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JULY 16TH NUCLEAR EXPLOSION: SOIL CORRECTION, ABSORPTION OF NEUTRONS IN SOIL, AND TIME DEPENDENCE OF SLOW-NEUTRON INTENSITY

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ABSTRACT [redacted]

This paper contains the solution of some theoretical problems which are of interest in connection with the Trinity Test. Section A is a continuation of LA-257 and solves rigorously the case, D, α, ρ and $\mathfrak{z}$ arbitary (for notation cf. LA-257). In Section B a fairly accurate expression for the absorption of the prompt neutrons in the earth as a function of time is derived; these prompt neutrons give rise to γ-rays which must be distinguished from the γ-rays coming directly from the gadget. Finally, in Section C, the problem of the time-decay of the slow-neutron intensity (averaged over all space) due to a pulse of fast neutrons is worked out to a good approximation; Fig. 3 contains the results for air.

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JULY 16TH NUCLEAR EXPLOSION:
SOIL CORRECTION, ABSORPTION OF NEUTRONS IN SOIL,
AND TIME DEPENDENCE OF SLOW-NEUTRON INTENSITY

SECTION A

Soil Corrections for Neutron Measurements

In LA-257, the slowing-down density of neutrons, due to a mono-energetic point source of fast neutrons located between two diffenent slowing down media, was obtained as a function of age for the following three cases:

(I) $\alpha = 1$, $z = 0$, D and ρ arbitrary

(II) $D = \infty$, $z = 0$, α and ρ arbitrary

(III) $D = \infty$, $\alpha = 1$, ρ and z arbitrary

In the above, ρ and z are the radial and lengthwise coordinates (cf LA-257),

$$D = \frac{\lambda_2^2}{a_2(1-b_2)} \Big/ \frac{\lambda_1^2}{a_1(1-b_1)}$$

and

$$\alpha = \frac{(1-b_2)}{(1-b_1)} \Big/ \frac{a_2}{a_1}$$

where λ, a, b are the mean free path, average logarithmic energy loss, and average cosine of the angle of deflection in a collision respectively; the subscript 2 denotes the medium with larger mean free path while 1 denotes the one with smaller mean free path. We have since worked out the case D, α, ρ and z arbitrary; this enables the results to be applied to Trinity earth which contains an appreciable amount of hydrogen, and to the mock-up experiments which were necessary to interpret

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the Trinity results.

We found in LA-257 that the Laplace transform (with respect to the age τ) of the slowing down density in the medium with larger mean free path is:

$$\phi(\rho, z, \eta) = \frac{\sqrt{D}}{2\pi}\int_0^\infty \frac{J_0(\lambda\rho)\, e^{-z\sqrt{\lambda^2+\eta}}\ \lambda\, d\lambda}{\left[\sqrt{\alpha(\lambda^2+D\eta)} + \sqrt{D(\lambda^2+\eta)}\right]} \tag{1}$$

To obtain the Laplace inverse of (1), we write:

$$\frac{e^{-z\sqrt{\lambda^2+\eta}}}{Q} = \int_0^\infty e^{-wQ - z\sqrt{\lambda^2+\eta}}\, dw \tag{2}$$

where

$$Q = \sqrt{\alpha(\lambda^2+D\eta)} + \sqrt{D(\lambda^2+\eta)}$$

Substituting into (1) and interchanging the order of integration, we get:

$$\mathcal{L}_\eta^{-1}\phi = \frac{\sqrt{D}}{2\pi}\int_0^\infty J_0(\lambda\rho)\,\lambda\, d\lambda \int_0^\infty \mathcal{L}_\eta^{-1}\left[e^{-wQ - z\sqrt{\lambda^2+\eta}}\right] dw \tag{3}$$

We may find the Laplace inverse of the exponential expression by taking the Laplace inverse of the component parts and using the convolution theorem.

Thus: $$\mathcal{L}_\eta^{-1} e^{-(w\sqrt{D}+z)\sqrt{\lambda^2+\eta}} = \left[\frac{w\sqrt{D}+z}{\sqrt{4\pi}}\right]\frac{e^{-\left[\frac{(w\sqrt{D}+z)^2}{4\tau}+\lambda^2\tau\right]}}{\tau^{3/2}} \tag{4}$$

$$\mathcal{L}_\eta^{-1} e^{-w\sqrt{\alpha(\lambda^2+D\eta)}} = \frac{w\sqrt{D\alpha}}{\sqrt{4\pi}}\ \frac{e^{-\left(\frac{\lambda^2\tau}{D}+\frac{w^2 D\alpha}{4\tau}\right)}}{\tau^{3/2}} \tag{5}$$

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and

$$\mathcal{L}_\eta^{-1}\left[e^{-w\rho - z\sqrt{\lambda^2+\eta}}\right] = \frac{(w\sqrt{D}+z)\,w\sqrt{D\alpha}}{4\pi}\int_0^\tau \frac{e^{-\left[\lambda^2\left(\frac{u}{D}+\tau-u\right)+\frac{w^2D\alpha}{4u}+\frac{(w\sqrt{D}+z)^2}{4(\tau-u)}\right]}}{u^{3/2}(\tau-u)^{3/2}}\,du \tag{6}$$

Inserting (6) into (3) and interchanging the order of integration gives:

$$\mathcal{L}_\eta^{-1}\,\phi = \frac{D\sqrt{\alpha}}{8\pi^2}\int_0^\tau \frac{du}{u^{3/2}(\tau-u)^{3/2}}\int_0^\infty w(w\sqrt{D}+z)\,dw\; e^{-\left[\frac{w^2D\alpha}{4u}+\frac{(w\sqrt{D}+z)^2}{4(\tau-u)}\right]}\cdot \int_0^\infty J_0(\lambda\rho)\,e^{-\lambda^2\left(\frac{u}{D}+\tau-u\right)}\lambda\,d\lambda \tag{7}$$

Evaluation of the integral over λ yields:

$$\int_0^\infty J_0(\lambda\rho)\,e^{-\lambda^2/S}\lambda\,d\lambda = \left(\frac{S}{2}\right)e^{-\rho^2/4\;S} \tag{8}$$

where

$$S = \frac{1}{\left(u/D+\tau-u\right)}$$

The evaluation of the integral over w is also straightforward and finally

leads to:

$$\int_0^\infty w(w\sqrt{D}+z)\,e^{-\left[\frac{w^2D\alpha}{4u}+\frac{(w\sqrt{D}+z)^2}{4(\tau-u)}\right]}\,dw = \tag{9}$$

$$\frac{2T^{1/2}e^{-\frac{z^2\alpha T}{4}}\sqrt{u(\tau-u)}}{D}\left\{e^{-\frac{z^2uT}{4(\tau-u)}}\,z\alpha\sqrt{Tu}\,(\tau-u)^{3/2} + \frac{\sqrt{\pi}}{2}\left[1-\mathrm{erf}\left(\frac{z\sqrt{Tu}}{2\sqrt{\tau-u}}\right)\right]\left[u(\tau-u)(2-z^2\alpha T)\right]\right\}$$

where

$$T = \frac{1}{\left[\alpha(\tau-u)+u\right]}$$

Combining results we get as the final expression for the slowing down density,

q,:

