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CRITICALITY OF THE WATER BOILER, NUMBER OF DELAYED NEUTRONS, AND DISPERSION OF THE NEUTRON EMISSION PER FISSION

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ABSTRACT

The calibration of the water boiler is made in such a way as to refer eventually to the multiplication of the primary-neutron source. The effective number γf of delayed neutrons in the water boiler is found to be 0.79 per cent. Here f is the fraction of delayed neutrons emitted on fission, and γ is the relative effectiveness of delayed and prompt neutrons in leading to further fission. This gives a value of $\overline{\nu^2} - \overline{\nu}$ equal to 4.4, where ν is the number of neutrons emitted per fission. The average time in the water boiler between fissions due to prompt neutrons is found to be 122 μs.

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CRITICALITY OF THE WATER BOILER, NUMBER OF DELAYED NEUTRONS, AND DISPERSION OF THE NEUTRON EMISSION PER FISSION

I. INTRODUCTION

It was shown in report LA-101 that in order to determine the quantity $\overline{\nu^2} - \overline{\nu}$, (i.e., the average of the square of the number of neutrons minus the average number of neutrons per fission,) a knowledge of the quantity γf, the effective number of neutrons delayed in the water boiler, was needed.

In the following both theory and experiments leading to a value of γf are described. The absolute calibration of the control rod on the water boiler is described and a value for the quantity τ_p, the average time between fissions due to prompt neutrons in the water boiler, is obtained. A slight correction to the value of Y (a measure of the fluctuation of the boiler) as obtained in LA-101 is reported and a value for the quantity $\overline{\nu^2} - \overline{\nu}$ is given. Some further discussion on the interpretation of the value of $\overline{\nu^2} - \overline{\nu}$ is presented.

The report throughout is divided into Sections A and B dealing with the determination of absolute criticality, and the determination of γf and τ_p respectively.

A table of notation is appended for the convenience of the reader.

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II. THEORY

Before discussing the theory underlying these experiments, it was felt that a clarification of the quantities K and K_p as used in connection with the water boiler was well advised.

Let us consider the water boiler in a subcritical state with reference to all neutrons, i.e., those promptly emitted plus those which are emitted delayed. Let us, for the time being, lump all delayed neutrons into one group with an average delay period. Let $\bar{\nu}$ be the average number of neutrons emitted per fission, that is, let it be the sum of those emitted promptly per fission, which we shall call $\bar{\nu}_p$[1], and those emitted delayed per fission, which we shall call $\bar{\nu}_d$. Further let us define the conditional probability P_p as the average probability which a neutron when born promptly has of eventually producing a fission[2]. Also let us define a corresponding conditional probability P_d for those which are born delayed. It is likely that P_d will be different from P_p since delays are born at a different energy. The ratio P_d/P_p shall be denoted by γ.

Now let us investigate what happens when we have a primary source of S fissions (or the equivalent of S primary fissions due to the insertion of a source of neutrons) and we ask ourselves what is the number of fissions

1) The reader should be careful to distinguish between the use of the symbol $\bar{\nu}_p$ as here defined and the symbol ν_p (i.e., unbarred) denoting the quantity $1 - K_p$, as was used in our previous report LA-101.

2) We shall neglect the effect of spatial distribution of fissions since if the boiler is running very near to critical a generation of fissions born will on the average have the same spatial distribution as the generation that preceeded it.

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that will occur in the boiler promptly. The number of fissions will first of all consist of the S fissions themselves; these in turn will produce $S\,\overline{\nu}_p$ prompt neutrons or $S\,\overline{\nu}_p P_p$ fissions; these in turn produce $S(\overline{\nu}_p P_p)^2$ fissions and so on. Hence the number of fissions produced by prompt neutrons is

$$S\left(1 + \overline{\nu}_p P_p + (\overline{\nu}_p P_p)^2 + (\overline{\nu}_p P_p)^3 \cdots\cdots\right) \tag{1}$$

Clearly the quantity in brackets is what is usually spoken of as the prompt multiplication. Since the boiler is subcritical, $\overline{\nu}_p P_p$ must be less than unity and we may write for the number of fissions produced by prompt neutrons the expression:

$$S/(1 - \overline{\nu}_p P_p). \tag{2}$$

It is usual to call the quantity $\overline{\nu}_p P_p$ by the notation K_p. Thus the prompt multiplication becomes $1/(1 - K_p)$.

Next let us investigate what the multiplication is when we consider all neutrons, i.e., we wait long enough so that delays can take effect - the usual situation in the operation of the water boiler. Let us consider what happens when S primary fissions occur in the boiler.

Of these S fissions $S\overline{\nu}_p$ prompt neutrons are born producing $S\overline{\nu}_p P_p$ fissions and also $S\overline{\nu}_d$ delayed neutrons producing $S\overline{\nu}_d P_d$ fissions. Thus the total number of fissions in the second generation[3] is $S(\overline{\nu}_p P_p + \overline{\nu}_d P_d)$. These in turn will produce $S(\overline{\nu}_p P_p + \overline{\nu}_d P_d)\overline{\nu}_p P_p$ fissions due to the prompt neutrons emitted and $S(\overline{\nu}_p P_p + \overline{\nu}_d P_d)\overline{\nu}_d P_d$ fissions due to the delayed neutrons emitted. Thus the third generation of fissions is $S(\overline{\nu}_p P_p + \overline{\nu}_d P_d)^2$ and so forth. The total number of fissions produced in the boiler is then

$$S\left(1 + (P_p\overline{\nu}_p + P_d\overline{\nu}_d) + (P_p\overline{\nu}_p + P_d\overline{\nu}_d)^2 + \cdots\cdots\right) \tag{3}$$

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3) The term generation as used in this case does not imply that all members of the (n + 1)st generation are later in time than those of the nth generation; it merely refers to the completion of a cycle of reproduction.

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Now since the boiler is subcritical, $(P_p\bar{\nu}_p + P_d\bar{\nu}_d) < 1$; therefore the number of fissions produced in the boiler is:

$$S\left(\frac{1}{1 - (P_p\bar{\nu}_p + P_d\bar{\nu}_d)}\right) = S\left(\frac{1}{1-K}\right) \tag{4}$$

Clearly then $1/\left[1 - (P_p\bar{\nu}_p + P_d\bar{\nu}_d)\right]$ is the multiplication of the boiler considering all neutrons. It is conventional to write

$$K = P_p\bar{\nu}_p + P_d\bar{\nu}_d \tag{5}$$

One might digress and note that we may if we wish alternately split up the delays into distinct groups each with a fraction $r_i\bar{\nu}_d$. and a probability $\gamma_i P_p$; then

$$P_d\bar{\nu}_d = P_p\bar{\nu}_d \sum_i r_i\gamma_i \tag{6}$$

Our arguments still hold using the method as outlined, so that we may if we wish write

$$K = P_p\bar{\nu}_p + P_p\bar{\nu}_d \sum_i r_i\gamma_i \tag{7}$$

Let us now return to equation (5) and find the relation between K and K_p. From equation (5) and since $K_p = P_p\bar{\nu}_p$

$$K_p = K\left[\bar{\nu}_p/(\bar{\nu}_p + \gamma\bar{\nu}_d)\right] \quad \text{use} \quad P_d = \gamma P_p \tag{8}$$

Let f denote the fraction delayed;

$$\text{then}\begin{cases}\bar{\nu}_d = \bar{\nu}f \\ \bar{\nu}_p = \bar{\nu}(1 - f)\end{cases} \tag{9}$$

$$\text{hence } \bar{\nu}_d = \bar{\nu}_p \; f/(1 - f) \tag{10}$$

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Remembering that $P_d = \gamma P_p$,

$$K = K_p \left[1 + f\gamma/(1 - f)\right] \qquad (11)$$

Expanding and neglecting second- and higher-order terms in f we obtain:

$$K_p = (1 - \gamma f) K \qquad (12)$$

i.e., the quantity $(1 - \gamma f)$ is a good approximation for the quantity $\bar{\nu}_p/(\bar{\nu}_p + \gamma \bar{\nu}_d)$. It should be noted that the quantity $(1 - K_p)$ at $K = 1$ is just γf.

It is evident from the above discussion that in order to determine anything about the quantity K_p we should first endeavour to obtain the quantity K for each setting of the control rod.

In order to obtain this calibration the following experiment was suggested and carried out.

A. The Boron-Bubble Experiment:

Consider a mock solution that has the same absorption for neutrons as our real 25 solution but does not give rise to fission. Imagine that one uniformly replaces 1/2 the volume of the boiler with this mock solution. Clearly only half as many fissionable nuclei are left and the probability P_p as well as P_d is cut in half. Since $K = P_p \bar{\nu}_p + P_d \bar{\nu}_d$, K also is halved.

Next consider the boiler running at critical for a certain control-rod setting $(CR)_A$. Then a volume ΔV of the mock solution is introduced uniformly for a like volume of 25 solution. The boiler is now subcritical and we raise our control rod to the position $(CR)_B$ to make the boiler critical again. By use of Fig. 1[4] we know that the raising of the control rod

4) This graph originally appeared in Report LA-134, where details of its construction may be found.

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corresponds to the addition of a certain number of grams of 25. Now let us examine what ΔK was produced by the addition of this mock solution before we raised the control rod. Originally $K = 1$; then we introduce ΔV mock solution leaving only $(V - \Delta V)/V$ as many fissionable nuclei as before so that

$$\frac{K_{\text{with mock solution}}}{K_{\text{original}} = 1} = \frac{V - \Delta V}{V} \qquad (13)$$

Let

$$\Delta K = 1 - K_{\text{with mock solution}}$$

then

$$\Delta K = \Delta V/V \qquad (14)$$

Hence the conversion from ΔM in gms of 25 to units of absolute ΔK has been established near critical; if we allow the assumption that in our experimental setup ΔK is proportional to ΔM.

Experimentally, of course, it is undesirable to dissolve the mock solution uniformly - instead it is introduced as an enclosed bubble of volume ΔV. Its size is so chosen that it does not perturb the prevailing neutron intensity greatly.

We then move this bubble radially and observe its effective ΔM in gms of 25 at each position through the control-rod-setting method (see Fig. 1). Then we take a volume average of ΔM and say that this average, $\overline{\Delta M}$, is the equivalent of dissolving the contents of the mock bubble completely:

$$\overline{\Delta M} = \frac{\int_0^r r^2 \Delta M dr}{\int_0^r r^2 dr} \qquad (15)$$

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Thus we have a certain $\overline{\Delta M}$ for a certain ΔK, where $\Delta K = V_{bubble}/V_{boiler}$ and have established the conversion. From now on, therefore, we may consider that we can measure K of any setup on the boiler. We first measure the control-rod settings, then convert these into ΔM in gms of 25 and finally convert ΔM into ΔK.

