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AUTHOR(S): T. A. Wareing
W. F. Walters
J. E. Morel

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Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

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DIFFUSION-ACCELERATED SOLUTION OF THE 2-D X-Y S_N EQUATIONS WITH LINEAR-BILINEAR NODAL DIFFERENCING

Todd A. Wareing, Wallace F. Walters, and Jim E. Morel
University of California
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

ABSTRACT

Recently a new diffusion-synthetic acceleration scheme was developed for solving the 2-D S_n Equations in x-y geometry with bilinear-discontinuous finite element spatial discretization, using a bilinear-discontinuous diffusion differencing scheme for the diffusion acceleration equations. This method differs from previous methods in that it is unconditionally efficient for problems with isotropic or nearly isotropic scattering. We have used the same bilinear-discontinuous diffusion scheme, and associated solution technique, to accelerate the x-y geometry S_n equations with linear-bilinear nodal spatial differencing. We find that this leads to an unconditionally efficient solution method for problems with isotropic or nearly isotropic scattering. Computational results are given which demonstrate this property.

I. INTRODUCTION

It has been shown that nodal S_n transport methods[1,2] are very accurate, especially for radiation shielding problems. For example, the 3-D transport code, TORT[3], uses nodal spatial differencing and is used extensively on many shielding applications. However, the efficiency of nodal S_n methods in multidimensional geometries can be significantly increased by means of effective acceleration techniques. The purpose of this work is to introduce a new diffusion-synthetic accelerated algorithm for solving the x-y geometry S_n equations with the linear-bilinear (L-BL) nodal spatial differencing introduced by Azmy[4]. Recently Morel, Dendy and Wareing[5] introduced a new diffusion-synthetic acceleration (DSA) scheme for solving the 2-D S_n equations in x-y geometry with bilinear-discontinuous (BLD) finite-element spatial discretization. The method of Morel-Dendy-Wareing is the first DSA method for the BLD S_n equations that is unconditionally efficient for problems with isotropic or nearly isotropic scattering. We have used the same BLD diffusion acceleration equations, and associated multilevel solution technique of Morel-Dendy-Wareing to accelerate the x-y geometry S_n equations with L-BL nodal spatial differencing. We have Fourier analyzed and computationally tested the accelerated L-BL nodal S_n solution method to find that it is unconditionally efficient for problems with isotropic or nearly isotropic scattering, and is just as efficient as the associated BLD S_n solution method. To our knowledge, this is the first DSA method for the L-BL nodal S_n equations to be unconditionally efficient for such problems.

II. THE DSA ALGORITHM

The DSA algorithm is essentially the same as the Morel-Dendy-Wareing method for the BLD S_n equations. This method is considered as a multilevel synthesis of the Adams-Martin[6] and Wareing-Larsen-Adams[7] methods, neither of which are unconditionally efficient. In particular, at the first level, the BLD S_n

equations are accelerated with a slightly modified version of the Adams-Martin BLD diffusion equation. At the second level, the BLD diffusion iterations are accelerated with a bilinear-continuous (BLC) diffusion equation that is equivalent to the Warcing-Larsen-Adams asymptotic diffusion equation. Finally at the third level, the BLC diffusion iterations are accelerated with Dendy's blackbox multigrid algorithm[8]. In our DSA method we simply replace the BLD S_n equations, in the multilevel DSA method of Morel-Dendy-Wareing, with the L-BL nodal S_n equations. We have performed a homogeneous infinite-medium Fourier analysis on the first level of the iteration process to find a worst case spectral radius of about 0.5. Morel, Dendy and Warein[7] show a worst case spectral radius of about 0.5 for the second level. Dendy's multigrid algorithm has previously been shown to have a worst case spectral radius of about 0.1. Unconditional efficiency for the overall DSA scheme follows directly from the unconditional efficiency achieved on each level.

There are two operational modes of our method. Operating in the first mode, the BLD diffusion solution and the BLC diffusion solution are iterated to convergence. This mode is consistent with our Fourier analysis for the first and second acceleration levels, which assume that the acceleration equations on these levels are solved exactly. Operating in the second mode, a set number of iterations are performed. In particular, the BLD diffusion solution is accepted after three iterations, and the BLC diffusion solution is accepted after one iteration. The second mode of operation is more efficient than the first and recommended for production use.