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$$g(\rho, z, \tau) \equiv \mathcal{L}_\eta^{-1}\phi = \frac{\sqrt{\alpha}}{8\pi^2}\int_0^\tau du\; e^{-\left[\frac{\rho^2 S}{4} + \frac{z^2\alpha T}{4}\right]}\, \frac{S\, T^{3/2}}{u^{1/2}} \cdot \tag{10}$$

$$\left\{ z\,\alpha(\tau-u)^{1/2}\, e^{-\frac{z^2 u T}{4(\tau-u)}}\, T^{1/2} + \frac{\sqrt{\pi u}}{2}\left[1-\mathrm{erf}\left(\frac{z\sqrt{Tu}}{2\sqrt{\tau-u}}\right)\right]\left[2-z^2\alpha T\right]\right\}$$

In limiting case $D = \infty$, (10) can be replaced by a simpler expression, namely:

$$g(\rho, z, \tau) = \frac{1}{4(\pi\tau)^{3/2}}\left\{ e^{-\frac{(\rho^2+z^2)}{4\tau}} + \sqrt{\alpha}\int_z^\infty \frac{1}{\sqrt{\rho^2+w^2}}\left[1-\frac{\left(\sqrt{\alpha}(w-z)+\sqrt{\rho^2+w^2}\right)^2}{2\tau}\right]\cdot e^{-\frac{\left[\sqrt{\alpha}(w-z)+\sqrt{\rho^2+w^2}\right]^2}{4\tau}}\, dw\right\} \tag{11}$$

The derivation of (11) has some mathematical interest and we shall give it briefly. When $D = \infty$, (1) reduces to:

$$\phi = \frac{1}{2\pi}\int_0^\infty \frac{J_0(\lambda\rho)\, e^{-\sqrt{\lambda^2+\eta}\; z}}{\left[\sqrt{\alpha\eta} + \sqrt{(\lambda^2+\eta)}\right]}\,\lambda\, d\lambda \tag{12}$$

To obtain the Laplace inverse of (1), we rewrite (12) in another form, namely:

$$\phi = \frac{1}{2\pi}\left\{\frac{e^{-\sqrt{\eta}\cdot\sqrt{\rho^2+z^2}}}{\sqrt{\rho^2+z^2}} - \sqrt{\alpha\eta}\; e^{\sqrt{\alpha\eta}\, z}\int_z^\infty \frac{e^{-\sqrt{\eta}\left[w\sqrt{\alpha}+\sqrt{\rho^2+w^2}\right]}}{\sqrt{\rho^2+w^2}}\, dw\right\} \tag{13}$$

We Show that (13) is equivalent to (12) as follows: for $\sqrt{\alpha} < 1$, $\eta, \lambda > 0$, the right hand side of equation (12) can be expressed in terms of an infinite series, thus:

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$$\phi = \frac{1}{2\pi}\int_0^\infty \frac{J_0(\lambda\rho)}{\sqrt{\lambda^2+\eta}}\, e^{-z\sqrt{\lambda^2+\eta}} \left\{\sum_{j=0}^{\infty} (-1)^j \left(\frac{\sqrt{\alpha\eta}}{\sqrt{\lambda^2+\eta}}\right)^j\right\} \lambda\, d\lambda \tag{14}$$

The integration over λ of each term in the infinite series can be performed and leads to the result:

$$I_j = \int_0^\infty \frac{J_0(\lambda\rho)\, e^{-z\sqrt{\lambda^2+\eta}}}{(\sqrt{\lambda^2+\eta})^{j+1}}\, \lambda\, d\lambda = \int_z^\infty dz_1 \int_{z_1}^\infty dz_2 \cdots \int_{z_{j-1}}^\infty dz_j\, \frac{e^{-\sqrt{\eta}\sqrt{\rho^2+z_j^2}}}{\sqrt{\rho^2+z_j^2}} \tag{15}$$

For $j = 0$, we get:

$$I_0 = \frac{e^{-\sqrt{\eta}\sqrt{\rho^2+z^2}}}{\sqrt{\rho^2+z^2}} \tag{15a}$$

Therefore:

$$\phi = \frac{1}{2\pi} \sum_{j=0}^{\infty} (-1)^j \, (\sqrt{\alpha\eta})^j \, I_j . \tag{16}$$

Now it is easy to prove - through successive integration by parts - that

$$\int_z^\infty \frac{e^{-[\sqrt{\eta}\sqrt{\rho^2+w^2}+w\sqrt{\alpha\eta}]}}{\sqrt{\rho^2+w^2}}\, dw = -\frac{e^{-\sqrt{\alpha\eta}\, z}}{\sqrt{\alpha\eta}} \sum_{j=1}^{\infty} (-1)^j (\sqrt{\alpha\eta})^j I_j \tag{17}$$

Substitution of (15a) and (17) into (16) leads to (13). Now (17) is valid for $\sqrt{\alpha} > 0$ while the series expansion (14) is only valid for $\sqrt{\alpha} < 1$; however, since (12) is an analytic function of $\sqrt{\alpha}$ and since there is a finite interval of overlap, by the process of analytic continuation we can

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use (17) for $\sqrt{a} > 1$ - which is the case we are interested in. The Laplace inverse of (13) is easy to find and we get (11).

It is fairly simple to derive the three special cases treated in LA-257 from either (10) or (11). As is evident from (10) or (11), one numerical integration is required to obtain number which can be used. Such numerical integrations have been performed for various values of D, α, ρ, and γ and we present the results in Tables I - III; R is always the ratio of the neutron intensity at the given point for the specified values of D and α to the neutron intensity at the same point for D = α = 1.

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TABLE I $(D = 4,\ a = 2,\ z = 0)$

$\frac{\rho}{2\sqrt{\tau}}$	R
0.50	1.22
0.71	.935
0.87	.733
1.00	.589
1.50	.287

TABLE II $(D = \infty,\ a = 2.25)$

$\rho/2\sqrt{\tau}$	$z/2\sqrt{\tau}$	R
0	0.5	1.33
0	1.0	.987
0	1.5	.893
0.5	0.5	1.026
0.5	1.0	.912
0.5	1.5	.862
1.0	0.71	.715
1.0	1.00	.765
1.0	1.73	.792
1.5	0.5	.480
1.5	1.0	.628
1.5	1.5	.700

TABLE III $(D = \infty,\ a = 4.5)$

$\rho/2\sqrt{\tau}$	$z/2\sqrt{\tau}$	R
0	0.5	1.165
0	1.0	.818
0	1.5	.727
0.5	0.5	.847
0.5	1.0	.744
0.5	1.5	.699
1.0	0.71	.570
1.0	1.0	.602
1.0	1.73	.628
1.0	3.18	.640
1.5	0.5	.351
1.5	1.0	.481
1.5	1.5	.538

It is seen from Tables II and III that R approaches an asymptotic value - independent of ρ - as z becomes large. This value depends on a and can be found directly from (11); it turns out to be $2/(1 + \sqrt{a})$.

