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METHOD FOR INEASURING FAST DECAY OF A NLAR_CEITICAL ASSEMBLY

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This paper contains a description of the apparatus and theories of the metnods successfully used for determining the fast decay periods of neer-oritical assembliea.

The methods desoribed are:
I. The modulation method.
II. The delayed coincidence or Rossi methodo
T..rificat:on whanged to rexclassified





METHODS FOR MEASURING FAST DECAY OF A NEAF-GRITICAL ASSEMBLY

The timendependence of a chain-reacting assembly is expressed by the factor $e^{a t}$. The quantity a onters into the expression for the efficience as the squarea It is therefore quite desirable to know a at the point winere the assembly goos of $S_{z}$ at say, around 3 orits. Then the measurements were made it was imposaible to measure a for more than one o ito what yes done was to measure a as a function of mass for near-oritical assemblies. This information could be used for extrapolating the a vse mass ourve and also as an integral oheok on the theory by which a is calculated.

The general features of the a ve. mass curve can be understood without any need of a precise theory. Fif. 1 shows

what may be expeoted. The insst part of the ourge will be characteristic of the tamper beause there is no aotive material preaent a will of course be negative. It will have a finite value winch is the reciprosal of the time of oapture of a neutron in the tamper materialo As active matorial is added, a will increase until it beoones equal to zaro. Thls corresponds to infinite multiplication and this mass is called the critical mass H If one were to oontinue to add material, a would continue to inorease but would eventually level off at a value which is charaoteristic only of the qotive material. This will ocour when nearly all the neutrons emitted by the fission prooess are captured in thęacisive mberghe locyithout going out into the tamper.


As soon as one goes into details, the situation is not so simple. We shall only mention one aspeot which bas an important bearing on our partiaular problom. We shall distinguish two kinds of $a^{\prime \prime} g_{g} a_{p}$ and $a_{t}{ }^{\circ} \alpha_{p}$ called aoprompt, is related to the fast multiplioation and is the a wioh is significant for the efficienoy caloulations. $a_{t}$ oalien a-total. is as80oiated with the total multiplicatione $a_{t}$ dirfers from $a_{p}$ in two respectse The small fraction of delayed neutrons enter into $a_{t}$, but not into $a_{p}$ " Also, booause of time absorption in the tamper, the tamper is less effece tive for the prompt multiplication. The problem disoussed here is hor to measure the slope of the $a_{p}$ ourve in the region wh on $a_{t}$ is slightly lesa than zero.

The results of these measurements on both 25 and 49 have been previously reported. ${ }^{1 .}$ Thia paper will describe the apparatus and the theory of the methods.

Two mathods have beon used successfully for determinhng ae They are the fastmodulation of the oyolptron and the delayed-coincidence or Rossi methodo The theory of these methods is given in the appendix. At this point it rd 11 suffioe to indicate briefly how these methods work to provide a background to botter understand the experiments and apparatuse

TRE MODULATION METHOD

The fundamental idoa behind this mothod is that the timomependenoe of the number of neutrons in ohain-reacting assembly oan be expressed as

$$
\begin{equation*}
N-A e^{\alpha \dot{\tau}} \tag{1}
\end{equation*}
$$



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where $A$ and $a$ are function of the geometry and physioal constants of the assembly. Theoretically this is not correot and the expression should be represented as a sum over a number of normal modes, ${ }^{2}$. i.e.o

$$
\begin{equation*}
N=\sum_{i=1}^{n} \quad A_{i} \oint^{\alpha_{i} t} \tag{2}
\end{equation*}
$$

However, it can be shom both thearetically and experimentally that if the assembly is close to critical and the method of excitation is reasonably chosen, then all the higher harmonios are negligible, and (i) is sufficiently aoourate for our purposes. 3 shall defer to the appendix the question of delayed neutrons and take (1) with $a_{p}$ as representing the time behavior of a nearmoritical assembly. If one introduces into such an assembly a source of noutrons $S(t)$, then the number of neutrons at time $t$ will be

$$
\begin{equation*}
N=e^{-a_{p} t} \int_{-\infty}^{t} e^{a_{p} t^{\prime}} s(t) d t^{\prime} \tag{3}
\end{equation*}
$$

If $S(t)$ is zero after some time $t_{1}$, then one has that $N$ is, aftar $t_{1}$, an exponentally deoaying number. The measurement of $a_{p}$ consist, then in reasuring tho decay period.

