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This document contains 23 pages

ELECTRIC AND THERMAL BEHAVIOR OF COMPLETELY IONIZED  
DEUTERIUM IN PRESENCE OF A MAGNETIC FIELD

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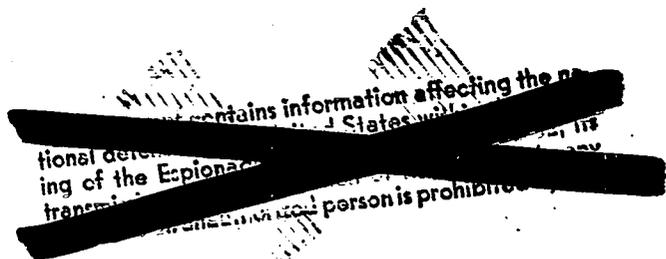
Rolf Landshoff

REPORT WRITTEN BY:

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ABSTRACT

Completely ionized deuterium has a large thermal conductivity

$$K_0 = 6.3 \times 10^{27} T_{(\text{Kev})}^{5/2} \text{ cm}^{-1} \text{ sec}^{-1} k$$

( $k$  is the Boltzmann constant). By applying a magnetic field  $H$  at right angles to the temperature gradient the conductivity reduces to  $K_H$ . In Fig. 5 the ratio  $K_H/K_0$  is plotted against  $H/H_0$  where  $H_0 = 2120 T_{(\text{Kev})}^{-3/2}$  kilogauss. This beneficial effect will disappear quickly because the magnetic lines of force tend to be squeezed out of the region of high temperature because of electric currents arising perpendicular to both the magnetic field and the temperature gradient. In a slab of thickness  $2a$  (cm), which has a temperature  $T_0$  at the center,  $H$  decreases at a rate

$$dH/dt = -46.6 \left[ T_0(\text{Kev})/a^2 \right] \text{ kilogauss}/\mu\text{sec}$$

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ELECTRIC AND THERMAL BEHAVIOR OF COMPLETELY IONIZED  
DEUTERIUM IN PRESENCE OF A MAGNETIC FIELD

Introduction

Completely ionized deuterium has a large thermal conductivity, which can be reduced by applying a magnetic field at right angles to the temperature gradient. There is a tendency for the magnetic lines of force to be squeezed out of the region of high temperature because of electric currents arising perpendicularly to both the magnetic field and the temperature gradient. It is of practical interest to know by how much the conductivity can be reduced and for how long a time the magnetic field can be kept high enough to keep the reduction effective. For this purpose we derive in Part I of this report general expressions for the "conductivity coefficients" which relate the electric and thermal currents to the electric field and the temperature gradient in the presence of a magnetic field. In Part II, we discuss the electric and magnetic field in the heated deuterium by entering the expressions for the electric currents derived in Part I into Maxwell's equations. We can thus determine the effective heat conductivity and the rate of decay of the magnetic field.

PART I -- Kinetic Theory

Let us assume a magnetic field  $H$  in the  $y$  - direction and an electric field and a temperature gradient both in the  $x - z$  plane. There will in general result electric and heat currents in the  $x$  and  $z$  directions.

$$\begin{aligned}
 j_x &= \sigma_{11} E_x - \sigma_{12} E_z + \tau_{11} \partial T / \partial x - \tau_{12} \partial T / \partial z \\
 j_z &= \sigma_{21} E_x + \sigma_{22} E_z + \tau_{21} \partial T / \partial x + \tau_{22} \partial T / \partial z \\
 q_x &= -\mu_{11} E_x + \mu_{12} E_z - \kappa_{11} \partial T / \partial x + \kappa_{12} \partial T / \partial z \\
 q_z &= -\mu_{21} E_x - \mu_{22} E_z - \kappa_{21} \partial T / \partial x - \kappa_{22} \partial T / \partial z
 \end{aligned}
 \tag{1}$$

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In the presence of a pressure gradient  $\underline{E}$  has to be replaced by  $\underline{E} + (kT/-e)(1/p)\nabla p$ , where  $-e = 4.8 \times 10^{-10}$  esu. These expressions will be obtained by extending the theory of LA-334 in the following manner:

1) for the electron distribution function we try the form:

$$\phi(x, z, \underline{v}) = f(x, z, \underline{v}) \left[ 1 + v_x h_x(\underline{v}) + v_z h_z(\underline{v}) \right] \quad (2)$$

2) In the Boltzmann equation, we include the magnetic field by using:

$$D(\phi) = v_x \frac{\partial \phi}{\partial x} + v_z \frac{\partial \phi}{\partial z} + \frac{e}{m} \left[ (E_x - \frac{v_z}{c} H) \frac{\partial \phi}{\partial v_x} + (E_z + \frac{v_x}{c} H) \frac{\partial \phi}{\partial v_z} \right] \quad (3)$$

It is no longer possible to completely replace  $D(\phi)$  by  $D(f)$ . If that were done,  $H$  would completely drop out of the equation. Instead one has to keep  $\phi$  in those parts of  $D$  which contain  $H$ . This is a consistent scheme inasmuch as it keeps just the terms which are linear in  $v_x$  and  $v_z$  in the equation. The significance of this approximation is, that electric fields and the gradients of pressure and temperature are considered as being small, whereas the magnetic field may have any value. Let us set:  $\epsilon = mv^2/2kT$ ,  $\omega = -eH/mc$  and put  $D(\phi)$  into the form

$$D(\phi) = \left( \frac{-eE_x}{kT} L_0(\epsilon) - \frac{1}{T} \frac{\partial T}{\partial x} L_1(\epsilon) - \omega h_z \right) v_x f + \left( \frac{-eE_z}{kT} L_0(\epsilon) - \frac{1}{T} \frac{\partial T}{\partial z} L_1(\epsilon) + \omega h_x \right) v_z f \quad (4)$$

(in presence of a pressure gradient its force must be added to that of the electric field). The  $L_n(\epsilon)$  are Laguerre polynomials of order  $3/2$ . We now expand:

$$h_x = \sum p_s L_s(\epsilon), \quad h_z = \sum q_s L_s(\epsilon) \quad (5)$$

and set

$$\begin{aligned} A_x &= eE_x/kT, & B_x &= (1/T)(\partial T/\partial x) \\ A_z &= eE_z/kT, & B_z &= (1/T)(\partial T/\partial z) \end{aligned} \quad (6)$$

and obtain:

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$$\begin{aligned}
 -D(\phi) = & \left[ A_x L_0(\epsilon) + B_x L_1(\epsilon) + \omega \sum q_s L_s(\epsilon) \right] V_x f \\
 & + \left[ A_z L_0(\epsilon) + B_z L_1(\epsilon) - \omega \sum p_s L_s(\epsilon) \right] V_z f
 \end{aligned} \tag{7}$$

On the right-hand side of the Boltzmann equation we must replace  $V_x h(v)$  as it was used in Eqs. (46) and (56) of LA-334 by  $V_x h_x(v)$  and then by  $V_z h_z(v)$  and add the expression obtained by these operations. We now multiply the Boltzmann equation on both sides in turn by  $(m/nkT) v_x L_r(\epsilon)$  and  $(m/nkT) v_z L_r(\epsilon)$  and integrate over  $dv$ . This procedure yields the two equations:

$$\begin{aligned}
 A_x \delta_{0r} + (5/2) B_x \delta_{1r} + \frac{\Gamma(r+5/2)}{\Gamma(r+1) \Gamma(5/2)} \omega q_r &= \sum p_s H_{rs} \\
 A_z \delta_{0r} + (5/2) B_z \delta_{1r} - \frac{\Gamma(r+5/2)}{\Gamma(r+1) \Gamma(5/2)} \omega p_r &= \sum q_s H_{rs}
 \end{aligned} \tag{8}$$

The matrix elements on the right hand side are defined by

$$H_{rs} = \nu h_{rs} \tag{9}$$

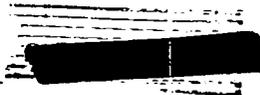
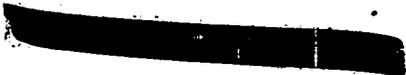
where

$$\nu = (4\sqrt{2\pi}/3) \lambda_n (e^2/kT)^2 (kT/m)^{\frac{1}{2}} \tag{10}$$

using  $\lambda_n$ ,  $h_{rs}^e$ ,  $h_{rs}^i$  as defined in Eqs. (46), (75), and (78) of LA-334.  $\nu^{-1}$  can essentially be interpreted as the time between collisions of the electron. It seems worth while to warn the reader that the  $H_{rs}$  as used in this report differ from those used in LA-334 by a factor  $m/nkT$ .