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ABSORPTION OF NEUTRONS IN SOIL AS FUNCTION OF TIME

SECTION B

When the neutrons emitted by the nuclear bomb strike the ground, they are captured by the nuclei contained in the ground, giving rise to γ-rays. Each neutron captured leads to at least on γ-ray emitted. The γ-rays produced as a result of this ground absorption of neutrons give an appreciable background to measurements of γ-rays coming directly from the gadget and must be corrected for. Once the absorption of neutrons by the ground is known as a function of time, it is possible to determine the number of γ-rays by multiplying by the average number of γ-rays produced per neutron captured and by taking into consideration the absorption of γ-rays in the soil (cf LA-250).

We treat the problem of the absorption of neutrons in the ground by a method of successive approximations. In first approximation we solve the time-dependent age equation for air with capture assuming that the soil is black, i.e. completely absorbing this overestimates the number of captured neutrons. In the second approximation, the current[1] flowing into the earth - as given by the "first approximation" solution - is regarded as a source for a stationary age problem with capture (since slowing down occurs almost instantaneously in the soil as measured in units of the air time scale) and the returning current is found. The net current flowing into the soil integrated over all ages represents the absorption as a function of time. This method can be continued to yield more and more accurate results but the second approximation is sufficiently good for most purposes.

1). In defining the current and age we assume that the total and transport mean free paths are identical; this is a good approximation for large mass of the scattering nucleus (the case of interest). However, the difference between the two mean free paths can easily be taken into account.

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To obtain the first approximation, we write down the time-dependent age equation with capture for air; we work with constant mean free path although the generalization to variable mean free path is also possible:

$$\frac{\partial^2 q}{\partial z^2} - \frac{3}{\ell v}\frac{\partial q}{\partial t} - \frac{3}{\ell \ell_c(\tau)} q = \frac{\partial q}{\partial \tau} - \delta(\tau)\,\delta(z - z_0)\,\delta(t) \qquad (18)$$

In equation (18), q is the slowing down density, τ the age, t the time, v the velocity of the neutron, ℓ the total mean path (which is essentially the scattering mean free path since the capture is assumed small), ℓ_c the capture mean free path, which is a function of τ, and finally z_o is the position of the plane source (cf. Fig. 1). It may appear strange that the age equation is written down for plane symmetry and not for spherical symmetry since the source of neutrons is a point source. The reason for this is that we are interested in the point-source solution integrated over a plane perpendicular to the z-direction i.e. the interface between air and soil. It is easy to show that $q_{pl.} = \int_0^\infty 2\pi\rho\, q_{pt.}\, d\rho$ where $q_{pt.}$ is the point-source solution and $q_{pl.}$ the plane-source solution of equation (18)

Equation (18) is to be solved subject to the condition that the medium adjacent to z = 0 (i.e. soil) is completely absorbing; this implies that the backgoing current is zero, i.e. :

$$q - \frac{2}{3}\ell\,\frac{\partial q}{\partial z} = 0 \quad \text{at} \quad z = 0 \qquad (19)$$

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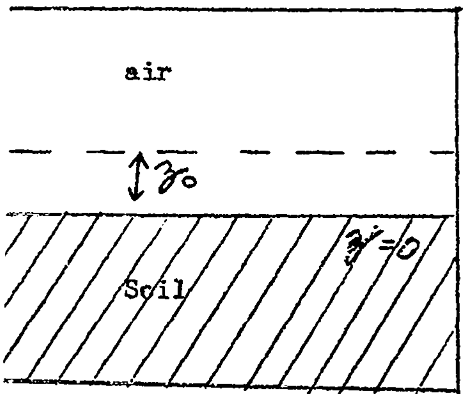

Fig. 1

The solution of equation (18) subject to the boundary condition (19) is:

$$\varphi = \left\{ \frac{e^{-\frac{(z-z_0)^2}{4\tau}}}{\sqrt{4\pi\tau}} + \frac{e^{-\frac{(z+z_0)^2}{4\tau}}}{\sqrt{4\pi\tau}} - \frac{3}{2\ell} e^{\frac{9\tau}{4\ell^2}+\frac{3(z+z_0)}{2\ell}} \cdot \left[1-\operatorname{erf}\left(\frac{3\sqrt{\tau}}{2\ell}+\frac{(z+z_0)}{2\sqrt{\tau}}\right)\right]\right\} e^{-\left(\frac{3}{\ell}\right)\int_0^{\tau}\frac{d\tau}{\ell_c(\tau)}} \cdot \delta\left[t-\frac{2\ell}{aV(0)}\left(e^{\frac{3a\tau}{2\ell^2}}-1\right)\right] \quad (20)$$

The solution (20) is arrived at by the use of Green's function[2] or by means of Laplace transforms. The latter method of deriving (20) seems sufficiently interesting to justify giving the details of the derivation (cf Appendix I). The rate per second at which neutrons are being absorbed by the soil (in this "black approximation) is given by:

$$A(t) = \int_0^{\infty} \frac{\partial\varphi(\tau)}{\partial z}\bigg|_0 d\tau = \frac{3}{2\ell}\left\{ \frac{e^{-\frac{z_0^2}{4\tau}}}{\sqrt{\pi\tau}} - \frac{3}{2\ell} e^{\frac{9\tau}{4\ell^2}+\frac{3z_0}{2\ell}}\left[1-\operatorname{erf}\left(\frac{3\sqrt{\tau}}{2\ell}+\frac{z_0}{2\sqrt{\tau}}\right)\right]\right\} e^{-\frac{3}{\ell}\int_0^{\tau}\frac{d\tau}{\ell_c(\tau)}} \quad (21)$$

2). Cf. Carslaw - "Conduction of Heat", Chapter X

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where

$$\bar{\tau} = \frac{2\ell^2}{3a} \log_e \left[1 + \frac{a t v(0)}{2\ell} \right] \tag{21a}$$

The fraction of neutrons absorbed by the ground up to the time t is $\int_0^t A(t')dt'$

The quantity $\int_0^t A(t')dt'$ is plotted up for air in Fig. 2 (curve I) as a function of t for $z_o = \ell = 25$ meters and an initial neutron energy of 200 ev (cf LAMS 250). In this calculation we have neglected the capture of neutrons (i.e. we have set $\ell_c = \infty$).

The solution given in equation (21) is a fist approximation to the correct answer. To improve our result we treat the "black" current $\frac{\ell}{3} \left.\frac{\partial q(\tau)}{\partial z}\right|_0$ as the source of neutrons slowed down in the earth. Since the soil density is so large compared to air, the mean time between two slowing down collisions in the soil is so much shorter than in air that we can - in good approximation - use the stationary age equation for the soil. This equation is:

$$\frac{\partial^2 q_s}{\partial z^2} - \frac{3}{\ell_s \ell_{sc}(\tau_s)} q_s = \frac{\partial q_s}{\partial \tau_s} \tag{22}$$

In equation (22), q_s is the slowing down density in the soil, ℓ_s the soil mean free path (assumed constant) ℓ_{sc} the capture mean free path in the soil (which may vary) and τ_s the age of neutrons in the soil. Since $\tau_s = \frac{\ell_s^2}{\ell^2}\tau$

We rewrite (22):

$$\frac{\partial^2 q_s}{\partial z^2} - \frac{3}{\ell_s \ell_{sc}(\tau)} q_s = \frac{\ell^2}{\ell_s^2} \frac{\partial q_s}{\partial \tau} \tag{23}$$