In practice this is acoomplished by modulating the oyolotron boam to give a pulse of neutrons approximately $0.5 \mu s$ soond long. When the beam is turned off, a gating oirouit is triggered wioh reoords the pulses from an ionization ohamber in ten definite tine intervals immediately after the noutron pulse, These data, after being corrected for backgrounds, will immediately give the period of decayo

THE ROSSI METHOD
The prinoiple behind the method is quite different from the modula-
 2. See: Effeot of tampris on the time gctio of suburitical assembiles LAvigh
produces ohoina. If an assambly is near aritioal and a neutron is introduced into it, it has a very good chance of producing a chain of other neum trons. Ltis, inflact, the production of these ohains which gives rise to the multiplication of the asserbly. If in such an assambly one looks for pairs of neutrons whioh are separated by a short tire interval, then it is highly probable that the two neutrons are related to each other by virtue of the fact that they are both members of the same chaine It is show in the appendix that the time distribution of suoh pairs is $e^{a}{ }^{t}$.

This distribution is measured in the following way. The assembly was provided with two ionization shambers. A natural source was used to introduce neutrons into the assembly. One of the chambers vias connected to the gating oirouit doseribod above, so that whenever a pulse was recorcied in it the oirouit was in a condition to measure tho time between this pulse and one recorded by the second ohambero

## RHE GATING CIRCUIT

The gating circuit is really the hoart of the equipmento It is shown diagramatioally in Fig. 2. Figas 3 to 7 give the oirouit constants for the component parts. Figs. 8 and 9 are photographs of the entire dem teoting equipment.

The principle of the oirouit is as follows: the pulze from diso criminator $B$ fires a blocking oscillator (b.0a) in Gato 1; the bao pulse oharges up a condenser in the cathode of $1 / 26$ SN7: tho b.o. pulse also goes down a delay Ine and fires b.0.2 in Gate 中2, the pulse from beo. 2 does tares things, (I) it shortis the cathode of the tubs in Gatemis thereby closiag Gatern, (2) it oharges up the gondenspri ing gitewn, and (3) it sends a pulse down the delay line tec continues dom the ohain of 10 gatese" fogimote generg s.n "uocession, but in

such a way that no two are open at the same time. The duration of the gate is determined by the length of the delay Iine. The pulses which are formed in the different gates are mixed in the ten coincidence oircuits with the pulse from discriminator A. To nake this a little clearer, suppose all the gates in Chain $B$ are 1 нseoond long. Suppose a pulse triggers discriminar tor $B$ and $31 / 2$ pseconds later a pulse triggers Discriminator A. In $31 / 2$ $\mu s e c o n d s$, the pulse in $B$ has reached Gate $4 s 0$ that when A is triggered a coincidence will be recorded in coincidence cirouit ${ }^{\prime} 4$ indiceting that the two pulses were between 3 and $4 \mu s e c o n d s$ aparto

The widths of the gates could be varied individually from 0 ol to 1 Hsecond, The resolving time of the component parts of the circuit was 1 Hseoond and it could be run at counting rates up to $50,000 / 500$ o

The circuit was also provided with a long gate (from 3 to 30 $\mu 890 a n d s)$. This could be triggered at any timo throughout the oycie and proved very convenient in determining backgrounds direotly.

## THE MOLULATION SQUIPMENT

The beam from the cyolotron was snouted and fooused on a small target about 10 feet outside of the water walls. The details of the magnetio rocusing of the beam have been reported el seriere, 3) and will not be given heres FHg . 10 shows the section of the snout between the target chame ber and the focusing chamber. This section of the tubs was provided at either end with adjustable slits. also provided nas a long set of deflecting platesc These plates were such that with 10 Kv across them the boam, winich was defined by the entrance slit, was completely defleoted across the exit siit. The nodulation was accomplisned by moglaitagilly:ifiespliting the demecting vol3o Ia Magnotic foousing.