III. COMPUTATIONAL RESULTS

In this section we give computational results that demonstrate the efficiency of our DSA method for the L-BL nodal S_n equations. We have performed three sets of calculations. All calculations were performed on a single-processor of a CRAY-YMP computer. The scalar flux in every calculation was subject to a pointwise relative convergence criterion of 10^{-4}.

The first set of calculations demonstrates the effectiveness of our DSA scheme in terms of error reduction per iteration. The geometry for the first set corresponds to a homogeneous rectangular region illustrated in Figure 1. This region has isotropic scattering, a scattering ratio of unity, and a constant isotropic distributed source. The rectangle has reflective boundaries on the bottom and left sides and vacuum boundaries on the top and right sides. There are 25 cells along the x-axis and 25 along the y-axis. In each calculations there is a single x-axis cell width and a single y-axis cell width, but these two widths are not necessarily the same. These widths vary between calculations. All of the calculations in this set were performed with an S_4 quadrature set.

The second set of calculations characterizes the overall efficiency of our scheme as a function of scattering ratio. The geometry for this set is identical to that of the first set. Both the x-axis and y-axis cell widths are fixed at 1.0 mean-free-paths, and an S_4 quadrature set is used in all of the calculations in this set. The scattering ratio is varied from 1.0 to 0.1 and each calculation is performed once without acceleration and one with acceleration.

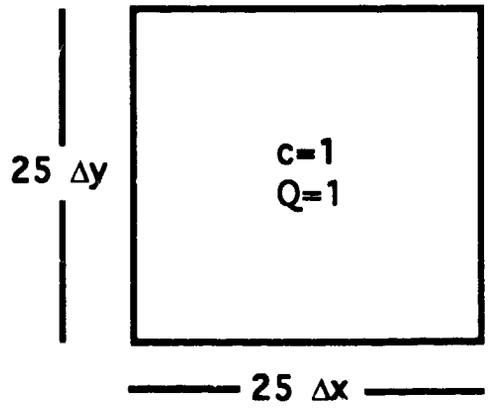


Figure 1. Geometry for Problem Set 1

The results for the first set of calculations are given in Table 1. The information appearing in this table consists of the x-axis cell width, the y-axis cell width, the total CPU time, the CPU time spent solving the DSA equations, the number of iterations required to

converge the S_n solution, and the estimated spectral radius. It can be seen from Table 1 that our method requires no more than 8 iterations for optically thick problems with a scattering ratio of one, including problems with high aspect-ratio spatial zoning. Optically thick problems with a scattering ratio of one cause difficulty for the Adams-Martin method, and problems with high aspect-ratio spatial zoning cause difficulty for the Wareing-Larsen-Adams method. About one-half of the total CPU time is spent solving the DSA equations. This fraction will rapidly approach zero as the quadrature order is increased.

Table 1. Results for Problem Set 1

Δx (mfp)	Δy (mfp)	Total CPU (s)	DSA CPU (s)	Iters.	Spect. Radius
0.1	0.1	1.12	0.54	6	0.22
0.1	1.0	1.33	0.62	7	0.35
0.1	5.0	0.95	0.45	5	0.21
0.1	10.0	0.95	0.45	5	0.16
0.1	100.	0.95	0.45	5	0.16
1.0	1.0	1.54	0.71	8	0.43
1.0	5.0	1.54	0.71	8	0.43
1.0	10.0	1.54	0.71	8	0.43
1.0	100.	1.54	0.71	8	0.43
5.0	5.0	1.09	0.54	6	0.26
5.0	10.0	1.09	0.54	6	0.26
5.0	100.	1.09	0.54	6	0.26
10.0	10.0	0.97	0.45	5	0.18
10.0	100.	0.97	0.45	5	0.18
100.	100.	0.97	0.45	5	0.18

