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CALCULATIONS OF THE CRITICAL MASS OF UF_6 AS A GASEOUS CORE, WITH REFLECTORS OF D_2O, Be AND C

by
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Numerical computations by
Max Goldstein and Group T-1

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ABSTRACT

A class of critical assemblies has been considered in which the core consists of an oralloy bearing gas at more or less standard pressure and temperature and the reflector is an efficient moderator with a small thermal capture cross section. Age and diffusion theory have been applied to compute the probability that a fast neutron is thermalized by the reflector and then captured by the core. It is first assumed that fast or epithermal neutrons do not interact with the core and it is later shown that this assumption should be a good one for many cases. The reflector may be a thick spherical shell.

It is shown that oralloy masses of a few kilograms can be critical within thick reflectors of D_2O, Be, C, or combinations thereof. Curves are attached which enable one to estimate critical configurations.

INTRODUCTION

It is well known that the critical mass of an untamped or inefficiently tamped gaseous core at ordinary gas densities is many orders of magnitude larger than the critical mass of a corresponding solid system. In fact, if one imagines that a solid or liquid critical system is uniformly decreased in density, the critical mass will vary as the reciprocal of the square of the density. Since gas densities are of the order of 10^{-3} x solid densities, this implies critical masses of the order of 10^{6} times as large for gaseous systems as for solid ones.

The above argument cannot, of course, be applied to a gaseous core surrounded by an efficient moderating reflector for in such a configuration it is only necessary that (1) the reflector thermalize and return to the core a sufficient fraction of the fission neutrons and (2) the core be of the order of a thermal neutron mean free path in thickness. The second condition is easily fulfilled by systems of moderate size since the mean free path for example of a thermal neutron in UF_6 at $70^{\circ}C$ is about 70 cm. The first condition can also be satisfied by a number of efficient moderators such as heavy water, beryllium, and graphite.

In this paper I have employed age and diffusion theory to calculate some critical configurations. I have assumed first that fast and epithermal neutrons do not interact with the core at all. The slowing down of neutrons in the reflector is treated according to

age theory and the source of thermal neutrons is taken from age theory. The diffusion of thermal neutrons is then calculated by use of diffusion theory.

In Section I, the problem is treated in plane geometry, with the reactive volume bounded by semi-infinite or finite slabs. I calculate the probability that a neutron which is born in the gas is thermalized by the moderator and captured in the gas. In Section II, the problem is treated in spherical geometry with the gas confined at the center of an infinite or finite sphere. Here we find that the radius of the sphere must be larger than the thermal diffusion length and/or square root of the age. The boundary condition at the core reflector interface is handled by means of an albedo.

In Section III, there are presented some very simple considerations regarding the angular distribution of thermal neutrons at the core-reflector interface and these are related to the core albedo. In Section IV, there is a discussion of the reactions of epithermal neutrons with the core. I conclude that for small spherical systems, such reactions are unlikely to change the reactivity by greater than about 1%.

SECTION I. PLANE GEOMETRY

A. INFINITELY THICK REFLECTORS

Consider first that we have a gas filled region bounded by two semi-infinite plane reflectors. Fission neutrons are born in the gas, thermalized in the reflectors and may be captured* by the gaseous fissionable material. Evidently departures from infinite plane geometry are likely to reduce the fraction of neutrons captured in the gas.

In one of the semi-infinite bounding planes, the slowing down density corresponding to a source of one fission neutron per second in the gas is

$$\chi(z, \theta) = \frac{e^{-z^2/4\theta}}{\sqrt{4\pi\theta}} \qquad (1)$$

where z is distance from the plane interface and θ is the age. The normalization is such that $\int_0^\infty \chi(z, \theta)dz = 1/2$. The slowing down density goes to zero as $z \longrightarrow \infty$ and has zero current at the gas-reflector boundary. This latter boundary condition is appropriate to our assumption that neutrons cannot react with the core while slowing down.

The slowing down density at the age to thermal forms a source of thermal neutrons. The thermal neutron density $\rho(z)$ then satisfies the diffusion equation

*I shall often refer to the disappearance of a neutron from the thermal group as "capture," but it should be understood that this may refer to either radiative capture or fission. Where the distinction is important it will be made.

$$\frac{\partial^2 \rho(z)}{\partial z^2} - \frac{\rho(z)}{L^2} = -\frac{3}{v\ell_t} \frac{e^{-z^2/4\theta}}{\sqrt{4\pi\theta}} . \qquad (2)$$

where v is the neutron velocity, ℓ_t the transport mean free path for thermal neutrons in the reflector, and L the thermal diffusion length (namely, $L = \sqrt{\frac{1}{3}\ell_t \ell_c}$ with ℓ_c the capture mean free path for thermal neutrons in the reflector). This equation must be solved subject to the boundary condition that as $z \to \infty$, $\rho \to 0$, and some other boundary condition at the gas reflector interface. In keeping with our optimistic assumptions we temporarily call this latter boundary condition: $\rho \to 0$ as $z \to 0$, while later on an albedo will be taken into account.