B. The Determination of γf and τ_p

As we saw in LA-101, and as derived on page 7 of this report, we essentially want to know the quantity $1 - K_p$ at $K = 1$ or γf. Perhaps the most obvious method that occurs to one is the following:

If the boiler is running at a known K and we suddenly introduce an absorber of a known ΔK into it, then the neutron-intensity curve should look somewhat as follows:

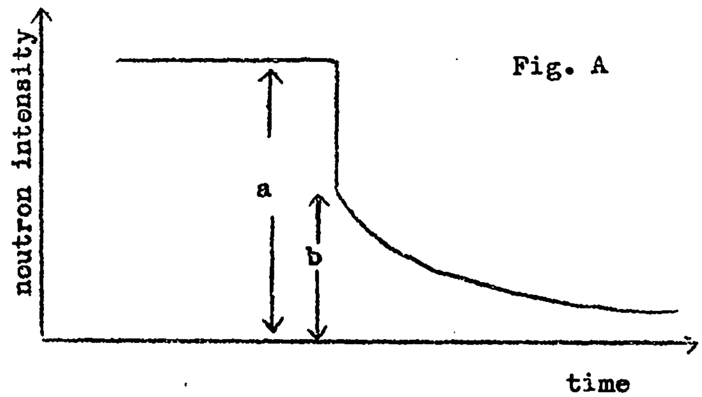

Fig. A

The ratio of a to b clearly depends in a simple manner on the effective fraction of neutrons delayed. Thus γf can be calculated if we know the old and new K as well as the ratio of counts a and b. The K we know absolutely by means of the boron-bubble experiment, but it turns out that experimentally it is extremely difficult to establish accurately the ratio of a to b, for in practice with, say, a BF_3 counter, it is impossible to count statistically

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significant numbers immediately after the kink in the curve. If we remember that a 0.7-second delay period has been found at Chicago, it is clear that an extrapolation to find the intersection b from points further on in time is a rather dangerous procedure. Following the general procedure outlined above, several experiments were attempted by using straight counting methods with existing BF_3 equipment, as well as some with oscillograph camera recording, but all tests were inconclusive because of the uncertainties of extrapolating back. They will therefore not be discussed further in this report.

Taking cognizance of the difficulties of extrapolating from a single experiment, it was decided to try to examine the fast drop in neutron intensity due to the presence of a piece of absorber by jerking it in and out with a fairly high repetition rate, such that the delays will not have time to decay and there will be merely a steady background of delays with a rapidly changing prompt-neutron intensity superimposed.

If we examine the neutron curve as sketched in Fig. A, we see that there should first be a sharp drop corresponding to the fast period and then a gradual one due to the emission of delayed neutrons. This fast drop will have a period of the order of 10^{-4} seconds as one can calculate from the concentration of the solution (neglecting tamper effects). If we run the repetition rate of the jerked absorber too high it is clear that the neutron level can not follow the absorber any more since there is a finite fast period which is sufficiently long so that it takes the boiler some time to take cognizance of the change in absorber. At a high repetition rate the boiler will thus lag behind.

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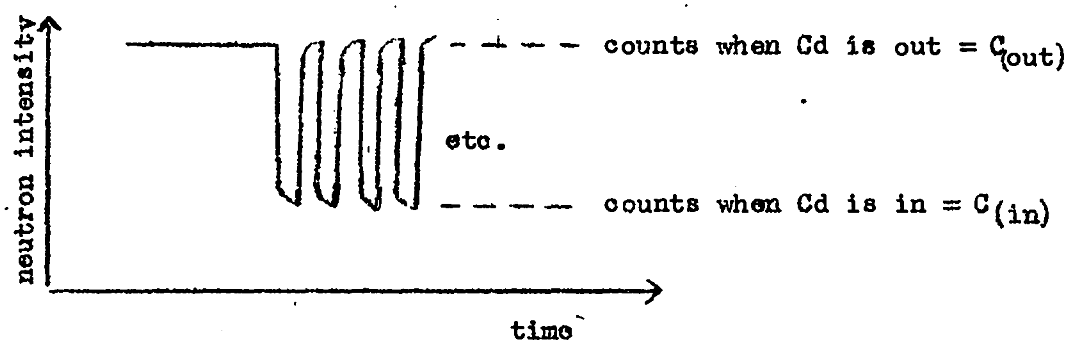
If, however, we are interested in a situation like this:

Fig.B



where the intensity hardly decays after the drop we must of necessity have not too low a repetition rate. If a repetition rate existed such that we could fulfill the condition of hitting a sufficiently high repetition rate so that the delayed neutrons form an average background and yet run slowly enough so that no lag occurs, then the following simple considerations will lead us to some expression of a function of time.

We again consider a source of S primary fissions per unit time. S is independent of time. Let us denote by F(t) the number of fissions per unit time actually occurring in the boiler at time t. Let there be N_d neutrons entering the boiler per unit time consisting of the delays being emitted from pregnant nuclei. In other words, our condition of a sufficiently high repetition rate means that the quantity N_d is approximately a constant with time. Thus the fission at time t consists of the S primary fissions multiplied up promptly plus the $N_d P_d$ fissions constantly being caused by delays also multiplied up promptly, i.e.,

$$F(t) = \frac{S}{1 - K_p(t)} + \frac{N_d P_d(t)}{1 - K_p(t)} \qquad (16)$$

Now

$$P_d(t) = \gamma P_p(t) = \gamma \; K_p(t)/\bar{\nu}_p$$

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so that

$$F(t) = \frac{S + N_d\,\gamma/\bar{\nu}_p}{1 - K_p(t)} - \frac{\gamma N_d}{\bar{\nu}_p} \tag{17}$$

Since $K_p(t) \approx 1$, it is clear that the first term of this expression is much the bigger, even if $S = 0$, so that

$$F(t) \cong \frac{S + N_d\gamma/\bar{\nu}_p}{1 - K_p(t)} \tag{18}$$

Now let us write expressions for $C(out)/C(in)$, i.e., the ratio of counts at the time when the absorber is all the way out to counts at the time the absorber is all the way in. Since the efficiency of the counter is constant we get:

$$\frac{C(out)}{C(in)} = \frac{1 - K_p(in)}{1 - K_p(out)} \tag{19}$$

$K_p = (1 - \gamma f) K$

Remembering equation (12) we get

$$\gamma f = 1 - \frac{\left(C(out)/C(in)\right) - 1}{\left(C(out)/C(in) - 1\right) K_{out} + (K_{out} - K_{in})} \tag{20}$$

This equation (20), however, is only true if a repetition rate with the required properties exists, which physically is unfortunately <u>not</u> the case although it is very closely so around 20 rpm. Let us now treat the question of varying multiplication in greater detail.

Before considering the problem of varying multiplication let us first consider a steady state in order to study more easily the quantities involved. Let us call $F(t)$ the fission rate at time t. This will depend on the fission rate at a previous time t'. First let us consider what the

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we shall say as before that P_p is its probability of causing a fission at any later time. If now we are interested to know what its chance is of producing a fission at time t we shall weigh this probability by the function $R(t - t')dt$; i.e., $P_p \cdot R(t - t')dt$ is the probability which a neutron born promptly at time t' has of producing a fission in the interval dt at time t, so that $\int_0^\infty R(t - t')dt$ is normalised to one.

Next let us consider a varying multiplication, i.e., the quantity $P_p \cdot R(t - t')$ is a function of t and not only of $(t - t')$. Considering P and R separately we first investigate P. This P will now be a function of some average time between t' and t. However, the fastest variation of our absorber was 384 rpm; i.e., one cycle lasted 6.7×10^{-2} seconds or longer. Since it will be shown that the average time between fissions is of the order of 10^{-4} seconds, it is clear that it is immaterial what average time within the range $t \rightarrow t'$ we chose since our absorber has moved a negligible amount within this time. For convenience sake, we shall choose the time t itself; i.e., we now consider $P_p(t)$.

Consider now the function $R(t - t')$ and its dependence on t. It is true that this function may vary slightly over a whole cycle; but since we are running always very close to critical this quantity will not vary by more than perhaps a percent or less. Since in all subsequent equations it does not appear as a difference term this quantity $R(t - t')$ will be assumed independent of t. It should be noted that such an argument could not be applied to P since this quantity does appear often as a difference term, i.e., we have occasion to subtract P from KP, so that a few percent

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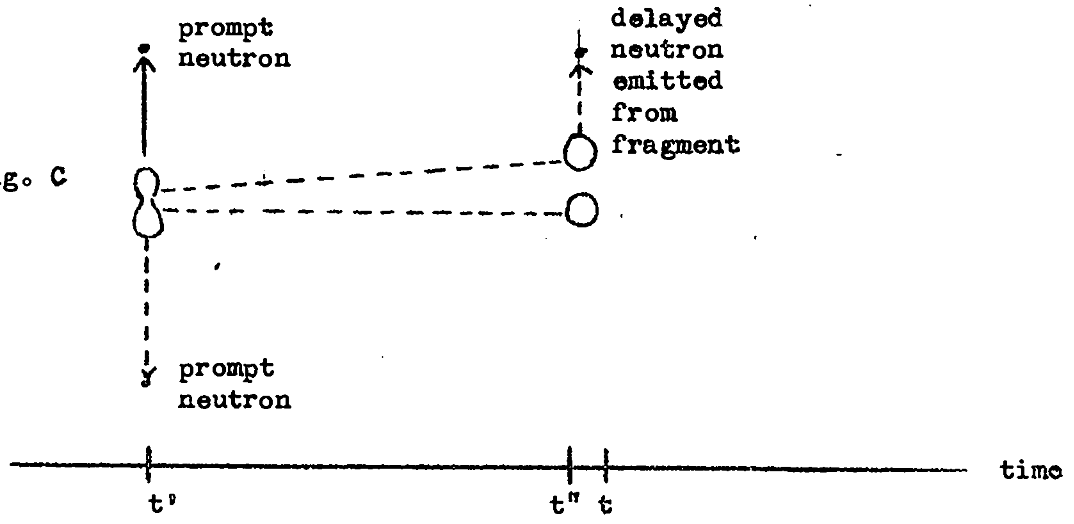
variation makes a large difference.

