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Parameter Study of a Long, Separated-Shock Θ Pinch with Superconducting Inductive-Energy Storage

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

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by

F. L. Ribe

PARAMETER STUDY OF A LONG, SEPARATED-SHOCK Θ PINCH

WITH SUPERCONDUCTING INDUCTIVE-ENERGY STORAGE

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ABSTRACT

Present Θ-pinch experiments involve high-β plasmas with area compression ratios of the order of 40, using high-voltage capacitors to provide both the early shock heating (during the first few tenths μsec) and the later adiabatic compression, in a single-turn coil. In the separated-shock concept[1,2] the shock heating is done by high-voltage circuits whose energy content is only a few percent of that of the total system. The compression magnetic energy, which is preponderant, is contained in a multiturn coil with slow rise time (of the order of milliseconds) appropriate to adiabatic compression of the plasma in long systems. The area compression ratios are less than 10. Physical parameters and costs of a large system, appropriate to a scientific feasibility experiment at $kT_i \approx 10$ keV and $n\tau \approx 10^{14}$ cm^{-3} sec are derived in this report. The shock heating is calculated on the basis of plasma sheaths driven by Blumlein transmission lines; although other electrical circuits might be used. The adiabatic compression field is assumed to be energized by a superconducting inductive energy source, switched by normal-going superconductors.

I. PLASMA HEATING

A. Shock Heating

In the model[3] taken here (Fig. 1) a sheath separating a magnetic field B_s and the plasma advances radially inward at a speed v_s, projecting ions ahead of it at a speed $2v_s$. The plasma filling density inside this "magnetic piston" is n_0, and the plasma ahead of the piston is assumed to be cold (~ 1 eV in practice). Only ions are assumed to gain energy, and the electrons remain cold. (In practice there is some joule heating of the electrons in the sheath; however, this heating is much less than that of the ions.) Plasma-simulation studies[4] show that the actual process whereby the ions are given a spread of energies from an ideal sheath is one of repeatedly bouncing off the sheath wall as they pass through the center of the discharge tube, leading to oscillations which finally (on a scale of 0.1 μsec) settle down to some sheath radius bx_{SH} at the end of the shock phase. Another likely physical process

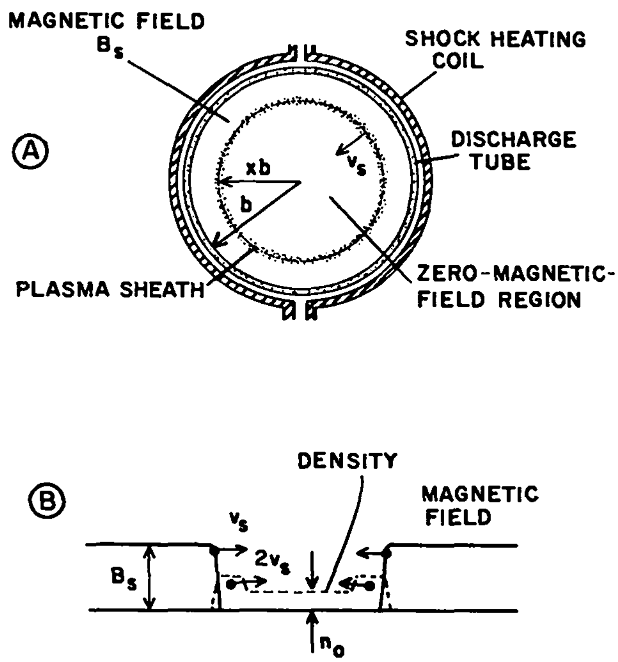

Fig. 1. Illustrating the quantities involved in a magnetically driven plasma sheath.

for imparting a spread of ion velocities is that the sheath becomes corrugated by Taylor or other instabilities so that ions bouncing off it are reflected in random directions, spreading energy into two or three degrees of freedom.

Simple dynamical considerations show that all of the ions projected from the sheath will hit it again after passing through the axis of the discharge when the sheath has advanced to $x_0 = 1/3$. As an average approximation to the actual train of events we assume that all of the ions inside the sheath at this instant thermalize and then push the sheath back to x_{SH}, doing an amount of work $(B_s^2/8\pi)\pi b^2[x_{SH}^2 - (1/3)^2]$ against the constant field B_s. Hence

$$\pi b^2 n_0[1 - (1/3)^2][1/2\ m_i(2v_s)^2] = 3/2\ \pi b^2 n_0 kT_{SH} + 1/8\ b^2 B_s^2[x_{SH}^2 - (1/3)^2] , \qquad (1)$$

where m_i is the ion mass. The momentum of the projected ions is related to B_s by

$$B_s^2/8\pi = 2m_i n_0 v_s^2 . \qquad (2)$$

After the piston-heated plasma becomes quiescent, pressure balance requires

$$(n_o/x_{SH}^2)kT_{SH} = B_s^2/8\pi . \qquad (3)$$

Note that $v_s = (kT_{SH}/2m_i)^{\frac{1}{2}}/x_{SH} = 2.20 \times 10^7 [kT_{SH}(keV)]^{\frac{1}{2}}$. Substituting (3) and (2) into (1), we find

$$x_{SH} = (2/5)^{\frac{1}{2}} = 0.632 . \qquad (4)$$

The initial back emf around the inside of the discharge tube is given by

$$V_s = 2\pi \times 10^{-8}\ bv_sB_s , \qquad \text{(Volts)} \qquad (5)$$

Substituting Eqs. (2) and (3),

$$V_s = 10^{-8}\left(\frac{\pi}{4m_i n_0}\right)^{\frac{1}{2}} bB_s^2 = 4 \times 10^{-8}\left(\frac{\pi^3 n_0}{m_i}\right)^{\frac{1}{2}} \frac{bkT_{SH}}{x_{SH}^2} . \qquad (6)$$

In more convenient units (for $m_i = 4.15 \times 10^{-24}$ g, an "average" D-T ion),

$$V_s(kV) = 0.517 \times 10^{-6}\ b\ B_s^2/p_0^{\frac{1}{2}} = 3.68\ p_0^{\frac{1}{2}} bkT_{SH}(keV) , \quad (x_{SH} = 0.632) \qquad (7)$$

where p_0 is the filling pressure in mT, b is in cm and B_s is in gauss.

B. Equilibration of Ions and Electrons

After the shock, but before the slow adiabatic compression has progressed very far, the electrons and ions will come to the same temperature by means of collisions. We assume this process to begin at $x = x_{SH}$, $T_i = T_{SH}$, $T_e = 0$ and to end at $x = x_E$, $T_i = T_E$, while the magnetic field maintains the constant value B_s. Thus $T_E = 1/2\ T_{SH}$, $x_E = x_{SH}$, and we write (3) as

$$2n_0 kT_E = x_{SH}^2(B_s^2/8\pi) . \qquad (8)$$

C. Isenthalpic Expansion

Before considering the adiabatic compression to the final plasma state, we consider the option of suddenly expanding the shock-heated equilibrated plasma without its doing work or exchanging heat with its surroundings. To accomplish this the field B_s would suddenly be removed for a time $\approx 0.3\ b/v_{th}$. Then a field $x_{SH}B_s$ would be reapplied. Here v_{th} is the ion thermal velocity. The plasma temperature remains constant at T_E, but the compression factor increases from x_{SH} to 1.