The special symmetry of problems in which a magnetic field is involved and which reflects itself both in Eqs. (1) and (8) suggests the introduction of complex quantities. Let us define

$$\left. \begin{aligned}
 j &= j_x + i j_z, & q &= q_x + i q_z \\
 E &= E_x + i E_z, & \nabla T &= (\partial T/\partial x) + i (\partial T/\partial z) \\
 \sigma &= \sigma_{||} + \sigma_{\perp}, & \tau &= \tau_{||} + i \tau_{\perp}
 \end{aligned} \right\} \tag{12}$$



$$\left. \begin{aligned} \mu &= \mu_{||} + i \mu_{\perp} & \kappa &= \kappa_{||} + i \kappa_{\perp} \\ A &= A_x + i A_z & B &= B_x + i B_z \end{aligned} \right\} \quad (12)$$

$$C_r = P_r + i Q_r$$

Then we can rewrite Eqs. (1) and (8) in much simpler forms, namely:

$$j = \sigma E + \nabla T; \quad q = -\mu E - \kappa \nabla T \quad (1a)$$

and

$$A \delta_{or} + (5/2) B \delta_{1r} - \frac{\Gamma(r+5/2)}{\Gamma(r+1) \Gamma(5/2)} i \omega C_r = \nabla \Sigma C_s h_{rs} \quad (8a)$$

More explicitly we can write:

$$\left\{ \begin{array}{cccc} \nu h_{00} + i \omega, & \nu h_{01} & \nu h_{02} & \dots \\ \nu h_{01} & \nu h_{11} + (5/2) i \omega, & \nu h_{12} & \dots \\ \nu h_{02} & \nu h_{12} & \nu h_{22} + (35/8) i \omega, & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{array} \right\} \left\{ \begin{array}{c} C_0 \\ C_1 \\ C_2 \\ \vdots \\ \vdots \\ \vdots \end{array} \right\} = \left\{ \begin{array}{c} A \\ (5/2) B \\ 0 \\ \vdots \\ \vdots \\ \vdots \end{array} \right\} \quad (13)$$

We want to solve this infinite system of equations for  $C_0$  and  $C_1$ , because these two coefficients yield  $j$  and  $q$  (see Eqs. (15) and (16) of LA-334). In order to do this we shall use only that part of the matrix actually written down in (13). This approximation was shown to be sufficiently accurate in absence of the magnetic field and we shall demonstrate its validity in the general case by checking the method for the case of negligible  $e - e$  scattering.

We can then write down immediately the solution of (13);

$$\begin{aligned} C_0 &= \nu^{-1} \left[ (\Delta_{00}/\Delta) A - (5/2) (\Delta_{01}/\Delta) B \right] \\ C_1 &= \nu^{-1} \left[ - (\Delta_{01}/\Delta) A + (5/2) (\Delta_{11}/\Delta) B \right] \end{aligned} \quad (14)$$

where

$$\Delta = \begin{vmatrix} (h_{00} + i \omega/\nu), & h_{01} & h_{02} \\ h_{01} & [h_{11} + (5/2) i \omega/\nu], & h_{12} \\ h_{02} & h_{12} & [h_{22} + (35/8) i \omega/\nu] \end{vmatrix} \quad (15)$$

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and  $\Delta_{00}$ ,  $\Delta_{01}$ ,  $\Delta_{11}$  are minors of the determinant  $\Delta$ . From (14) we derive further that:

$$\sigma = \frac{ne^2}{m\nu} \frac{\Delta_{02}}{\Delta}, \quad \tau = \frac{(-e)nk}{m\nu} \frac{5}{2} \frac{\Delta_{01}}{\Delta} \quad (16)$$

$$\mu = \frac{(-e)nkT}{m\nu} \frac{5}{2} \left( \frac{\Delta_{00} + \Delta_{01}}{\Delta} \right), \quad \kappa = \frac{nk^2T}{m\nu} \frac{25}{4} \left( \frac{\Delta_{01} + \Delta_{11}}{\Delta} \right)$$

In Part II we shall see that the effective heat conductivity is obtained by giving E that value for which  $j = 0$ , namely  $E = -(\tau/\sigma)\Delta T$ . This leads to

$$q = -\kappa_{\text{eff}} \nabla T \quad (17)$$

with

$$\kappa_{\text{eff}} = \kappa - \tau\mu/\sigma \quad (18)$$

$$= \frac{25}{4} \frac{nk^2T}{m\nu} (\Delta_{11} \Delta_{00} - \Delta_{01}^2)$$

This can be transformed just as it was done in Eq. (24) of LA-334 by applying Sylvester's theorem, and leads to

$$\kappa_{\text{eff}} = \frac{25}{4} \frac{nk^2T}{m\nu} \frac{h_{22} + (35/8) i(\omega/\nu)}{\Delta_{00}}$$

In particular we want the real part of this expression which represents the conductivity in direction of the temperature gradient. We will denote this real part by  $\kappa_H$ . Numerical calculations were carried out for  $f_{\sigma_{11}}$  if  $\sigma_L = \Delta_{00}/\Delta$ ,  $\Delta_{01}/\Delta$ , and  $\Delta_{11}/\Delta$  as well as for the derived quantities

$$f_{\tau_H} + if_{\tau_L} = (5/2)(\Delta_{01}/\Delta), \quad f_{\mu_{11}} + if_{\mu_L} = (5/2) (\Delta_{00} + \Delta_{01})/\Delta$$

$$f_{\kappa_{11}} + if_{\kappa_L} = (25/4) (\Delta_{01} + \Delta_{11})/\Delta$$

and

$$f_{\kappa_H} = \text{real part of } (25/4) \left[ \frac{h_{22} + (35/8) i(\omega/\nu)}{\Delta_{00}} \right]$$

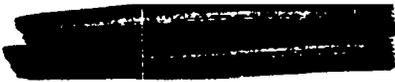


TABLE I

$\omega/\nu$	$f_{\sigma_{11}} + if_{\sigma_{12}} = \Delta_{00}/\Delta$	$\Delta_{01}/\Delta$	$\Delta_{11}/\Delta$
0	1.950	.5546	.6636
.2	1.558 - .7290 i	.3783 - .2834 i	.5278 - .2563 i
.5	.8601 - .9186 i	.1040 - .2713 i	.2737 - .3027 i
1.0	.3603 - .7025 i	-.00809 - .1454 i	.1208 - .2201 i
2.0	.1246 - .4192 i	-.02389 - .06047 i	.05364 - .1328 i
4.0	.04274 - .2287 i	-.01703 - .01926 i	.02507 - .07933 i
6.0	.02226 - .1581 i	-.01089 - .008093 i	.01467 - .05803 i
asympt.	$(\omega/\nu)^{-2} - (\omega/\nu)^{-1}i$	$-.6(\omega/\nu)^{-2} - 2.64(\omega/\nu)^{-3}i$	$.746(\omega/\nu)^{-2} - .4(\omega/\nu)^{-1}i$

