. This can be alleviated by vaporizing the $\mathrm{H}_{2}$ in the reflector and/or keeping the pressure above the critical value ( 188 psi ) to avoid two-phase flow. It might be possible to use the positive hydrogen reactivity as a con-
trol technique coupled directly to the power demand. Other techniques that have been shown to be neutronically feasible in providing control include movement of the inlet reflector, control drums in the side reflector, movement of fuel out of the core, and boron control rods in the core.

Fast reactors characteristically have high uranium investments compared to thermal assemblies of comparable size. However, for power levels of the order of 1000 NW , the requirements ( 200 to 500 lb of highly enriched $\mathrm{U}^{235}$ ) are similar to those of graphite reactors which are neutronically limited (small flow areas) and rather intermediate in their energy spectra for this size. The fast reactor fissile mass increases with power level, whereas that for graphite and other moderated systems decreases with size and power level. Thus, for large, high power reactors, thermal systems may be more economical, and due to the lower density of the moderator, can even have lower reactor weights per unit power where heat transfer considerations are limiting. For test purposes, one could make very small, low power UC reactors with 50 to 100 lb of enriched $\mathrm{U}^{235}$, but the long term usefulness of such reactors is not clear.

Power distributions will vary considerably with reactor and reflector sizes and compositions. One desires to have the radial power as flat as possible, and this is typically so within $\pm 10 \%$. Thermalizing reflectors (Be, C) lead to a power spike at the core surface, (Fig. 8, curve a), but this can be eliminated or reduced by putting
some absorbing (or fissile) material in the reflector (curve b). Generally the larger the core, the more it will approach a cosineshaped distribution. The axial distribution will be close to a cosine for a 4 ft core, whereas it will be considerably truncated (as shown in Fig. 9) for a small (19 in. long) core. Such a shape is generally desirable, since it has high power density at the cold inlet and tails off where the gas is hot, although high power density at the cold end can lead to fuel stress and heat transfer problems.

On a weight basis, Be is the best reflector material. A direct replacement of a 12 cm Be reflector would require over twice the thickness of graphite. In a typical case, this could be reduced to 18 cm by increasing the core diameter $15 \%$, which changes the weight, power level, power distribution, etc. For these fast spectra, heavy element reflectors are feasible, and Ni is almost as good as Be on a volume basis. An equivalent ( 14 cm ) Ni reflector would weigh about six times as much as Be. If gamma shielding were a consideration, the Ni reflector would act as a shield as well and, being closer to the core, would be more efficient. Beryllium would be better for slowing neutrons, and some composite might be optimum. Loading part of the reflector with fissile material would improve the core power distribution and supply a portion of the temperature rise of the $H_{2}$, which would improve control and heat transfer at the core inlet.

The small physical size of the fast reactors leads to much smaller shield weights than for less dense (e.g., graphite) reactors. Table

VIII illustrates relative values for three types of reactors with two types of shield. The shadow shield covers the inlet end of the reactor and lies between it and the propellant tankage and payload. It gives an attenuation of $\sim 5000$ for 1 Mev garma rays. The 5 in. circumferential shield gives an attenuation of $\sim 70$, and such a shield might be required for rendezvous operations. Even with the reactor shut down, the fission product decay gammas represent an appreciable source. We do not imply that the specific shield weights given are either necessary or sufficient; only the relative values are significant. However, a 10 in. lead shadow shield has been estimated to be sufficient for a biological shield under certain conditions, and a circumferential shield with a factor of 70 attenuation would permit debarkation from a nuclear vehicle a short time after landing.

## CONCLUSIONS

A number of conclusions can be drawn from these calculations, and only the more important will be mentioned. First; fast reactors offer possibilities both for lightweight nuclear rocket engines and for very high temperature engines. Second, there is a wide variety of reactor weights, sizes, and compositions from which to choose. Cermets of $\mathrm{U}^{235} \mathrm{O}_{2}-\mathrm{W}$ require about $40 \mathrm{vol} \%$ loading to be superior to intermediate, heavily loaded graphite reactors. Uranium carbide offers a spectrum of possibilities ranging from very small, lightweight ( $\sim 300 \mathrm{lb}$ ) pure UC reactors with low exit gas temperature $\left(\leq 2000^{\circ}\right.$ ), through moderate size
and power UC-ZrC reactors, to heavier UC-HfC reactors of high ( $>3000^{\circ}$ C) temperature potential. Uranium-233 may be used to lower weights or increase gas temperatures. The small physical size of fast reactors (small mass, high density) is attractive where considerable shielding is required (such as for manned orbital ferries). The increase in fissile mass with size of fast reactors makes them less competitive with moderated systems for very high power applications.

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TABLE I
NEUTRON ENERGY GROUPS AND SPECTRA

|  |  |  | 50-50 vol \% UC-ZrC, Be-reflected reactor |  |  | Oy sphere $+2 \mathrm{in} . \mathrm{Be}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group | $\begin{array}{r} \text { Upper } \\ \text { energy } \end{array}$ |  | Core spectrum <br> (\%) | Reflector spectrum <br> (\%) | Fissions/group $(\%)$ | Fissions/group $(\%)$ |
| 1 | 10 | Mev | 3.3 | 1.7 | 2.2 | 5.4 |
| 2 | 3.7 | Mev | 15.3 | 8.2 | 10.5 | 23.0 |
| 3 | 1.3 | Mev | 28.1 | 16.6 | 17.9 | 27.6 |
| 4 | 0.5 | Mev | 26.2 | 17.3 | 18.2 | 18.8 |
| 5 | 0.183 | 3 Mev | 19.2 | 16.6 | 20.0 | 11.9 |
| 6 | 24 | kev | 5.8 | 12.7 | 12.0 | 5.0 |
| 7 | 3 | kev | 1.5 | 8.9 | 6.4 | 2.4 |
| 8 | 0.45 | kev | 0.4 | 6.1 | 4.5 | 1.9 |
| 9 | 61 | ev | 0.08 | 4.0 | 2.6 | 1.0 |
| 10 | 8 | ev | 0.10 | 2.7 | 1.2 | 0.7 |
| 11 | 1 | ev | 0.025 | 1.8 | 1.5 | 0.7 |
| 12 | 0.1 | ev | 0.005 | 1.2 | 1.0 | 0.5 |
| 13 | 0.025 | ev | 0.006 | 2.2 | 1.8 | 1.0 |

## TABLE II

PROPERTIES OF MOLYBDENUM AND TUNGSTEN

| Element | $\begin{gathered} \text { M.p. } \\ \left(o_{C}\right) \end{gathered}$ | $\rho_{\text {theor }}$ <br> $\mathrm{g} / \mathrm{cc}$ | $10^{24}$ atoms/cc | $\begin{gathered} \sigma_{\mathrm{tr}}(1 \mathrm{Mev}) \\ \text { (barns) } \end{gathered}$ | $\begin{gathered} \sigma_{\text {in }}(1 \mathrm{Mev}) \\ \text { (barns) } \end{gathered}$ | $\begin{gathered} \sigma_{a} \\ \text { (barns) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mo | 2620 | 10.2 | 0.0640 | 4.2 | 1.1 | 0.02 |
| W | 3370 | 19.3 | 0.0632 | 5.2 | 2.0 | 0.05 |

TABLE III
REPRESENTATIVE $\mathrm{UO}_{2}$ REACTORS

29

| Metal | $\ell$ | Wt (Ib) |  |  |  |  | Core diam. <br> (in.) | Void frac. | Flow area$\left(f t^{2}\right)$ | Thrust ${ }^{\text {a }}$ <br> ( 1 b ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Vol $\% \mathrm{UO}_{2}$ <br> in core | $\mathrm{UO}_{2}$ | Metal | Be | Reactor |  |  |  |  |
| W | 0.4 | 0.3 | 730 | 2060 | 530 | 3320 | 20.4 | 0.25 | 0.577 | 57,700 |
| W | 0.4 | 0.25 | 1030 | 2720 | 720 | 4470 | 24.4 | 0.375 | 1.18 | 118,000 |
| W | 0.5 | 0.4 | 550 | 970 | 380 | 1800 | 16.2 | 0.2 | 0.29 | 29,000 |
| W | 0.5 | 0.25 | 1030 | 1700 | 720 | 3550 | 24.4 | 0.5 | 1.57 | 157,000 |
| Mo | 0.4 | 0.3 | 495 | 730 | 430 | 1655 | 17.7 | 0.25 | 0.428 | 42,800 |
| Mo | 0.5 | 0.25 | 885 | 870 | 560 | 2315 | 21.2 | 0.5 | 1.23 | 123,000 |

$3_{\text {Ihrust }}$ based on $2000 \mathrm{Mw} / \mathrm{ft}^{2}$ of flow area ( 50 lb thrust/Mw).