Equation (2) may be solved by use of integral transforms[1], or by means of the appropriate Green's function, $e^{-|z-z'|/L}$. At any rate the solution is found to be:

$$\rho(z) = A\, e^{-z/L} + \frac{3L}{4v\ell_t} e^{\theta/L^2} \left[e^{-z/L} \left\{ \operatorname{erf}\left(\frac{z}{2\sqrt{\theta}} - \frac{\sqrt{\theta}}{L} \right) + 1 \right\} + e^{z/L} \left\{ 1 - \operatorname{erf}\left(\frac{z}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{L} \right) \right\} \right] \qquad (3)$$

where A is to be chosen to satisfy $\rho(0) = 0$ and

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du.$$

1) viz. Sneddon, Fourier Transforms, McGraw-Hill (1951), Chapter 6.

It follows that

$$A = \frac{3L}{2v\ell_t}\, e^{\theta/L^2}\left\{\operatorname{erf}\left(\frac{\sqrt{\theta}}{L}\right) - 1\right\}.$$

The current into the gas, which is equal to half the probability of capture in the gas, is given by

$$\frac{v\ell_t}{3}\left(\frac{\partial \rho}{\partial z}\right)_{z=0} = -\frac{v\ell_t}{3L}\, A$$

so that the probability that a neutron is captured in the gas, Po, is:

$$Po = e^{\theta/L^2}\left[1 - \operatorname{erf}\frac{\sqrt{\theta}}{L}\right]. \qquad (4)$$

Note that Po is a function only of the ratio of the distance that a neutron goes while slowing down in the reflector, $\sqrt{\theta}$, to the distance it goes after thermalization, L. Evidently if a reflector does not have $Po \gtrsim 0.50$ it is inefficient and large critical masses are likely to result. This condition implies $\sqrt{\theta} \lesssim 0.77L$.

I have taken the constants for a number of materials from Glasstone and Edlund[2] and computed Po, with the results noted in Table I.

TABLE I

Material	$\theta(cm^2)$	L(cm)	$\sqrt{\theta}/L$	Po eq. (4)
H_2O	33	2.88	2.00	0.255
D_2O	120	100	0.110	0.887
Be	98	23.6	0.420	0.659
C	350	50.2	0.373	0.687

[2] Glasstone and Edlund, Elements of Nuclear Reactor Theory, D. Van Nostrand Co. (1952), pp. 127, 183.

We see that heavy water returns almost 89% of the fission neutrons as thermals and is extremely efficient from this point of view.

Our interior boundary condition can be improved upon by taking the density to vanish at an extrapolated boundary. Suppose that the gas region is of finite thickness and there is a probability that a neutron entering the gas from one semi-infinite reflector will cross the gas region without interacting. We call this probability the albedo, β. (This consideration ignores elastic scattering in the gas, which is usually unimportant.) We may then take the interior boundary condition to be (viz. ref. 2) :

$$\left(\frac{\partial \rho}{\partial z}\right)_{z=0} = \Gamma \rho(0) \tag{5a}$$

with

$$\Gamma = \frac{3}{2\ell_t} \frac{1-\beta}{1+\beta} . \tag{5b}$$

Substituting eq. (3) in (5a), we find

$$-\frac{1}{L} A = \Gamma (A - A_o)$$

where A_o is that value of A which satisfied the previous boundary condition, i. e., $\Gamma = \infty$. It follows that the probability of capture $Po(\beta)$ is now given by

$$P_o(\beta) = \frac{Po}{1 + \frac{1}{L\Gamma}} . \tag{6}$$

We can see how bad an error was made by using Po for the black core by looking at $Po(\beta)$ for $\beta = 0$. A comparison is made in Table II.

One can also use the above equations to estimate how thin a slab of material can be made critical. For albedos near unity the angular distribution of neutrons emerging from a surface is presumably nearly proportional to $\cos\theta$ with θ the angle between neutron direction and normal to the surface. (Viz., Section IV.) In such a case $\beta \simeq 2\, E_3(d/\lambda)$ where d/λ is the thickness of the gas region in capture mean free paths. If capture of at least 48% of the neutrons is needed for criticality, the minimum thickness can be found by setting $Po(\beta) = 0.48$. Results are shown in Table II.

TABLE II

Material	λ_t(cm)	Po (Table I)	Po (0) Eq. (6)	Minimum d/λ
D_2O	2.40	0.887	0.873	.020
Be	2.10	0.659	0.622	.183
C	2.71	0.687	0.663	.091

If we are considering UF_6 one atmosphere and 70°C, λ is about 70 cm, so that the minimum thicknesses become 1.40, 12.8, and 6.4 cm for D_2O, Be, and C.