$\omega/\nu$	$f_{\sigma_{11}}$	$f_{\sigma_{12}}$	$f_{\mu_{11}}$	$f_{\mu_{12}}$	$f_{\kappa_{11}}$	$f_{\kappa_{12}}$	$f_{\kappa_{13}}$
0	1.386	0	6.261	0	7.614	0	3.1614
.2	.9458	-.7085	4.841	-2.531	5.663	-3.373	2.7742
.5	.2600	-.6782	2.410	-2.975	2.361	-3.588	1.7374
1.0	-.0202	-.3635	.8805	-2.120	.7044	-2.284	.8477
2.0	-.0597	-.1512	.2518	-1.199	.1859	-1.208	.3871
4.0	-.0426	-.0481	.0643	-.6199	.0502	-.6162	.1739
6.0	-.0272	-.0202	.0284	-.4176	.0236	-.4133	.09806
asympt.	$-1.5(\frac{\omega}{\nu})^{-2}$	$-6.6(\frac{\omega}{\nu})^{-3}$	$(\frac{\omega}{\nu})^{-2}$	$-2.5(\frac{\omega}{\nu})^{-1}$	$.913(\frac{\omega}{\nu})^{-2}$	$-2.5(\frac{\omega}{\nu})^{-1}$	$4.664(\frac{\omega}{\nu})^{-2}$

In the following section we check the validity of our approximation by treating the electron scattering as negligible. The Boltzmann equation simplifies to:

$$D(\phi) = -4\pi \lambda n (e^2/mv^2)^2 V(v_x h_x + v_z h_z) f \tag{19}$$

(See Eq. (56) of LA-334).



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Introducing  $\nu$  and  $\epsilon$  and dividing by  $f$  this can be written as

$$\begin{aligned} \left[ A_x L_0(\epsilon) + B_x L_1(\epsilon) + \omega h_x \right] V_x + \left[ A_z L_0(\epsilon) + B_z L_1(\epsilon) - \omega h_x \right] V_z \\ = (3\sqrt{\pi}/4) \nu e^{-3/2} (V_x h_x + V_z h_z) \end{aligned} \quad (20)$$

This splits into the two equations for the factors of  $V_x$  and  $V_z$  which can be recombined by the use of complex notation

$$A L_0(\epsilon) + B L_1(\epsilon) - i\omega h(\epsilon) = (3\sqrt{\pi}/4) \nu e^{-3/2} h(\epsilon) \quad (21)$$

where

$$h = h_x + i h_z \quad (22)$$

This is just an algebraic equation for  $h$  and leads at once to

$$h(\epsilon) = \frac{A L_0(\epsilon) + B L_1(\epsilon)}{(3\sqrt{\pi}/4) \nu e^{-3/2} + i\omega} \quad (23)$$

The electric and thermal current can be found by using the expressions

$$j = (e/3) \int v^2 h f d\underline{v} \quad \text{and} \quad q = (m/6) \int v^4 h f d\underline{v} \quad (24)$$

Now

$$f d\underline{v} = 2\pi n \pi^{-\frac{1}{2}} e^{-\epsilon} e^{\frac{1}{2}} d\epsilon \quad (25)$$

So that we have

$$\begin{aligned} j &= \frac{4}{3\sqrt{\pi}} \frac{nekT}{m} \int_0^\infty e^{3/2} e^{-\epsilon} h(\epsilon) d\epsilon \\ q &= \frac{4}{3\sqrt{\pi}} \frac{n(kT)^2}{m} \int_0^\infty e^{5/2} e^{-\epsilon} h(\epsilon) d\epsilon \end{aligned} \quad (26)$$

In order to carry out these integrations we set

$$\xi = \left( \frac{3\sqrt{\pi}}{4} \frac{\nu}{\omega} \right)^{2/3} \quad (27)$$

so that

$$\begin{aligned} h &= \omega^{-1} \frac{e^{3/2} [A L_0(\epsilon) + B L_1(\epsilon)]}{\xi^{3/2} + i e^{3/2}} \\ &= \omega^{-1} \frac{[\xi^{3/2} e^{3/2} + i e^3] [A + B(5/2 - \epsilon)]}{\xi^3 + e^3} \end{aligned} \quad (28)$$

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We now introduce the integrals:

$$I_n(\xi) = \int_0^{\infty} \frac{e^{\eta} e^{-\epsilon}}{\xi^3 + \epsilon^3} d\epsilon \quad (29)$$

and express  $j$  and  $q$  in terms of these integrals.

$$j = \frac{4}{3\sqrt{\pi}} \frac{n e k T}{n \omega} \left[ \left( A + \frac{5}{2} B \right) (\xi^{3/2} I_3 - i I_{4.5}) - B (\xi^{3/2} I_4 - i I_{5.5}) \right]$$

$$q = \frac{4}{3\sqrt{\pi}} \frac{n (k T)^2}{m \omega} \left[ \left( A + \frac{5}{2} B \right) (\xi^{3/2} I_4 - i I_{5.5}) - B (\xi^{3/2} I_5 - i I_{6.5}) \right] \quad (30)$$

This leads at once to the relations

$$f_{\sigma_{11}} = \frac{4}{3\sqrt{\pi}} \frac{\nu}{\omega} \xi^{3/2} I_3 = \left( \frac{\nu}{\omega} \right)^2 I_3$$

$$f_{\sigma_{12}} = - \frac{4}{3\sqrt{\pi}} \frac{\nu}{\omega} I_{4.5}$$

$$f_{\sigma_{21}} = \left( \frac{\nu}{\omega} \right)^2 \left( I_4 - \frac{5}{2} I_3 \right)$$

$$f_{\sigma_{22}} = \frac{4}{3\sqrt{\pi}} \left( \frac{\nu}{\omega} \right)^2 \left( \frac{5}{2} I_{4.5} - I_{5.5} \right)$$

$$f_{\mu_{11}} = \left( \frac{\nu}{\omega} \right)^2 I_4 \quad (31)$$

$$f_{\mu_{12}} = - \frac{4}{3\sqrt{\pi}} \left( \frac{\nu}{\omega} \right) I_{5.5}$$

$$f_{\mu_{21}} = \left( \frac{\nu}{\omega} \right)^2 \left( I_5 - \frac{5}{2} I_4 \right)$$

$$f_{\mu_{22}} = \frac{4}{3\sqrt{\pi}} \left( \frac{\nu}{\omega} \right) \left( \frac{5}{2} I_{5.5} - I_{6.5} \right)$$

which permit us to check the  $f$  values found by the matrix method. The integrals  $I_n$  are evaluated in the appendix. We notice in both theories the relationship

$$f_{\mu} = (5/2) f_{\sigma} + f_{\xi} \quad (32)$$

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therefore we do not have to check the components of one of them, say  $f_{\epsilon_H}$  and  $f_{\epsilon_L}$ . Figs. 1 and 2 show that the agreement is very good for  $f_{\sigma_H}$ ,  $f_{\sigma_L}$ ,  $f_{\mu_H}$ , and  $f_{\mu_L}$ . For  $f_{K_H}$  (Fig. 3) which changes sign, the agreement is not very good. Of particular interest is of course  $f_{K_H}$  (Fig. 4) for which the agreement is fair. In all diagrams the full-line curves correspond to the matrix theory and the broken-line curves to the differential-equation theory.