TABLE IV
METAL CARBIDE PROPERTIES

| Compound | M.p. <br> $\left(\mathrm{o}_{\mathrm{C}}\right)$ | $\rho_{\text {theor }}$ <br> $\mathrm{g} / \mathrm{cc}$ |
| :--- | :---: | :---: |
| UC | 2450 | 13.5 |
| ZrC | 3500 | 6.9 |
| NbC | 3500 | 7.8 |
| TaC | 3800 | 14.5 |
| HfC | 3800 | 12.7 |
| WC | 2777 | 15.7 |
| MoC | 2570 | 8.48 |
| $\mathrm{~B}_{4} \mathrm{C}$ | 2450 | 2.54 |

TABLE V
REPRESENTATIVE UC REACIORS

|  | Fuel | Wt (lb) |  |  |  |  | Void <br> frac. | Core <br> diam. <br> (in.) | Flow area$\left(f t^{2}\right)$ | $\begin{gathered} \text { Thrust }{ }^{\text {a }} \\ \text { (lbs) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | UC | ZrC | W | Be | Reactor |  |  |  |  |
|  | 50\% UC-50\% ZrC | 212 | 107 | 90 | 227 | 648 | 0.2 | 11.8 | 0.152 | 15,200 |
|  | 50\% UC-50\% ZrC | 436 | 220 | 323 | 445 | 1424 | 0.5 | 18.1 | 0.893 | 89,300 |
|  | 30\% UC-70\% ZrC | 257 | 303 | 182 | 327 | 1069 | 0.2 | 15.0 | 0.224 | 22,400 |
| W | 30\% UC-70\% 2rC | 524 | 615 | 645 | 655 | 2439 | 0.5 | 23.2 | 1.42 | 142,000 |
|  | 100\% UC | 104 | --- | $50^{\text {b }}$ | 114 | 268 | 0.2 | 7.1 | 0.055 | 5,500 |
|  | 100\% UC | 266 | - | $83^{\text {b }}$ | 213 | 562 | 0.5 | 11.3 | 0.35 | 35,000 |

Innust based on $2000 \mathrm{Mw} / \mathrm{ft}^{2}$ of flow area ( 50 lb thrust/Mw).
${ }^{\mathrm{b}}$ Support plate at exit end, otherwise 10 vol $\%$ of core.

## TABLE VI

CRITICAL RADII OF Be-REFLECTED UC-HfC ASSEMBLIES

| Vol $\%$ in core | Loading ${ }^{\text {a }}$ | $\%$ Void | Core rad. (in.) |
| :--- | :--- | :--- | :--- |
| $15 \% 0 y C, 35 \% \mathrm{HfC}$ | 0.3 | 50 | 33.5 |
| $15 \% 0 \mathrm{yC}, 35 \% \mathrm{TaC}$ | 0.3 | 50 | 66 |
| $15 \% 0 y C, 60 \% \mathrm{HfC}$ | 0.2 | 25 | 42 |
| $10 \% 0 \mathrm{yC}, 40 \% \mathrm{HfC}$ | 0.2 | 50 | 59 |

avol \% Oyc in fuel

TABLE VII
$U^{233}$ REACTORS

| Wt (lbs) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Fuel | Void | $\mathrm{U}^{233}$ | Core | Reactor | Diam. (in.) | $\begin{gathered} \mathrm{A}_{f} \\ \left(\mathrm{ft}^{2}\right) \end{gathered}$ | Thrust ${ }^{\text {a }}$ | Wt of equivalent ${ }^{b}$ $v^{235}$ reactor |
| $40 \% \mathrm{UO}_{2}$ in W | 0.375 | 150 | 640 | 920 | 13.4 | 0.37 | 37,000 | 2,900 |
| $40 \% \mathrm{UO}_{2}$ in W | 0.625 | 420 | 1310 | 2395 | 22.1 | 1.65 | 165,000 | 5,300 |
| 22\% UC in ZrC | 0.5 | 183 | 860 | 1305 | 18.1 | 0.893 | 89,300 | 1,424 |
| W $50 \%$ UC in ZrC | 0.3 | 70 | 142 | 282 | 8.65 | 0.12 | 12,000 | 620 |
| $50 \%$ UC in ZrC | 0.5 | 150 | 355 | 615 | 12.9 | 0.42 | 42,000 | 930 |
| 20\% UC in HfC | 0.5 | 850 | 4290 | 5300 | 29 | 2.28 | 228,000 | 20,000 |
| 20\% UC in HfC | 0.25 | 450 | 2270 | 2820 | 20.5 | 0.57 | 57,000 | --- |
| 15\% UC in Hfc | 0.2 | 1230 | 8230 | 9330 | 31 | 1.05 | 105,000 | --- |

- Based on $2000 \mathrm{Mw} / \mathrm{ft}^{2}$ flow area.
$\mathrm{b}_{\text {Equivalent }}$ means same $\%$ composition and flow area.


## TABLE VIII

RELATIVE SHIELDING REQUIREMENTS

| Reactor | $\begin{gathered} \text { Diameter } \\ \text { (in.) } \end{gathered}$ |  | $\begin{aligned} & \text { Wt } \\ & \text { (lb) } \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Core | Reflector (Be) | Engine | Lead ab | elds |
|  |  |  |  | 10 in. shadow | 5 in. circum. |
| 200 Mw UC | 10 | 18 | 500 | 1200 | 2,700 |
| 1000 Mw UC-ZrC | 18 | 26 | 5000 | 2600 | 5,200 |
| 1000 Mw Graphite | 36 | 48 | 5500 | 9000 | 16,000 |



FIG. 1. The geometry used for several one-and two-dimensional criticality calculations. The core consists of $20 \mathrm{vol} \% \mathrm{UC}, 20 \mathrm{val} \% \mathrm{ZrC}, 10 \% \mathrm{Mo}$, $50 \%$ void. The reflector is full density Be, 9 cm thick.
a) One-dimensional (spherical) case. Computed with 13 group cross sections ( $k=1.000$ ) and with a collapsed set of 4 groups which gave good agreement ( $k=1.006$ ), and was used for two-dimensional computations.
b) Two-dimensional (finite circular cylinder) case, completely reflected, $k=1.038$.
c) Similar to b) with exit reflector replaced by 12 cm of core, $\mathrm{k}=1.033$.
d) Similar to c) with inlet reflector moved 9 cm from core for control purposes. $\Delta k=0.062 \approx 1.0 \$ / \mathrm{cm}$.


FIG. 2. Critical radii of $\mathrm{VO}_{2}-\mathrm{W}$ cores reflected by 12 cm of Be at 75\% of full density.
a) $0 y 0$ fuel ( $93 \%$ enriched $v^{235}$ ), 50 vol $\% \mathrm{~W}$ in fuel.
b) $0^{23 \xi} o_{2}$ fuel, 60 vol $\%$ win fuel element. Other calculations indicate the radii to be relatively insensitive to the amount of W in the core.