D. Adiabatic Compression

During this process the plasma behaves as a fluid, with specific-heat ratio $\gamma = 5/3$, for which $(kT)\ x^{2(\gamma - 1)} = \text{const}$. Let subscript f refer to the final, compressed plasma state. Then

$$T_f = T_E\left(\frac{x_{SH}}{x_f}\right)^{2(\gamma - 1)} \qquad (9)$$

$$n_f = n_0/x_f^2 . \qquad (10)$$

Pressure balance, $2nkT = B^2/8\pi$ then requires

$$B_f/B_s = (x_{SH}/x_f)^\gamma = (T_f/T_E)^{\frac{\gamma}{2(\gamma - 1)}} , \qquad (11)$$

where the final compression field is given by

$$B_f(kG) = 2.386\ \frac{[p_0 kT_f(keV)]^{\frac{1}{2}}}{x_f} . \qquad (12)$$

Substituting (6) and (8) into (9), we obtain (for $\gamma = 5/3$)

$$kT_f = \frac{10^8}{8\pi^{3/2}} \frac{x_{SH}^{2\gamma}}{x_f^{2(\gamma - 1)}} \frac{m_i^{\frac{1}{2}}}{n_0^{\frac{1}{2}}} \frac{V_s}{b} . \qquad (13)$$

$$kT_f(\text{keV}) = 0.0737 \frac{V_s(\text{kV})}{bp_0^{\frac{1}{2}} x_f^{4/3}} . \qquad (14)$$

Figure 2 is a graph of the various plasma and field parameters for $kT_f = 5$ keV and $b = 8$ cm.

For the case (primed) with added isenthalpic expansion we set $x = 1$ after equilibration so that

$$B_f'/B_s = x_f^{-\gamma} \qquad (15)$$

$$T_f'/T_E = x_f^{-2(\gamma - 1)} . \qquad (16)$$

Then

$$kT_f' = \frac{10^8}{8\pi^{3/2}} \frac{x_{SH}}{x_f^{2(\gamma - 1)}} \frac{m_i^{\frac{1}{2}}}{n_0^{\frac{1}{2}}} \frac{V_s}{b} , \qquad (17)$$

$$kT_f'(\text{keV}) = 0.136 \frac{V_s(\text{kV})}{bp_0^{\frac{1}{2}} x_f^{4/3}} . \qquad (18)$$

E. Free Expansion of the Ions[5]

In order to obtain lower compression (larger x_{SH}) for a given B_s and V_s, a magnetic field having the time history shown in Fig. 3 may be used. An initial piston field of magnitude $2^{-\frac{1}{2}} B_s$ is suddenly removed at time t_0 when the sheath has arrived at $x_0 = 1/3$. A larger field is reapplied at $3/2\ t_0$, when all ions have advanced to the opposite wall, and reflects them all at velocity $2v_s$. In contrast to expression (2), we now have

$$B_s^2/8\pi = 4n_0 m_i v_s^2 . \qquad (19)$$

We denote the temperature of the thermalized ions

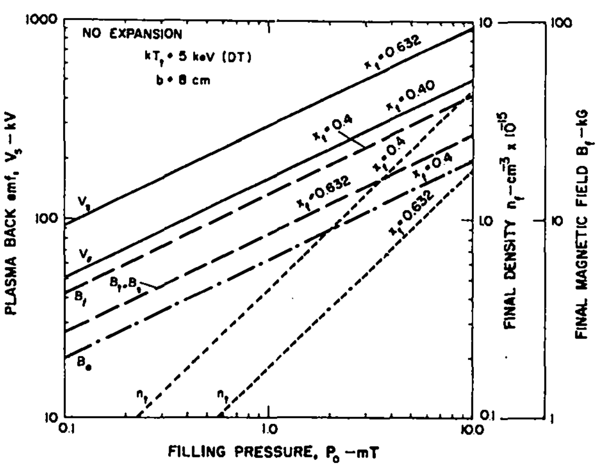

Fig. 2. Plasma heating parameters versus filling pressure for various compression ratios x_f.

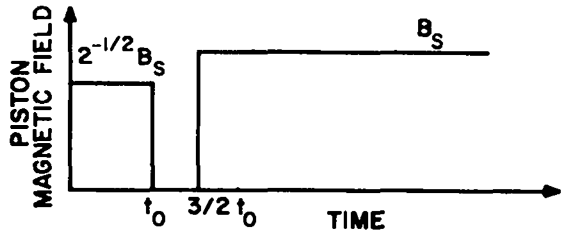

Fig. 3. Time history of magnetic field for free expansion of the ions following the initial implosion.

after the free expansion and reflection by T_{SH}''. Thus

$$(x_{SH}'')^2 B_s^2/8\pi = n_0 kT_{SH}'' . \qquad (20)$$

Note that $v_s = (kT_{SH}''/4m_i)^{\frac{1}{2}}/x_{SH}'' = 1.29 \times 10^7 [kT_{SH}''(\text{keV})]^{\frac{1}{2}}$. Corresponding to (1) we now have

$$\pi b^2 n_0 [1 - (1/3)^2][1/2\ m_i (2v_s)^2]$$
$$= 3/2\ \pi b^2 n_0 kT_{SH}'' - 1/8\ b^2 B_s^2 (1 - x_{SH}''^2) , \qquad (21)$$

where the last term represents work done on the plasma after the reversing reflection. Solving (21), (20) and (19) yields

$$x_{SH} = (26/45)^{\frac{1}{2}} = 0.76 . \qquad (22)$$

Since the ions are accelerated by $2^{-\frac{1}{2}} B_s$, we have

$$V_s'' = 2^{\frac{1}{2}} \pi \times 10^{-8}\ bv_s B_s$$
$$= 10^{-8} \left(\frac{\pi}{16 m_i n_0}\right)^{\frac{1}{2}} bB_s^2 ; \qquad (23)$$

$$V_s''(\text{kV}) = 0.259 \times 10^{-6}\ bB_s^2/p_0^{\frac{1}{2}}$$
$$= 1.275\ p_0^{\frac{1}{2}} bkT_{SH}''(\text{keV}). \quad (x_{SH}'' = 0.76) \qquad (24)$$

After adiabatic compression

$$kT_f'' = \frac{10^8}{4\pi^{3/2}} \frac{(x_{SH}'')^{2\gamma}}{x_f^{2(\gamma - 1)}} \frac{m_i^{\frac{1}{2}}}{n_0^{\frac{1}{2}}} \frac{V_s''}{b} ; \qquad (25)$$

$$kT_f''(\text{keV}) = 0.272 \frac{V_s''(\text{kV})}{bp_0^{\frac{1}{2}} x_f^{4/3}} . \quad (x_{SH}'' = 0.76) \qquad (26)$$

Figure 4 is a graph of the quantities B_s, B_f, n_f and V_s'' versus filling pressure p_0 for the final compression factor $x_f = 0.3$ and 0.4. A final D-T ion temperature $kT_f'' = 10$ keV is assumed.

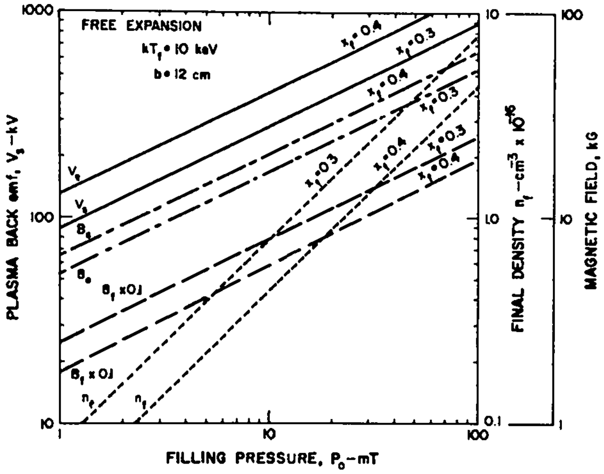

Fig. 4. Plasma heating parameters versus filling pressure for various compression ratios x_f. Free expansion of the ions after the initial implosion is assumed.