## PART II

Let us consider a slab of deuterium parallel to the  $y - z$  plane which is placed in a homogeneous magnetic field in the  $y$  direction. This slab is suddenly heated and subsequently maintained at a temperature, which we assume to be a known function  $T(x)$ , and which is of the order of magnitude of several Kev. This situation will, of course, give rise to electric and thermal currents in the  $x$  and  $z$  direction. The electric current in the  $x$  direction dies out very rapidly as we can see from the following argument. Let us call the current due to temperature and pressure gradients  $j_{x0}$ . The current will then establish an electric field  $E_x$  by the relation:

$$\partial E_x / \partial t + 4\pi j_x = 0$$

( $\partial H / \partial z = 0$ , because of the symmetry of the problem) where

$$j_x = \sigma_{||} E_x + j_{x0}$$

Thus we have

$$\partial E_x / \partial t + 4\pi(j_{x0} + \sigma_{||} E_x) = 0$$

which means that  $E_x$  will approach a value for which  $j_x = 0$  during a time of the order of magnitude  $1/4\pi\sigma_{||}$ . By Eqs. (10) and (16) and using  $\lambda = 9.67$  as computed from Eq. (46) of LA-334 for a temperature of around 30 Kev and a density of  $4 \times 10^{22}$  electrons/cm<sup>3</sup> we find  $\sigma_{||} = 3.3 \times 10^{21} (kT/mc^2)^{3/2} f_{\sigma_{||}} \text{ sec}^{-1}$ . The time of adjustment

thus turns out to be  $2.5 \times 10^{-23} (kT/mc^2)^{-3/2} f_{\sigma_{11}}^{-1}$  sec, which is an exceedingly short time. We may therefore say that the electric field  $E_x$  immediately counteracts any effects which tend to cause a current in the x-direction. That means that we can set the first of the Eqs. (1) equal to zero and use this relation to eliminate  $E_x$  from the other ones. Applying this procedure for the second equation we obtain:

$$j_z = \sigma E_z - \tau dT/dx \quad (33)$$

if we set

$$\sigma = \sigma_{11} + \sigma_{\perp}^2/\sigma_{11} \quad \text{and} \quad \tau = \sigma_{\perp} \tau_{11}/\sigma_{11} = \tau_{\perp} \quad (34)$$

( $\partial T/\partial z = 0$  by assumption).

The sudden heating will therefore give rise to a current:

$$j_{z0} = - \tau dT/dx \quad (35)$$

which will be established in a time of the order of magnitude of the time between atomic collisions that is about  $10^{-11}$  sec. This current tends to push the magnetic lines of force out, which induces a field  $E_z$ , which in turn slows down the process of pushing out the field. In order to investigate this sequence let us solve

$$\partial H/\partial x = (4\pi/c)(j_{z0} + \sigma E_z) + (1/c) \partial E_z/\partial t \quad (36)$$

$$\partial E_z/\partial x = (1/c) \partial H/\partial t \quad (37)$$

subject to the condition that at  $t = 0$ ,  $\partial H/\partial x = 0$  and  $E_z = 0$ . The extremely large value of  $\sigma$  inside the slab will make the term  $(1/c) \partial E_z/\partial t$  effectively drop out of the equation and of course remove the condition  $E_z = 0$ . In order to show this we must show that during the time that  $E_z$  is "switched on"  $\partial H/\partial x$  remains sufficiently small so that it can still be regarded as zero. Let us for the moment make this assumption. We can then solve (36) and obtain (regarding  $\sigma$  as constant)

$$E_z = - (1/\sigma) j_{z0} (1 - e^{-4\pi\sigma t})$$

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The transient term disappears when, let us say,  $4\pi\sigma t_1 = 10$ . From Eq. (37) we find

$$\frac{\partial}{\partial t} \frac{\partial H}{\partial x} = c \frac{\partial^2 E_z}{\partial x^2} = -\frac{c}{\sigma} j_{z0}'' (1 - e^{-4\pi\sigma t})$$

and

$$\frac{\partial H}{\partial x} = -\frac{c}{\sigma} j_{z0}'' \left( t - \frac{1 - e^{-4\pi\sigma t}}{4\pi\sigma} \right)$$

At  $t = t_1$

$$\frac{\partial H}{\partial x} = -\frac{c}{\sigma} j_{z0}'' \frac{g}{4\pi\sigma}$$

This should be compared to  $(4\pi/c)j_{z0}$  and is found to be smaller by a factor  $g(c/4\pi\sigma)^2 j_{z0}''/j_{z0}$  which is of the order of magnitude  $10^{-20}$  or less. This shows that our assumption is self consistent. During the switching on period  $j_z = j_{z0} + \sigma E$  drops to zero,

This is of importance for the calculation of the initial value of the effective heat conductivity. It means that we can set  $j = j_x + ij_z = 0$  and eliminate  $E$  from (1a) as it was done in Eqs. (17) and (18). By taking the real part of  $K_{\text{eff}}$  we obtain the conductivity in direction of the temperature gradient,

$$K_H = (nk^2 T/m\nu) f_{K_H} \quad (38)$$

where  $f_{K_H}$  is tabulated in Table I.

In the case of liquid deuterium with a density of  $n = 4.2 \times 10^{22} \text{ cm}^{-3}$  the critical value of the magnetic field at which  $\omega = \nu$  is

$$H_c = 2120 T^{-3/2} \text{ kilogauss} \quad (39)$$

(Kov)

In the absence of a magnetic field the thermal conductivity is

$$K_0 = 6.3 \times 10^{27} T^{5/2} \text{ Kev} \text{ cm}^{-1} \text{ sec}^{-1} \cdot k \quad (40)$$

( $k =$  Boltzmann constant). In Fig. 5 the ratio  $K_H/K_0$  is plotted against  $H/H_0$ . We see from this graph that one can obtain a considerable reduction of heat losses by means of a sufficiently strong magnetic field.

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If  $j_z$  is not taken to be zero the heat current can be expressed by the equation

$$q_x = -K_H \frac{dT}{dx} + \frac{\sigma_{11} \mu_{12} - \sigma_{12} \mu_{11}}{\sigma_{11}^2 + \sigma_{12}^2} j_z$$

By inspection of Table I we see that the factor of  $j_z$  is negative. Now  $j_z$  tends to have the opposite sign as  $dT/dx$  so that the gradual appearance of  $j_z$  tends to decrease the thermal flow. This beneficial effect, however, is insignificant because as the current  $j_z$  appears, the magnetic lines of force are squeezed out of the deuterium.