B. REFLECTOR OF FINITE THICKNESS

Suppose that we now have a region of gas bounded by two reflectors of finite thickness. The age equation is now to be solved subject to the conditions that the slowing down density vanish at the external boundary of a reflecting slab and that it have zero slope at the gas-reflector boundary. Consider one of the reflecting slabs which lies

[redacted]

between the planes $z = 0$ and $z = D$. A solution to the age equation having the above properties is

$$\chi(z, \theta) = \frac{1}{\sqrt{4\pi\theta}} \left[e^{-z^2/4\theta} - \frac{e^{-(z-2D)^2/4\theta} + e^{-(z+2D)^2/4\theta}}{1 + e^{-2D^2/\theta}} \right] \qquad (7)$$

where the plane $z = 0$ divides gas and reflector. This solution applies only for the region $0 \leq z \leq D$.

When the diffusion equation is solved with the source given by Eq. (7), we obtain

$$\rho(z) = A\, e^{-z/L} + B\, e^{z/L} + \frac{3L}{4v\ell_t}\, e^{\theta/L^2} \left[e^{-z/L} \left\{ \operatorname{erf}\left(\frac{z}{2\sqrt{\theta}} - \frac{\sqrt{\theta}}{L} \right) + 1 \right\} \right.$$

$$+ e^{z/L} \left\{ 1 - \operatorname{erf}\left(\frac{z}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{L} \right) \right\} - \frac{1}{1 + e^{-2D^2/\theta}} \left\{ e^{-z/L}\, e^{2D/L} \left\{ 1 + \right. \right.$$

$$\left. \operatorname{erf}\left(\frac{z}{2\sqrt{\theta}} - \frac{D}{\sqrt{\theta}} - \frac{\sqrt{\theta}}{L} \right) \right\} + e^{z/L}\, e^{-2D/L} \left\{ 1 - \operatorname{erf}\left(\frac{z}{2\sqrt{\theta}} - \frac{D}{\sqrt{\theta}} + \frac{\sqrt{\theta}}{L} \right) \right\}$$

$$+ e^{-z/L}\, e^{-2D/L} \left\{ \operatorname{erf}\left(\frac{z}{2\sqrt{\theta}} + \frac{D}{\sqrt{\theta}} - \frac{\sqrt{\theta}}{L} \right) + 1 \right\}$$

$$\left. + e^{z/L}\, e^{2D/L} \left\{ 1 - \operatorname{erf}\left(\frac{z}{2\sqrt{\theta}} + \frac{D}{\sqrt{\theta}} + \frac{\sqrt{\theta}}{L} \right) \right\} \right\} \right] . \qquad (8)$$

For boundary conditions we may take $\rho(D) = \rho(0) = 0$. From the condition at $z = D$, we find

[redacted]

$$A\, e^{-D/L} + B\, e^{D/L} = -\frac{3L}{4v\ell_t}\, e^{\theta/L^2}\left[\frac{e^{-2D^2/\theta}}{1 + e^{-2D^2/\theta}}\left\{e^{-D/L}\left(\operatorname{erf}\left(\frac{D}{2\sqrt{\theta}} - \frac{\sqrt{\theta}}{L}\right) + 1\right)\right.\right.$$

$$\left. + e^{D/L}\left(1 - \operatorname{erf}\left(\frac{D}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{L}\right)\right)\right\} - \frac{1}{1 + e^{-2D^2/\theta}}\left\{e^{-3D/L}\left(\operatorname{erf}\left(\frac{3D}{2\sqrt{\theta}} - \frac{\sqrt{\theta}}{L}\right) + 1\right)\right.$$

$$\left.\left. + e^{3D/L}\left(1 - \operatorname{erf}\left(\frac{3D}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{L}\right)\right)\right\}\right] = Q. \qquad (9)$$

The boundary condition at $z = 0$ gives

$$A + B = -\frac{3L}{4v\ell_t}\, 2e^{\theta/L^2}\left[1 - \operatorname{erf}\frac{\sqrt{\theta}}{L} - \frac{1}{1 + e^{-2D^2/\theta}}\left\{e^{2D/L}\left(1 - \operatorname{erf}\left(\frac{D}{\sqrt{\theta}} + \frac{\sqrt{\theta}}{L}\right)\right)\right.\right.$$

$$\left.\left. + e^{-2D/L}\left(1 + \operatorname{erf}\left(\frac{D}{\sqrt{\theta}} - \frac{\sqrt{\theta}}{L}\right)\right)\right\}\right] = M. \qquad (10)$$

The current at $z = 0$ is given by

$$\left.\frac{\ell_t v}{3}\,\frac{\partial \rho}{\partial z}\right)_{z=0} = \frac{\ell_t v}{3}\,\frac{B - A}{L}. \qquad (11)$$

This, when multiplied by 2, is the probability of capture in the gas.