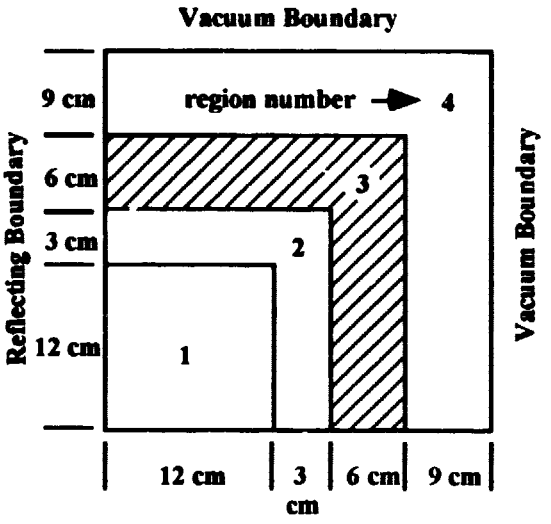
The results for the second set of calculations are given in Table 2. The information appearing in the table consists of the scattering ratio, the number of iterations required to converge the S_n solution, and the total CPU time required. Each problem is performed once with acceleration and once without acceleration. It can be seen from Table 2 that our DSA scheme is extremely efficient for problems with scattering ratios near unity. For instance, when the scattering ratio is unity, DSA reduces the total CPU time by a factor of roughly 253. The scheme remains efficient for all scattering ratios greater than roughly 0.2. The scattering ratio at which DSA becomes inefficient will rapidly approach zero as the quadrature order is increased.

Table 2. Results of Problem Set 2

Scattering Ratio	Iterations		Total CPU (s)	
	Unacc.	Acc.	Unacc.	Acc.
1.0	3752	8	389.8	1.54
0.9	89	7	9.25	1.35
0.8	43	6	4.45	1.16
0.7	27	6	2.80	1.16
0.6	20	5	2.08	0.97
0.5	15	5	1.56	0.97
0.4	12	4	1.25	0.77
0.3	9	4	0.93	0.77
0.2	7	4	0.73	0.77
0.1	5	3	0.52	0.56

The final problem set demonstrates the efficiency of our DSA method for heterogenious problems and shows the accuracy of the L-BL nodal method. We have performed a set of calculations on a one group, isotropic scattering, iron-water shield problem shown in Figure 2. Our DSA method was performed using the second mode of operation..

The results of the iron-water shield problem are given in Table 1. Here we give, for a variety of cell widths, the number of iterations to convergence, the total CPU time, the CPU time spent for the DSA, the average flux in the four regions, and the total leakage. We see that the number of accelerated iterations is always less than 9. For a scattering ratio of 0.994, one would expect approximately 1500 unaccelerated iterations to converge to a convergence criterion of 10^{-4}. Also, with acceleration one would expect the CPU time to be two orders of magnitude less than the unaccelerated CPU time for this problem. Finally, we see that the L-BL nodal method is very accurate. Here we see that the average flux in each region and total leakage have nearly converged to the continuous solution even for a maximum cell width of about five mean free paths.



Composition	Σ_t (cm^{-1})	c	Source ($cm^{-3}\ s^{-1}$)
1 (water)	3.33	0.994	1.0
2 (water)	3.33	0.994	0.0
3 (iron)	1.33	0.831	0.0
4 (water)	3.33	0.994	0.0

Figure 2. Geometry and Material Properites for the Iron-Water Shield Problem

IV. CONCLUSIONS

We have tested our method on a variety of problems in addition to those presented here. We have found that our method is highly efficient for problems with scattering ratios near unity and moderately anisotropic scattering. To our knowledge, no previous DSA scheme for the 2-D x-y geometry S_n equations with L-BL nodal spatial differencing has been unconditionally efficient. Also, the L-BL nodal method is very accurate even for thick mesh cells and highly suitable for shielding applications.

Table 2. Results of Iron-Water Shield Problem

Number of X-Y mesh cells	8x8	16x16	32x32	64x64
Maximum cell width (mfp)	20.0	10.0	5.0	2.5
Number of Iterations	9	9	9	8
Total CPU (s)	0.504	0.972	2.62	8.22
DSA CPU (s)	0.410	0.577	1.04	2.65
Average flux by regions ($cm^{-1}\ s^{-1}$)				
1	[a]1.28-3	1.88-3	1.91-3	1.91-3
2	40.09	40.73	40.90	40.93
3	10.20	9.52	9.35	9.31
4	0.250	0.234	0.230	0.229
Total Leakage (s^{-1})	2.02-4	5.45-4	6.00-4	6.04-4

[a] Read as 1.28×10^{-3}

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