It is a critical question if the field can be maintained for a sufficiently long time to be effective. Since the rate at which the field decays has the largest value right at the beginning, we can answer this question by calculating this initial rate of decay. From Maxwell's equations (e.g., 37) we deduce that:

$$\partial H / \partial t = c (\partial E_z / \partial x)$$

but we have seen that initially

$$E_z = - (1/\sigma) j_{z0} = (\tau/\sigma) (dT/dx) \tag{42}$$

so that

$$\frac{\partial H}{\partial t} = c \frac{\partial}{\partial x} \left( \frac{\tau}{\sigma} \frac{dT}{dx} \right) \tag{43}$$

At the center of the slab  $dT/dx = 0$  so that

$$\partial H / \partial t = c (\tau/\sigma) (d^2 T / dx^2) \tag{44}$$

We now introduce

$$\frac{\tau}{\sigma} = \frac{\tau_{11} \sigma_{12} - \tau_{12} \sigma_{11}}{\sigma_{11}^2 + \sigma_{12}^2} = \frac{k}{-e} f \tau / \sigma \tag{45}$$

where the expression

$$f \tau / \sigma = \frac{f \tau_{11} f \sigma_{12} - f \tau_{12} f \sigma_{11}}{f \sigma_{11}^2 + f \sigma_{12}^2} \tag{46}$$

has been computed and represented graphically as a function of  $H/H_0$  in Fig. 6. If we assume  $T$  to be of the form  $T = T_0 [1 - (\pi/a)^2]$  we obtain;

$$\frac{\partial H}{\partial t} = -2 \frac{ckT_0}{ca^2} f_{\pi}/\sigma \quad (47)$$

We see from the graph of  $f_{\pi}/\sigma$  that it does not show much variation with  $H/H_0$  in the important region,  $H/H_0 > 1$ , and we shall use the maximum value  $f_{\pi}/\sigma = .233$ . This yields;

$$\frac{\partial H}{\partial t} = -46.6 \frac{T_0 \text{ Kev Kilogauss}}{a^2 \mu\text{sec}} \quad (48)$$

One can easily show that in case of a cylinder of radius  $a$  one has to multiply this result by 2.

Up to now, we have not included any hydrodynamics in the problem. But it is clear that the high pressure inside the material will bring about an expansion. In absence of a magnetic field the kinetic theory is completely independent of the material velocity. If a magnetic field is present, however, this is no longer the case. If we designate the material velocity in the  $x$  direction with  $u$  we find that  $E_z$  has to be replaced by  $E_z + (u/c)H$ . Let us disregard for the discussion of this effect the current  $j_{20}$  whose importance in producing a decay of the magnetic field was discussed in the previous paragraph. We have the equations

$$\frac{\partial H}{\partial x} = \frac{4\pi\sigma}{c} \left( E + \frac{u}{c} H \right) \quad (49)$$

$$\frac{\partial E}{\partial x} = \frac{1}{c} \frac{\partial H}{\partial t} \quad (50)$$

We combine them to

$$\frac{\partial}{\partial x} \left( \frac{c^2}{4\pi\sigma} \frac{\partial H}{\partial x} \right) = \frac{\partial H}{\partial t} + \frac{\partial}{\partial x} (uH) \quad (51)$$

The left hand side of this equation would be of importance only if our time scale was of the order of magnitude of  $a^2\sigma/c^2$  which is obviously not the case. We can

therefore set

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x} (uH) = 0 \quad (52)$$

In the material coordinate system we can write this as

$$dH/dt = -H \, du/dx \quad (53)$$

At the same time we have the equation of continuity

$$d\rho/dt = -\rho \, du/dx \quad (54)$$

and therefore

$$\frac{1}{H} \frac{dH}{dt} = \frac{1}{\rho} \frac{d\rho}{dt} \quad (55)$$

Integrating this we get

$$H/H_0 = \rho/\rho_0 \quad (56)$$

which means that the lines of force are carried along with the material. At the same time we know from Eq. (10) that  $\nu$  and therefore the critical field strength  $H_c$  are proportional to the density, so that the important ratio  $H/H_c$  is unaffected.

APPENDIX

The integrals  $I_n(x) = \int_0^\infty \frac{y^n e^{-y}}{x^3 + y^3} dy$  obey the recursion formula

$$I_{n+3} = n! - x^3 I_n \quad (57)$$

For  $n = (0, 1, 2)$  they can be evaluated by the following method. Set

$$\epsilon = \frac{1}{2} (-1 + i\sqrt{3}) = e^{2\pi i/3}$$

then:

$$\frac{3x^{2-n} y^n}{y^3 + x^3} = \frac{a_n}{y+x} + \frac{b_n}{y+\epsilon x} + \frac{\bar{b}_n}{y+\bar{\epsilon}x}$$

with  $a_n = (1, -1, 1)$ ,  $b_n = (\epsilon, -\bar{\epsilon}, 1)$ . We obtain

$$3x^{2-n} I_n(x) = a_n \int_0^\infty \frac{e^{-y}}{y+\epsilon} dy + 2\text{Re} \left( b_n \int_0^\infty \frac{e^{-y}}{y+\epsilon x} dy \right) \quad (58)$$

(Re stands for real part of). The first integral is simply

$$\int_0^{\infty} \frac{e^{-Y}}{Y+x} dY = e^x E_1(x)$$

The second integral can be transformed to:

$$\int_0^{\infty} \frac{e^{-Y}}{Y+\epsilon x} dY = e^{\epsilon x} \int_{\epsilon x}^{\infty} \frac{e^{-t}}{t} dt = e^{\epsilon x} \left( \int_{\epsilon x}^x \frac{e^{-t}}{t} dt + E_1(x) \right) \quad (59)$$

The integral from  $\epsilon x$  to  $x$  can be carried along the circular arc and if we set

$t = xe^{i\phi}$ ,  $dt/t = i d\phi$  we obtain:

$$\int_{\epsilon x}^x \frac{e^{-t}}{t} dt = i \int_{2\pi/3}^0 e^{-xe^{i\phi}} d\phi = -\frac{2\pi i}{3} - (1-\epsilon) \alpha(x) + (1-\bar{\epsilon}) \beta(x) \quad (60)$$

where

$$\alpha(x) = x - x^4/4.4! + x^7/7.7! - + \dots \quad (61)$$

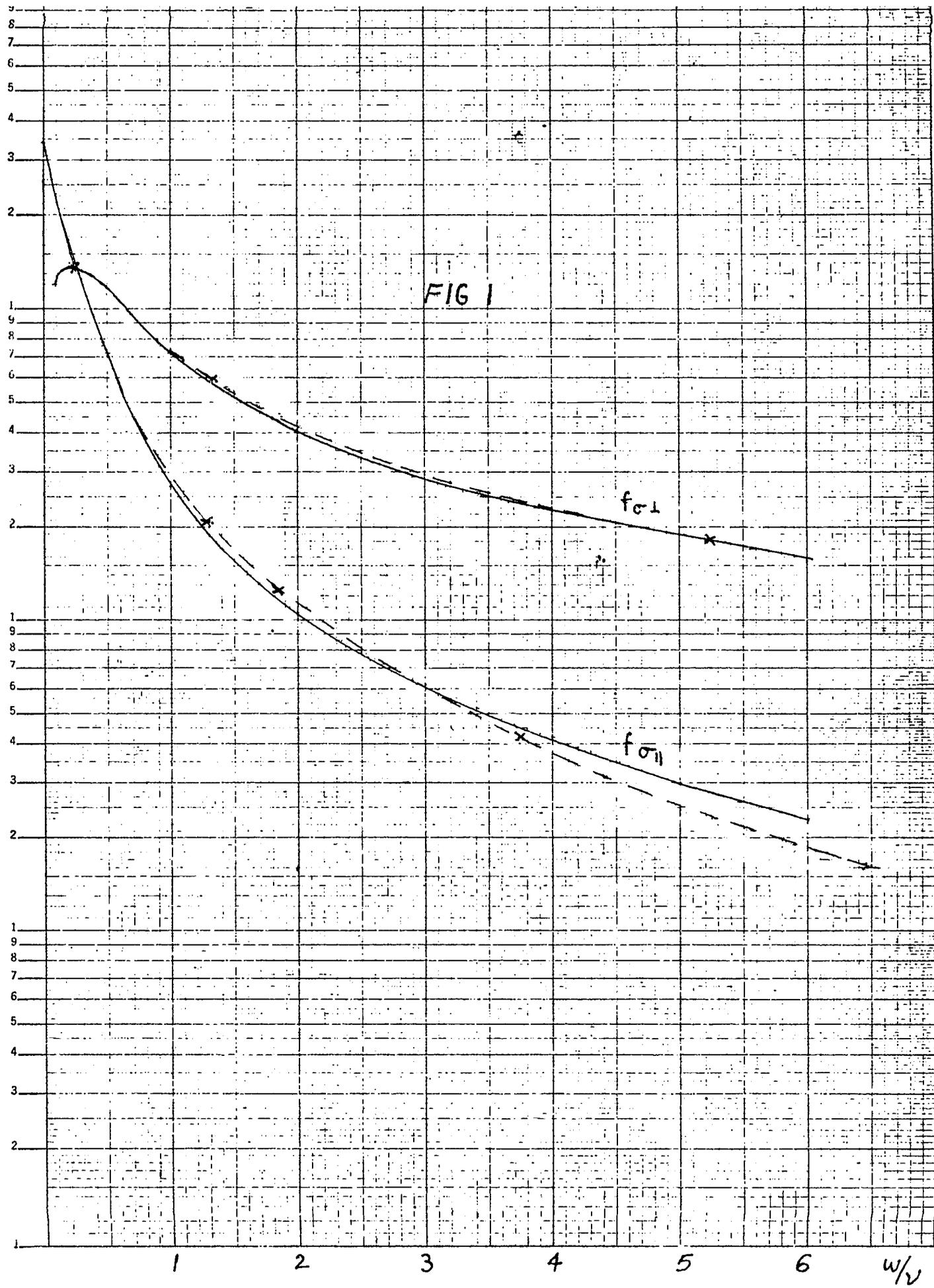
$$\beta(x) = x^2/2.2! - x^5/5.5! + x^8/8.8! - + \dots \quad (62)$$