FIG. 3. Mass vs. cross sectional flow area in core for $\mathrm{UO}_{2}$-W reactors. $\ell$ (fuel loading) $=$ vol $\% \mathrm{UO}_{2}$ in fuel, $u=\mathrm{vol} \% \mathrm{UO}$ in core. Heat transfer and fluid flow analyses indicated up to 2600 Mw (100,000 lb thrust) per $1 \mathrm{ft}^{2}$ of flow area appear feasible.


FIG. 4. Melting point of UC-ZrC solid solutions. Single phase solid solutions occur also for UC in TaC and in HfC.


FIG. 5. Critical radii of UC-ZrC cores reflected with 12 cm of $75 \%$ dense Be .
a) Fuel consists of $100 \%$ UC.
b) $50 \mathrm{vol} \% \mathrm{UC}-50 \mathrm{val} \% \mathrm{ZrC}$ fuel.
c) 30 vol of UC - 70 vol o ZrC. "Max" indicates the point where core void is reduced to zero.


FLOW AREA $\mathrm{ft}^{2}$
FIG. 6. Mass vs. cross sectional flow area in core for carbide reactors reflected by 12 cm of $75 \%$ dense Be .
a) $100 \%$ UC fuel leads to very small reactors.
b) Fuel camposed of $50 \% \mathrm{UC}-50 \% \mathrm{ZrC}$, no structure in core.
c) The effect upon b) of adding $10 \%$ by volume of the core of W structure. The flow area is reduced and the core mass increased.
d) A more dilute fuel, $30 \% \mathrm{UC}-70 \% \mathrm{ZrC}, 10 \% \mathrm{~W}$ structure included.


FIG. 7. Critical core radii of various carbide cores reflected by 12 cm of $75 \%$ dense Be. Curves labeled by vol \% of nonfissile metal carbide in fuel.


FIG. 8. Radial power distribution for typical fast reactors.
a) Be reflector, no poison.
b) Be reflector with $\mathrm{B}^{10}$.
c) Ni reflector.


FIG. 9. Axial power distribution for 19 in. long reactor, reflected at inlet end.

## Appendix A

## DETAIIS OF CALCULATIONAL TECHNIQUES AND RESULIS

## 1. CROSS SECTION

The set used for most of the calculations was a 13 group set of SNG transport cross sections originated by C. B. Mills. This was intended to be a subset of a "universal" 25 group set, but when the Mills set was combined with that of G. Hansen (16 groups), an 18 group set with some new group energy limits resulted. Since the author had been checking critical assemblies for 2 years with the 13 group set and had developed cross sections for several elements not included in the others, he continued with the 13 group spacing, and occasionally transcribed cross sections from the Hansen-Mills 18 group set. In particular the $U^{235}$ set was so transcribed, rechecked on fast critical assemblies, and used in all calculations reported here. A description of the cross sections follows, including complete IBM listings. Table Al lists some checks against $O y$ spheres reflected by materials of interest.

Subsequent to the work described here, a number of experimental critical assemblies consisting of mixtures of Oy and metal plates were stocked by G. Jarvis of N-2. Calculational checks with the 13 group cross sections generally showed good agreement ( $\leq 2 \%$ error in radius) with two exceptions, Ni and Ta . The Ni cross sections gave too low a critical mass, owing primarily to lack of inclusion of a recently observed n-p reaction at high energies. The Ta cross sections gave too
high a critical mass, which resulted entirely from a conservative choice of the high energy absorption cross section. Weighting the BNL energy-dependent cross sections by the typical fast reactor spectrum (instead of choosing the maximum $\sigma_{a}$ in each energy group as was done for Ta Cl) eliminated the discrepancy between experiment and calculation.

## 2. REACTOR CALCULATIONS

Tables $A 2$ to A9 contain details of many criticality calculations of Be-reflected fast cores. They were compiled over a period of two years, using several coding versions of spherical $S_{4}$ transport theory and the 13 group cross sections previously discussed. Most calculations used two regions (core and reflector) with 10 equally spaced points in each, but for examination of the power spike at the core surface, additional regions or varied point spacing were used. The results are tabulated separately on the basis of the core composition. Each reactor is described by some main features such as materials, (material percentages indicate vol $\%$ in core), loading (vol \% of fissionable compound in the fuel), and void percentage. Next the core composition is given exactly as used in the calculation, where $N$ is the density in atoms/cc $\times 10^{24}$. This is followed by the core radius $R_{c}(c m)$ as found by the calculation and by reflector density $N$ and radius $R_{r e f}(\mathrm{~cm})$ reflector. Since the earlier codes did not allow for variation of the core radius with a fixed reflector thickness, and since several reflector densities were used, we also include the re-
flector thickness in $10^{24}$ atoms $/ \mathrm{cm}^{2}$, the product of thickness and atomic density. We also list at the bottom of the tables calculations for other cases where one or two changes have been made, giving the base calculation, the change, and the resulting new core and reflector radii ( $R_{c}$ and $R_{r e f}$ ).

BRIEF DESCRIPTIION OF 13 GROUP CROSS SECTIONS
Al Cl Aluminum - derived from BNL , unchecked.
B5
Be C3

C-Cl Carbon - based on BNL 325 with slight changes in high energy transport to check with graphite reflected $U^{235}$ spherical assemblies.

Cd Cl Cadmium - from BNL 325, unchecked, for use in dilute concentrations.

Fe C3 Iron - transcribed from Hansen's 16 group set and checked against an iron-reflected $U^{235}$ sphere. Gadolinium - for control rods only - groups 1-4 have zero cross section.

H4M
H(G+1) Hydrogen - basically from the above set, with some cross section moved from $\sigma_{g \rightarrow g+1}$ to $\sigma_{g g}$ such that $\sum_{g g}>0$ for $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{CH}_{2} \cdot \mathrm{k}_{\text {eff }}$ for this set is usually $1 \%$ lower than for $H 4 \mathrm{M}$ in wellmoderated systems ( $\mathrm{H} / \mathrm{U}>100$ ).

Hf Cl Hafnium - BNL 325 data plus cloudy crystal ball model and Topsy replacement data at high energies. Inelastically scattered neutron distribution by the statistical model.

| Mo C4 | Molybdenum - same basis as Hf plus G. Bell's resonance absorption and scattering for homogeneous dilute concentration. |
| :---: | :---: |
| $\mathrm{Ni} \mathrm{C3}$ | Nickel - from BNL 325 and Topsy replacement data $\mathrm{k}_{\text {eff }}=$ 0.993 for $4^{\prime \prime} \mathrm{Ni}$ reflected $\mathrm{U}^{235}$ sphere. |
| $\mathrm{Ni} \mathrm{C4}$ | Ni C3 plus high energy $n-p$ reaction. |
| 0 x | Oxygen - BNL 325, no clear check. |
| Pu | Plutonium - from Hansen's 16 group set. No independent check. |
| Ta Cl | Tantalum - same basis as Hf, but conservative values used for high energy absorption. |
| Ta C 2 | Tantalum - Ta Cl with high energy absorption weighted by fast reactor spectrum. |
| U233 Cl | $\mathrm{u}^{233}$ - from Hansen's 16 group set. |
| U235 Cl | $U^{235}$ - from Hansen's 16 group set, plus $G$. Bell's dilute homogeneous resonance cross sections. $\mathrm{k}_{\text {eff }}=0.9994$ for $\mathrm{U}^{235}$ bare sphere. |
| U238 cl | $U^{238}$ - from Hansen's 16 group set plus $G$. Bell's dilute homogeneous resonance cross sections. $k_{\text {eff }}=1.002$ for $2^{\prime \prime} U^{238}$ reflected $U^{238}$ sphere. |
| 28 C 2 | $U^{238}$ - with smaller high energy absorption to agree with low enrichment fast spectrum experiments. |
| WC2 | Tungsten - same basis as Mo C4. |
| Zr C3 | Zirconium - from Hansen's 16 group set. |

The group number for the 13 group cross section data is centered over the column.