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tage and permitting the beam to pass through the exit slit and strike the target. Fig. Il shows the external part of the snout with the target ime bedded in the WC tamper of the 25 assembly.

The modulation of the deflecting plate was acoomplished by shorting the deflecting roltage to ground through a pair of 304 Til's. The 304 's were driven by an 329 as a cathode follower which was in turn driven by a 828 blocking oscillator. The width of the pulse could be varied from 0.2 to 0.6 Hsec. by varying the capacity in the grid airouit of the blocking oscillatoro

## OTHER BLECTRONIC EQUIPMENT

CrouchøElmore amplifiers were used very suocessfully. A delay line clipper in the grid of the finst stage of the amplifiers permitted olipping the pulses to $0.3 \mu s e c o n d$.

The disoriminators, Figo 3, weretriggered uniovibrators. Their resolving time was lo5 $\mu$ seconds. However, it was possible to remove the seoond tube thus converting them into the more conventional fijp-flop disoriminator which had a resolvency time of $.6 \mu s e c o n d s$. The reason for using them as uni-vibrators was to make certain that the discriminators wore the slowest part of the entire circuit.

Tho overall timing for the modulation was usually in a state of flux. The final and most satisfactory arraggement is shown in Fig. 12o The quadruple pulser was a selfotriggered affair whose frequency could be varied from 10 to 10,000 oycles/sec. The four pulses could be arbitrarily phased with respect to each other. One of the pulses was used to trigger a sweop cirouit, a second trifgered the long getey A thisd triggered the azo modulator of the eyolotrone 4)


The fourth pulse triggered a delay time-controlled double pulser. The two pulses from this triggered the beam modulator and the $B$ discriminator respeotively, This latter puleer was necessary because any phase jitter at this point would have been quite serious. while it was not so important for the other parts of the setup.

None of this latter equipment was, of course, neoessary when one used the Rossi method. Instead, another ionization chamber and amplifier were used to trigger $B$ discriminatoro

## CALIBRATIAIS OF GATING CIRCUTT

It was necessary to know accurately not only the widths of the gate8 but also the time when each gate opened and closed relative to the time when Gate 1 opened. This calibration was made with a precision double pulser which was aocurate to o01 Hsoconds. This pulser mas calibrated arainst a orystal oscillator. Knowledge of the width of the gates was not sufficient since there was a littie overlap in the gates. However, the widths were determined independently by counting random pulses and were compared with the widhs as determined by the atendard pulsero The agreement ws quite satisfactory and served as a useful cheak on the overall behavior of the apparatus.

## CHAMBERS

The ohamber problem was greatly simplified since we are interested only in obtaining timing pulses indicating when an event is taking placeo This means that the chambers did not.hafe to heze.glateaus. All that was


The chambor used for the rise gircmones cit $25^{\circ}$ jis shown in Figo 13

and 14. The square design was for convenience in stanking in the assembly. As shown in Fig. 13. it consiated of two separate ohambers which could be used independently for the Rossi method or connected together and used as a single chamber for the modulation work. All the grounded parts of the charn ber were coated with $80 \% 25$ to a thickness of $1.3 \mathrm{mg} / \mathrm{cm}^{2}$. The pulses were taken off the high voltage. It was operated at 101 bso gauge pressure with the Argonocon mixtureo

The chambers used for the 49 work are shown in Figs. 15 and 16. Their design was also conditioned by the geometry of the assemblyo It oono sisted of stacks of eleotrodes which are alternately grounded and at high voltage, the spacing between the eleatrode was 30 mils . Both aets of electrodes were coated wi th $80 \% 25$ to a thiakness of $2 \mathrm{mg} / \mathrm{cm}^{2}$ 。 Each ohamber oontained approximately 1 gm of oxide. It was operated at a pressure 100 lbs of argon and $\mathrm{CO}_{2}$. The pulses were also taken off the high voltageo