are two very rapidly converging series obtained by integrating the power expansion of  $e^{-xe^{i\phi}}$ . We can now enter the integrals (59) and (60) into (58) and obtain  $I_0$ ,  $I_1$ , and  $I_2$  and from those by means of (57) also the  $I_n$  for higher integral values of  $n$ . For half-integral values of  $n$  we obtained the  $I_n$  by interpolation. The results of these calculations are collected in Table II.

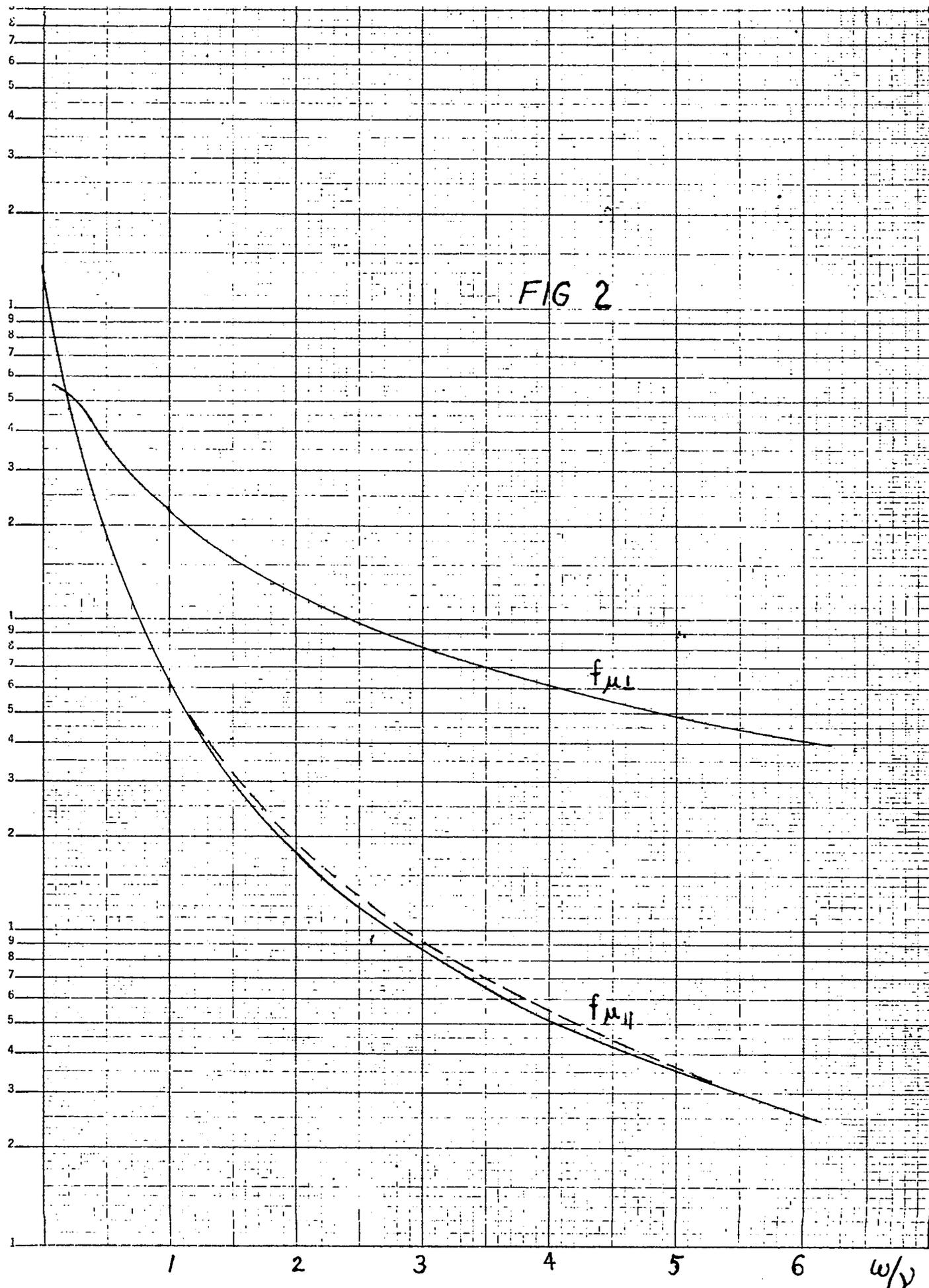
TABLE II

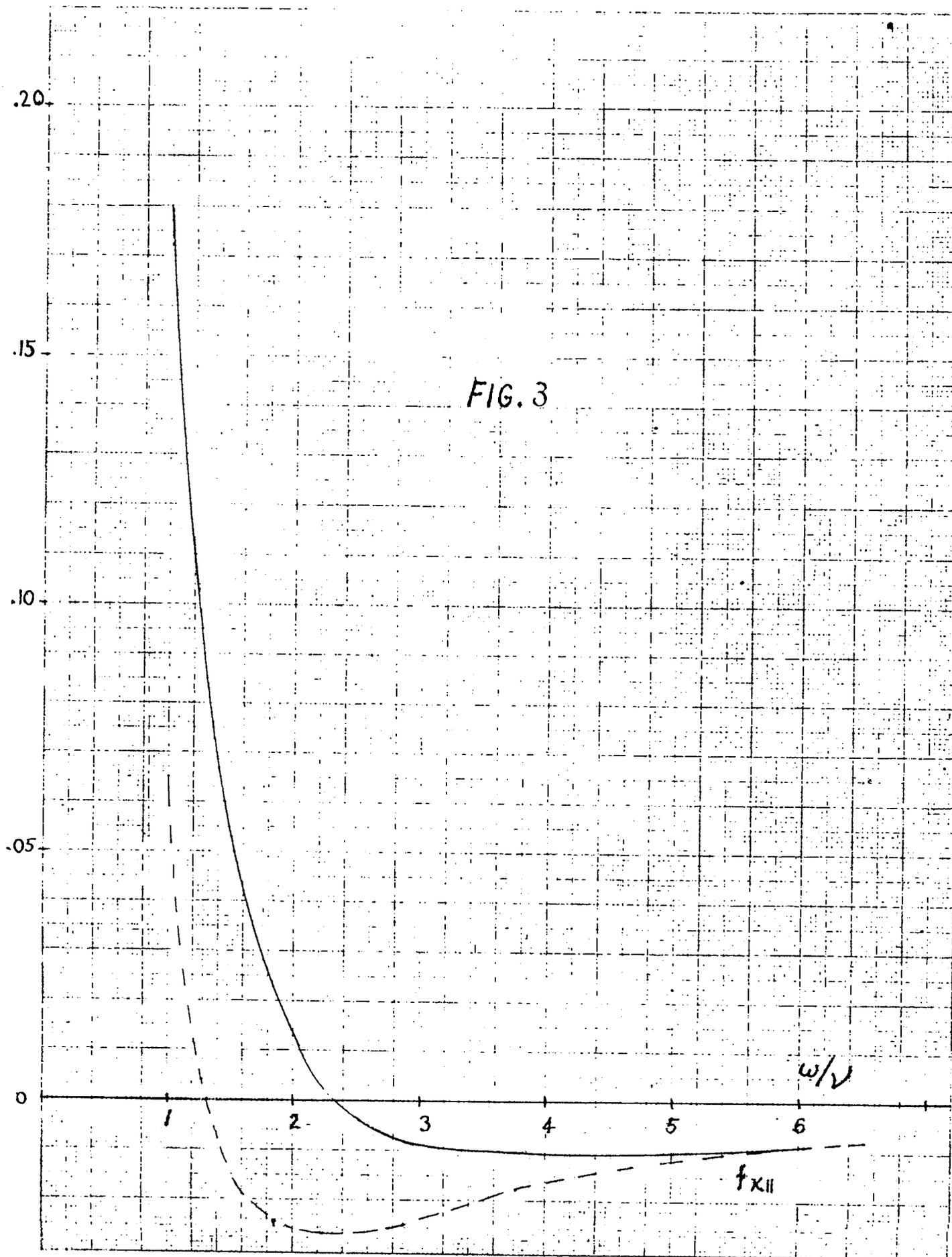