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To write down the solution of (23) with $\frac{\ell}{3}\frac{\partial q}{\partial z}\Big|_0$ as the source, we note first that equation (23) can be transformed into an ordinary heat equation by means of the substitution:

$$q_s = Q_s\, e^{-\frac{3}{\ell_s}\int_0^{\tau}\frac{d\tau}{\ell_{sc}(\tau)}}$$

In terms of Q_s, (23) thus becomes:

$$\frac{\partial^2 Q_s}{\partial z^2} = \frac{\ell^2}{\ell_s^2}\frac{\partial Q_s}{\partial \tau} \tag{24}$$

Secondly, we note that the incoming current from the air into the soil is

$\frac{\ell}{3}\frac{\partial q(\tau)}{\partial z}\Big|_0$ (with $\frac{\partial q(\tau)}{\partial z}\Big|_0$ given by differentiating (20), whereas, expressed in terms of q_s, it is

$$\frac{\ell_s}{2\ell}\left[\frac{\ell_s}{3}\frac{\partial q_s(\tau)}{\partial z}\Big|_0 + \frac{q_s(\tau)}{2}\right]. \quad (*)$$

Equating the two expressions for the incoming current and rewriting in terms of Q_s, yields:

$$\frac{\partial Q_s(\tau)}{\partial z}\Big|_0 + \frac{3}{2\ell_s}Q_s(\tau) = \frac{2\ell^2}{\ell_s^2}\frac{\partial q(\tau)}{\partial z}\Big|_0\, e^{\frac{3}{\ell_s}\int_0^{\tau}\frac{d\tau}{\ell_{sc}(\tau)}} \tag{25}$$

(*) We assume that the average logarithmic energy loss per collision is the same in air and soil, and that the average of the cosine of the angle of deflection per collision is zero for both. These assumptions, however, are not essential to the subsequent argument.

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Equation (24) is to be solved subject to the boundary condition:

$$-\frac{\partial Q_s}{\partial z}\Big|_0 + h\,Q_s\Big|_0 = h\,\phi(\tau) \tag{26}$$

where $$h = -\frac{3}{2\ell_s} \tag{26a}$$

$$h\,\phi(\tau) = -\frac{2\ell^2}{\ell_s^2}\,\frac{\partial \varphi(\tau)}{\partial z}\Big|_0\; e^{\frac{3}{\ell_s}\int_0^\tau \frac{d\tau}{\ell_{sc}(\tau)}} \tag{26b}$$

The solution of (24) with boundary condition (26) is:[3]

$$Q_s(z,\tau) = \frac{(-h)}{\sqrt{\pi}}\cdot\frac{\ell_s}{\ell}\int_0^\tau\left[e^{-\frac{z^2\ell^2}{4\ell_s^2(\tau-u)}} + h\int_0^\infty e^{h\xi-\frac{(-z+\xi)^2\ell^2}{4\ell_s^2(\tau-u)}}\,d\xi\right]\frac{\phi(u)\,du}{\sqrt{\tau-u}} \tag{27}$$

and hence:

$$\varphi_s(z,\tau) = \frac{2\ell}{\sqrt{\pi}\,\ell_s}\, e^{-\frac{3}{\ell_s}\int_0^\tau\frac{d\tau}{\ell_{sc}(\tau)}}\int_0^\tau\left[e^{-\left(\frac{z\ell}{2\ell_s}\right)^2\frac{1}{(\tau-u)}} - \frac{3}{2\ell_s}\int_0^\infty e^{-\frac{3\xi}{2\ell_s}-\frac{(-z+\xi)^2\ell^2}{4\ell_s^2(\tau-u)}}\,d\xi\right]\cdot \tag{28}$$

$$\frac{\partial\varphi(u)}{\partial z}\Big|_0\;\frac{e^{\frac{3}{\ell_s}\int_0^u\frac{d\tau}{\ell_{sc}(u)}}}{\sqrt{\tau-u}}\,du$$

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3). Cf. Carslaw, "Conduction of Heat", Sect. 83; the integral depending on the internal sources is zero since $Q_s = 0$ for $\tau = 0$ and all $z \leq 0$. Moreover, $(-h)$ replaces h because z is negative.

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$$= \frac{3}{\sqrt{\pi}\,\ell_s} \cdot e^{-\frac{3}{\ell_s}\int_{\bar\tau}^{\tau}\frac{d\tau}{\ell_{sc}(\tau)}} \left[e^{-\left(\frac{z\ell}{2\ell_s}\right)^2 \frac{1}{(\tau-\bar\tau)}} - \frac{3}{2\ell_s}\int_0^\infty e^{-\frac{3\xi}{2\ell_s} - \frac{(-z+\xi)^2\ell^2}{4\ell_s^2(\tau-\bar\tau)}}\, d\xi\right] \cdot \frac{1}{\sqrt{\tau-\bar\tau}} \cdot$$

$$e^{-\frac{3}{\ell}\int_0^{\bar\tau}\frac{d\tau}{\ell_c(\tau)}} \cdot \left\{\frac{e^{-\frac{z_0^2}{4\bar\tau}}}{\sqrt{\pi\bar\tau}} - \frac{3}{2\ell}\, e^{\frac{9\bar\tau}{4\ell^2}+\frac{3z_0}{2\ell}}\left[1-\mathrm{erf}\left(\frac{3\sqrt{\bar\tau}}{2\ell}+\frac{z_0}{2\sqrt{\bar\tau}}\right)\right]\right\} \quad \text{for } \tau > \bar\tau \tag{28a}$$

$$= 0 \quad \text{for } \tau \leq \bar\tau \tag{28b}$$

with $\bar\tau$ defined by (21a)

To determine the rate at which neutrons are absorbed in the soil, we must evaluate:

$$B(\tau) = \int_0^\infty \frac{\partial q_s}{\partial z}\bigg|_0 d\tau = \int_0^\infty d\tau \left[\frac{2\ell^2}{\ell_s^2}\,\frac{\partial q(\tau)}{\partial z}\bigg|_0 - \frac{3}{2\ell_s}\, q_s(0,\tau)\right]$$

$$= \frac{3\ell}{\ell_s^2}\, q(0,\bar\tau) - \frac{3}{2\ell_s}\int_{\bar\tau}^\infty q_s(0,\tau)\, d\tau \tag{29}$$

where:

$$q(0,\bar\tau) = \left\{\frac{e^{-\frac{z_0^2}{4\bar\tau}}}{\sqrt{\pi\bar\tau}} - \frac{3}{2\ell}\, e^{\frac{9\bar\tau}{4\ell^2}+\frac{3z_0}{2\ell}}\left[1-\mathrm{erf}\left(\frac{3\sqrt{\bar\tau}}{2\ell}+\frac{z_0}{2\sqrt{\bar\tau}}\right)\right]\right\} e^{-\frac{3}{\ell}\int_0^{\bar\tau}\frac{d\tau}{\ell_c(\tau)}} \tag{29a}$$

$$q_s(0,\tau) = \frac{3}{\sqrt{\pi}\,\ell_s}\, e^{-\frac{3}{\ell_s}\int_{\bar\tau}^{\tau}\frac{d\tau}{\ell_{sc}(\tau)}} \left\{\frac{1}{\sqrt{\tau-\bar\tau}} - \frac{3\sqrt{\pi}}{2\ell}\left[1-\mathrm{erf}\left(\frac{3\sqrt{\tau-\bar\tau}}{2\ell}\right)\right]\right\} \cdot$$

$$e^{-\frac{3}{\ell}\int_0^{\bar\tau}\frac{d\tau}{\ell_c(\tau)}} \left\{\frac{e^{-\frac{z_0^2}{4\bar\tau}}}{\sqrt{\pi\bar\tau}} - \frac{3}{2\ell}\, e^{\frac{9\bar\tau}{4\ell^2}+\frac{3z_0}{2\ell}}\left[1-\mathrm{erf}\left(\frac{3\sqrt{\bar\tau}}{2\ell}+\frac{z_0}{2\sqrt{\bar\tau}}\right)\right]\right\} \tag{29b}$$