## METIOD OF TAKING AND BANDIING DATA

The general method for taking data by ach method has already been indicated. There is little more that can be said. The oounts in the ten different channels were individually correoted for backgrounds. These backgrounds were figured on the basis of the frequency and the total counts in A diseriminator for the modulation method or on the total count in $A * B$ disoriminators and the time for the Rossi method. These oounts were then reduced to count per unit gate width and plotted on semi-1og paper against the time when aach paricular gate was opene These points always lay on a streight line within the statistics. The slope of these lines determined $a_{p}$.

These measurements of $a_{p}$ weseigiase fot gifiefitly different massea. The partioular methods for chancing ${ }^{\circ}$ ine
5. 10c. ait. footnote 1.

Will not be given here。 From this data it was possible to calculate the slope of the a vi. mass ourve at a point near oritioal.

## DISCUSSION

Tais report hes dealt principally with the methods for measuring $a_{p}$ rather than with the more important question of the measurement of dap/dM. While this does not belie the title, it does represent a siort. ooming, \%e have remarked rather glibly that we measure $a_{p}$ for different maseso The difficulty is not in measuring the $a_{p}$ or the different mass, but rather knowing just what the ohange in mass meanso In the laboratory one ohanges the mass by actually removing some small piece of the active materialo Tho problem, which is ossontially a theoretioal one of evaluating the offeco tiveness of suoh a removal, is one of extensive difficulty and well beyond the scope of this paper.

The picture is much brightor on the question of determing $a_{p}$ o The two methods described have been checked against each other and agree within the statistical errors. Each method has its advantages and dicadvantages to recommend it.

The advantage of the modulation method is its greater intensity. This is partioularly true when one backs off from critiaal by changing the mass. It can be shown that the counting rate varies linearly with total multiplication for the modulation method winie it varies as the culon of the multiplication for the Rossi mothodo

The Rossi method excels in its simplicity. Although the detecting equipment is the samo in both cases the simplicity of a natural source compared to a modulated eyolotron cannot be ovenamghasisech: : The Rossi method is also free of some spurious effects such ais.mainlotaiokscikyounds which cen cause

trouble with the modulated beam. Finally there is something inherently very nice about measuring a "dynamic" quantity by a "static" experimento

## Appendix $I$.

THEORY OF THE MODULATION EXPERIMENT
We shall start with the differential equation expressing the rate of change of the number of neutrons $N$ in an assembly and taking into account explicitly the delayed neutrons and an external source.

$$
\begin{equation*}
\frac{d N}{d t}=-\frac{n}{\tau_{0}}+\frac{K(1-f)}{\tau_{0}} N+\frac{K I \beta}{\tau_{0}} \int_{-\infty}^{t} N\left(t^{\prime}\right) a^{-\beta\left(t-t^{\prime}\right)} d t^{\prime}+s \tag{A1}
\end{equation*}
$$

Bere:
 of four parts:
(1) The neutrons lost by capture; (2) the prompt neutrons produced by captiare; (3) the delayed neutrons produced by oapture; and (4) the neutrons supplied by tho source. Upon multiplying the equation by ot differentiating, and rearranging torms, the following results:

$$
\begin{equation*}
\frac{d^{2} N}{d t^{2}}+\left[\beta+\frac{1-K(2-f)}{\tau_{0}}\right] \frac{d N}{d t}+\left[\frac{1-K}{\tau_{0}}\right] \beta N=\beta s+\dot{S} \tag{A2}
\end{equation*}
$$