$x$	$I_3$	$I_4$	$I_{4.5}$	$I_5$	$I_{5.5}$
0	1.0000	1.0000	1.3293	2.0000	3.3233
.5	.5854	.8702	1.2366	1.9234	3.2516
1	.3717	.6882	1.0557	1.7201	3.0269
2	.1725	.4130	.6976	1.2454	2.3448
3	.09070	.2523	.4564	.8643	1.7188
4	.05214	.1599	.3024	.5992	1.2372
5	.03211	.1053	.2059	.4209	.8969
6	.02088	.07175	.1437	.3012	.6565
large $x$	$\frac{6}{x^3 + 120}$	$\frac{24}{x^3 + 210}$	$\frac{52.344}{x^3 + 268.125}$	$\frac{120}{x^3 + 336}$	$\frac{287.892}{x^3 + 414.375}$

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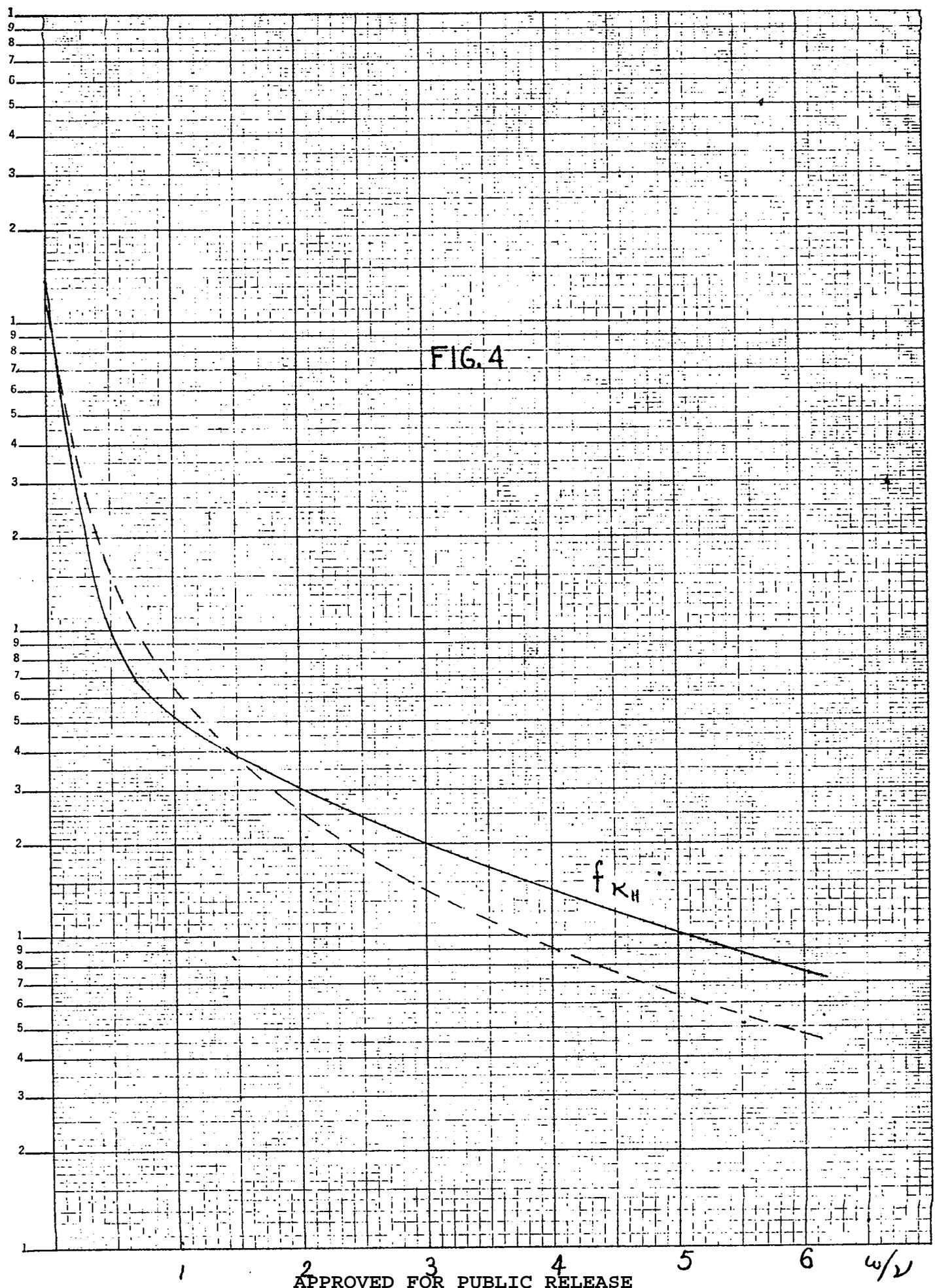