II. A SHOCK-HEATING CIRCUIT

Figure 5 shows a shock-heating coil with a Blumlein transmission line connected at each of two feed slots. The charge voltage V_{BL} is that between each half of each Blumlein line. The total voltage developed by the lines, considering the coil as an open circuit, is

$$V_0 = 2N_{BL}V_{BL} , \quad (1)$$

where N_{BL} is the total number of transmission lines. The characteristic impedance of each half of each line is

$$Z_{BL} = 0.4\pi \times 10^{-8} \, ct_{BL}/\epsilon^{\frac{1}{2}} w$$
$$= 377 \, t_{BL}/\epsilon^{\frac{1}{2}} w , \quad \text{(ohms)} \quad (2)$$

where ϵ is the dielectric constant of the insulator (for Mylar $\epsilon = 3$), t_{BL} is the spacing of a half Blumlein line, w is its width, and c is the speed of light in cm/sec.

When the shock-heating switches are closed, a voltage $2V_{BL}$ is applied after one wave transit time over the length ℓ_{BL}. It lasts for a time

$$\tau_{BL} = \frac{2\ell_{BL}}{v_{BL}} , \quad (3)$$

where

$$v_{BL} = c/\epsilon^{\frac{1}{2}} \quad (4)$$

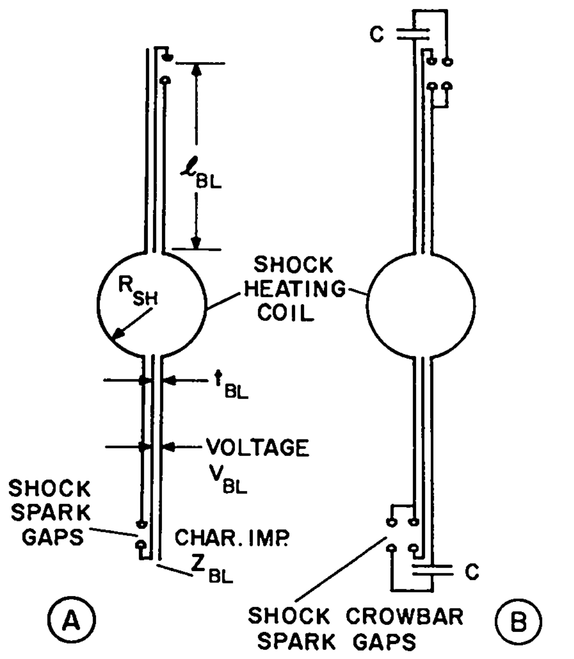

Fig. 5. Blumlein-line driven shock heating coil switch capacitor crowbar.

is the phase velocity in the line. The rise time of the current in the coil (when there is no plasma) is

$$\tau_r = \frac{L_{SH}}{2Z_{BL}} . \quad (5)$$

The coil current rises according to the relation

$$I = I_{BL}(1 - e^{-t/\tau_r}) \quad (6)$$

to a final value

$$I_{BL} = \frac{V_{BL}}{Z_{BL}} . \quad (7)$$

This must match the current necessary to produce the shock magnetic field B_s in length ℓ_{SH} of the shock coil. If we specialize to the case of Mylar insulation whose dielectric strength is taken as 1.18×10^6 V/cm (3 kV/mil), we have

$$t_{BL} = 0.85 \times 10^{-6} \, V_{BL} . \quad (8)$$

Using (2), we derive the width of the transmission lines

$$w = 1.85 \times 10^{-4}\, I_{BL} \,. \qquad (cm) \qquad (9)$$

III. COUPLING OF THE PLASMA AND THE SHOCK-HEATING CIRCUIT

Having established the parameters of the shock-heating circuit, we now give an approximate treatment of the effect of the plasma back emf on the current driven by the Blumlein lines. We consider the situation at early times, $x = t/\tau_r \lesssim 1$, when the sheath has not moved appreciably from its initial position x = 1. Using (I.6), (I.7) and (IV.3), the plasma back emf is given by

$$V_s = AI^2 \,, \qquad (volts) \qquad (1)$$

where I is the shock-coil current in amperes, and

$$A = (0.4\pi)^2 \times 10^{-5} \left(\frac{\pi}{4m_i n_0}\right)^{\frac{1}{2}} \frac{b}{\ell_{SH}^2}$$

$$= 0.973 \times 10^{-3} \frac{b}{p_0^{\frac{1}{2}}\, \ell_{SH}^2} \,. \qquad (V/A^2) \qquad (2)$$

A diagram of the shock circuit is given in Fig. 6. Thus

$$2N_{BL}Z_{BL}I + I\dot{L}_{SH} = 2N_{BL}V_{BL} - AI^2 \,. \qquad (3)$$

Setting $\alpha = AI_{BL}^2/2NV_{BL} = AI_{BL}/2N_{BL}Z_{BL}$ and $y = I/I_{BL}$, this reduces to

$$dy/dx = 1 - y - \alpha y^2 \,. \qquad (4)$$

For y = 0 at x = 0 the solution is

$$\tanh^{-1} \frac{1 + 2\alpha y}{(1 + 4\alpha)^{\frac{1}{2}}} - \tanh^{-1} \frac{1}{(1 + 4\alpha)^{\frac{1}{2}}}$$

$$= (1 + 4\alpha)^{\frac{1}{2}}\, x \,, \qquad (5)$$

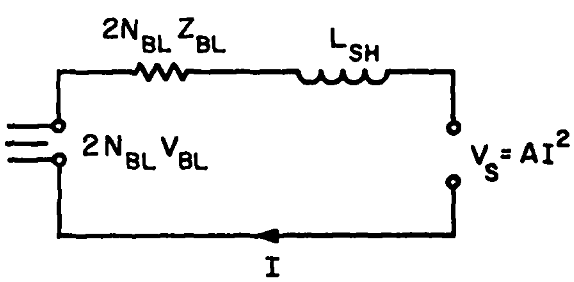

Fig. 6. Circuit diagram of the shock-heating circuit.

with the limit for $\alpha \to 0$ given by $\log(1 - y) = x$. Graphs of I/I_{BL} and $\alpha y^2 = V_s/2N_{BL}V_{BL}$ versus t/τ_r are given in Fig. 7. It is seen that the back emf V_s reduces the attainable currents and coil voltages from their values I_{BL} and $2N_{BL}V_{BL}$ in the simple line-driven circuit.

IV. THE MAGNETIC COMPRESSION CIRCUIT, LAYOUT AND NOTATION

A schematic arrangement of the magnetic compression circuit is shown in Fig. 8. The superconducting storage coil is assumed to consist of stabilized NbTi windings at 4° K, surrounded by a secondary at room temperature, or possibly 78° K. The superconducting switch R_{SC}, on going normal, reaches a value R_{SW}, much greater than that of the ballast resistor R_B, which therefore dissipates most of the energy involved in the resistive magnetic energy transfer.