Within the block listing, cross section data cover the following:
Row No. $1 \quad \sigma_{\text {absorption }}$
Row No. 2 No ${ }_{\text {fission }}$
Row No. $3 \quad \sigma_{\text {transport }}$
Row No. 4
${ }_{g g}$
Row NO. 5
$\sigma_{g-1} \rightarrow g$
Row No. 6
$\sigma_{g}-2 \rightarrow g$
Row No. $7 \quad \sigma_{g}-3 \rightarrow g$
Row No. 8
$\sigma_{g-4} \rightarrow g$
ROW NO. $9 \quad \sigma_{g}-5 \rightarrow g$

[^0]2
$0.000000+00$
$0.000000+00$ $0.000000+00$
$1.600000+00$ $1.600000+00$ $1.500000+00$
$1.000000-01$
$0.000000+00$
$0.000000+00$ $0.000000+00$ $0.0 C 0000+00$

10
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3
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11
6.000000-02
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## R. COOPER 13 GRP

$\frac{1}{0.000000+00}$ $0.000000+00$ $1.000000+00$ 7.999999-01 1. $000000+00$ -. $000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ 9
$2.62000 c+01$ $0.000000+00$ $2.970000+01$ 3. $200000+00$ 3. 000000-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

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## 6

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7
$3.600000+00$ $0.000000+00$ $6.700000+00$ $2.900000+00$ 2.000000-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

# 8 

$9.600000+00$ $0.000000+00$ 1. $310000+01$ 3. $200000+00$ 2.000000-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

|  | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $-2.600000-01$ | $-8.000000-02$ | $1.000005-02$ | $0.000000+00$ | $0.000000+00$ |
| 2 | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
| 3 | $7.899999-01$ | $1.140000+00$ | $3.000000+00$ | $3.810000+00$ | $5.390000+00$ |
| 4 | $2.300000-01$ | $5.999999-01$ | $2.459999+00$ | $3.209999+00$ | $4.790000+00$ |
| 5 | $0.000000+00$ | $5.199999-01$ | $6.199999-01$ | $5.299999-01$ | $5.999999-01$ |
| 6 | $0.000000+00$ | $0.060000+00$ | $3.000000-01$ | $0.000000+00$ | $0.000000+00$ |
| 7 | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
| 8 | $0.000000+00$ | $0.000000+00$ | $0.000060+00$ | $0.000000+00$ | $0.000000+00$ |
| 9 | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
|  | 9 | 10 | 11 | 12 | 13 |
| 1 | $3.000498-04$ | $1.000046-03$ | $2.500057-03$ | $6.400049-03$ | $9.999990-03$ |
| 2 | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
| 3 | $5.460300+00$ | $5.461000+00$ | $5.462500+00$ | $5.466400+00$ | $5.560000+00$ |
| 4 | $4.860000+00$ | $4.860000+00$ | $4.860000+00$ | $4.860000+00$ | $5.550000+00$ |
| 5 | $5.999949-01$ | $5.999999-01$ | $5.999999-01$ | $5.949999-01$ | $5.999999+01$ |
| 6 | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
| 7 | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
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| $4.340000+00$ | $4.390000+00$ |
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| 1 | $2.999998-03$ | $0.0 C 0000+00$ |
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| 3 | $1.703000+00$ | $2.400000+00$ |
| 4 | $6.799999-01$ | $1.580000+00$ |
| 5 | $0.000000+00$ | $1.020000+00$ |
| 6 | $0.000000+00$ | $0.000000+00$ |
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| 9 | $0.000000+00$ | $0.000000+00$ |
| 1 | $6.999943-02$ | $2.000000-01$ |
| 2 | $0.000000+00$ | $0.000000+00$ |
| 3 | $1.137000+01$ | $1.129999+01$ |
| 4 | $1.089000+01$ | $1.070000+01$ |
| 5 | $4.000000-01$ | $4.100000-01$ |
| 6 | $0.000000+00$ | $0.000000+00$ |
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| $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
| $5.599999-01$ | $9.299999-01$ | $1.590000+00$ |
| $-4.900000-01$ | $-8.399999-01$ | $-1.440000+00$ |
| $0.000000+00$ | $6.699999-01$ | $1.119999+00$ |
| $0.000000+00$ | $0.000000+00$ | $2.400000-01$ |
| $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
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| $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
| 9 | 10 | 11 |
| $1.100006-02$ | $3.000003-02$ | $8.199999-02$ |
| $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
| $6.670999+00$ | $6.820000+00$ | $8.082000+00$ |
| $-1.979999+00$ | $-2.020000+00$ | $-2.370000+00$ |
| $7.479999+00$ | $7.479999+00$ | $7.630000+00$ |
| $1.160000+00$ | $1.160000+00$ | $1.160000+00$ |
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| $0.000000+00$ | $0.000000+00$ | $6.000018-02$ |
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$2.000000+00$ $6.100000+00$ $4.500000+00$ $1.500000+00$ 1. $500000+00$ 2. $500000-0$ . $000000+00$ $0.000000+0$ $0.000000+00$ $0.000000+00$ 10
$1.600000+01$ $4.400000+01$ $2.600000+01$ 1.000000+01 . $000000+00$ $0.000000+00$ . $000000+00$ . $00000+00$ $0.000000+00$

3
$1.850000+00$ $5.300000+00$ $5.250000+00$ $.500000+00$ -500000+00 .499999-01 5.999999-0 $.000000+00$ . $000000+00$ $.000000+00$ 11
. $280 \mathrm{C00+03}$ $2.300000+03$ 1.290000+03 $.000000+01$ $0.000000+00$ $0.000000+00$ $.000000+00$ $. .000000+00$ . $000000+00$
$1.850000+00$
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$7.250000+00$ $5.000000+00$ 5.000000-01 4.000000-01 $4.00000-01$ . $000000+00$ . $000000+00$ 12

## . $400000+02$

 $.615000+03$ $8.500000+02$ 1. $000000+01$ $0.000000+00$ $0.000000+00$ $.00000+00$ . $000000+00$ . $000000+00$
# 5 

$2.200000+00$
$5.300000+00$
$1.070000+01$ $8.500000+00$ .000000-01 .000000-01 4.000000-01 4.999999-02 $0.000000+00$ 13
$.025000+03$ $2.160000+03$ $1.035000+03$ $1.000000+01$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$
 $8.700000+00$ 1. $500000+01$ $1.000000+01$ -. $000000+01$ $.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$
$0.000000+00$ $0.000000+00$
$6.000000+00$

1. $160000+01$
$1.600000+01$
$1.000000+01$ . $0.00000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

8
2. $400000+01$ $4.400000+01$ $3.400000+01$ $1.000000+01$ $0.000000+00$ $0.000000+00$ $0.00000+00$ $0.000000+0$ $0.000000+00$
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7
$6.000000+00$ $6.000000+00$ $0.000000+00$ 2. $200000+01$ 1. $600000+01$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

8
$1.000000+01$ $0.000000+00$ $3.000000+01$ $2.000000+01$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$
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Ta C2
COOPER

1
3.000000-02 $3.000000-02$
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9
3. $000000+01$ $0.000000+00$ $5.000000+01$ $2.000000+01$ . $0.000000+00$ . $0.000000+00$ . $000000+$ CO $0.000000+00$ $0.000000+00$ $0.000000+00$

2
7.000002-02 $0.000000+00$ $3.770000+00$ $3.170000+00$
$2.100000+00$ $7.994999-01$ $7.994999-0$ $0.000000+0$ $0.000000+00$ $0.000000+00$ $0.060000+00$ 10
3. $000000+01$ $0.000000+00$ $3.600000+01$ $6.000000+00$ $0.000000+00$ . $000000+00$ -. $000000+00$ . $00000+00$ $0.000000+00$

1 $.800000+00$ . $394999+00$ 4. 250000+00 1. 200000+00 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ 9
$8.895000+01$ $1.940 C 00+02$ 9. $900000+01$ 1.000000+01 4. $999994-02$ . $999994-02$ -. 000000+00 $0.000000+00$ $0.000000+00$ $0.000000+00$