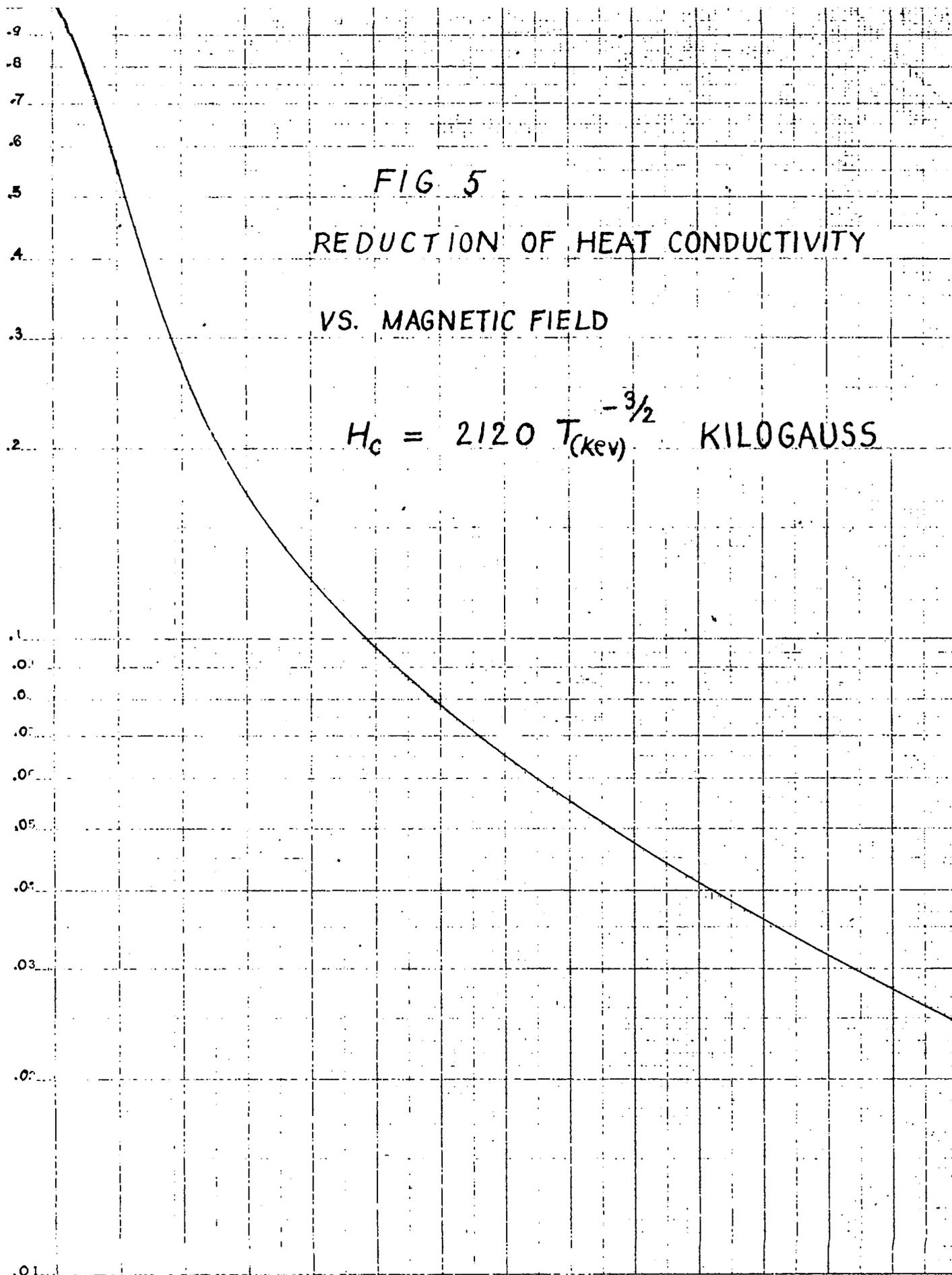
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Semi-Logarithmic, 10 cycles in the freq. range 100-100,000 Hz



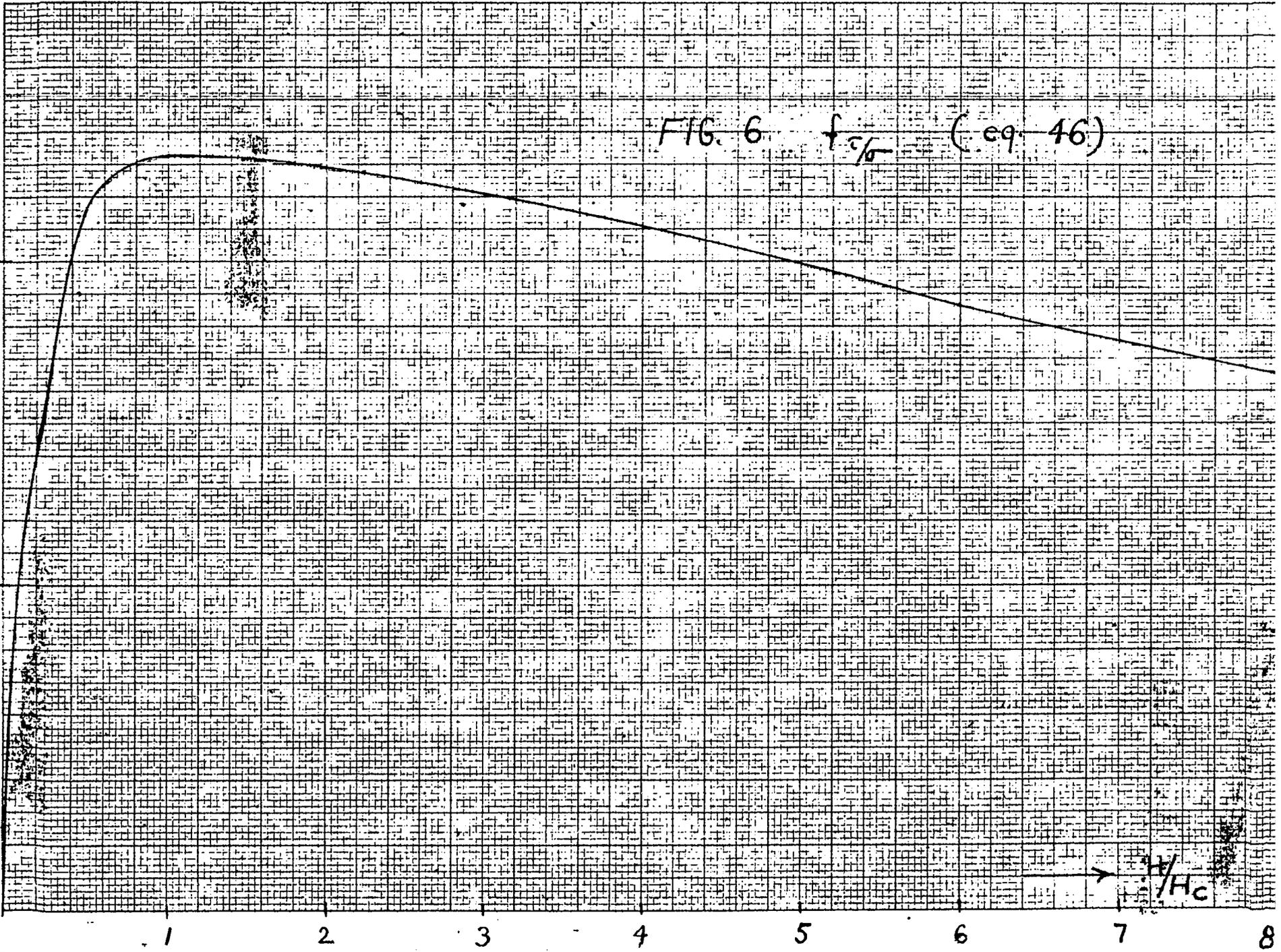


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H/Hc





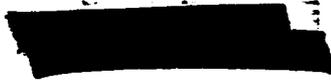
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