We have come now to a point where we may examine the life story of a delayed neutron. Let us first study the steady state again and for the sake of mathematical simplicity let us consider the following model:

Fig. C



Note: $t - t'' \ll t'' - t'$

We have our fission occurring at time t' giving rise to some prompt neutrons emitted at t'. In addition the excited fragments may give rise to the emission of a delayed neutron at time t''. Such a delayed neutron has a probability P_d of causing a fission at any later time. We shall again weigh it by some function $R'(t - t'')dt$ such that $P_d \cdot R'(t - t'')^{dt}$ is the probability which a delayed neutron when born at time t'' has of producing a fission at time t. Now we know that the probability of emission at time t'' due to a fission at time t' may be expressed as $e^{-(t''-t')/\tau_i} \; dt''/\tau_i$, where τ_i is the delay period. Four such periods τ_i and their relative fractions r_i of all those delayed are approximately known to us. The average delay period shall be τ_d. We may therefore write that the probability

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that a delayed neutron due to a fission at time t' will cause fission at time t is: $\left[e^{-(t''-t')/\tau_i}\ dt''/\tau_i\right] P_d \cdot R'(t - t'')dt$. If we examine the relative magnitude of these quantities we note that τ_i is on the average of the order of 10 seconds so that any extra effect due to the function R' whose decay is in the 10^{-4} seconds is entirely negligible, i.e., for this purpose we set

$$\left[e^{-(t''-t')/\tau_i}\quad dt''/\tau_i\right] P_d \cdot R'(t - t'')dt = \left[e^{-(t-t')/\tau_i}\ dt/\tau_i\right]P_d \qquad (21)$$

From now on we shall call $a_i = 1/\tau_i$.

For varying multiplication we go through the same reasoning concerning P_d as we did for the case of P_p so that we choose $P_d(t)$. We shall further assume that $P_d(t)$ is independent of the particular delay period that is, it is independent of i.

We are now ready to write down the equation governing the variation of F(t) in the case of our experiment and under the limitations as explained:

$$F(t)dt = Sdt + \int_{-\infty}^{t} F(t')dt'\bar{\nu}_p P_p(t)\ R(t - t')dt + \sum_i \int_{-\infty}^{t} F(t')dt'\bar{\nu}_{di} e^{-(t-t')a_i} a_i dt P_d(t) \qquad (22)$$

The term on the left of equation (22) is the number of fissions occurring at the time t in the interval dt. This will be made up of the following three terms: I) the steady primary fissions Sdt; II) those fissions at time t which are initiated by prompt neutrons that have been born at time t'; since t' can occur at any previous time to t we shall integrate from $-\infty$ to t; III) those fissions at time t which are initiated by delayed neutrons which in turn are due to fissions at time t'. Again in order to

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sum all such processes we integrate from $-\infty$ to t. If we divide through by dt, use equations (8) through (12), and call $\bar{\nu}_{di} = r_i \bar{\nu}_d$ we get:

$$F(t) = S + (1-\gamma f)K(t)\int_{-\infty}^{t} F(t')R(t-t')dt' + \gamma f K(t) \sum_i a_i r_i \int_{-\infty}^{t} F(t')e^{-(t-t')a_i}dt' \qquad (23)$$

Let us now examine the quantity $\int_{-\infty}^{t} F(t')\, R(t - t')dt$ more closely. Set $t - t' = x$; then the integral becomes

$$\int_{0}^{\infty} F(t - x)R(x)dx$$

expanding $F(t - x)$ in a Taylor series around t we get

$$\int_{-\infty}^{t} F(t')R(t-t')dt = F(t)\int_{0}^{\infty} R(x)dx - F'(t)\int_{0}^{\infty} xR(x)dx + \frac{F''(t)}{2}\int_{0}^{\infty} x^2R(x)dx---- \qquad (24)$$

By definition of the function R, the integral $\int_{0}^{\infty} R(x)dx$ is unity; it is also clear that the physical meaning of the next term, namely, $\int_{0}^{\infty} xR(x)dx$, is merely the average time from one fission to the next due to prompt neutrons. We shall denote it by the quantity τ_p. The next term, namely, $\left[F''(t)/2\right]\int_{0}^{\infty} x^2R(x)dx$ is of order $(\tau_p)^2$ and since from physical arguments we know that τ_p is of the order of 10^{-4} seconds we shall neglect this second term and all higher terms. Collecting now the two terms in F(t) we may write:

$$F(t)\left\{1 - (1-\gamma f)K(t)\right\} = S - (1-\gamma f)K(t)F'(t)\tau_p + K(t)\gamma f \sum_i a_i r_i \int_{-\infty}^{t} F(t')e^{-(t-t')a_i}dt' \qquad (25)$$

This equation (25) is essentially our fundamental equation. Let us now solve equation (25) considering various speeds of F(t), which we shall consider as having an angular speed ω, by two different methods, i.e., the reproduction method (case I and II) and the direct analysis.

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TREATMENT OF EQUATION (25): REPRODUCTION METHOD

Case I: If $\omega \overline{\tau_d} \gg 1$, i.e., we run at a high repetition rate (e.g., our experimental 884-rpm case) let us examine the various terms on the right of equation (25). Term I, i.e., S, will be a small constant term if we run near critical, with terms II and III outweighing it by far since the number of primary fissions is very small compared to the number of fissions occurring in the boiler. Term I denoting the contribution from promptly caused fissions cannot be simplified. In term III we note that if the function $F(t')$ has a high repetition rate, the quantity $e^{-(t-t')a}$ has hardly decayed during a cycle; thus, term III is essentially a constant which we may call S_d'. It expresses mathematically our physical situation of a steady background of delays. Combining $S_d' + S = S_d$ we make equation (25) read:

$$F(t)\left\{1 - (1 - \gamma f)K(t)\right\} = S_d - F'(t)\tau_p K(t)(1 - \gamma f) \tag{26}$$

Since $K(t)\ (1 - \gamma f) \approx 1$ we may write

$$F'(t)\tau_p + F(t)\left\{1 - (1 - \gamma f)K(t)\right\} = S_d \tag{27}$$

Since this is a linear differential equation of first order, it may be integrated and yields, considering our periodicity conditions:

$$F(t) = (S_d/\tau_p)\ e^{-\int_0^t \frac{1-(1-\gamma f)K(\xi)}{\tau_p} d\xi} \int_{-\infty}^{t} e^{+\int_0^t \frac{1-(1-\gamma f)K(\xi)}{\tau_p} d\xi} dt \tag{28}$$

Now let us mention that experimentally, as discussed on page 27, we really measure the quantity $F(t)/F(t + n)$ and are thus not interested in absolute values of $F(t)$. Our lack of knowledge of the quantity S_d is therefore no handicap. If we choose the correct γf and τ_p we should be able to reproduce

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...ed curve of $F(t)/F(t + n)$. This trial-and-error method of reproducing $F(t)/F(t + n)$ is thus one way of establishing γf and τ_p from the 884-rpm data.

Case II: $\omega\overline{\tau_d}$ is comparable to one and delay terms can no longer be set constant; also, $\omega\tau_p \ll 1$. Such cases are given experimentally by our 20- and 155-rpm cases. Since we are very near critical we set $S = 0$. Since $\omega\tau_p \ll 1$ we may as a first approximation set $F'(t)\tau_p = 0$ so that equation (25) becomes

$$F(t)\left\{1 - (1 - \gamma f)K(t)\right\} = K(t)\gamma f \sum_i a_i r_i \int_{-\infty}^{t} F(t') e^{-(t-t')a_i} dt' \tag{29}$$

Let $H(t) = K(t)\gamma f/\left[1 - (1-\gamma f)K(t)\right]$

then

$$F(t) = H(t) \sum_i a_i r_i \int_{-\infty}^{t} F(t') e^{-(t-t')a_i} dt' \tag{30}$$

so that if we choose an appropriate γf, i.e., the right $H(t)$, we can reiterate $F(t')$ which converges rapidly if we assume the values of a_i and r_i known from previous measurements at Chicago.

Note that if we simplify $H(t)$ by setting $K(t)$ in the numerator equal to 1, and in the denominator if we write $K(t)$ is equal to $1 + c_1\Delta M$ then we get

$$H(t) = \frac{\gamma f}{1 - (1 + c_1\Delta M)(1 - \gamma f)} \tag{31}$$

where c_1 is the conversion factor from ΔM in gms of 25 to ΔK supposedly known from the boron-bubble experiment. Since both c_1 and γf are small we may write

$$H(t) = \frac{\gamma f}{\gamma f - C_1\Delta M}$$

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thus if we express γf in terms of gms of 25 the function $H(t)$ becomes independent of c_1. The whole iteration process thus becomes independent of c_1.

If we wish to improve on equation (30) let us carry out a few iterations of $F_n(t)$ and write for $F(t)$ by using equation (29) setting $S = 0$ again,

$$F(t)\left\{1-(1-\gamma f)K(t)\right\} = -(1-\gamma f)K(t)F_n'(t)\tau_p + K(t)\gamma f \sum_i a_i r_i \int_{-\infty}^{t} F_{n-1}(t')e^{-(t-t')a_i}dt' \tag{32}$$

i.e.,

$$F(t) = \left\{-H(t)/\gamma f + H(t)\right\} F'_n(t)\tau_p + H(t)\sum_i a_i r_i \int_{-\infty}^{t} F_{n-1}(t')e^{-(t-t')a_i}dt' \tag{33}$$

so that

$$F(t) = \left\{- H(t)/\gamma f + H(t)\right\} F'_n(t)\tau_p + F_n(t) \tag{34}$$

Since γf is of the order of 10^{-2} and $H(t)$ of the order 1

$$F(t) = - (H(t)/\gamma f) F'_n(t)\tau_p + F_n(t) \tag{35}$$

Thus

$$F(t) = F_n\left(t - H(t)\tau_p/\gamma f\right) \tag{36}$$

as can be verified by expanding the right side of this equation (36) in a Taylor series and neglecting higher terms. In other words, we apply a small phase shift to the curve obtained by reiterating $F(t)$ without the correction. It is permissible to apply this correction at this late stage; applying it throughout would, of course, only yield an additional second-order correction.

This method may be used to reproduce the correct $F(t)/F(t + n)$

[redacted]

[redacted]

for a particular value of γf and τ_p, and this theoretical curve may be compared with the experimental curve. This, then, is a method of determining γf from the 20- and 155-rpm curve by trial and error of a particular value of γf. It is clear that since τ_p enters only as a correction, this method will not determine the value of τ_p since it is entirely insensitive to it.