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2
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$1.860000+00$
$.860000+00$ $.000000+00$ $4.500000+00$ . $560000+00$ $2.000000-01$
$0.000000+00$ $0.000000+00$
$0.000000+00$ . $000000+00$ $0.000000+00$ $0.000000+00$ 10
2. $269500+02$ $4.900000+02$ $2.370000+02$ 1. $000000+01$ 1. $000000+01$ . $000000+00$ $0.000000+00$ . $000000+00$ . $000000+00$ $0.000000+00$

3
1.500000-01 $0.000000+00$ $5.150000+00$ $3.800000+00$ 3.8000449-01 7.999949-01 7.994999-0 $0.000000+00$ $0.000000+00$ $0.000000+00$ 11
$5.000000+00$ $0.000000+00$ $1.000000+01$ $5.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

. 600000-01 $0.000000+00$ $6.360000+00$ 5. $800000+00$ 7. $999999-01$ 5.999999-01 4. 000000-01 . 000000-0 $0.000000+00$ 0.000 12 1. $000000+01$ $0.000000+00$ . $500000+01$ 5. $000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

5
.000000-01 $.000000+00$ $9.500000+00$ $9.000000+00$ 3.000000-01 $4.000000-01$ 2.000000-01 . $000000+00$ $.000000+00$ 13
$100000+01$
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$0.000000+00$ $.000000+00$ $2.600000+01$ . $000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

6
$1.000000+00$ $0.000000+00$ . $300000+01$ 1. $200000+01$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $6.000000+00$ $0.000000+00$ $2.200000+01$ 1. $600000+01$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

8
$1.000000+01$ $0.000000+00$ $3.00 \mathrm{C000+01}$ $2.000000+01$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

$$
3
$$

$2.030000+00$ $5.000000+00$ $5.000000+00$
$5.300000+00$ $5.300000+00$ 2.770000+00 5.999999-01 6.999999-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ 11 $1.829500+02$ 3. $940000+02$ $1.930000+02$ $1.000000+01$ 4.999999-02 . $0.009000+00$ -. $00000+00$ . $000000+00$ 0.000000+00 $0.000000+00$

| 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: |
| $2.400000+00$ | $3.350000+00$ | $5.550000+00$ | $1.020000+01$ |
| $5.500000+00$ | $7.300000+00$ | $1.400000+01$ | $2.250000+01$ |
| $7.800000+00$ | $1.060000+01$ | $1.560000+01$ | $2.025000+01$ |
| $5.350000+00$ | $7.200000+00$ | $1.000000+01$ | $1.000000+01$ |
| $4.000000-01$ | $4.999999-02$ | $4.999999-02$ | $4.999999-02$ |
| $3.000000-01$ | $1.000000-01$ | $0.000000+00$ | $0.000000+00$ |
| $3.000000-01$ | $1.800000-01$ | $0.000000+00$ | $0.000000+00$ |
| $0.000000+00$ | $4.999999-02$ | $0.000000+00$ | $0.000000+00$ |
| $0.000000+00$ | $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |

8
. $740000+01$ $6.000000+01$ 3.745000+01 $1.000000+01$ 4.999999-02 $0.000000+00$ $0.000000+00$ $0.000000+00$ $. .000000+00$
$1.239999+00$
$3.090000+00$
$4.040000+00$
$5.599999-01$
$0.000000+00$
$0.000000+00$
$0.000000+00$
$0.00000 c+00$
$0.000000+00$
9
$1.026000+02$
$1.480000+02$
$1.133000+02$
$1.06580 c+01$
$4.200000-02$
$0.000000+00$
$0.000000+00$
$0.000000+00$
$0.000000+00$

2
.300
$200000+00$
$3.209999+00$ $4.500000+00$ $1.800000+00$ 1. $800000+00$ $1.000000+00$ . $00000 \mathrm{C}+00$ $0.000000+00$ $0.0 \mathrm{COOOO+00}$ $0.000000+00$ 10 $3.980000+01$
$6.100000+01$ $6.100000+01$ $5.000000+01$ $1.015800+01$ 4. 200000-02 $0.000000+00$ $0.000000+00$ - $00000 \mathrm{C}+00$ $0.0 \mathrm{COOOO}+00$

## 3

1. $320000+00$ $3.050000+00$ $5.350000+00$ $3.440000+00$ 9.499999-01 9.499999-01 8.799999-01 . $0.000000+00$ $0.000000+00$ . $000000+00$ 11
$1.430000+02$ $2.950000+02$ $1.530000+02$ $9.958000+60$ 4. 200000-02 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$
2. $520000+00$ $3.340000+00$ $1.520000+00$ $5.790000+00$ 4.400000-01 3. 500000-01 2. 600000-01 2.600000-01 $0.000000+00$
$\qquad$ 12
$4.340000+02$ $9.000000+02$ $4.440000+02$ 9.958000+00 4.200000-02 $0.000000+00$ U. $000000+00$ $0.000000+00$ $0.000000+00$
5
$2.800000+00$ $4.950000+00$ $1.200000+01$
$9.160000+00$
1.500000-01
1.200000-01
1.000000-01
1.000000-01 $0.000000+00$ 13
$6.940000+02$ $1.440000+03$ $7.040000+02$ $1.000000+01$ 4. 200000-02 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

6
$5.400000+00$ $9.850000+00$ $1.540000+01$ $9.959999+00$ 4.000000-02 3.500000-02 3.000000-02 $0.000000+00$ $0.000000+00$

7
. $420000+01$ $2.020000+01$ $2.420000+01$ $9.959999+00$
4.000000-02
9.999999-09
1.500000-02
$0.000000+00$
$0.000000+00$

8
$799800+01$ . $900000+01$ $3.800000+01$ $9.959999+00$ 4.000000-02 9.999999-09 9.999999-09 9.999999-03 $0.000000+00$

| 6 | 7 | 8 |
| :---: | :---: | :---: |
| $6.500000-01$ | $2.600000+00$ | $2.000000+01$ |
| $0.060000+00$ | $0.000000+00$ | $0.000000+00$ |
| $1.375000+01$ | $1.550000+01$ | $3.029000+01$ |
| $1.300000+01$ | $1.280000+01$ | $1.020000+01$ |
| $9.000000-02$ | $1.000000-01$ | $1.000000-01$ |
| $1.000000-01$ | $0.000000+00$ | $0.000000+00$ |
| $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
| $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |
| $0.000000+00$ | $0.000000+00$ | $0.000000+00$ |

5
3.000000-01
$0.000000+00$
$1.099000+01$
$1.060000+01$
5.499999-01
3.500000-01
3.500000-01 2.900000-01 2. $500000+00$ 13
2. $800000+00$ $0.000000+00$ $1.180000+01$ $9.000000+00$ 8.000000-02 $0.000000+00$ $0.000000+00$
$0.000000+00$
$0.000000+00$
$5.800000+01$ $0.000000+00$ 6. $700000+01$ 8.919999+00 9.000000-02 $0.000000+00$ $0.000000+00$ . $0.000000+00$ . . 000000+00 $0.000000+00$

2


3
1.100000-01 $0.000000+00$ $5.090000+00$ 3. $800000+00$ 9.499999-01 9.499999-0 8.999999-0 . $0.00000+00$ $0.000000+00$ 11
7. 000000-01 $0.000000+00$ 9. 700000 +00 8.919999+00 8.000000-02 $0.000000+00$ $0.000000+00$ $0.000000+00$ -. $000000+00$
1.300000 $0.000000+00$ 7.180000+00 $6.400000+00$ $6.400000+0$ 8.299999-0 7.999999-0 6.999999-0 $0.000000+00$ 12 12

1. $000000+00$ $0.000000+00$ 1. $000000+01$ 8.919999+00 8.000000-02 $0.000000+00$ $0.00000 c+00$ $0.000000+00$ $0.000000+00$