In the following the subscripts SH, C, P, and S refer to the shock, compression, primary and secondary coils, respectively. The subscript 0 refers to conditions before switching the magnetic energy. A radial section through a coil is shown in Fig. 9:

ΔR = actual thickness of coil	(cm)		
ν = no. of coil layers in ΔR			
λ = coil volume filling factor			
ℓ = length of coil subdivision	(cm)		
$N = \ell n \nu$ = total number of turns in one coil subdivision*			
$N_1 = N/\ell$ = no. of turns per cm	(cm^{-1})		
ΔR_{SC} = thickness of superconducting material in ΔR_P	(cm)		
B = magnetic induction in a solenoid	(G)		
R_0 = mean coil radius	(cm)		
$W = 1.25\, B^2R^2 \times 10^{-8}$ = magnetic energy per unit length of solenoid	(J/cm)	(1)	
$W_\ell = \ell W$ = magnetic energy per coil subdivision	(J)	(2)	
k = coupling coefficient from primary to secondary of storage coil			

*In the case of a one-turn coil, as in a conventional Θ pinch, N = 1 is associated with the length ℓ.

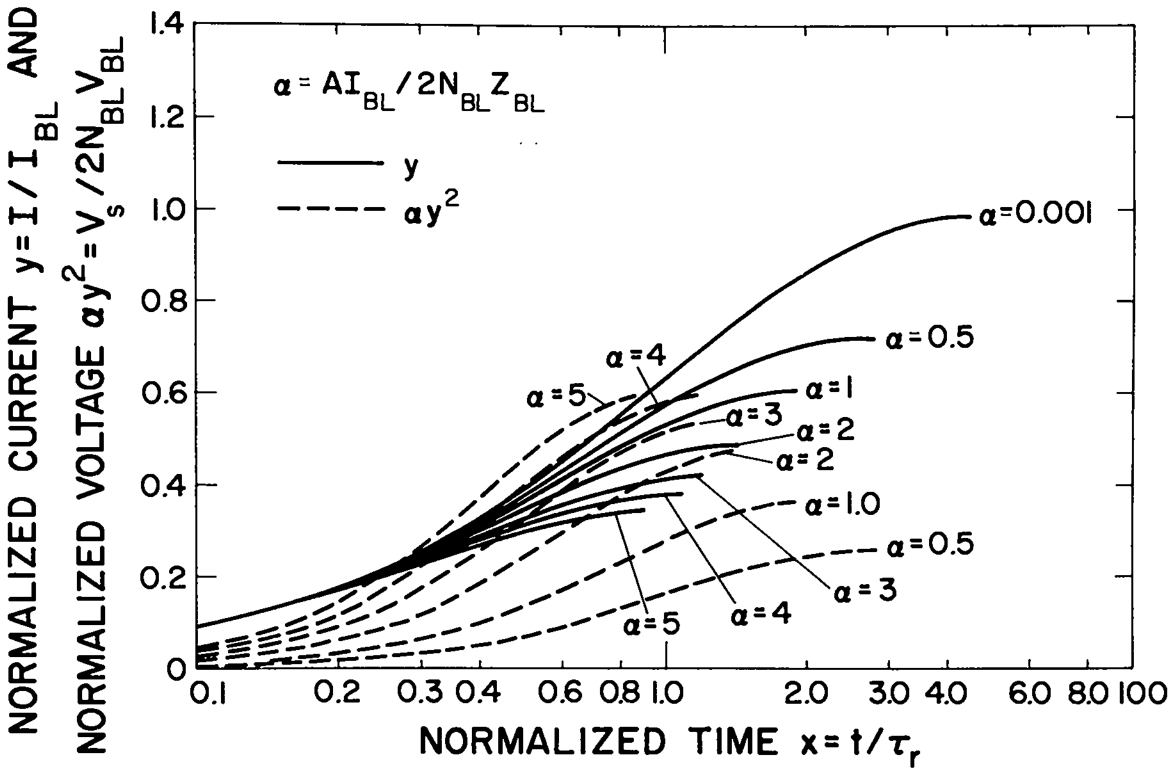

Fig. 7. Shock-coil current and voltage as functions of time for various plasma-sheath back-emf parameters.

$I_1 = B/0.4\pi$ = current per unit length of solenoid (A/cm) (3)

$I = \ell I_1/N = I_1/N_1$ = current per turn (A) (4)

$j = \lambda B/0.4\pi\Delta R = \lambda I_1/\Delta R$ = average current density of solenoid (A/cm^2) (5)

$j_{SC} = I_{1P}/\Delta R_{SC}$ = superconducting current density in primary solenoid (A/cm^2) (6)

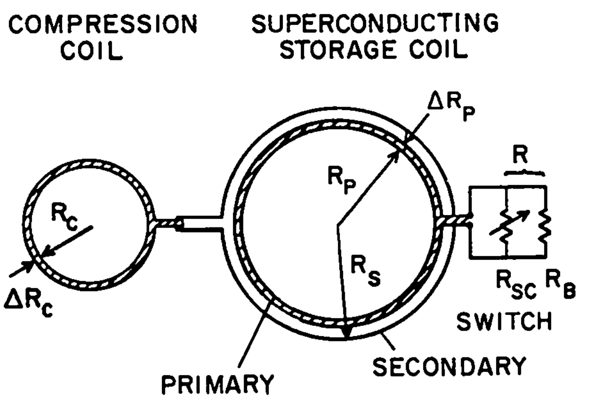

Fig. 8. Schematic diagram of the compression-field circuit of a separated-shock Θ pinch with cryogenic inductive energy storage.

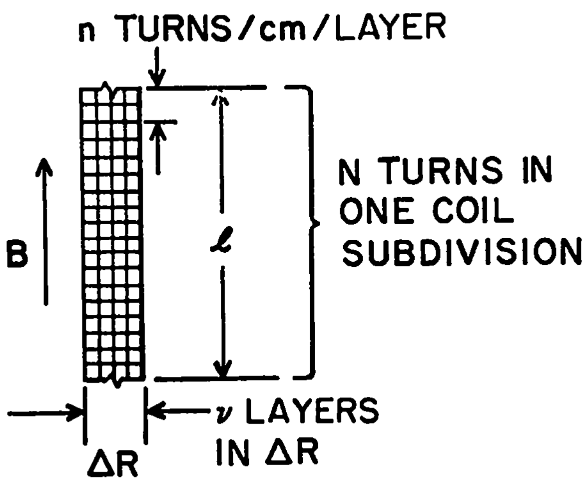

Fig. 9. Radial section of compression coil.

$L_1 = 4\pi^2R^2N_1^2 \times 10^{-9}$ = inductance per unit length (H/cm) (7)

$L = (4\pi^2R^2N^2/\ell) \times 10^{-9}$ = inductance of one coil subdivision (H) (8)

τ = containment time of plasma

τ_s = switching (L/R) time of current from storage to compression coil (rise time) (sec)

η = resistivity (ohm-cm)

η_{SC} = normal resistivity of super-conductor (ohm-cm)

η_{com} = resistivity of normal superconductor and stabilizing matrix

ϖ = fraction of initial primary energy W_{PO} dissipated in the superconducting switch

$x = L_c/L_s$ = ratio of inductances of compression coil and inductive-storage secondary

d = thickness of winding $\approx \Delta R/\nu\lambda^{\frac{1}{2}}$

a = ratio of area of superconductor plus stabilizing matrix to area of superconductor in switch material

V. RESISTIVE SWITCHING OF MAGNETIC ENERGY WITH MUTUAL INDUCTANCE

Figure 10 shows the circuit of the inductive-storage circuit after the resistance of the transfer element has changed from zero to R. Let I_0 be the initial current in L_P, which is the storage inductor whose corresponding initial stored energy is $W_{PO} = \frac{1}{2}L_PI_0^2$. After the switching has taken place

$$I_S = \left(\frac{L_P}{L_S}\right)\frac{k^{\frac{1}{2}}}{(1 + x)} I_0(1 - e^{-t/\tau_s}) , \quad (1)$$

$$I_P = I_{PO}\, e^{-t/\tau_s} , \quad (2)$$

where

$$\tau_s = \frac{L_P}{R}\,\frac{(1 + x) - k}{1 + x}$$

is the rise time of the compression field, and

$$x = L_C/L_S \quad (4)$$

$$k = M^2/L_SL_P . \quad (5)$$

The energy dissipated in R is

$$W_R = \frac{1 + x - k}{1 + x} W_{PO} . \quad (6)$$

The energy transferred to the load L_C is

$$W_C = \frac{xk}{(1 + x)^2} W_{PO} . \quad (7)$$

The energy remaining in L_S (and filling L_P) is

$$W_S = \frac{k}{(1 + x)^2} W_{PO} . \quad (8)$$

The transferred energy is a maximum when $x = 1$. For $k = 1$ one half of the energy W_{PO} is dissipated and the secondary and load inductors each get one quarter.