TREATMENT OF EQUATION (25): DIRECT ANALYSIS OF EXPERIMENTAL CURVES

Assume for a moment that the experimental curves for F(t) at a known ω are given.

Recalling our fundamental equation (25) which reads:

$$F(t)\left\{1-(1-\gamma f)K(t)\right\} = S-(1-\gamma f)K(t)F'(t)\tau_p + K(t)\gamma f\sum_i a_i r_i\int_{-\infty}^{t} F(t')e^{-(t-t')a_i}dt' \quad (25)$$

Now set S = 0 since we are at critical, and rewrite as

$$F(t)\left\{1-K(t)\right\} = -F(t)\gamma f K(t) - (1-\gamma f)K(t)F'(t)\tau_p + \gamma f K(t)\sum_i a_i r_i\int_{-\infty}^{t} F(t')e^{-(t-t')a_i}dt' \quad (37)$$

Next express F(t) as a Fourier series, i.e.,

$$\text{let} \qquad F(t) = \sum_n a_n e^{+i\omega nt} \quad (38)$$

Hence

$$F(t)\left\{1-K(t)\right\} = -\gamma f K(t)\sum_n a_n e^{+i\omega nt} - (1-\gamma f)K(t)\tau_p i\omega n\sum_n a_n e^{+i\omega nt} + \sum_n a_n e^{+i\omega nt}\,\gamma f\sum_i \frac{a_i r_i}{a_i+i\omega n} \quad (39)$$

Thus if we call $D(\omega n) = \sum_i a_i r_i/(a_i + i\omega n) = \sum_i r_i/(1 + i\omega n\tau_i)$

[redacted]

[redacted]

and apply Fourier's theorem to equation (39) and change sign on both sides we get

$$\frac{\int_0^{\infty}\{K(t) - 1\}F(t)\, e^{-i\omega nt}\, dt}{\int_0^{\infty}K(t)\, F(t)\, e^{-i\omega nt}\, dt} = \gamma f + (1-\gamma f)\, i\omega n\tau_p - \gamma f \sum_n D\ (\omega n) \qquad (40)$$

Thus a complete knowledge of F(t) and D(ωn) would enable us to find γf and τ_p. An additional simplification is to set K(t) = 1 in the lower integral and to set $K(t) = 1 + c_1\Delta M(t)$ in the upper one where c_1 again is the conversion factor from ΔM in gm of 25 to ΔK, supposedly known from the boron-bubble experiment. Equation (36) then becomes:

$$\frac{c_1 \int_0^{\infty}\Delta M(t)\, F(t)\, e^{-i\omega nt} dt}{\int_0^{\infty}F(t)\, e^{-i\omega nt}\, dt} = \gamma f + (1-\gamma f)\, i\omega n\tau_p - \gamma f D(\omega n) \qquad (41)$$

Now expressing both $\Delta M(t)F(t)$ and $F(t)$ as:

$$\Delta M(t)F(t) = \sum_n \ (A_n \cos n\omega t + B_n \sin n\omega t) \qquad (42)$$

$$F(t) = \sum_n \ (M_n \cos n\omega t + L_n \sin n\omega t) \qquad (43)$$

we get

$$c_1 \left\{\frac{M_n - iL_n}{A_n - iB_n}\right\} = \gamma f + (1-\gamma f)\, i\omega n\tau_p - \gamma f D(\omega n) \qquad (44)$$

Separating into real and imaginary parts and setting $D(\omega n) = d_r(\omega n) + i\ d_s(\omega n)$ we get from the real part:

$$c_1 \left(\frac{M_nA_n + L_nB_n}{A_n^2 + B_n^2}\right) = \gamma f - \gamma f d_r(\omega n) \qquad (45)$$

[redacted] [redacted]

and from the imaginary part:

$$c_1\left(\frac{M_nB_n - L_nA_n}{A_n^2 + B_n^2}\right) = (1 - \gamma f)\omega n \tau_p - \gamma f d_s(\omega n) \tag{46}$$

This method has the added advantage that the final equations contain c_1 as a direct factor and thus any error that may be made in c_1 shows up as a direct factor in γf or τ_p; thus both can be expressed as gram equivalents of 25 independent of the value of c_1.

The only drawback to this method is that experimentally $F(t)/F(t + \pi)$ is known and not $F(t)$. If we call $F(t)/F(t + \pi) = X(t)$ it can however be verified that it is a good approximation to say that:

$$\frac{\int_0^\infty \Delta M(t) F(t)\ e^{-i\omega nt}\,dt}{\int_0^\infty F(t)\ e^{-i\omega nt}\,dt} \approx \frac{\int_0^\infty \Delta M(t)\sqrt{X(t)}\ e^{-i\omega nt}\,dt}{\int_0^\infty \sqrt{X(t)}\ e^{-i\omega nt}\,dt} \tag{47}$$

provided the modulation of $F(t)$ is shallow. By means of using $\sqrt{x(t)}$ in this method we can thus get very nearly correct values of γf and τ_p from different speeds of ω. This will give a good starting point for the assumption of a γf and τ_p in the more complete reproduction method as described in cases I and II.

[redacted] [redacted]

III. EXPERIMENTAL SETUP

[redacted]

A The Boron-Bubble Experiment

The mock solution consisting of an appropriate mixture of depleted 28 sulfate and H_3BO_3 in water was prepared by G. Friedlander. Our thanks are due to him for carrying out this chemical work and also for his kind assistance in the experimental work itself.

In Table I will be found a tabulation of the composition of the mock solution as well as the regular 25 solution and the cross sections used. This composition of 25 solution was the same as that used when the measurements on fluctuations as reported in LA-101 were made.

The bubble itself was made by joining the hemispherical ends of two centrifuge tubes by means of Duce cement and by filling them with normal or mock soup respectively by the use of a syringe after which the small hole was again sealed by the use of Duce. This bubble was attached to a long lucite rod so that the bubble could be submerged into the actual sphere and so its position from the center of the sphere was known accurately. The bubble could be moved along the entire vertical diameter of the sphere.

First a bubble of volume 14.73 cc containing 19.85 gms of normal 25 solution was moved along the radius and the control rod adjusted at each position such that the boiler was running critical again. Then this procedure was repeated with a bubble of volume 15.17 cc containing 19.80 gms of mock solution. The volume of the sphere proper was known to be 15 liters from previous measurements.

B. The γf and τ_p Experiment:

The concentration of 25 solution in the boiler when this experiment

[redacted]

[redacted] [redacted]

was carried out was identical with that at the time the boron-bubble experiment was carried out.

Fig. 2 shows the setup. The absorber, a piece of Cd, labelled A in Fig. 2 moved horizontally in and out of a slot in the tamper. It was moved back and forth by means of the connecting rod attached to wheel B. The wheel was driven by a motor which could drive the wheel at several predetermined speeds. The control rod was so adjusted that the boiler was critical when the absorber was moving back and forth.

Counts were taken by means of a BF_3 chamber (active volume, 520 cc and pressure 60.1 cm of Hg) which was located outside the tamper and was surrounded by paraffin. It was connected to the discriminator through a preamplifier. The discriminator was permanently hooked up to scaler No. 3, which thus counted total number of pulses from the BF_3 chamber.

In addition, a mechanism for recording counts from the same chamber over only a 12° interval of the cycle was available as follows: the light source L.S. was focused through a lens and mirror at a point just in front of each of the two phototubes so that a shutter fixed to the rim of the wheel B could cut off the light sharply for 12° of the cycle.

Let us consider what happens for instance to phototube 1 when the shutter passes. When the light is cut off in phototube 1 it actuates the gate circuit No. 1 which lets pulses from the discriminator pass through to scaler No. 1 as long as the light is off, i.e., for 12°. The equivalent happens 180° later with circuit No. 2.

The shutter could be moved along the rim of the wheel so that the circuits were actuated with a certain phase lag or lead with respect to the motion of the cadmium. Tubes 1 and 2, however, were still actuated 180° apart.

Scaler No. 4 served to count the number of revolutions the wheel B had made; i.e., how many cycles of motion the cadmium had completed.

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IV. EXPERIMENTAL RESULTS

A. Boron-Bubble Experiment

When the bubble containing normal 25 solution was moved along the radial position no change in criticality could be detected. The control-rod reading as noted in Table II was 7.130 inches. This showed that the lucite rod and the shell of the bubble presented negligible absorption. Thus the fact that the volume of the mock bubble was not exactly that of the normal 25 bubble did not affect our work.

The effect of the mock-solution bubble on the control position for criticality is tabulated in Table II and the conversion to Δ gm of 25 carried out. The results are plotted on Fig. 3. We note that the curve is very symmetrical. From this curve we compute by means of equation number (15)

$$\overline{\Delta M} = \frac{\int_0^r \Delta M\, r^2 dr}{\int_0^r r^2 dr} = 1.98 \text{ gms of } 25 \qquad (15)$$

It is to be considered that we assume that the volume of the boiler is just 15 liters. Actually, of course, there are an additional 400 cc or so of solution in the pipes leading to the sphere. How active this amount of solution is, is somewhat doubtful; measurements of criticalness versus solution level in the larger pipe indicate that less than a percent of this volume might effectively be added to the 15 liters.

We have assumed so far that our mock solution really matches perfectly, but let us examine this question in more detail. In order to see how well the absorption of the mock solution matched that of the normal 25 solution we turn now to an examination of the composition of the two solutions as given

[redacted] [redacted]

sections as indicated except for the boron cross section. Let us examine the effectiveness of boron as a substitute for 25, and let us assume that we have a spectrum that follows the $E^{-3/2}$ law and then joins onto a constant thermal one. That the $E^{-3/2}$ law is valid comes from the fact that we deal with UH_{500} and can almost consider the higher-energy spectrum to be one of pure hydrogen. To test the assumption of a constant spectrum at thermal energies we calculated what the effect of assuming a Maxwellian distribution with peaks at 200°K, 300°K, and 400°K weighed by the σv of B would do. This gave only a change of 4 per cent over the whole range from 200°K to 400°K since the σv of B has too shallow a slope at thermal energies to give any marked effect[5]. This gave confidence in the assumptions made which resulted in an answer stating that the effective B cross section as compared to that of 25 would have to be raised by 2.5 per cent because of the high-energy effects. In this way the term "effective cross section" for boron in Table I is explained.