If we use tho following abbyevinterns $:$

$$
\begin{aligned}
& \text { !: : :.. : - :.. : : : - : }
\end{aligned}
$$

$$
\begin{align*}
& a_{p}=\frac{1-K\left(l_{\odot}\right)}{\tau_{0}}, \quad a_{d}=\frac{1-K}{1-K(1-f)} \beta \tag{A3}
\end{align*}
$$

then, since $\beta(\sim 1)$ is negligible compared to $\alpha_{p}\left(\sim 10^{G}\right)$ and, in our experiment $a_{d} \sim 1 \ll a_{p}[$ see next page $]$, we may write:

$$
\begin{equation*}
\left[\frac{d}{d x}+a_{p}\right]\left[\frac{\dot{\alpha}}{d \dot{t}}+a_{d}\right] N=\beta s+\dot{s} \tag{1}
\end{equation*}
$$

The particular solution of this equation is:

$$
\begin{align*}
& \mathrm{N}=\frac{1}{a_{d} a_{p}} e^{\infty a_{p} t} \int_{-\infty}^{t} e^{a_{p} t^{\prime}}\left[\beta s\left(t^{\prime}\right)+\dot{s}\left(t^{\prime}\right)\right] d t^{\prime} \\
&+\frac{1}{a_{d} a_{p}} e^{-a_{d} t^{\prime}} \int_{-\infty}^{t} e^{a_{d} t^{\prime}}\left[\beta s\left(t^{\prime}\right)+s\left(t^{\prime}\right)\right] d t^{\prime} \tag{A5}
\end{align*}
$$

This can be simplified by removing the $\stackrel{\circ}{ }$ terms from under the integral sign by an integration by parts. In the range of $K$ which is interesting (K nearly 1) and since $f \sim$.007, $a_{d}$ is negligible compared to $a_{p}$ 。 Using this fact we can write as a final result:

$$
\begin{equation*}
N=e^{-a_{p} t} \int_{-\infty}^{t} e^{a_{p} t^{\prime}} \quad s\left(t^{\prime}\right) d t^{\prime} \operatorname{ri}^{-} \frac{\frac{x}{f} \beta}{a_{p} C_{0}} e^{-a_{d} t} \int_{-\infty}^{t} e^{a_{d} t^{\prime}} s\left(t^{\prime}\right) d t^{\prime} \tag{AG}
\end{equation*}
$$

The result consists of one term with a very short period (approximately $1 \mu s e c o n d$ ) with a coefficient of unity plus another term with a long period (approximately 1 seoond) with a very small ooefficient ( $\alpha_{p}^{2} \tau_{0} \sim 10^{4}$ ). This second term will integrate the source for a period of $1 / a_{d}$ o

Consider what happens if $S$ is a periodic square pulse of duration $\delta \sim 1 / a_{p}$ and unit intensity whose period if $T \ll 1 / a_{d}$. We shall assume that $S$ has bean on for infinitely many oycles in the past and will measure time from the beginning of some arbitrary pulse. If $T \gg 1 / a_{p}$ then the first integral will only contribute from tre entodiato gialio under considerationo The last integral will average the sfapse igntisisution for a time of $1 / a_{d}$, the result is, then


What we have，then，is a rapidly decaying exponential superimposed on a continuous background．This background is ce．used by the delsyed neutronso It is desirable that the background be sufficiently small for a good measurement of $a_{p}$ ．We might take as a criterion that the background be oneatenth of the counting rate of the exponential pulse after three periods．

This should permit a good measurement．Assume $\beta=01 \mathrm{sec}{ }^{-1} \mathrm{t}_{0}=10^{08} \mathrm{sec}$ 。 and $a_{d}=.01880^{-1}$ ．This gives $K=09992 ; a_{p}=8.10^{5}$ ；and $\delta=1.2 \cdot 10^{-6}$ sec。 Our oriterion is

$$
\frac{\left(\pi \pm / \delta / a_{p} \tau_{0}\right)\left(\delta / a_{d} T\right)}{\left(e^{C_{p}^{\delta}}-1\right) e^{-3}}=0.1
$$