5
3.000000-01 $0.000000+00$ $1.099000+01$ $1.060000+01$ 5.499999-0 5.499999-0 3.500000-0 2.900000-0 2.500000-01 . $000000+00$ 13 2. $800000+00$ $0.000000+00$ 1. 180000+01 $9.000000+00$ 8.000000-02 $0.000000+00$ $0.000000+00$ $0.000000+00$ . $000000+0$
$\qquad$ 6.500000-01 $0.000000+00$ $1.375000+01$ 1. $300000+01$ 9.000000-0 9.000000-02 $1.000000-01$ $0.000000+00$ $0.000000+00$ $0.000000+00$

- 7
$2.600000+00$ $0.000000+00$ $1.550000+01$ . $280000+01$ 1.000000-01 . $000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$
${ }^{8}$

2. $000000+01$ $0.000000+00$ $3.029000+01$ $1.020000+01$ 1.000000-01 $.000000+00$ - $000000+00$ $.000000+00$ . $000000+00$ $0.000000+00$
2.999998-02 $0.000000+00$ 3. $330000+00$ .330000+00 7.999999-01 $0.000000+100$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $\stackrel{y}{4}$
3. $500 \mathrm{COO}+01$ $0.000000+00$ $5.500000+01$ 2. $000000+01$ $2.000000+01$ . $000000+00$ . $000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

| 2 | 3 |
| :---: | :---: |
| $3.0 c 0000-02$ | $5.000001-02$ |
| $0.000000+00$ | $0.000000+00$ |
| $3.930000+00$ | $5.150000+00$ |
| $2.200000+00$ | $4.600000+00$ |
| $1.000000+00$ | $1.300000+00$ |
| $0.000000+00$ | $1.000000+00$ |
| $0.000000+00$ | $0.000000+00$ |
| $0.000000+00$ | $0.000000+00$ |
| $0.000000+00$ | $0.000000+00$ |
| 10 | 11 |
| $3.000000+01$ | $3.999999+00$ |
| $0.000000+00$ | $0.000000+00$ |
| $3.600000+01$ | $9.100000+00$ |
| $6.000000+00$ | $5.100000+00$ |
| $0.000000+00$ | $0.000000+00$ |
| $0.000000+00$ | $0.000600+00$ |
| $0.000000+00$ | $0.000000+60$ |
| $0.000000+00$ | $0.000000+00$ |
| $0.000000+00$ | $0.000000+00$ |

4
7.999998-02 $0.000000+00$ $6.180000+00$ $6.180000+00$ $6.100000+00$ 5.000000-01 4.000000-01 5.000000-01 $0.000000+00$ $0.000000+00$ 12
$1.210000+01$ $0.000000+00$ $1.720000+01$ 5. $100000+00$ $0.000000+00$ $0.000 \times 00+00$ $0.000000+00$ $0.000000+00$ $0.00000+00$

5
1.999999-01
$0.000000+00$ $9.200000+00$ . $200000+00$ $9.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ 13
1.770000+01 $0.000000+00$ 2. 280000+01 $5.100000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$
$0.000000+00$
$7.000000-01$
$0.000000+00$
$1.329999+01$
$1.260000+01$
$0.000000+00$
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7

1. $200000+00$ $0.000000+00$ $1.780000+01$ $1.660000+01$ $1.060000+01$ $0.000000+00$ $0.000000+00$ $0.000000+00$
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## 1

$0.000000+00$ $0.000000+00$ 2. $500000+00$ 1. 02000c+00 $0.000000+00$ $0.000000+00$ . $000000+00$ . $000000+00$ $0.000000+00$ 9
$0.000000+00$ $0.000000+00$ . $200000+00$ 6. $200000+00$ . $060000+00$ 1.399999-01 $0.000000+00$
$0.000000+00$ $0.000000+00$
$0.000000+00$ $0.000000+00$

2
$0.000000+00$ $0.000000+00$ $3.400000+00$ $2.680000+00$ $6.899999-01$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.010000+00$ 10
$0.000000+00$ $0.000000+00$ $6.200000+00$ $6.060000+00$ 1.399994-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.0 C 000 C+00$

3

## $0.000000+00$

$0.000000+00$
$4.900000+00$ $4.860000+00$ 4. 700000-0 6.099999-0 $0.000000+00$ $0.000000+00$ $0.000000+00$ 11
5.000007-02 .000007-02 $0.000000+00$ $6.250000+00$ $6.060000+00$
$1.399999-01$ $0.000000+00$ $0.000 \mathrm{co0}+00$ $0.000000+00$ $0.000000+00$
0.4

## $0.000000+00$

## $0.000000+00$

 $7.300000+00$ $7.100000+00$ 4.000000-02 2.500000-01 1.800000-01 $0.000000+00$ . 12 121.200000-01 $0.000000+00$ $6.320000+00$ $6.050000+00$ 1.399999-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$
0.05
$0.000000+00$ $0.000000+00$ . $000000+00$ $8.800000+00$ $8.610000+00$ 2.000000-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ 13

1. 800000-01
$1.800000+01$
$0.000000+00$
$0.000000+00$
$6.380000+00$
$6.380000+00$
$6.200000+00$ $6.200000+00$ 1.500000-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

- ${ }^{6}$


## $0.000000+00$

## $0.000000+00$

 8. $000000+00$ $7.820000+00$ 1. 900000-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$7
$0.000000+00$ $0.000000+00$ $6.400000+00$ $6.260000+00$ 1.800000-01 $0.000000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$
$0.000000+00$

TABLE AI
CRITICALITY CHECKS OF MATERTALS REFTECT


62

$* N=$ atoms $/ c c \times 10^{-24}$

TABLE A2
$\mathrm{U}^{235} \mathrm{O}_{2}$ MEIAL REACTORS

| Description | W-U ${ }^{235}$ | Mo-U ${ }^{235}$ | $\mathrm{UO}_{2}-\mathrm{MO}$ |  | $\mathrm{UO}_{2}-\mathrm{W}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vol $\%$, core |  |  |  |  |  |  |  |
| $\mathrm{UO}_{2}$ | ( $\mathrm{O}_{2}$ neglected) |  | 16 | 16 | 20 | 30 | 40 |
| Metal |  |  | 24 | 64 | 20W | 30W | 40W |
| Void |  |  | 60 | 20 | 60 | 40 | 20 |
| Case No. | 241 | 242 | 255 | 255a | 410 | 411 | 412 |
| Core |  |  |  |  |  |  |  |
| N(235) | 0.004 | 0.0017 | 0.0034 | 0.0034 | 0.0042 | 0.0063 | 0.0082 |
| N(238) |  |  |  |  | 0.0003 | 0.00045 | 0.0006 |
| N(W) | 0.020 |  |  |  | 0.01205 | 0.0181 | 0.0241 |
| N (Mo) |  | 0.019 | 0.019 | 0.038 |  |  |  |
| N(0) |  |  | 0.0068 | 0.0068 | 0.009 | 0.0135 | 0.018 |
| $\mathrm{R}_{\mathrm{c}}$, cm | 41.85 | 62.86 | 42.57 | 40.60 | 37.12 | 26.08 | 20.75 |
| Reflector |  |  |  |  |  |  |  |
| N ( Be ) | 0.11 | 0.11 | 0.21 | 0.11 | 0.0927 | 0.0927 | 0.0927 |
| Radius, cm | 50.22 | 75.44 | 51.08 | 48.71 | 49.12 | 38.08 | 32.75 |
| $\mathrm{Nt} 10^{24}$ atom $/ \mathrm{cm}^{2}$ | 0.921 | 1.384 | 0.936 | 0.892 | 1.112 | 1.112 | 1.112 |

Other cases:
415 Similar to 411. $30 \%$ Mo ( $N=0.0183$ ) replacing $W, R_{c}=22.6, R_{r e f}=34.6$
411a Similar to 411. $30 \% \mathrm{UO}_{2}$, no W $\quad R_{c}=26.21, R_{r e f}=38.21$
411b Similar to 411. $30 \% \mathrm{UO}_{2}, 50 \% \mathrm{~W} \quad \mathrm{~N}=0.0302, \mathrm{R}_{\mathrm{c}}=26.30 \quad \mathrm{R}_{\mathrm{ref}}=38.30$