[redacted]

In equation (29), (29a), (29b) $\overline{\tau}$ is found from (21a). The fraction of neutrons absorbed by the ground up the time t is $\int_0^t B(t')dt'$. The integral $\int_0^t B(t')dt'$ has been computed for air for several values of t and the results are given in Curve II of Fig. 2. The distance of the source from the interface between air and soil was again chosen as 25 meters (i.e. $z_0 = \ell = 25$ meters). The capture mean free paths in air and soil were assumed to be proportional to the velocities with the lifetime in air taken as .07 seconds and that in soil, twice as large (for density equal to that of air).

Formula (29) was used to compute points on Curve II up to t = .001 seconds. Beyond this time, the "second" approximation (29) gives too high a value for the neutrons absorbed by the ground and it is more sensible to calculate the point $t = \infty$ and to interpolate for the remaining times. The calculation of the point $t = \infty$ is equivalent to solving the stationary problem for the absorption of neutrons in one semi-infinite capturing medium due to a plane source of neutrons in an adjacent semi-infinite capturing medium. If the lifetimes in the two media are identical, (we assume capture cross-sections varying inversely with the velocity and normalization to equal density) the problem can be solved rigorously with the following result for the absorption in the medium which does not contain the source:

$$\frac{z_0}{4\sqrt{\pi}} \int_0^\infty \frac{\exp.\left[-\frac{z_0^2}{4\tau} - \frac{3}{\ell}\int_0^\tau \frac{d\tau'}{\ell_c(\tau')}\right]}{\tau^{3/2}} \, d\tau \qquad (30)$$

All quantities in (30) have their usual significance. The value of (30) for $Z_0 = \ell = 25$ meters and the air lifetime is .41; for the same z_0 and the soil lifetime (twice as large) the result is .45. The correct result lies between these values and has been chosen as .43 (cf. Fig. 2).

 [redacted]

It is possible to continue our scheme of approximation by treating the current returning to the air as a new source for equation (18) and so on. However, for our purposes the "second" approximation is sufficiently accurate.

[redacted]

TIME DECAY OF SLOW-NEUTRON INTENSITY DUE TO A PULSE OF FAST NEUTRONS.

SECTION C

In this section we treat the problem of the time decay of the slow-neutron intensity due to a pulse of fast neutrons. This problem is of interest in connection with the delayed-neutron measurements at Trinity (cf. LAMS 250) where it is necessary to know how soon the prompt neutrons from the bomb die out.

Let us consider a plane source of mono-energetic fast neutrons in an infinite slowing-down medium; if the pulse of fast neutrons occurs at time $t = 0$, the transport equation becomes:

$$\frac{\partial N}{\partial t}(z,\mu,u,t) + V\mu\frac{\partial N}{\partial z} + \frac{N_v}{\ell(v)} = \int_0^u du'\int d\Omega'\,\frac{N(z,\mu',u',t)v'}{\ell(v')}\,f(\mu_0,u-u') + \frac{\delta(z)\,\delta(u)\,\delta(t)}{4\pi} \tag{31}$$

where

$$f(\mu,u) = \frac{(M+1)^2}{8\pi M}\,e^{-u}\,\delta\left\{\mu - \frac{1}{2}\left[(M+1)e^{-\frac{u}{2}} - (M-1)e^{\frac{u}{2}}\right]\right\}$$

In equation (31), $N(z, \mu, u, t)\,dz\,d\mu\,du\,dt$ is the number of neutrons between z and $z + dz$, μ and $\mu + d\mu$, etc., $u = \log(E_0/E)$ where E_0 is the primary energy and E the energy of interest, v is the velocity of the neutron and $\ell(v)$ the mean free path for scattering corresponding to the velocity v. If we take the zero-moment (with respect to μ) of equation (31) and integrate over all space, we get:

$$\frac{\partial M_0(u,t)}{\partial t} + \frac{M_0 V}{\ell(v)} = \int_0^u du'\,\frac{M_0(u',t)\,V'}{\ell(V')}\,f_0(u-u') + \delta(u)\,\delta(t) \qquad \text{for } u < q$$

$$\frac{\partial M_0(u,t)}{\partial t} + \frac{M_0 V}{\ell(v)} = \int_{u-q}^u du'\,\frac{M_0(u',t)\,V'}{\ell(V')}\,f_0(u-u') \qquad \text{for } u > q \tag{32}$$

[redacted]

where

$$g = \log\left(\frac{M+1}{M-1}\right)^2$$

$$f_0(u) = \frac{(M+1)^2}{4M} e^{-u}$$

$$M_0(u,t) = \int_{-\infty}^{\infty} dz \int N(z,\mu,u,t)\, d\Omega$$

If the mass, M, of the scattering nucleus is large compared to one and we are interested in the value of M_0 (u,t) for large u (slow neutron intensity) we can neglect the first of equations (32).

We now take the Laplace transform of the second of equations (32) to obtain:

$$\psi(u,s)\left(s+\frac{v}{\ell}\right) = \int_{u-g}^{u} du'\, \psi\,\frac{(u',s)}{\ell(v')}\, v' f_0(u-u') \tag{33}$$

where

$$\psi(u,s) = \int_0^{\infty} e^{-st} M_0(u,t)\, dt$$

Assuming that the mean free path is constant and using essentially the reciprocal velocity as the independent variable, enables us to rewrite (33) as:

$$\phi(w) = \frac{2}{(1-r^2)w^2} \int_{rw}^{w} \frac{w'\phi(w')}{1+w'}\, dw' \tag{34}$$

 [redacted]

where

$$\phi(w) = \frac{s\psi}{w}(u,s) \quad (1+w)$$

$$w = \frac{\ell s}{V}, \qquad r = \left(\frac{M-1}{M+1}\right)$$

Placzek has solved equation (34) in the form of a power series[4], namely:

$$\phi(w) = \sum_{n=0}^{\infty} \beta_n w^n, \qquad \beta_0 = 1 \tag{35}$$

where $\beta_n = (-1)^n \lambda_n \prod_{\ell=1}^{n} (1-\lambda_\ell)^{-1}$

with $\lambda_n = \frac{2}{n+2}\left[\frac{1-r^{n+2}}{1-r^2}\right]$

The desired solution is the Laplace inverse of $w\,\phi(w)/s(1+w)$.