Putting in the numbers and solving for $1 / T$ given a frequency of $\sim 300$ cyoles $/ \mathrm{sec}$ ．
One might estimate the counting rates approximately as follows：if one assumes the deuteron bean to be 14 amp，which yields $10^{10}$ noutrons／seo， then one calaulates from the a bove expression assuming a repetition rate of 300 cyoles／sec that there should be $1.5010^{5}$ neutrons present in the assembly after throe periods．Since the oritical mass of the assembly is $\sim 3 \cdot 10^{4}$ gms and the chamber contains one cram then there should be $1.5 \cdot 10^{5} / 3010^{4}=5$ neutrons per second making fissions in the ohambero

Aotually these condibions are quite idealized．There $\mathrm{is}_{0}$ besides the background disoussed above，the general background from the tank wall，etco This necessitates lowering the frequency somewhat．Also，since the chamber was placed in the tamer，the estimate of the effioienoy is certainly optimistic． However，the estimated counting rates are so good that one can afford to take，a bit of a licking on these other thires．


APPLINDIX II

## The Rosbi Nethod

The Rossi method represents a different approach to this probleme It is essentially statistical in nature Let $P(t)$ be the probability thato If a nautron exists at time $t=0$ then a neutron will exist at time $t=t$. We may calculate $P(t)$ directlyo Suppose the initial $t$ is subdivided into a large number of small Intervals dte Mie asko what is the probable number of neutrons after a time dt? Tins probable change in the number of neutrons is given by the produot of $\mathrm{dt} / \tau_{0}$, the probability of a captive in the interval $d t$, and $\{-1+K(1-1)\}$ (ve are considering only prompt neutrons), the change in the number of noutrons if a capture takes placeo Therefore, the probable number after a time dt will be the number existing at the beginning of the interval plus the probable change in the number during the intervalo

$$
\text { Probable number after } d t=2+\left(-1+K(I-f) \quad d t / \tau_{0}=1-a_{p} d *\right.
$$

The probable number after a time $t$ will be the product of the probable numbers of all the subintervalso. There are $t / d t$ of these intervalso Therefore 0

$$
\begin{align*}
P(t) & =\left(1-a_{p} d t\right)^{t / d t} \\
& =\left[\left(1-a_{p} d t\right)^{-1 / a_{p} d t}\right]^{-a_{p} t} \tag{A8}
\end{align*}
$$

In the limit of dt approashing zero, this beoomes simply

$$
\begin{equation*}
p(t)=e^{-a_{p} t} \tag{A9}
\end{equation*}
$$

From this wo immediately have, for the probability of a noutron produciag another neutron at a time tater and being captured in the time interval dt,

$$
\begin{equation*}
e^{-a_{p}{ }^{t}=d t / \tau_{0}} \quad \because: \because \because: \because \because:! \tag{A10}
\end{equation*}
$$

since the probability that a noutron will be captured is just dt/ oo


As an imediate application of (AlO) we may calculate the total number of neutrons that will be captured because a neutron exists at time $t=00$ ihnis requires summing (AlO) over all times.

$$
\begin{equation*}
\int_{0}^{-\infty} a_{p} t d t / \tau_{0}=1 / a_{p} \tau_{0} \tag{All}
\end{equation*}
$$

The reason for emphasizing the capture of neutrons is beaause this is the same as detecting then. The deteoted neutrons are, among others, noutrons that are removed from the system.

Now, let $D$ be the reciprocal period for the assembly including the delays. Then, in analogy to (Al0), we have that the probability of producing a neutron at timg tin the interval dt ia $\beta e^{-D t}$ dt where $\beta$ is the reoiprocal 1 ifetine for the delayed neutrons. He get then that the total probability of producing delayed neutrons is $\beta / D$ in analogy to (A11). Now since it is impossible to distinguish a delayed neutron from a source neutrono these delayod neutrons will be nultiplied promptly according to (Ail). Therco fore the total multiplication will be

$$
\begin{equation*}
(\beta / D) \quad\left(1 / a_{p} \tau_{0}\right) \tag{A12}
\end{equation*}
$$