VI. RELATIONS BETWEEN THE INDUCTANCES IN MAGNETIC ENERGY STORAGE

In the following we take $L_C = L_S$ $(x = 1)$. Since $W_C = (k/4)W_{PO}$ we have

$$R_P = (2/k^{\frac{1}{8}})R_CB_C/B_{PO} , \quad (1)$$

where R_C and B_C are given. We also take R_S/R_P (≈ 1.1) as given, thus determining $k = (R_P/R_S)$. The field B_{PO} will be taken as an independent variable. We shall also take the initial current I_{PO} in a primary-coil subdivision as an independent variable. We then find

$$N_P = B_{PO}\ell_P/0.4\pi I_{PO} . \quad (2)$$

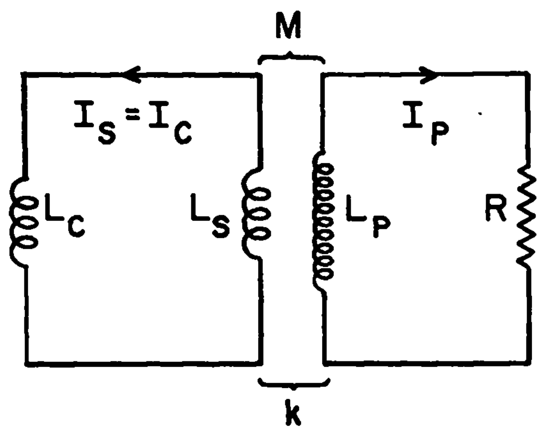

Fig. 10. Diagram of inductively driven circuit.

From the condition $L_{S1} = L_{C1}$, we have

$$N_{1C} = R_S/R_C \ . \tag{3}$$

We note that

$$I_C = B_C \ell_C / 0.4\pi N_C \ . \tag{4}$$

VII. RELATIONS IN THE SUPERCONDUCTING CIRCUIT

A. The Superconducting Switch

From (V.7) we have, for $x = 1$,

$$W_{PO} = \tfrac{1}{2} L_{1P} I_{PO}^2 = \frac{4}{k} W_C \ . \tag{1}$$

From (V.3), for $x = 1$,

$$R = \frac{L_{1P}\ell_P}{\tau_s}\left(1 - \frac{k}{2}\right) , \tag{2}$$

$$R = \frac{4(2 - k)}{k} \frac{W_C \ell_P}{\tau_s I_{PO}^2} \ . \tag{3}$$

The voltage across the resistors shortly after the instant of switching is

$$E_0 = R I_{PO} = \frac{4(2 - k)}{k} \frac{W_C \ell_P}{\tau_s I_{PO}} \ . \tag{4}$$

Referring to Fig. 8, $R = R_B R_{SC}/(R_B + R_{SC})$. We shall assume that $R_{SC} \gg R_B$ so that $R \approx R_B$. In order that only a fraction φ of the energy W_{PO} of the superconducting coil be dissipated in R_{SC}, and hence appear as a load on the refrigerator, we set

$$R_{SC} = \frac{2 - k}{2\varphi} R$$

$$= \frac{2(2 - k)^2}{k\varphi} \frac{W_C \ell_P}{\tau_s I_{PO}^2} \ . \tag{5}$$

Let A_{SC} be the cross sectional area of superconductor carrying I_{PO} in R_{SC} before the switching takes place and let ℓ_{SC} be the length of superconductor. Let j_{SC} be the (near-critical) current density of the superconductor before switching. Then from (5)

$$\eta_{com} \frac{\ell_{SC}}{a A_{SC}} = \frac{2(2 - k)^2}{k\varphi} \frac{W_C \ell_P}{\tau_s j_{SC}^2 A_{SC}^2} ,$$

where a is the area ratio of stabilizing matrix material plus superconductor to superconductor. It is estimated[6] that values of a as small as 1.5 can be attained. The volume of superconductor in cm^3 is[7]

$$V_{SC} = \ell_{SC} A_{SC} = \frac{2(2 - k)^2}{k\varphi} \frac{a W_C \ell_P}{\tau_s \eta_{com} j_{SC}^2} \ . \tag{6}$$

In terms of the switching voltage E_0, the area of the superconductor is gotten from (4):

$$A_{SC} = \frac{4(2 - k)}{k} \frac{W_C \ell_P}{\tau_s E_0 j_{SC}} \ . \tag{7}$$

The length of superconductor is V_{SC}/A_{SC}:

$$\ell_{SC} = \frac{2 - k}{2\varphi} \frac{a E_0}{\eta_{com} j_{SC}} \ . \tag{8}$$

In order to express in terms of presently available, fabricated, stranded superconductor we note the empirical value $j_{SC} = 5 \times 10^5$ A/cm^2 for braided NbTi switches[8] and estimate[6] the value $\eta_{com} = 5 \times 10^{-5}$ ohm-cm for NbTi stabilized in a CuNi or German-silver matrix.

B. The Superconducting Primary Solenoid

In order to express in terms of presently available, fabricated superconductor we note the following empirical relationship between j_{SC} (in A/cm^2) and B_{PO} in G for $B_{PO} \leq 50$ kG:

$$B_{PO} j_{SC} = 6 \times 10^9 \ \mathrm{AG/cm^2} \ . \tag{9}$$

Let ΔR_{SC} be the thickness of superconductor in the primary solenoid. Using (9):

$$\Delta R_{SC} = \frac{B_{PO}}{0.4\pi j_{SC}} = 1.33 \times 10^{-10} B_{PO}^2 \ . \tag{10}$$

The volume of superconductor in length ℓ_P is then

$$V_{SC} = 2\pi R_P \ell_P \Delta R_{SC} = 1.67 \times 10^{-9} \ell_P R_C B_{PO} B_C / k^{\frac{1}{2}} , \tag{11}$$

where we have used (VI.1).

VIII. PARAMETERS OF A 10-KEV SCIENTIFIC FEASIBILITY EXPERIMENT

A. Choice of Shock and Compression-Coil Dimensions and Fields

We start with a considerably larger discharge tube (about a factor of 2) than those in previous experiments and proceed outward.

Inner radius of discharge tube = b = 12 cm.

Inner radius of shock heating coil = $x_c b$ = 13 cm.

Thickness of shock heating coil = 0.5 cm.

Outer radius of shock heating coil = 13.5 cm.