Since it is impossible to find any analytic expression for the Laplace inverse, we have recourse to the following approximation method. We observe that:

$$(t^n)_{av} = (-1)^n \left.\frac{\partial^n \psi(u,s)}{\partial s^n}\right|_{s=0}$$

where $(t^n)_{av} = \int_0^\infty t^n M_0(u,t)\,dt$

But

$$\psi(u,s) = \frac{w\,\phi(w)}{s(1+w)} = \frac{\ell}{V}\,\frac{\phi(w)}{(1+w)}$$

so that

$$\left.\frac{\partial^n}{\partial s^n}\psi(u,s)\right|_{s=0} = \left(\frac{\ell}{V}\right)^{n+1} \left.\frac{\partial^n}{\partial w^n}\left[\frac{\phi(w)}{1+w}\right]\right|_{w=0}$$

4). Cf. A-4, pp. 30-31

[redacted]

 [redacted]

Placzek[5] has also calculated

$$\frac{\partial^n}{\partial w^n}\left[\frac{\phi(w)}{1+w}\right]\Bigg|_{w=0}$$

using his result, we get:

$$(t^n)_{av} = \left(\frac{\ell}{v}\right)^{n+1} n! \prod_{k=1}^{n} (1-\lambda_k)^{-1} \tag{36}$$

Since the moments determine the generating function, it is clear that

$M_0(u,t) = (\ell/v)/f(x)$. where $x = vt/\ell$.

We must now obtain an approximate expression for $f(x)$. We first notice from the moments that for large x, $f(x)$ behaves like $e^{-x}\, x^{2/(1-r^2)}$.

This follows from the fact that:

$$\log \prod_{k=1}^{n} (1-\lambda_k)^{-1} = \sum_{k=1}^{n} \log(1-\lambda_k) \approx \sum_{k=1}^{n} \log\left[1 - \frac{2}{(k+2)(1-r^2)}\right]^{-1}$$

$$\approx \sum_{k=1}^{n} \frac{2}{(k+2)(1-r^2)} \underset{\text{asymp.}}{\approx} \frac{2}{(1-r^2)} \log n = \log n^{\frac{2}{1-r^2}}$$

and

$$(x^n)_{av} \approx \int_0^\infty e^{-x}\, x^{\frac{2}{(1-r^2)}}\, x^n\, dx = \Gamma\left(\frac{2}{1-r^2}+n+1\right) \approx n!\; n^{\frac{2}{(1-r^2)}}$$

Secondly, we notice that the integral equation (34) is equivalent to a differential equation with an essential singularity. The simplest essential singularity (at least for integration purposes) is exhibited by $e^{-b/x}$ (b a constant). We therefore try:

$$f_E(x) = A\, e^{-\left(\frac{b}{x}+x\right)}\, x^{\frac{2}{(1-r^2)}} \tag{37}$$

5). Cf. A-25, pp. 16-17 [redacted]

 [redacted]

where A is a normalization constant. The advantage of (37) is that integrals of the form

$$\int_0^\infty e^{-\left(\frac{b}{x}+x\right)} x^{n+\frac{1}{2}}\, dx$$

(n an integer) are readily evaluable.

We now specialize the above to the case M = 15 (air) although the procedure is capable of obvious generalization. In this case $2/(1-r^2) = 8.533$ so that we choose n = 8. To determine b, we maximize $f_E(x)$ at the point $x = x_{av} = 16.364$; we find b = 128.132. Computing the higher moments by recurrence relations, we find empirically:

$$\frac{\int_0^\infty f_E(x)\, x^n\, dx}{\int_0^\infty f(x)\, x^n\, dx} = 1 + a(n+1) \tag{38}$$

where a varies monotonically from .054 to .063 as n = 1, 2, --- 30. Therefore, choosing A = .058, we can rewrite (38) after integrating by parts):

$$\int_0^\infty f_E(x)\, x^n\, dx = \int_0^\infty f(x)\, x^n\, dx - a\int_0^\infty x^{n+1} f'(x)\, dx \tag{39}$$

we can satisfy (39) if f(x) satisfies the differential equation:

$$f - a x f' = f_E \tag{40}$$

with the boundary condition $f(\infty) = 0$. The solution of (40) is:

$$f(x) = \frac{1}{a} x^{\frac{1}{a}} \int_x^\infty \frac{f_E(x')}{x'^{\left(1+\frac{1}{a}\right)}}\, dx' \tag{41}$$

[redacted]

[redacted]

If (41) is normalized, we obtain for the final result:

$$f(x) = 17.241 \; x^{17.241} \int_x^\infty x'^{-9.741} e^{-x' - \frac{128.132}{x'}} \, dx' \tag{42}$$

It is clear that the value of $f(x)$ at $x = x_{av}$ is equal to that of $f_E(x)$; this follows from (40) since $f^1(x_{av}) = 0$. If further accuracy is desired, one can express:

$$\frac{\int_0^\infty f_E(x)\, x^m \, dx}{\int_0^\infty f(x)\, x^m \, dx} = 1 + a_1(m+1) + a_2(m+1)(m+2) + \cdots \tag{43}$$

and obtain higher order differential equations. These higher order equations become increasingly laborious although they are always of Cauchy type and thus soluble.

The ratio, r_n, of the n^{th} moment obtained by means of (42) to the n^{th} moment defined by $n! \prod_{k=1}^{m} (1 - \lambda_k)^{-1}$ (cf. (36)) is given in Table IV for the first thirty moments. It is seen that the deviation from one is never greater than .046. The distribution function given by (42) should therefore be quite accurate up to fairly large values of x. Fig. 3 contains a log-log plot of $f(x)$ as a function of x up to x = 35.

[redacted]

[redacted]

TABLE IV

RATIO OF MOMENTS OF EMPIRICAL FUNCTION TO MOMENTS OF ACTUAL DISTRIBUTION FUNCTION

n	r_n
0	1
1	.999
2	.998
3	.997
4	.996
5	.996
6	.996
7	.996
8	.996
9	.997
10	.999
11	1.000
12	1.002
13	1.004
14	1.006
15	1.008
16	1.011
17	1.013
18	1.016
19	1.018
20	1.021
21	1.024
22	1.026
23	1.029
24	1.032
25	1.034
26	1.037
27	1.039
28	1.042
29	1.044
30	1.046

[redacted]

 [redacted]

SOLUTION OF AGE EQUATION FOR SEMI-INFINITE MEDIUM WITH PLANE SOURCE AND "BLACK" BOUNDARY CONDITION.