If we now take equation (A6) of Appendix I and interpolate it over all time assuming the source $s$ to be unity, we find that, after dividing by $\tau_{0}$ the number of neutrons present is. $1 /(1-K)$. If we equate this to the oxpression (A12) and solve for $D$, we find

$$
\begin{equation*}
D=\frac{\beta(1-K)}{a_{p} \tau_{0}}=a_{d} \tag{A13}
\end{equation*}
$$

This establishes the equivalence between the significant quantities of the Modulation and the Rossi Methodso


We shall now investigate the probabilifises of counting pairs of

neutrons separatad by a definite cime interval T. We shall try tó, make this ides.

somewhat clearer. Looking at the accompanying figure, suppose at time $t=0$ a neutron exists. This neutron gives rise to a chain. There is a certain probability that, because of the existence of the neutron at $t=0$, a neutron will be captured in the time interval o at $t=t^{\prime \prime}$ Similarly, there is a ohanca for one to be captured during the interval $\delta_{2}$ at time $T=t^{\prime}+$ o we wish to calculate the probability for counting the two neutrons separated by the interval $T$ for all values of t'o We can simplify the caloulation by caloulating the probabinity that a aeutron is captured at time $t$ ' and in the inters val $\delta_{1}+\delta_{2}$ at time tit To The result is

$$
\begin{equation*}
e^{\infty a_{p} t^{\prime}} \frac{d t^{t}}{\tau_{0}} \quad e^{\infty a_{p}\left(t^{\prime}+T\right)} \frac{1+\frac{1}{2},{ }^{2}+2}{2} \tag{A14}
\end{equation*}
$$

If we integrate this over all times $t$ ' we obtain the required
resulto

$$
\begin{equation*}
\frac{0^{\infty} a_{p} t}{2 a_{p} \tau_{0}^{2}}\left(\delta_{1}+\delta_{2}\right) \tag{A15}
\end{equation*}
$$

6) 
6. This expression is inacourate in one respect. Fe have omittod any diacussion of the fact that the number of neutrons emitted during fission Iluctuate. A more detailed analysis of the theory of ohain production shows that equation $A l 5$ should be multiplied by a fictor usually called X phere


Hence, if one obtains the curve for the probability of ountine pairs as a function of $T$, then this ourve gives us a mothod of determining $a_{p}$ o

If one uses a source of strength $S$, then the counting rate, for pairs will bo

$$
\begin{equation*}
\frac{s}{1 \cos } \quad \frac{e^{\infty a_{p} i}}{2 a_{p} t_{0} L} \quad\left(\delta_{1}+\delta_{2}\right) \quad E_{1} E_{2} \tag{A15}
\end{equation*}
$$

When $E_{1}$ and $E_{2}$ are the respective efficiencies of the two chamberso The total counting rate in one chamber will be

$$
\frac{S}{I-K} \quad E_{1} \circ
$$

Hence the background of accidental counts will be

$$
\begin{equation*}
\left(\frac{S}{1-K}\right)^{2} \quad E_{1} \quad E_{2} \quad\left(\delta_{1}+\delta_{2}\right) \tag{A16}
\end{equation*}
$$

We may estinate the expectod results here as we did on the modulation methodo :We will first find what value of $S$ we can use to keep the background down to onectenth afier three periodso That is

$$
\frac{[S /(1-\pi)] / e^{-3}}{2 a_{p} \tau_{0}^{2}}=01
$$

If we use the values of $K, a_{p}$, and $\tau_{0}$ of Appendix $I_{0}$ then wo find that $S=10^{4}$ Fistion/seo which is easily obtainable with a natural sourceo If in (AI部) we again take $E_{1}=E_{2}=1 /(3 \times 104)$ and $\delta_{1}=\delta_{2}=501007 \mathrm{sec}$ as reasonabls gate width $S$ we find that the pairs/sec of three periods separation are counted at the rate of $1 / \mathrm{seco}$




Fig 3
DISCRIMINATOR









Fig. 13



GStarige oltand yos ainoyady



Fula
SCale


Fig. 15



## DOCLINENT P.OOM

pec. F7: \& M. M.... DATE ... 2.12-47 IEC. MO.F.S.