In choosing the inner radius $R_c = 16$ cm of the compression coil we leave space for shock-circuit connections and insulation around the shock-heating coil. We assume room-temperature copper ($\eta = 1.73 \times 10^{-6}$ ohm-cm) and a containment time of 42 ms or greater.

As a convenient value we choose the compression-coil thickness $\Delta T_c = 2.5$ cm. The L/R decay time of the compression coil is then

$$\tau_c = 2\pi R_c \Delta R_c \times 10^{-9}/\eta = 0.15 \text{ sec} . \qquad (1)$$

The skin depth in the copper is approximately

$$\delta_E = 1.1 \times 10^4 \, \eta^{\frac{1}{2}} \, \tau_c^{\frac{1}{2}} . \qquad (2)$$

We choose a rise time $\tau_s = 2$ ms. It is shown elsewhere[9] that eddy-current loss W_{EE} represents approximately the following fraction of the "d.c." joule loss W_E during the pulse of compression field:

$$W_{EE}/W_E = \frac{\tau}{\tau_s} \frac{\lambda^2 \Delta R_c^2 d^2}{3.68 \, \delta_E^4} . \qquad (3)$$

If we choose a 5-layer coil ($\nu = 5$) with a filling factor $\lambda = 0.7$, the winding thickness if

$$d \approx \Delta R_c / 5\lambda^{\frac{1}{2}} = 0.6 \text{ cm} , \qquad (4)$$

corresponding to $W_{EE}/W_E \approx 0.01$.

B. Plasma and Magnetic-Field Parameters

From the curves of Fig. 4 we see that a D-T ion temperature $kT_f = 10$ KeV (8.9 keV for D-D) can be obtained at a filling pressure p_0 of about 3 mT and a final radial compression factor $x_f = 0.3$. The corresponding final magnetic field at $\beta = 1$ is 43 kG. Allowing for losses and experimental latitude, we choose

$B_f = 60$ kG

$B_s = 9.2$ kG

$n_f = 3.0 \times 10^{15} \text{ cm}^{-3}$.

Correspondingly, the maximum plasma back emf $V_s = 153$ kV. The shock and pre-compression temperatures are given by

$$kT_{SH} = 2kT_E = 5.8 \text{ keV},$$

and the shock velocity is

$$v_s = 3.11 \times 10^7 \text{ cm/sec} .$$

We note that the shock-coil inductance outside the initial plasma sheath, for $\ell_{SH} = 100$ cm, is

$$4\pi^2 \times 10^{-9}(x_c^2 - 1)b^2/\ell_{SH} = 9.8 \text{ nH} .$$

This corresponds to 4.9 nH for each of the two Blumlein feeds. We assume 10 nH total parasitic inductance. Thus we take the initial inductance in series with the plasma sheath at one feed slot to be

$$L_{SH} = 10 \text{ nH} .$$

The transit time of the plasma sheath to the center of the shock coil is approximately 12 cm/3.1 x 10^7 cm/sec = 0.39 μsec. We assume that a sufficient approximation to ideal behavior can be achieved with a rise time $\tau_r = 0.10$ μsec. From (III.5) the the characteristic impedance is

$$Z_{BL} = L_{SH}/2\tau_r = 0.050 \text{ ohm} .$$

If the current $I_{BL} = V_{BL}/Z_{BL}$ were completely effective in producing B_s we should require 0.73 MA/m. However Fig. 7 shows that the plasma back emf has the effect of raising this by a factor of about 2.2($\alpha \approx 2.2$). Thus we have

$I_{BL} = 1.6$ MA

$V_{BL} = 80$ kV .

From Fig. 7 (for $\alpha = 2.2$) we find that at $t \approx \tau_r$ the plasma back emf is

$$V_s = 0.45(2N_{BL}V_{BL}) = 145 \text{ kV} .$$

This is approximately consistent with the value (153 kV) obtained from Fig. 4. The length of the line is given by

$$\ell_{BL} = 2/3 \frac{bc}{v_s \varepsilon^{\frac{1}{2}}} = 45 \text{ meters} .$$

From (II.8) the thickness of mylar insulation must be

$$t_{BL} = 0.68 \text{ mm} .$$

From (II.9) the width of the Blumlein line must be

$$w = 3.0 \text{ meters} .$$

C. The Inductive Storage Secondary Circuit

This circuit consists of the compression coil and the single-turn secondary of the inductive energy store (Fig. 8). It must receive an amount of energy equal to that of the compression coil. From (IV.1) for $B_C = B_f = 60$ kG

$$W_C = 1.15 \text{ MJ/m} .$$

We take a magnetic field

$$B_{PO} = 20 \text{ kG}$$

in the primary, superconducting storage coil and assume a coupling coefficient k = 0.85. Using (VI.1), we derive for the primary and secondary coil radii:

$$R_P = 104 \text{ cm}$$

$$R_S = 122 \text{ cm} .$$

The number of turns per unit length of compression coil is given by (VI.3):

$$N_{1C} = 7.6 \text{ cm}^{-1} .$$

For $\nu = 5$ layers, we arrive at

$$n_C = 1.5 \text{ turns/cm/layer} .$$

The current per turn is

$$I_{1C} = \frac{B_C}{0.4\pi N_{1C}} = 6.24 \text{ kA} .$$

As a check we note that the inductances of the secondary and compression coil per unit length are the same:

$$L_{1C} = 4\pi^2 R_C^2 N_{1C}^2 \times 10^{-9} = 0.58 \text{ mH}$$

$$L_{1S} = 4\pi^2 R_S^2 \times 10^{-9} = 0.58 \text{ mH} .$$

The linear current density in the secondary is 6.24 kA/cm, and the secondary magnetic field is

$$B_S = 0.4\pi \times 6.24 = 7.84 \text{ kG}.$$

From (IV.1) the secondary-coil magnetic energy is

$$W_{MS} = 1.15 \text{ MJ/m}$$

(the same as W_{MC}). The maximum voltage on the secondary is approximately

$$V_S = 10^{-8} \pi R_S^2 \frac{B_S}{\tau_s} = 1.8 \text{ kV} .$$

D. The Superconducting Primary Circuit

From (VII.4) the maximum primary voltage over length ℓ_P is related to I_{PO} by the relation

$$E_0 I_{PO}/\ell_P = 3.11 \times 10^7 .$$

For $B_{PO} = 20$ kG the ampere turns per cm are

$$N_{1P} I_{PO} = B_{PO}/0.4\pi = 1.59 \times 10^4 .$$

We assign as a maximum value, $V_P = 40$ kV. The "charging" current I_{PO} is assigned the value 25 kA. Thus the module length is $\ell_P = 32$ cm, and $N_{1P} =$ 0.63 turns/cm. Thus a braided superconducting cable would have a width of about 1.6 cm. From (VII.9) and (VII.10) the superconductor thickness is

$$\Delta R_{SC} = 0.053 \text{ cm} ,$$

and the volume per meter length is

$$V_{SC} = 3.48 \times 10^3 \text{ cm}^3/\text{m} .$$

E. The Superconducting Switches

From (VII.10) the volume of switch superconductor is

$$V_{SC}^{SW} = \frac{214}{\varphi} , \qquad \text{cm}^3/\text{cm}$$

where η_{com} is taken as 5.5×10^{-5} ohm-cm, a = 1.5 and $\tau_s = 2 \times 10^{-3}$ sec. If we take the volume of switches to be 1.6 times that of the windings, we find

$$\varphi = 214(1.5 \times 3480) = 0.038 .$$

The energy dissipated in the superconducting switch is

$$W_{DISS} = (4\varphi/k)W_C = 0.18 W_C = 2.05 \times 10^5 \text{ J/m} .$$

If there is one discharge every 15 minutes the power to be removed by the refrigerator at 4°K is

$$P_{DISS} = 230 \text{ watts/m} .$$

Using (VII.8), the length of superconductor in each switch is

$\ell_{SC} = 268$ meters .