APPENDIX I

The time-dependent and capture terms in equation (18) merely introduced into the solution (20) the factors

$$e^{-\frac{3}{\ell}\int_0^{\tau}\frac{d\tau}{\ell_c(\tau)}}\quad \delta\left[t-\frac{2\ell}{a v(0)}\left(e^{\frac{3a\tau}{2\ell^2}}-1\right)\right]$$

The essential features of a Laplace transform solution of the problem of a semi-infinite medium with plane source may therefore be seen from the stationary age equation without capture. The latter equation is:

$$\frac{\partial^2 q}{\partial z^2} = \frac{\partial q}{\partial \tau} - \delta(z-z_0)\,\delta(\tau) \tag{44}$$

We must solve (44) subject to the boundary condition-

$$q - \frac{2}{3}\ell\frac{\partial q}{\partial z} = 0 \qquad \text{at } z = 0$$

We take the Laplace transform of both sides of (44) and of the boundary condition. We assume constant mean free path and measure lengths in terms of it. We get:

$$\frac{\partial^2 \phi}{\partial z^2} = \eta\,\phi - \delta(z-z_0) \tag{45}$$

subject to: $\phi - \frac{2}{3}\frac{\partial \phi}{\partial z} = 0 \quad \text{at } z = 0$

[redacted]

 [redacted]

We distingquish ~~two~~ regions, $z > z_0$ and $z < z_0$ ($z = 0$ is the edge of the semi-infinite medium); we have:

$$\phi_I = C(\eta)\, e^{-(z-z_0)\sqrt{\eta}} \qquad z > z_0 \tag{46a}$$

$$\phi_{II} = A(\eta)\, e^{-(z-z_0)\sqrt{\eta}} + B(\eta)\, e^{(z-z_0)\sqrt{\eta}} \qquad z < z_0 \tag{46b}$$

At $z = z_0$:

$$\phi_I = \phi_{II}$$

$$\frac{\partial \phi_I}{\partial z} - \frac{\partial \phi_{II}}{\partial z} = -1 \tag{47}$$

At $z = 0$:

$$\phi_{II} - \frac{2}{3}\frac{\partial \phi_{II}}{\partial z} = 0 \tag{48}$$

Substituting (46a) and (46b) into (47) and (48) yields:

$$\phi_I = -\frac{1}{2\sqrt{\eta}}\left[\left(\frac{1-\frac{2}{3}\sqrt{\eta}}{1+\frac{2}{3}\sqrt{\eta}}\right)e^{-2z_0\sqrt{\eta}} - 1\right] e^{-(z-z_0)\sqrt{\eta}} \tag{49a}$$

[redacted]

[redacted]

$$\Phi_{II} = \frac{1}{2\sqrt{\eta}}\, e^{+(z-z_0)\sqrt{\eta}} - \frac{1}{2\sqrt{\eta}}\left(\frac{3/2-\sqrt{\eta}}{3/2+\sqrt{\eta}}\right) e^{-(z+z_0)\sqrt{\eta}} \qquad (49b)$$

The Laplace inverse of ϕ_{II} (required for Section B) is obtained most simply by making use of the theorem[6] that if $\mathcal{L} f(\tau) = g(\eta)$, then

$$\mathcal{L}\left[\frac{1}{\sqrt{\pi\tau}}\int_0^\infty e^{-\frac{x^2}{4\tau}} f(x)\, dx\right] = \frac{g(\sqrt{\eta})}{\sqrt{\eta}}$$

Let us write $\Phi_{II} = \dfrac{g(\sqrt{\eta})}{\sqrt{\eta}}$; therefore:

$$g(\eta) = \frac{1}{2}\, e^{(z-z_0)\eta} - \frac{1}{2}\left(\frac{3/2-\eta}{3/2+\eta}\right) e^{-(z+z_0)\eta}$$

The Laplace inverse of $g(\eta)$, namely $f(\tau)$, is:

$$f(\tau) = \frac{1}{2}\,\delta\left[\tau-(z_0-z)\right] - \frac{3}{4}\, e^{-\frac{3}{2}\left[\tau-(z_0+z)\right]}$$
$$+\frac{1}{2}\,\delta\left[\tau-(z_0+z)\right] - \frac{3}{4}\, e^{-\frac{3}{2}\left[\tau-(z_0+z)\right]}$$

$$\text{for } \tau \le (z_0+z) \qquad (50a)$$

$$f(\tau) = 0 \qquad \text{for } \tau > (z_0+z) \qquad (50b)$$

Equations (50a) and (50b) are derived by making use of the two simple identities:

$$\mathcal{L}^{-1} e^{-k\eta} = \delta(\tau-k)$$

$$\mathcal{L}_\eta^{-1}\left[\frac{1}{a+\eta}\right] = e^{-a\tau}$$

6) McLachlan and Humbert - "Tables of Laplace Transforms".

[redacted]

[redacted]

and of the convolution theorem. Now $q_{II} = \mathcal{L}_{\eta}^{-1}\,\phi_{II}$

and hence

$$q_{II} = \frac{1}{\sqrt{\pi\tau}}\int_0^\infty e^{-\frac{x^2}{4\tau}} f(x)\,dx \tag{51}$$

Substituting for f(x) into (51) yields:

$$q_{II} = \frac{e^{-\frac{(z-z_0)^2}{4\tau}}}{\sqrt{4\pi\tau}} + \frac{e^{-\frac{(z+z_0)^2}{4\tau}}}{\sqrt{4\pi\tau}} - \frac{3}{2}\,e^{\frac{3}{2}(z+z_0)+\frac{9}{4}\tau}\left[1-\operatorname{erf}\left(\frac{z+z_0}{2\sqrt{\tau}}+\frac{3\sqrt{\tau}}{2}\right)\right] \tag{52}$$

Equation (52) is the desired result (cf. equation (20).

It is interesting to note that the usual relation between the plane-source solution and the point-source solution, i. e.

$$q_{pl}(z,\tau) = \int_0^\infty 2\pi\rho\, q_{pt}(z,\rho,\tau)\,d\rho$$

holds for a semi-infinite medium with "black" boundary condition. As an illustration of this point and for some applications, we have evaluated the point-source solution due to a point-source of fast neutrons located at z = o. Using equation (9) of LA-257 and the boundary condition $q-(2/3)\,\partial q/\partial z = 0$ $(z=o)$, we obtain by methods discussed in Section A:

$$q(z,\rho,\tau) = \frac{e^{\frac{3}{2}z-\frac{\rho^2}{4\tau}+\frac{9\tau}{4}}}{(4\pi\tau)^{3/2}}\left\{\frac{3\sqrt{\pi\tau}}{2}\left[1-\operatorname{erf}\left(\frac{z}{2\sqrt{\tau}}+\frac{3\sqrt{\tau}}{2}\right)\right] - e^{-\frac{(z+3\tau)^2}{4\tau}}\right\} \tag{53}$$

Substitution of (53) into the integral $\int_0^\infty 2\pi\rho\, q(z,\rho,\tau)\,d\tau$ leads immediately to (52) with $z_0 = 0$

[redacted]