Using these cross sections one obtains the following results:

	Scattering Cross Section per cc	Absorption Cross Section per cc
Mock Solution	3.034 cm^2	0.0893 cm^2
Normal 25 Solution	3.044 cm^2	0.0905 cm^2
Ratio:	0.997	0.987

See LA 183A

5) See LA-140, graph 5.

[redacted]

[redacted]

[redacted]

We see that the scattering cross section is almost the same, and any effects due to it may be neglected. As far as absorption goes, our bubble was less effective than it should have been; consequently it gave lower values of ΔM. Hence

$$\overline{\Delta M}_{\text{corrected}} = 1.98/0.997 = 2.00 \text{ gms of } 25$$

The ΔK by equation (14) is

$$\Delta K = \frac{\text{Volume of bubble}}{\text{Volume of boiler}} = \frac{15.17 \text{ cm}^3}{15 \times 10^3 \text{ cm}^3} = 1.01 \times 10^{-3}$$

with the same uncertainty in the volume of the boiler.

Thus we get that:

1 gm of 25 is equivalent to $1.01 \times 10^{-3}/2.00$ units of ΔK

1 gm of 25 = 5.05×10^{-4} ΔK

i.e., calling the conversion factor c_1:

$$c_1 = 5.05 \times 10^{-4} \ \Delta K/\text{gm}$$

B. The γf and τ_p Experiment

Fig. 4 shows the effect of the Cd at various static positions in gms of 25 equivalent. This enables us, using the conversion factor c_1, to establish K for each position of the Cd sheet.

The actual quantity measured during a run was the ratio of counts on scaler 2 to those on scaler 1 which we denote by X. The gate circuits were calibrated by a pulse generator before and after each run to check the ratio of gate 2 to gate 1. All data were corrected by the average of the correction factor before and after the run. Note that by taking the ratio X over only 180° of [redacted]

[redacted]

[redacted]

the cycle we could obtain the X's for the rest of the cycle by using the measured value and plotting the reciprocal 180° further on.

On Figs. 5, 6 and 7 results for speeds of 20, 155, and 884 rpm respectively are plotted. The probable error was calculated from the statistical Poisson error which was multiplied by the appropriate quantity γ[6] to allow for fluctuations. The curves are the best curves through the experimental points.

A simple evaluation according to formula (20), assuming that 20 rpm is the necessary ideal speed, yields $\gamma f = 0.0076$.

Next, all data were first evaluated according to the direct experimental harmonic-analysis method. A 12-point analysis according to the scheme of Runge[7] was performed throughout. Only the first harmonics were found to be of use since the experimental results were not accurate enough to yield significant results from the higher ones. The function $D(\omega n)$ was computed in two ways: 1) by using the data of Snell[8] and 2) by using the data of Nagle[9] concerning the period τ_i and relative fraction r_i of the delayed neutrons as follows:

Snell = S		Nagle = N	
τ_i	r_i	τ_i	r_i
0.7 seconds	0.167	1.6 seconds	0.414
6.5 "	0.620	6.5 "	0.310
34 "	0.188	32.5 "	0.240
83 "	0.025	79.3 "	0.036
	1.000		1.000

6) See LA-101 for explanation and evaluation of γ.
7) See Scarborough, Numerical Mathematical Analysis, page 396
8) Communication from Fermi
9) See CP-2317

[redacted]

[redacted]

This yields the following values:

Speed in rpm	20		155		884	
γf in gms	S	N	S	N	S	N
	15.52	14.86	15.22	15.15	13.27	13.27

Let us first examine the 20- and 155-rpm cases. In order to see how much of a correction this method needed, an arbitrary value (γf = 14.75 gms) was chosen and the reproduction-method Case II applied. This yielded the curves shown in Figs. 5 and 6 using the Snell and Nagle delay data, respectively. These curves were then subjected to the direct harmonic-analysis method and a γf extracted as follows:

Speed in rpm	20		155	
	S	N	S	N
γf extracted	14.76	14.44	14.11	14.12
correction factor	1.0	1.022	1.045	1.045

It is seen that the γf obtained by the harmonic analysis is too low or too high; the correction factor is then obtained in the 20-rpm Nagle case for instance by saying that the γf should be multiplied by 14.75/14.44 = 1.022 thus obtaining an appropriate correction factor. Any finer correction would have been out of place since for instance our resolution is only 12° of a cycle. This gives the following corrected γf values:

Speed in rpm	20		155	
	S	N	S	N
Corrected γf in gms	15.52	15.19	15.90	15.83

[redacted] [redacted]

For the 884-rpm case, application of the reproduction-method Case I showed that the curve was very insensitive to the value of γf. A γf of 13.27 gms as obtained from the direct analysis of the experimental curve as well as a γf of 14.75 were tried. Both lie well within experimental error, as shown on Fig. 7, (both curves are calculated with $\tau_p = 122$ microseconds and $c_1 = 5.05 \times 10^{-4}$ ΔK/gms) showing that at this high speed the curves are very insensitive to γf; we shall, therefore, not weigh in this result.

The τ_p on the other hand can be obtained from the 884-rpm curve with some accuracy since the phase lag is large giving

$$\tau_p = 122 \text{ microseconds}$$

The curves for both 20 and 155 rpm are extremely insensitive to the value of τ_p, as is physically clear from the small phase shift, and thus give no useful results for τ_p.

The γf as averaged from the 20- and 155-rpm data using both Snell and Nagle delayed-neutron data is

$$\gamma f = 15.6 \text{ gms equivalent.}$$

i.e., if $c_1 = 0.000505 \Delta K/\text{gm}$, $\gamma f = 0.0079$.

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[redacted]

V. DISCUSSION OF RESULTS

The value of c_1 should be fairly good and applicable to the case of the γf and τ_p experiment since the concentration of 25 in the boiler was identical. Thus no change in c_1 due to that effect is expected. The cross sections used are known to within a few percent and better.

In the γf and τ_p experiment it can be seen that larger errors must be expected. If, however, we take the value of $\gamma f = 0.79$ per cent at face value we may draw the following conclusions:

Assuming $\gamma = 1.3$[10] we get $f = 0.61$ per cent which is in good agreement with values previously measured at Chicago.

From LA-101 we see that

$$\overline{\nu^2} - \overline{\nu} = Y(\overline{\nu}_p)^2 \ (\gamma f)^2/\varepsilon$$

Y was found to be 4.17. A preliminary value of $\varepsilon = 3.69 \times 10^{-4}$ was given in LA-101. Since then a better value of the efficiency of the 25 chamber[11] has been obtained giving the BF_3-chamber efficiency as $\varepsilon = 3.51 \times 10^{-4}$. If we take $\overline{\nu}$ to be 2.47, then $\overline{\nu}_p = \overline{\nu}\,(1 - f) = 2.47 \cdot 0.9939 = 2.45$. Hence

$$\overline{\nu^2} - \overline{\nu} = 4.4$$

It would be a mistake to infer anything very definite regarding the actual number of neutrons emitted from each fragment. Surely the limit of error is large enough so as not to exclude the possibility $\overline{\nu^2} - \overline{\nu} = 4$; i.e., an even split between two and three giving a $\overline{\nu}$ of 2.5.

10) Obtained by using Chicago data on the age in water.
11) See LA-101, page 11, where $\varepsilon = 2.54 \times 10^{-7}$ counts/fission was given. The value should be 2.42×10^{-7} counts/fission.

[redacted]

It is also not fair to deduce anything regarding the question of immediate versus evaporated emission of neutrons on fission. It can be shown that if one assumes that neutrons evaporate from each fragment, i.e., 1.25 neutrons per fragment on the average, we get values of $\overline{\nu^2} - \overline{\nu}$ very close to those expected from direct splitting.

The value of $\overline{\nu^2} - \overline{\nu}$ should thus be used only as an entity in itself for such calculations as the probability of predetonation where it is needed.

The value of $\tau_p = 122$ microseconds is interesting as a differential quantity of the particular water boiler since it confirms theoretical calculations as to its order of magnitude.

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[redacted]

[redacted]

TABLE I.

A. Boron-Bubble Experiment

Composition of Mock Solution

and of Normal 25 Solution at

39° C

Element	No. of gm/cm^3 in normal 25 solution	No. of gm/cm^3 in mock solution	Absorption Cross section used barns	Scattering Cross section used barns
25	0.03878	0.0000933	653[12]	8.2
28	0.2256	0.2257	2.56	-
B	none	0.001591	721[13]	-
H	0.1172	0.1168	0.3	41
O	0.9307	0.9308	0.0015	4.2
S	0.0357	0.0304	0.47	1.5

12) See LA-158
13) Effective cross section including effect of high-energy neutrons, page 26.

[redacted]

[redacted] [redacted]

TABLE II

A. Boron-Bubble Experiment

Vertical Radial Position of Bubble from Center of Sphere (inches)	Control-Rod Position for Criticality with Bubble at Indicated Position[14] (inches)	Equivalent ΔM of 25[15] (grams)
-5.25	9.317	-1.56
-4.75	9.760	-1.93
-3.75	10.468	-2.53
-2.75	11.112	-3.10
-1.75	11.652	-3.58
- .75	11.965	-3.87
+ .25	12.000	-3.90
+1.25	11.765	-3.69
+2.25	11.309	-3.28
+3.25	10.683	-2.73
+4.25	10.074	-2.19
+5.25	9.270	-1.53

14) Control-rod position for criticality without bubble: 7.130 inches.
15) Computed from difference in control-rod setting by use of Fig. 1.

[redacted] [redacted]

TABLE OF NOTATION

[redacted]

A_n = cosine coefficients in the expansion of $\Delta M(t)\,F(t)$

B_n = sine coefficients in the expansion of $\Delta M(t)\,F(t)$

$C(\text{out})$ = counting rate at time when the aborber is all the way out

c_1 = $\Delta K/\text{gm}$ of 25

$$D(\omega n) = \sum_i \frac{r_i}{1 + i\omega n \tau_i}$$

d_r = real part of $D(\omega n)$

d_s = imaginary part of $D(\omega n)$

f = fraction of neutrons delayed

$F(t)$ = number of fissions per unit time occurring in the boiler at time t

$$H(t) = \frac{K(t)\gamma f}{1 - K(t)(1 - \gamma f)}$$

$$K = P_p \bar{\nu}_p + P_d \bar{\nu}_d$$

$$K_p = \bar{\nu}_p P_p$$

L_n = sine coefficients in the expansion of $F(t)$

M_n = cosine coefficients in the expansion of $F(t)$

$\overline{\Delta M}$ = average grams of 25 equivalent of boron bubble

N_d = number of neutrons entering steady-state boiler per unit time consisting of the delays being emitted from pregnant nuclei.