## TABLE A3

LOW C/U SYSTEPMS

|  | Description |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | c/u | 1.78 | 7.3 | 0.9 | 2 | 4 | 8 |
|  | Reflector t ( cm ) | 6.96 | 10.1 | 7.15 | 15.2 | 15.2 | 15.2 |
|  | Case No. | 271 | 272 | 273 | 281 | 282 | 283 |
| $\stackrel{9}{F}$ | Core |  |  |  |  |  |  |
|  | N(235) | 0.0225 | 0.009 | 0.03 | $3.27 \times 10^{-3}$ | $3.07 \times 10^{-3}$ | $2.76 \times 10^{-3}$ |
|  | N(238) | 0.0015 | 0.0006 | 0.002 | $2.12 \times 10^{-4}$ | $2.0 \times 10^{-4}$ | $1.8 \times 10^{-4}$ |
|  | N(C) | 0.0412 | 0.066 | 0.0274 | $6.54 \times 10^{-3}$ | $1.23 \times 10^{-2}$ | $2.21 \times 10^{-2}$ |
|  | $\mathrm{R}_{\mathrm{c}}, \mathrm{cm}$ | 9.90 | 15.2 | 8.16 | 28 | 28 | 28 |
|  | Reflector |  |  |  |  |  |  |
|  | $\mathrm{N}(\mathrm{Be})$ | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 |
|  | $\mathrm{R}_{\text {ref }}, \mathrm{cm}$ | 16.86 | 25.3 | 15.31 | 43.2 | 43.2 | 43.2 |
|  | $\mathrm{Nt} 10^{24}$ atom $/ \mathrm{cm}^{2}$ | 0.835 | 1.212 | 0.856 | 1.823 | 1.823 | 1.823 |

> TABLE A4
> $\mathrm{U}^{235} \mathrm{C}$ REACTORS

| Description |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vol \%, core |  |  |  |  |  |  |  |
| UC | 80 | 60 | 40 | 20 | 30 | 100 | 67 |
| W | 0 | 0 | 0 | 0 | 10 | 0 | 0 |
| Void | 20 | 40 | 60 | 80 | 60 | 0 | 33 |
| Case No. | 246 | 247 | 248 | 249 | 250 | 400A | 400B |
| Core |  |  |  |  |  |  |  |
| $\begin{array}{lllllllll}\text { ¢ } \\ \mathrm{J}(235) & 0.0236 & 0.0176 & 0.0118 & 0.0059 & 0.00885 & 0.0272 & 0.0186\end{array}$ |  |  |  |  |  |  |  |
| N(238) | 0.0016 | 0.0012 | 0.0008 | 0.0004 | 0.0006 | 0.002 | 0.001 |
| $N(C)$ | 0.0252 | 0.0188 | 0.0126 | 0.0063 | 0.00945 | 0.0292 | 0.0196 |
| N(W) |  |  |  |  | 0.0063 |  |  |
| $\mathrm{R}_{\mathrm{c}}$, cm | 9.14 | 11.96 | 16.30 | 26.96 | 18.86 | 9.95 | 13.23 |
| Reflector |  |  |  |  |  |  |  |
| $\mathrm{N}(\mathrm{Be})$ | 0.123 | 0.123 | 0.123 | 0.123 | 0.123 | 0.10 | 0.10 |
| $\mathrm{R}_{\text {ref }}$, cm | 18.29 | 19.94 | 24.46 | 35.95 | 28.29 | 14.95 | 19.84 |
| $\mathrm{Nt} 10^{24}$ atom $/ \mathrm{cm}^{2}$ | 1.124 | 0.98 | 1.00 | 1.10 | 1.16 | 0.5 | 0.661 |

TABLE A5
$\mathrm{U}^{235} \mathrm{C}$-ZrC REACTORS

## Description

Vol $\%$, core

| UC | $\sim 40$ | $\sim 27$ | 20 | 15 | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ZrC | $\sim 40$ | $\sim 53$ | 20 | 25 | 30 |
| Mo | 0 | 0 | 10 | 10 | 10 |
| Void | 20 | 20 | 50 | 50 | 30 |
| Case No. | 253 | 254 | 256 | 257 | 258 |
| Core |  |  |  |  |  |
| N(235) | . 0129 | . 0088 | . 00588 | . 00441 | . 00883 |
| N(238) | . 0009 | . 0006 | . 0004 | . 0003 | . 0006 |
| $\mathrm{N}(\mathrm{C})$ | . 0276 | . 0282 | . 0139 | . 0142 | . 0208 |
| $N(\mathrm{Zr})$ | . 0138 | . 0188 | . 0076 | . 00952 | . 0114 |
| N(Mo) |  |  | . 0064 | . 0064 | . 0064 |
| $\mathrm{R}_{\mathrm{c}}, \mathrm{cm}$ | 12.84 | 15.97 | 22.92 | 26.9 | 17.0 |
| Reflector |  |  |  |  |  |
| $\mathrm{N}(\mathrm{Be})$ | . 123 | . 123 | . 12 | . 12 | . 12 |
| $\mathrm{R}_{\text {ref }}$, cm | 21.4 | 23.95 | 32.08 | 35.9 | 25.5 |
| $\mathrm{Nt} 10^{24}$ atom/ $\mathrm{cm}^{2}$ | 1.05 | . 980 | 1.10 | 1.08 | 1.018 |

259 Similar to 257. Without reflector. $R_{\text {core }}=43.05$
257a Similar to 257. $10 \% \mathrm{~W}$ in place of $10 \% \mathrm{Mo}, \mathrm{R}_{\text {core }}=27.89$,

$$
R_{\text {ref }}=37.19
$$

2570 Similar to $257.5 \% \mathrm{~W}$ in place of $10 \% \mathrm{Mo}, \mathrm{R}_{\text {core }}=27.61$,

$$
R_{\text {ref }}=36.81
$$

253a. Similar to 253. (UC) $)_{1}(\mathrm{ZrC})_{1}, 60 \%$ void, $R_{\text {core }}=18.66, R_{\text {ref }}=31.1$
254a Similar to 254 . (UC $)_{1}(\mathrm{ZrC})_{2}, 60 \%$ void, $R_{c}=23.47, R_{r e f}=35.2$
256a Similar to 256. $10 \% \mathrm{~W}$ in place of $\mathrm{Mo}, \Delta R_{\text {core }}=+0.43 \mathrm{~cm}$,

$$
R_{c}=23.35, \quad R_{r e f}=32.69
$$

256b New Mo(C4) and $\operatorname{Zr}(\mathrm{C} 2) \sigma$ $\mathrm{\sigma}^{\prime} \mathrm{s} \cdot \mathrm{R}_{\mathrm{c}}=22.73, \mathrm{R}_{\text {ref }}=31.82$

TABIE A6
$\mathrm{U}^{235}$ C-METAL CARBIDE REACTORS
Description
Vol $\%$, core

| UC | 15 | 10 | 15 | 20 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Metal Carbide | 35 HfC | 40 HfC | 35 TaC | 20 ZrC | 60 |
| W | 0 | 0 | 0 | 10 | 0 |
| Void | 50 | 50 | 50 | 50 | 25 |
| Case No. | 321 | 322 | 325 | 400 | 420 |