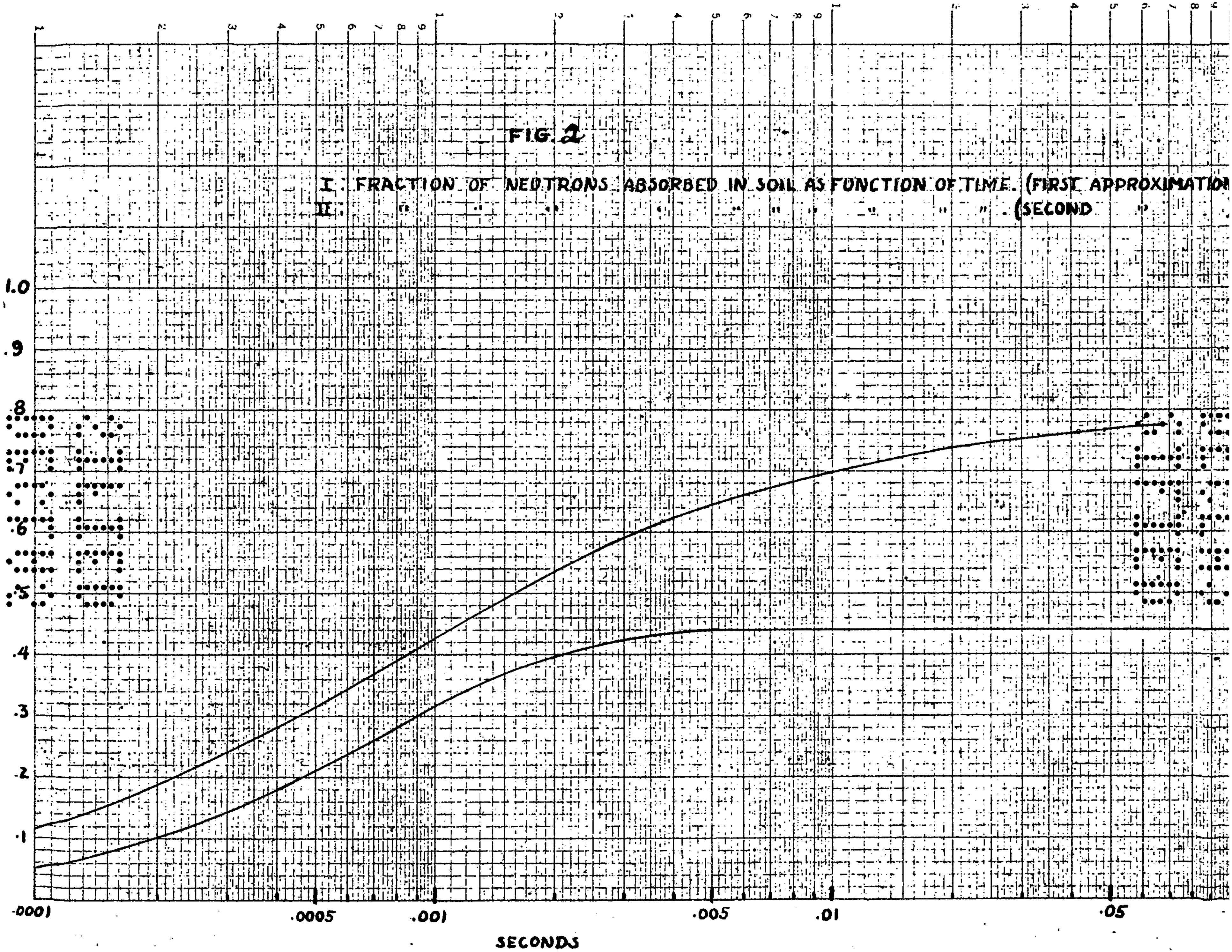

FIG. 2

I. FRACTION OF NEUTRONS ABSORBED IN SOIL AS FUNCTION OF TIME. (FIRST APPROXIMATION

II. " " " " " " " " " . (SECOND "

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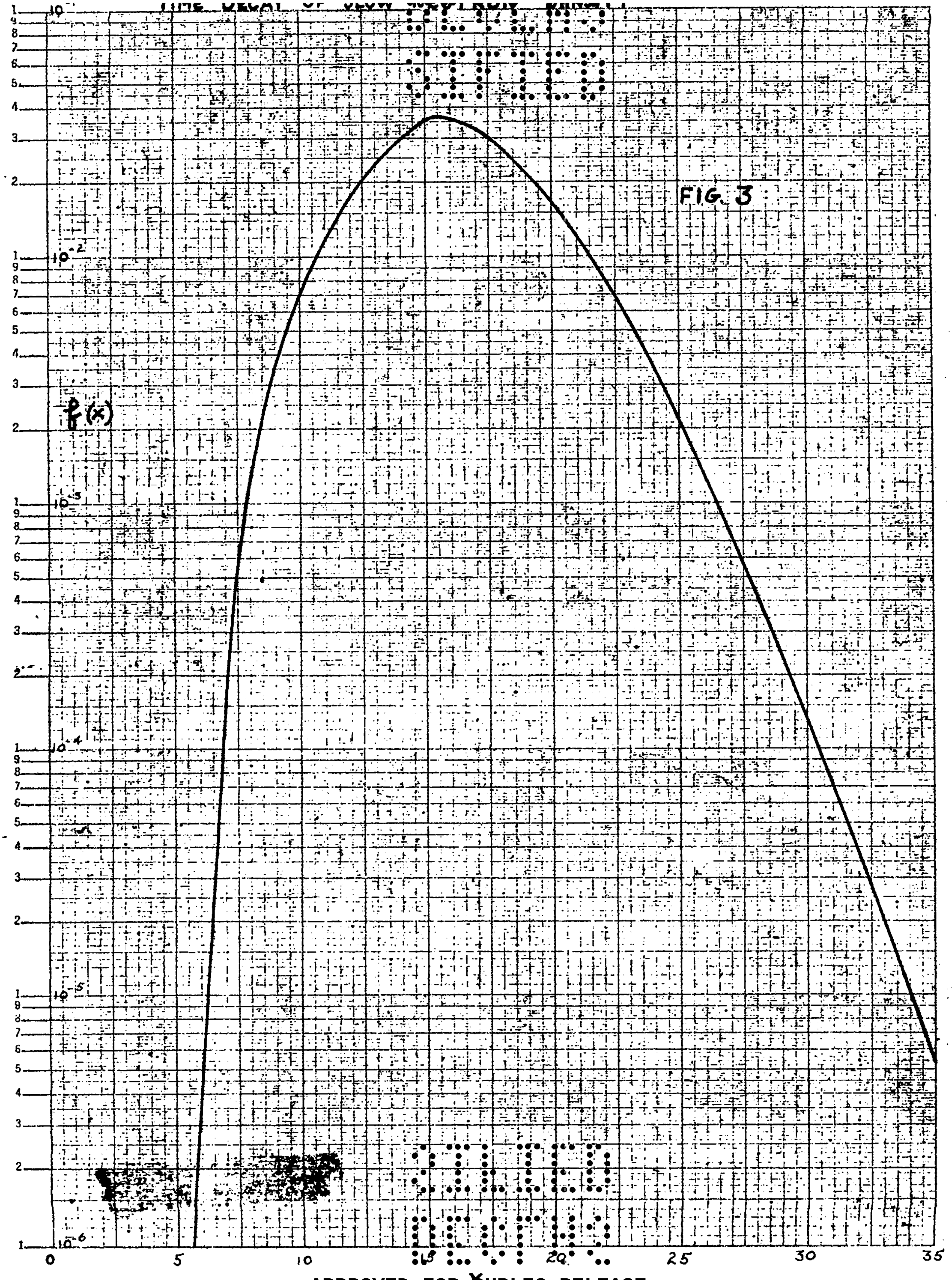

FIG. 3

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