P_d = average probability which a neutron when born delayed has of eventually producing a fission

P_p = average probability which a neutron when born promptly has of eventually producing a fission

$R(t - t')$ is defined so that $P_p \cdot R(t - t')dt$ is the probability which a neutron born promptly at time t' has of producing a fission at time t.

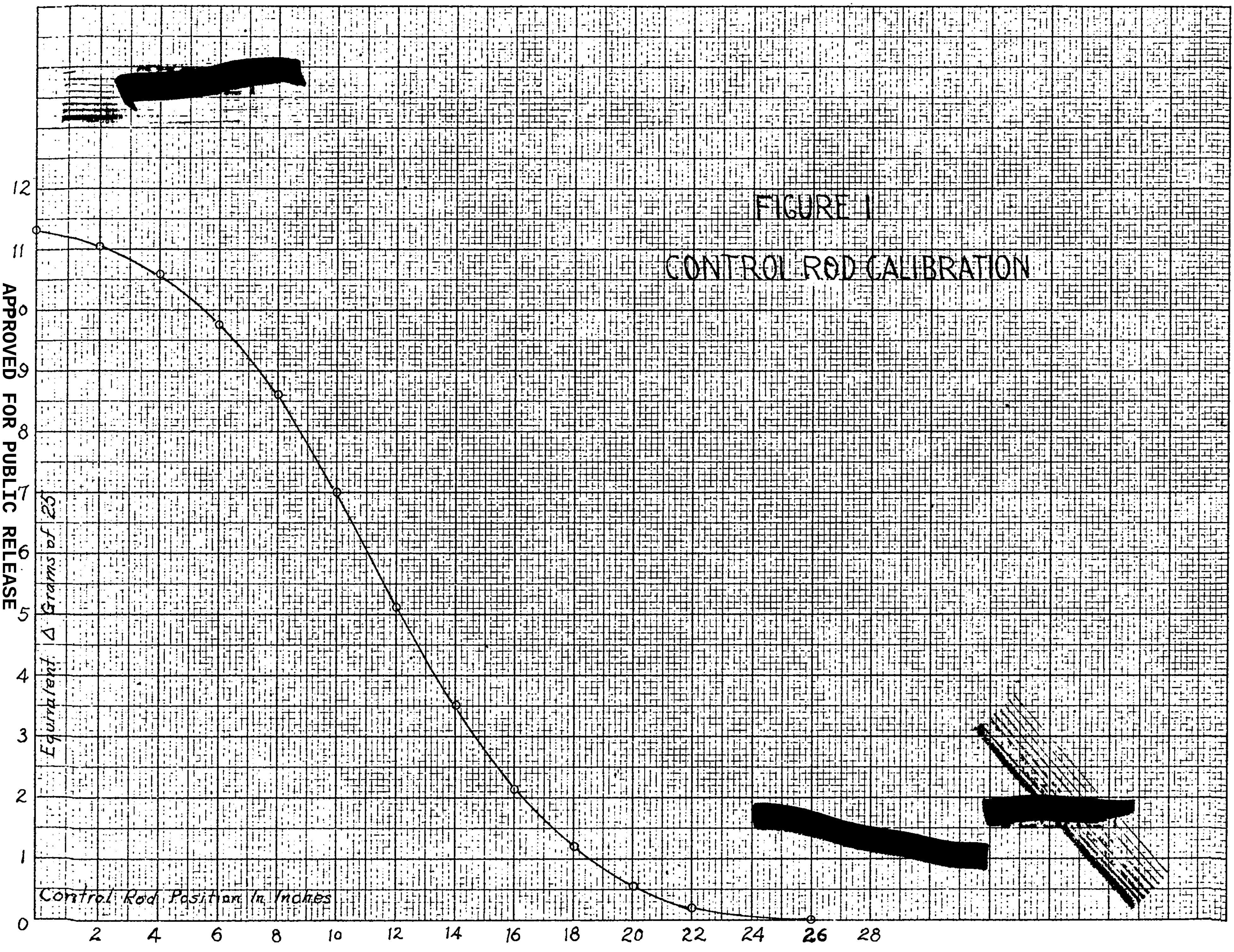
[redacted]

[redacted]

[redacted]

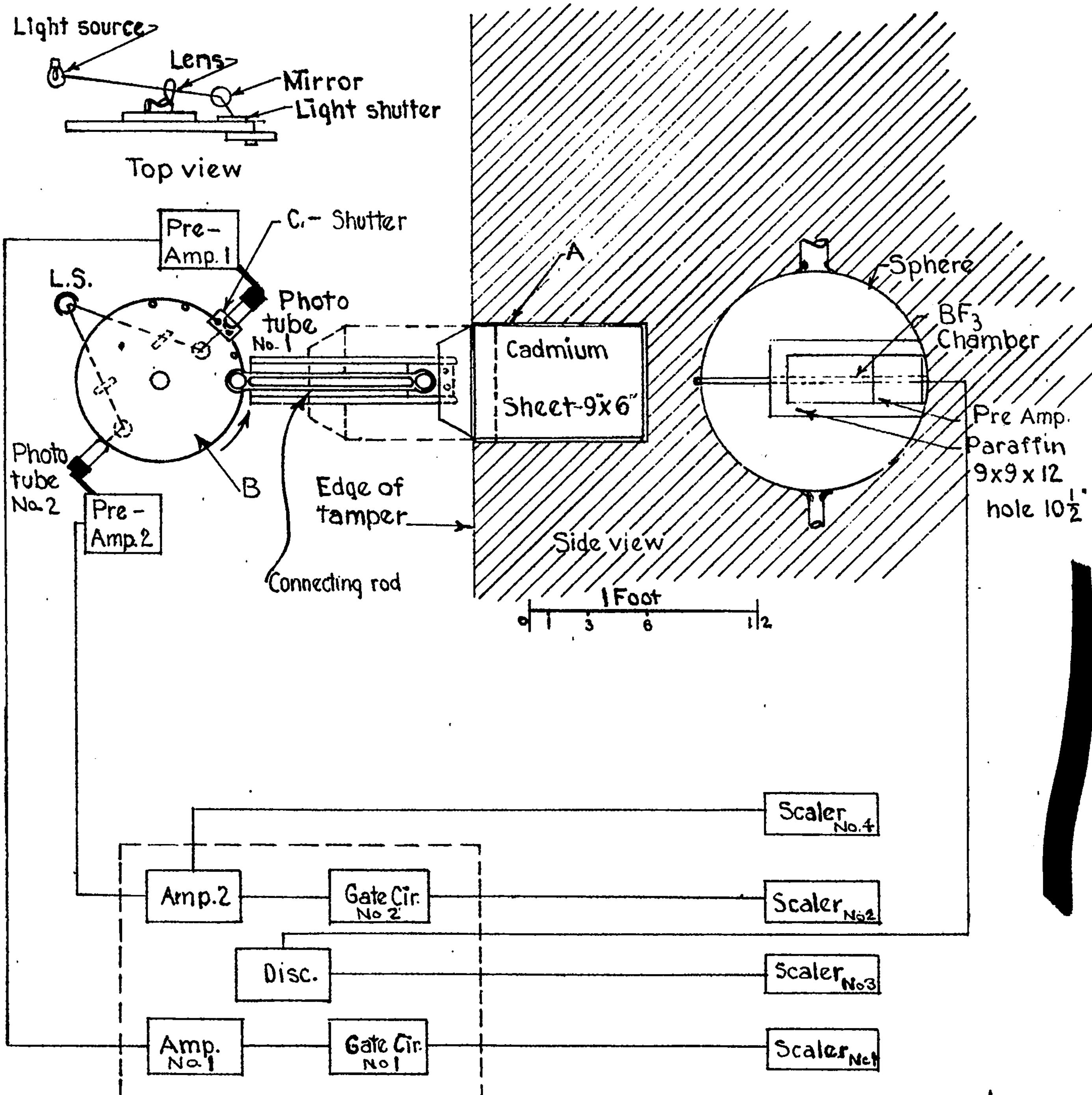
$R'(t - t'')$		is defined so that $P_d \cdot R'(t - t'')dt$ is the probability which a delayed neutron when born at time t'' has of producing a fission at time t.
r_i	=	relative fraction of total delays with period i.
S	=	number of primary fissions in the boiler
S_d	=	$S_d' + S$
S_d'	=	$K(t)\ \gamma f \sum_i \alpha_i r_i \int_{-\infty}^{t} F(t')e^{-(t-t')\alpha_i}dt'$ if $F(t')$ has a high repetition rate so that the quantity has hardly decayed during a cycle
V	=	volume of the boiler
ΔV	=	volume of the bubble
$X(t)$	=	$F(t)/F(t + \pi)$
Y	=	$\dfrac{\overline{c^2} - (\overline{c})^2}{\overline{c}} - 1$ where c is the number of counts per gate. See LA-101, page 9
α	=	$1/\tau_i$
γ	=	ratio P_d/P_p
ϵ	=	counts/fission; i.e., efficiency of chamber
$\overline{\nu}$	=	average number of neutrons emitted per fission
$\overline{\nu}_p$	=	average number of neutrons emitted promptly per fission
$\overline{\nu}_d$	=	average number of neutrons emitted delayed per fission
$\overline{\nu}_{di}$	=	$r_i \overline{\nu}_d$
τ_p	=	$\int_0^{\infty} x\, R(x)\, dx$; i.e., the average time from one fission, emitting prompt neutrons, to the next fission due to these prompt neutrons
τ_i	=	delay period of a class of delayed neu[redacted]
$\overline{\tau_d}$	=	average delay period
ω	=	angular repetition rate of the cadmium in radians per second

[redacted]


[redacted]
FIGURE I
CONTROL ROD CALIBRATION
Equivalent Δ Grams of 25
Control Rod Position In Inches
0
1
2
3
4
5
6
7
8
9
10
11
12
2
4
6
8
10
12
14
16
18
20
22
24
26
28
[redacted]
[redacted]