Core

| N(235) | . 00443 | . 00295 | . 001443 | . 00588 | .00443 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| N(238) | . 0003 | . 0002 | . 0003 | . 0004 | . 0003 |
| C | . 01823 | . 0186 | . 02003 | . 0139 | . 0279 |
| Zr |  |  |  | . 0076 |  |
| Hf | . 0135 | . 01545 |  |  | . 0232 |
| Ta |  |  | . 0153 |  |  |
| W |  |  |  | . 00625 |  |
| Mo |  |  |  |  |  |
| $\mathrm{R}_{\mathrm{c}}, \mathrm{cm}$ | 42.48 | 76.6 | 71.6 | 24.27 | 53 |
| Reflector |  |  |  |  |  |
| $\mathrm{N}(\mathrm{Be})$ | . 092 | . 092 | . 092 | . 1 | . 0927 |
| Rref, cm | 54.48 | 88.6 | 83.6 | 36.27 | 65 |
| Nt $10^{24} \mathrm{atom} / \mathrm{cm}^{2}$ | 1.104 | 1.104 | 1.104 | 1.20 | 1.11 |

\#400 as $k_{\text {eff }}=.99913,401-405$ changed element densities in inner 22.7 cm of core. $401 \Delta N(235)=+.001, k=1.0567, \Delta k=0.707(\mathrm{~kg} \text { mole })^{-1}=0.003 / \mathrm{kg}^{-1}$. $402 \Delta N(C)=+.005, k=1.0083, \Delta k=0.0226 " \mathrm{n} \quad \mathrm{n}=0.0019 / \mathrm{kg}^{-1}$. $403 \Delta N(\mathrm{Zr})=+.005, k=1.0179, \Delta k=0.0462 " \quad " \quad "=0.00051 / \mathrm{kg}^{-1}$. $404 \Delta N(W)=+.005, k=.99319, \Delta k=-0.0148 " \mathrm{n} \quad \mathrm{k}=-0.000076 / \mathrm{kg}^{-1}$. $405 \Delta N H \quad=+.001, k=1.0082, \Delta k=0.112 \quad$ " $\quad \mathrm{k} H \quad \mathrm{k}=0.056 / \mathrm{kg}^{-1}$. 406 lo\% Mo ( $N=.0004$ ) replacing $W, R_{c}=20.47$, $R_{\text {ref }}=34.47$. 407 Similar to 400. Ref. of Graphite at $N=.067, t_{\text {ref }}=34.6 \mathrm{~cm}$. 408 Similar to 400. Ref. of Ni at $\mathrm{N}=.073$, $\mathrm{t}_{\text {ref }}=14.02$.

> TABLE A7
> $\mathrm{U}^{233} \mathrm{O}_{2}-\mathrm{W}$ REACTORS

Description
Vol \%, core

| $\mathrm{UO}_{2}$ | 20 | 32 | 15 | 10 | 21.7 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| W | $\sim 30$ | $\sim 48$ | 40 | 30 | 42 |
| Void | $\sim 50$ | $\sim 20$ | 35 | 60 | 36.3 |
| Case No. | 314 | 315 | 413 | 414 | 419 |

Core

| $\mathrm{U}^{233}$ | .00488 | .00782 | .00338 | .00225 | .00488 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | .00976 | .0156 | .00676 | .00450 | .00976 |
| W | .0190 | .0303 | .0271 | .0181 | .0281 |
| $\mathrm{R}_{\mathrm{c}}, \mathrm{cm}$ |  |  |  |  |  |
| Reflector | 21.39 | 14.82 | 28.89 | 41.47 | 21.02 |
| $\mathrm{~N}(\mathrm{Be})$ |  |  |  |  |  |
| $\mathrm{R}_{\mathrm{ref}}, \mathrm{cm}$ | .092 | .092 | .0927 | .0927 | .0927 |
| $\mathrm{Nt} \mathrm{l0} 24 \mathrm{atom} / \mathrm{cm}^{2}$ | 1.08 | 0.962 | 1.11 | 1.11 | 1.11 |

422 Similar to 413, half as much $W, N=.0136, R_{\text {core }}=28.40$,

$$
R_{r e f}=40.40
$$

4l3a Similar to 413, half as much $0, N=.00338, R_{\text {core }}=29.17$,

$$
R_{r e f}=41.17
$$

414a Similar to 413, half as much $0, N=.00225, R_{\text {core }}=41.98$

$$
R_{\text {ref }}=53.98
$$

TABLE A8
$\mathrm{U}^{233}$-METAL CARBIDE REACTORS
Description

|  | UC | 8.8 | 20 | 30 | 8 | 10 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Metal Carbide | 31.2 zrc | 20 zrC | 30 zrC | 30 ZrC | 40 HfC | 72 HfC |
|  | W | 10 | 10 | 10 | 10 | 0 | 0 |
|  | Void | 50 | 50 | 30 | 52 | 50 | 20 |
|  | Case No. | 310 | 311 | 312 | 313 | 323 | 421 |
| Core |  |  |  |  |  |  |  |
| 8 | N(233) | . 00262 | . 0063 | . 0094 | . 0026 | . 00315 | . 00252 |
|  | C | . 0139 | . 0139 | . 0208 | . 0139 | . 0186 | . 0303 |
|  | Zr | . 01128 | . 0076 | . 0114 | . 0113 |  |  |
|  | Mo | . 0064 |  |  |  |  |  |
|  | W |  | . 0064 | . 0064 | . 0064 |  |  |
|  | Hf |  |  |  |  | . 01545 | . 0278 |
|  | $\mathrm{R}_{\mathrm{c}}, \mathrm{cm}$ | 22.73 | 16.43 | 12.93 | 25.43 | 35.86 | 69.4 |
|  | Reflector |  |  |  |  |  |  |
|  | $\mathrm{N}(\mathrm{Be})$ | . 12 | . 092 | . 092 | . 092 | . 092 | . 0927 |
|  | $\mathrm{R}_{\text {ref }}, \mathrm{cm}$ | 31.82 | 28.0 | 22.7 | 40.68 | 47.86 | 81.4 |
|  | Nt $10^{24}$ atom $/ \mathrm{cm}^{2}$ | 1.09 | 1.064 | 0.90 | 1.40 | 1.104 | 1.11 |

## TABLE A9

## $\mathrm{Pu}_{-1} \mathrm{O}_{2}$ W REACTORS

| Description |  |  |  |
| :---: | :---: | :---: | :---: |
| Vol $\%$, core |  |  |  |
| Pu $\mathrm{O}_{2}$ | 16 | 10 | 7 |
| W | 30 | 30 | 28 |
| Void | 54 | 60 | 65 |
| Case No. | 316 | 317 | 318 |
| Core |  |  |  |
| Pu | . 004 | . 0024 | . 0017 |
| 0 | . 008 | . 0048 | . 0034 |
| W | . 019 | . 019 | . 018 |
| $\mathrm{R}_{\mathrm{c}}, \mathrm{cm}$ | 24.8 | 39.6 | 55.8 |
| Reflector |  |  |  |
| $\mathrm{N}(\mathrm{Be})$ | . 092 | . 092 | . 092 |
| $\mathrm{R}_{\text {ref }}, \mathrm{cm}$ | 38.3 | 53.2 | 69.3 |
| Nt $10^{24}$ atom/ $\mathrm{cm}^{2}$ | 1.24 | 1.25 | 1.24 |

317 a No reflector, $\mathrm{R}_{\text {core }}=55.1 \mathrm{~cm}$.


[^0]:    1
    $0.000000+00$
    $0.000000+00$
    $1.400000+00$

    1. $300000+00$
    $0.000000+00$
    $0.000000+00$
    $0.000100+00$
    $0.000000+00$
    $0.000000+00$
    9
    $0.000000+00$
    $0.000000+00$
    $0.000000+00$
    $1.400000+00$
    $1.349999+00$
    4.999999-02
    $0.000000+00$
    $0.000000+00$
    $0.000000+00$
    $0.000000+00$
[^1]:    5
    $0.000000+00$
    $0.000000+00$
    $0.000000+00$
    4. $660000+00$
    $-1.390000+00$ 6.999999-01 2.400000-01 $0.000000+00$ $0.000000+00$ 13
    300001-01 $3.300001-01$
    $0.000000+00$ $0.000000+00$ $1.731000+01$ $1.698000+01$ $1.440000+01$ $1.390000+00$ $0.000000+00$ $0.000000+00$ $0.000000+00$