The superconductor volume is 5.6×10^3 cm^3/m.

IX. COST ESTIMATE OF A 10-KEV SCIENTIFIC FEASIBILITY EXPERIMENT

A. Superconductor Circuit

For each meter of length we have found volume of coil and switch superconductor = 9.1×10^3 cm^3/m.

Cost at \$1.9/cm^3 for fabricated coil wire \$17.2 K/m

The coil form is backed by an outer shell of stainless steel with a working stress $\sigma = 18.8$ kpsi. Its thickness is

$$t = 0.59 \times 10^{-6} B_{PO}^2 R_P/\sigma = 1.4 \text{ cm} .$$

Its volume for one meter length is

$$V_{SS} = 200\, \pi R_P t = 0.95 \times 10^5 \text{ cm}^3/\text{m} .$$

Its weight is

$$7.8 \times 0.95 \times 10^5/4.54 \times 10^2 = 1632 \text{ lb/m}.$$

Cost at \$2/lb for fabricated coil backing \$ 3.3 K/m

Cost of plastic inside coil form, 2 cm thick at \$1.5/lb.	\$ 0.9 K/m
Labor for fabrication and assembly of one-layer coil	\$ 1.7 K/m
Total cost of primary coil plus switch	\$23.1 K/m

B. Primary-coil Dewar

The cost is scaled from an existing 13-foot Dewar, 30 feet long, assuming that the joints of a long system will cost the same as the ends of the present Dewar, which has a 1.25" Al inner vessel and a 3/8" S.S. outer vessel. The cost of the present Dewar is about \$13 K/meter, including supports. As a basic scaling we assume

$$\text{cost} \propto (\text{volume})^{0.8} = \$1550\, D^{1.6} ,$$

where D is the diameter in meters. Thus

Cost of 2.5-m diameter Dewar:	\$ 6.7 K/m
Installation at 4.7 man-day /m(\$100/man-day)	\$ 0.45K/m
Dewar cost with installation	\$ 7.2 K/m

C. Refrigeration[10]

(1.) Radiation and support loss at 4°K. Experience[10] indicates a figure of 0.026 watt/ft^2, which corresponds to

$$Q = 0.9\, D_{in}, \text{ watts } (4^\circ K)/m$$

where D_{in} is the inner diameter of the Dewar in meters. In our case $Q \approx 2.2$ w/m and is neglible.

(2.) Lead-in loss at 4°K.[6] Experience indicates a figure of 2 w/kA. The ballast resistors connected every 32 cm could cause an appreciable heat loss. However we propose to make this neglible by using pneumatically driven "banana-plug" switches (at 25 kA pulse rating) to break thermal contact during the times between pulses.

(3.) Dissipation in the superconducting switches. In Section (VIII.E) we found a refrigeration requirement of 230 watts/m, assuming 15 minutes between discharges. From Strobridge's[11] and Fraas's[12] curves for refrigeration capacities in the 10^5 to 10^6-watt (300°K) range we find

$$\mu = \text{fraction of Carnot efficiency} = 0.33 .$$

Thus the input power to the refrigerator is given by

$$\frac{\text{input power}}{4^\circ K \text{ power}} = \frac{1}{\mu}\, \frac{300 - 4}{4} = 222 .$$

In our case the refrigeration power is thus

$$P_{in} = 222 \times 230 = 51.1 \text{ kW/m} .$$

For a 100-m length we are in the range of 5000 kW total. From Strobridge's Fig. 4 we find a unit cost of \$380/kW. Thus we have:

Refrigerator cost at \$380/kW \$19.4 K/m

For a labor cost we assume 4 man-days per meter at \$100/man-day. Thus

Refrigerator installation	\$ 0.4 K/m
Total refrigerator cost (15 min)	\$19.8 K/m

D. The Primary Charging Circuit

Figure 11 shows the charging circuit with its generator. Each primary module of length ℓ_P has its superconducting switch in the normal state and the voltage V_G is increased until the current I_{PO} is built up. The superconducting switches are brought to their superconducting state, and the generator current I_G is reduced to zero. The isolating switches are then opened, leaving the circulating currents I_{PO} in each module loop as shown. We assume that all primary modules are charged in series. Thus for a device 100 meters in length the total primary inductance, from (IV.7) with $N_{1P} = 0.63$ turns/cm and

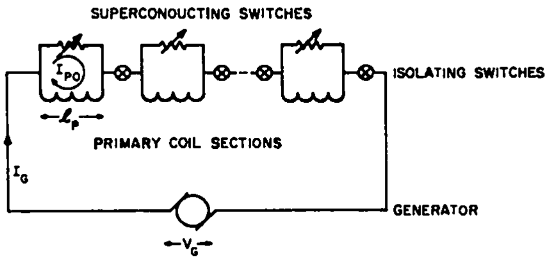

Fig. 11. Charging circuit for superconducting primary coils.

R_p = 104 cm, would be

$$L_p = 1.7 \text{ H} .$$

Assuming a charge time of 300 sec to 25,000 A, we arrive at an average charging voltage of 71 volts. This indicates that rectifier supplies would be appropriate for this application. Their rating would be in the range of 1800 kVA. We take a unit cost of \$0.21 K/kVA and arrive at

cost of charging generators at \$0.21 K/kVA	\$ 3.6 K/m

To this we add the cost of the isolation switches (3 per meter), rated at 40 kV, 25 kA when closed. Each is estimated to cost \$0.5 K. Thus

Cost of isolating switches	\$ 1.5 K/m
Cost of primary charging circuit	\$ 4 K/m

E. The Secondary Coil

We take a single turn of radius R_s = 122 cm, one cm thick for self support, since there is no appreciable magnetic pressure. At \$0.56/lb the material cost for 550 lb is ≈ \$300/m. Fabrication is taken as \$300. Installation at 2 man-days is \$200.

Cost of secondary	\$ 1 K/m

F. The Ballast Resistors

Each must carry a peak current of 25 kA and dissipate 2.3/4 ≈ 0.6 MJ. Thus, for a temperature rise of 20°C, electrolytic resistors, each with a volume of about 13 liters, are suitable. These are estimated to cost \$40 each including labor. With allowance for the banana contactors of \$70 per pair we arrive at

Cost of ballast resistors and contactors with labor	\$ 1 K/m

G. Unit Energy Cost of the Superconducting Inductive Energy Source

We neglect the cost of triggering, since pulses to drive the superconducting switches normal can probably be derived from the shock-heating circuit. Adding the above figures and allowing 10% for contingencies, we arrive at a total cost of \$58 K/m. Since we deliver 1.15 x 10^6 joules to the compression coil, this gives a unit cost of 5 ¢/J.

H. The Compression Coil

We assume water-cooled square copper stock wound into pancakes in a conventional manner. The cost of material, fabrication and potting and miscellaneous fittings is taken as \$2.0 K/m. Installation at 4 man days per meter is \$400. Thus

Cost of compression coil	\$ 2 K/m

I. Compression-Coil Backing

We consider of high strength stainless-steel backing of both the 37-cm coil and its horizontal feed. Material cost of 1550 lbs at \$0.60/lb = \$928/m. Fabrication and miscellaneous fitting = \$1700/m. Installation (2 man days/m) = \$200. Thus

Cost of compression-coil backing	\$ 3 K/m

J. Shock-Heating Apparatus

We estimate this item at the total cost per joule of the Scyllac device, exclusive of the building. The figure \$0.6/J includes materials, parts and labor.