[redacted] FIGURE 2

B.) THE γf AND Tp EXPERIMENT



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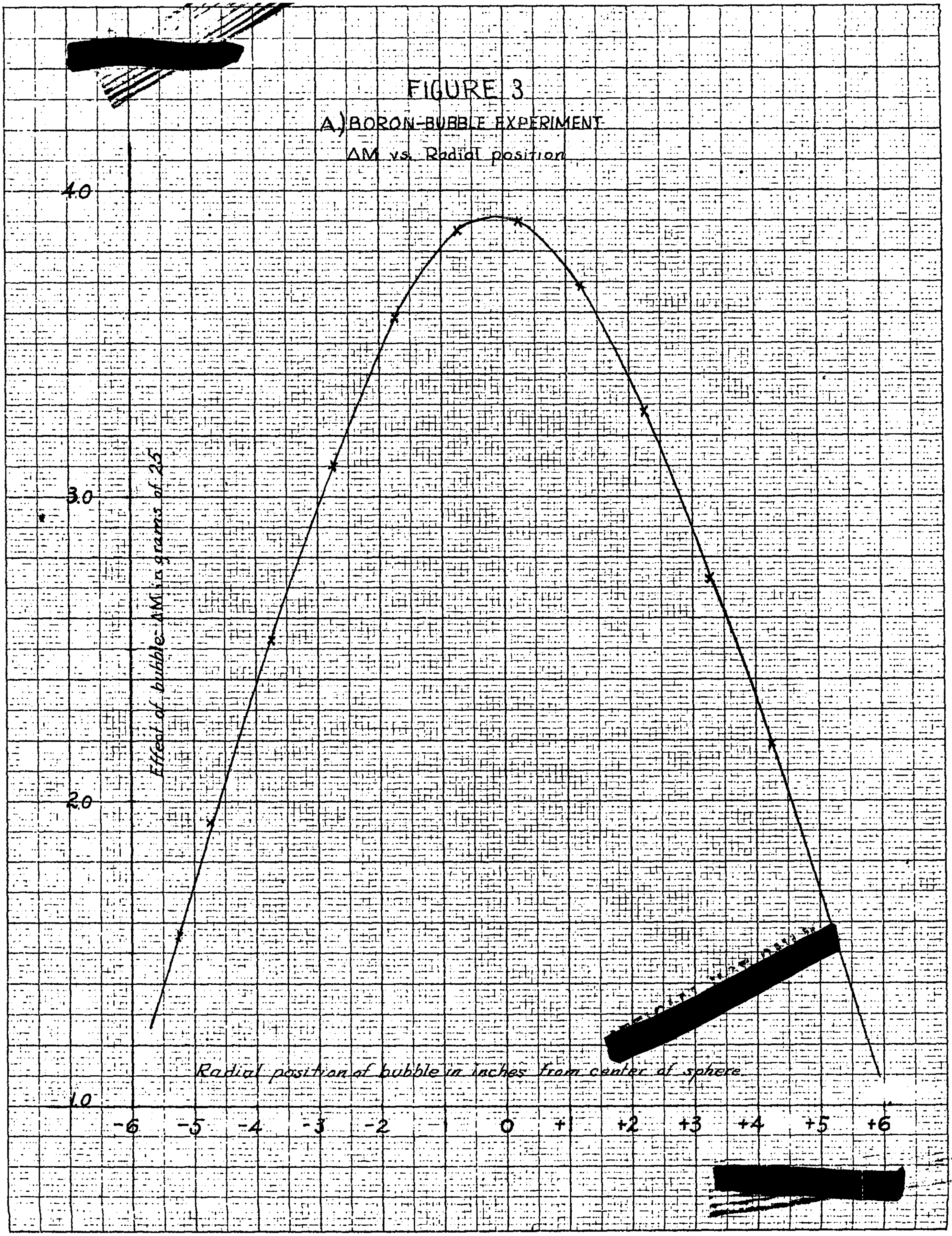
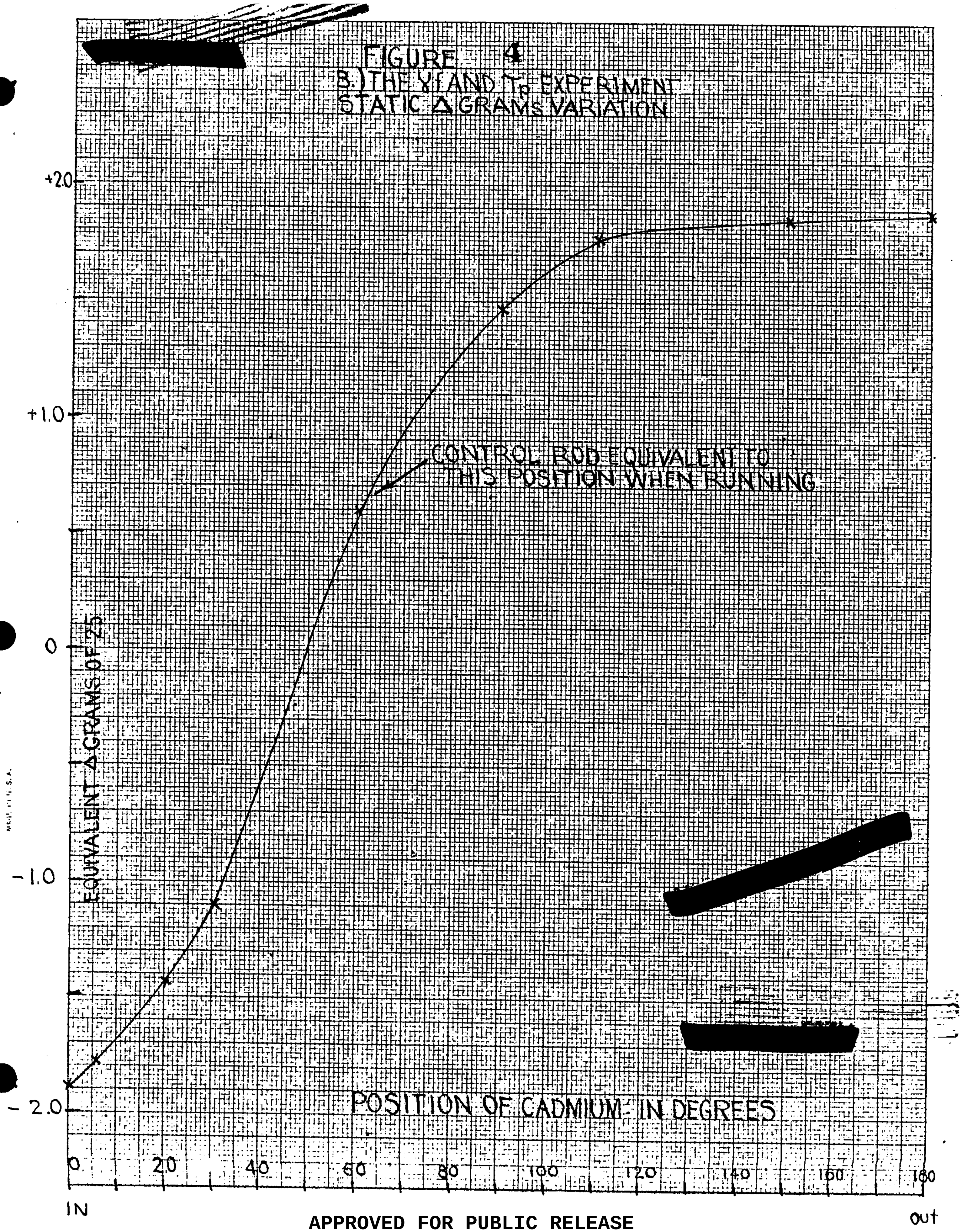
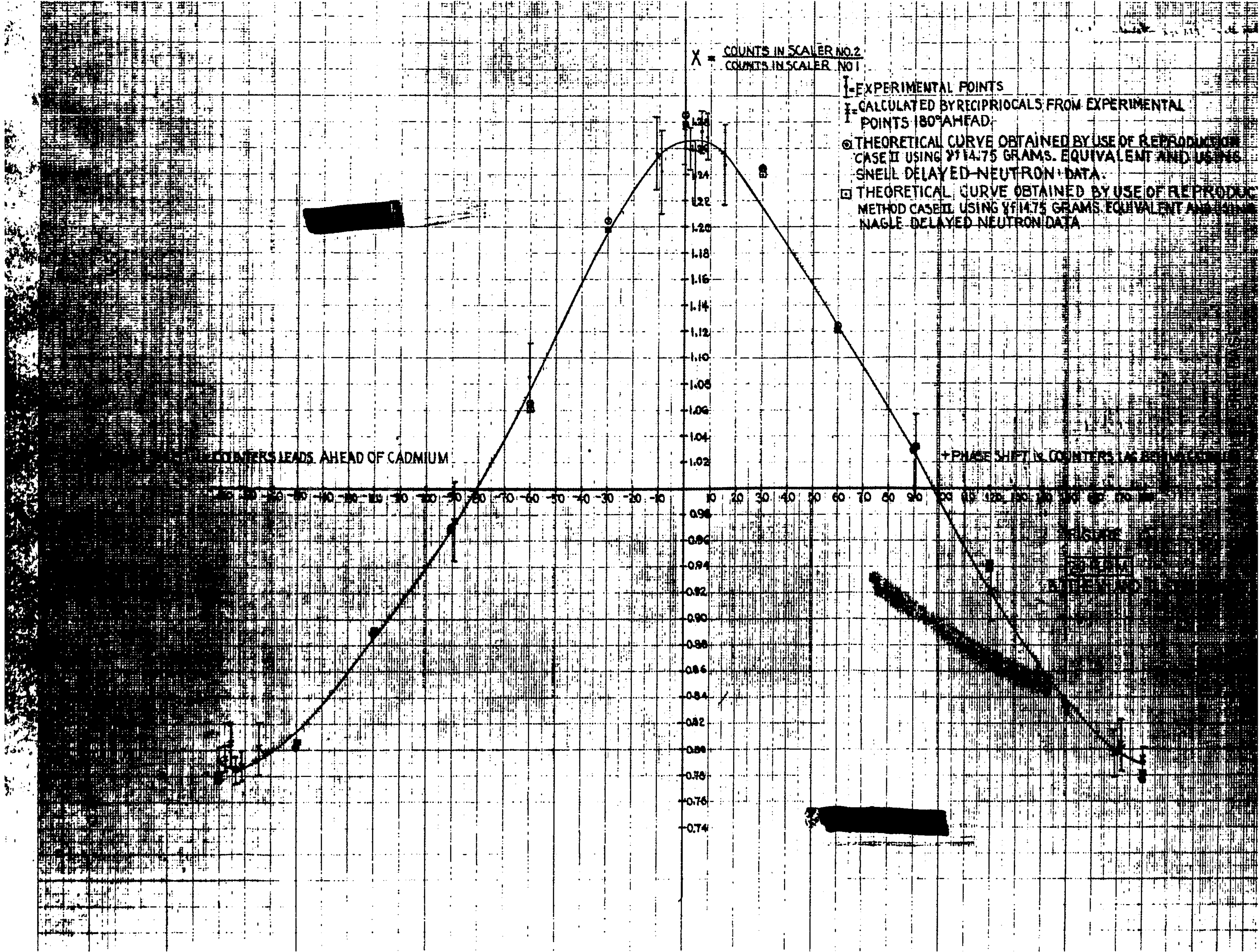

[redacted]
FIGURE 3
A) BORON-BUBBLE EXPERIMENT
ΔM vs. Radial position
4.0
3.0
2.0
1.0
Effect of bubble ΔM in grams of 25
Radial position of bubble in inches from center of sphere
-6
-5
-4
-3
-2
-1
0
+1
+2
+3
+4
+5
+6
[redacted]
[redacted]



APPROVED FOR PUBLIC RELEASE


X = COUNTS IN SCALER NO.2 / COUNTS IN SCALER NO 1
=EXPERIMENTAL POINTS
=CALCULATED BY RECIPRIOCALS FROM EXPERIMENTAL POINTS 180° AHEAD
THEORETICAL CURVE OBTAINED BY USE OF REPRODUCTION CASE II USING 14.75 GRAMS. EQUIVALENT AND USING SNELL DELAYED NEUTRON DATA.
THEORETICAL CURVE OBTAINED BY USE OF REPRODUC METHOD CASE II USING 14.75 GRAMS EQUIVALENT AND USING NAGLE DELAYED NEUTRON DATA
COUNTERS LEADS AHEAD OF CADMIUM
+PHASE SHIFT

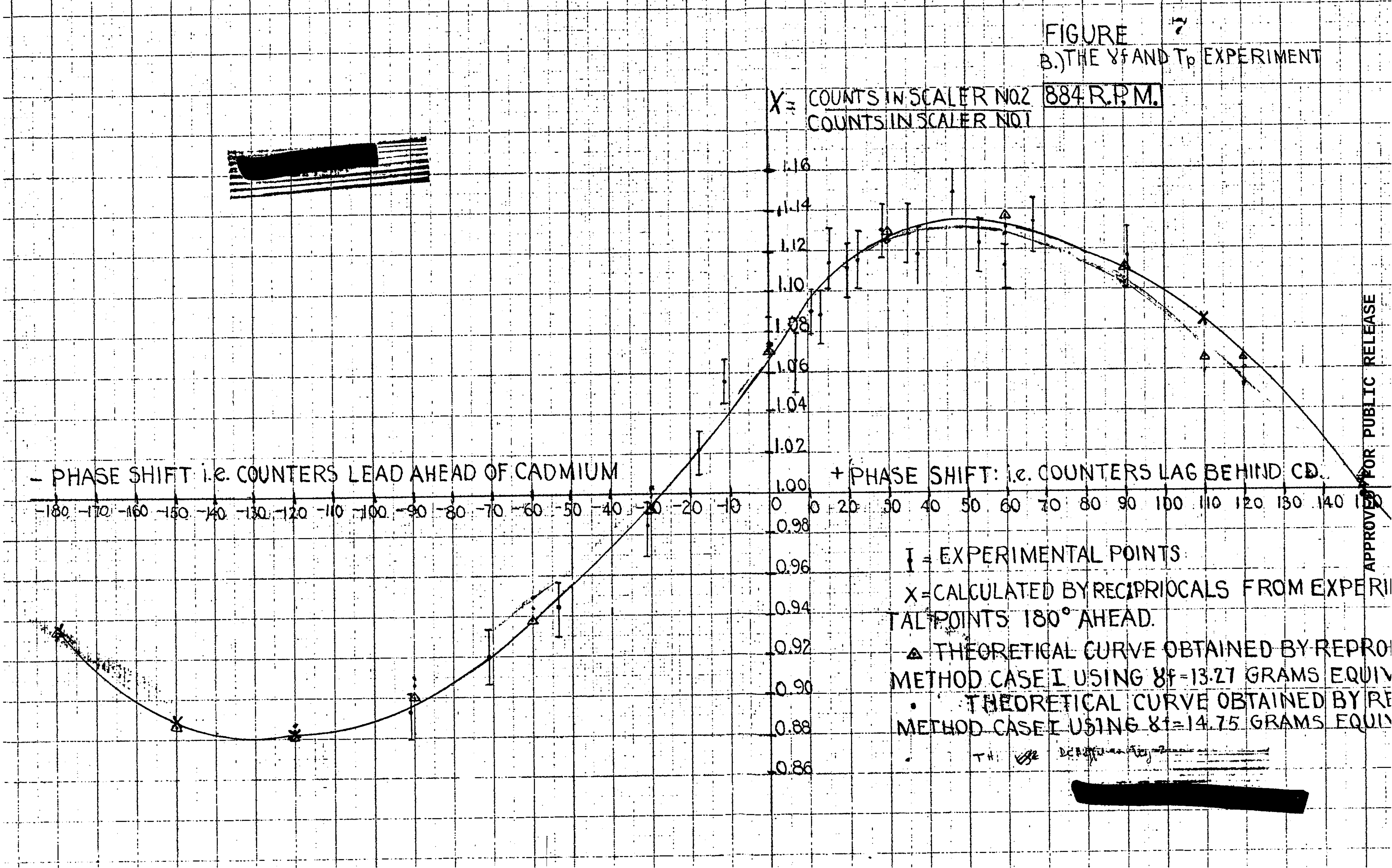


FIGURE 7
B.) THE γf AND Tp EXPERIMENT
X = COUNTS IN SCALER NO.2 / COUNTS IN SCALER NO.1
884 R.P.M.
[redacted]
1.16
1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.86
– PHASE SHIFT i.e. COUNTERS LEAD AHEAD OF CADMIUM
+ PHASE SHIFT: i.e. COUNTERS LAG BEHIND CD.
-180 -170 -160 -150 -140 -130 -120 -110 -100 -90 -80 -70 -60 -50 -40 -30 -20 -10 0
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150
I = EXPERIMENTAL POINTS
X = CALCULATED BY RECIPRIOCALS FROM EXPERI
TAL POINTS 180° AHEAD.
△ THEORETICAL CURVE OBTAINED BY REPRO
METHOD CASE I USING γf = 13.27 GRAMS EQUIV
• THEORETICAL CURVE OBTAINED BY RE
METHOD CASE I USING γf = 14.75 GRAMS EQUI
[redacted]

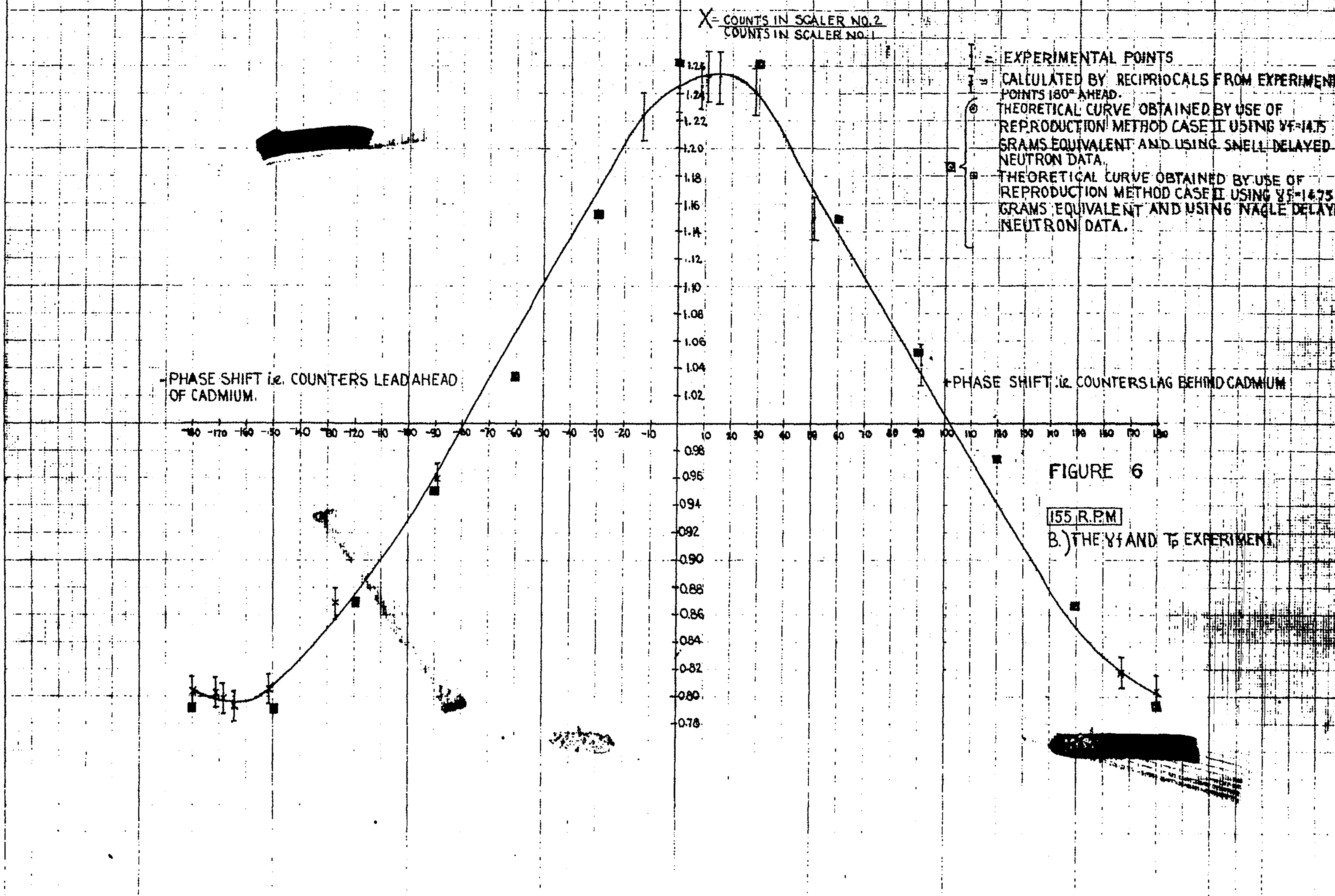


FIGURE 6

155 R.P.M

B.) THE ν_f AND T_p EXPERIMENT

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REC. FROM ef

DATE 12/13/44

REC. ___ NO. REC. 1

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