In the present experiment the shock magnetic field is 9.3 kG (Fig. 4 at 3 mT with x_f = 0.3). The energy in the 12-cm radius coil is thus 16 kJ/m. Assuming equal energies in the Blumlein-line and the capacitive crow bar circuit (32 kJ/m in all), we arrive at

Cost of shock-heating apparatus	\$19 K/m

K. Unit cost of the overall system

Superconducting coil and switch	\$23 K/m
Dewar	\$ 7 K/m
Refrigeration	\$20 K/m
Primary charging circuit	\$ 4 K/m
Secondary coil	\$ 1 K/m
Ballast resistors and contactors	\$ 1 K/m
Compression coil and backing	\$ 5 K/m
Shock-heating apparatus	\$19 K/m
Contingency for miscellaneous items (20%)	\$16 K/m
Total cost	\$95 K/m

Total energy in compression and shock systems 1180 Kj/m

Unit energy cost of Θ pinch 8 ¢/J

L. Total Cost of the Θ Pinch System

For a toroidal system we take an estimated circumference of 180 meters (see Section X below), arriving at a total Θ pinch cost of $17 M. The present low compression design is not suitable for a linear experiment, since the low density and correspondingly long containment time would require an excessive length (Section X below).

M. Building Costs

For the toroidal experiment we consider a building roughly 230-ft square at $105/ft^2, giving a building cost of about $5.6 M.

X. THE OVERALL SYSTEM

A conceptual view of a portion of a separated-shock Θ pinch, based on the foregoing parameter study, is shown in Fig. 12. Here the Blumlein lines are shown folded to accommodate their length. In addition each shock heating coil is shown as fed from its two ends by two Blumlein lines. This arrangement presents insulation and parastic inductance difficulties which may be better met by other geometrical configurations. In any case the high voltage pulse from the shock-heating circuit must not be allowed to couple to the slow compression circuit. This can be accomplished by surrounding the shock-heating coil by thin cylinders of (for example) stainless steel which return the magnetic flux of these coils inside the compression coils. The rise times of the shock and compression circuits are greatly different. Therefore one can choose the thickness of these cylinders to lie between the skin depths corresponding to these times in order to provide this isolation with little joule-heating loss as the compression field penetrates the cylinders.

The length of a toroidal system is determined by considerations of toroidal equilibrium and MHD stability. We assume the ℓ = 1, 0 helical system presently being tested in the Scyllac experiment.[13] The minimum aspect ratio is set by considerations of the strength and wavelength of the helical fields and the plasma radius. The ratio of shock-coil radius to plasma radius, for wall stabilization of the m = 1 MHD mode, will be important. On the basis of present theoretical and experimental indications a rough estimate of the diameter a toroidal device is from 50 to 70 meters. A conceptual drawing of the main components of a toroidal device is shown in Fig. 13.

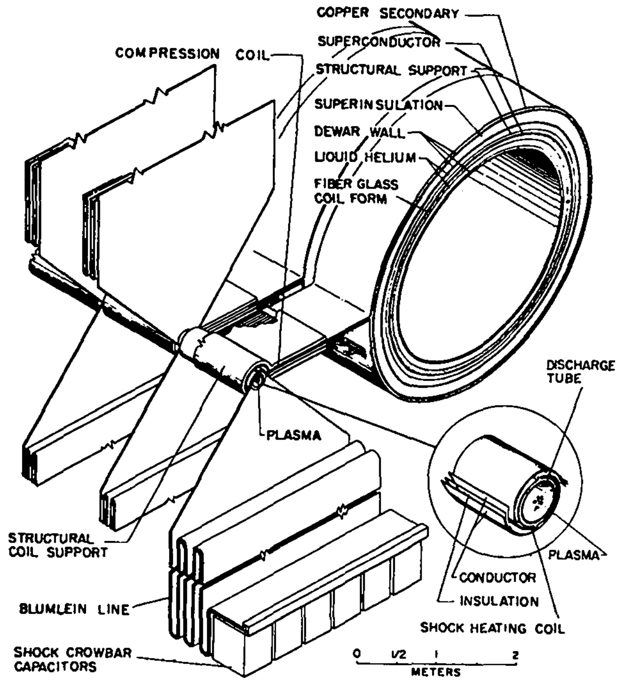

Fig. 12. Conceptual view of a portion of a separated-shock Θ pinch.

The length ℓ_{Th} of a linear system is determined by loss of the ions at their thermal speed through ends constricted by mirrors and is given by[14]

$$\ell_{Th} = (2/\pi^{\frac{1}{2}})(\tau_0 v_{Th}/\rho_M) \,, \qquad (1)$$

where $v_{Th} = (2kT/m_i)^{\frac{1}{2}} = 0.9 \times 10^8$ cm/sec and $\tau \approx$ 0.042 sec is the burning time of the D-T plasma. The factor ρ_M is the high-β mirror ratio. We assume β = 0.9 and vacuum mirrors of 2.5, leading to ρ_M = 9. Thus a device about 5-km long is indicated. A conceptual diagram is given in Fig. 14.

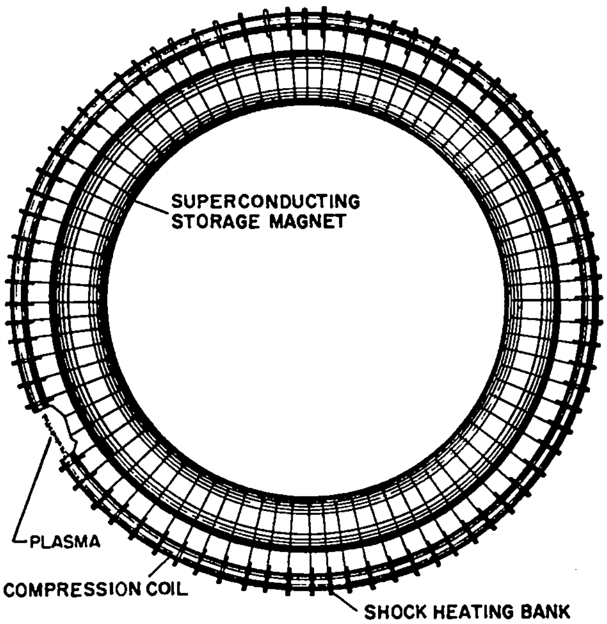

Fig. 13. Conceptual view of a toroidal separated-shock Θ pinch.

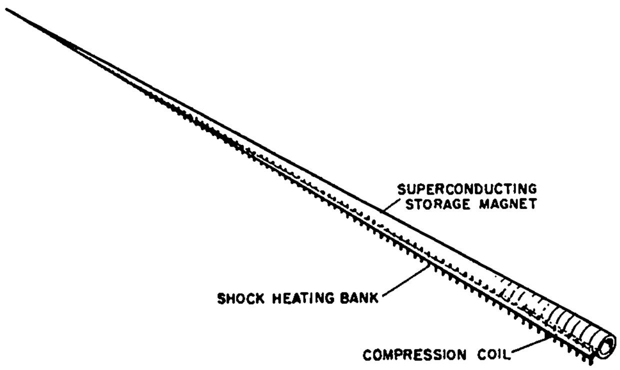

Fig. 14. Conceptual view of a linear separated-shock Θ pinch.

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