We may solve equations (9) and (10) for B-A. Thus from Eq. (9):

$$B = -A\, e^{-2D/L} + Q\, e^{-D/L}.$$

Substituting in (10) gives

$$A = \frac{M - e^{-D/L}\, Q}{1 - e^{-2D/L}},$$

so that

$$B - A = - M \frac{1 + e^{-2D/L}}{1 - e^{-2D/L}} + Q \frac{2e^{-D/L}}{1 - e^{-2D/L}} . \qquad (12)$$

If we substitute this result into Eq. (11), we find that the probability of capture is a complicated function of D, θ, and L. However, we are mostly interested in values of D^2/θ considerably larger than unity. For such values, we may expand the error integrals (except for erf $\frac{\sqrt{\theta}}{L}$) in Eqs. (9) and (10). Assuming $e^{-D^2/\theta} << 1$, and $D >> \frac{2\theta}{L}$, is equivalent to equating the erfs to unity. We then find

$$P(D) \simeq \frac{1 + e^{-2D/L}}{1 - e^{-2D/L}} \; e^{\theta/L^2} \left(1 - \text{erf} \left(\frac{\sqrt{\theta}}{L} \right) - 2 \frac{e^{-2D/L}}{1 + e^{-2D/L}} \right) . \text{ Large D.} \qquad (13)$$

This is the probability for slabs of thickness D to thermalize a fission neutron and return it to the gas region. P(D) as computed from Eq. (13), is plotted in Figs. 1 for D_2O, Be, and C. P(D) has also been computed from Eqs. (9) - (12) for some values of $\frac{D}{\sqrt{\theta}} \geq 2.0$. A comparison of the results with those of Eq. (13) indicates that Eq. (13) is accurate to better than 1% for all values of $P(D) \gtrsim 0.45$.

If we take as the interior boundary condition that

$$\left(\frac{\partial \rho}{\partial z} \right)_{z = 0} = r \rho(0),$$

we find

$$A = \frac{rQ e^{-D/L} - Mr - \frac{Q e^{-D/L}}{L}}{r - r e^{-2D/L} + \frac{1 + e^{-2D/L}}{L}}$$

[redacted]

whence

$$P(\beta, D) = \frac{P(D)}{1 + \frac{1}{L\Gamma} \frac{(1 + e^{-2D/L})}{(1 - e^{-2D/L})}}, \qquad (14)$$

where Γ is given by Eq. (5b).

[redacted]

SECTION II. SPHERICAL GEOMETRY

A. INFINITELY THICK REFLECTOR

We now consider a spherical cavity which is surrounded by a sphere of very large radius. Fission neutrons are born in the gas which is contained in the cavity, and thermalized in the reflector. We seek the probability that a fission neutron is captured by the gas in the cavity.

The slowing down problem for a source in a spherical cavity in an infinite sphere has been solved by age theory, viz. ref. 1), page 229 ff. Let the radius of the cavity be a. Then the slowing down density at distance r from the center and at age θ is

$$\chi(r, \theta) = \frac{1}{4ar\left(\pi^3\theta\right)^{1/2}} \left[e^{-(r-a)^2/4\theta} - \frac{\sqrt{\pi\theta}}{a} e^{r-a/a} e^{\theta/a^2} \left(1 - \operatorname{erf}\left(\frac{r-a}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{a}\right)\right)\right] \tag{15}$$

This is a solution of the age equation satisfying the boundary conditions $\chi(r, \theta) \xrightarrow[r\to\infty]{} 0$ and $\left(\frac{\partial \chi}{dr}\right)_{r=a} = 0$. It is normalized so that

$$\int_a^\infty \chi(r, \theta)\, 4\pi r^2 dr = 1;$$

thus one neutron per sec is born in the cavity.

We assume that the density of thermal neutrons, $\rho(r)$, satisfies the diffusion equation:

$$\frac{\partial^2(r\,\rho)}{\partial r^2} - \frac{1}{L^2}(r\,\rho) = -\frac{3}{r\ell_t}\, r\,\chi(r, \theta), \tag{16}$$

[redacted]

where θ is here the age to thermal. The solution for $r\rho$ may again be found by appropriate use of the Green's function $e^{-|r-r'|/L}$. I obtain for the quantity $r\rho(r) = \psi(r)$,

$$\psi(r) = A e^{-(r-a)/L} + B e^{r-a/L} + \frac{3L}{8\ell_t v \pi a^2}\left[\frac{2a^2L}{L^2-a^2} e^{\theta/a^2} e^{r-a/a}\left\{1 - \right.\right.$$

$$\left.\left.\operatorname{erf}\left(\frac{r-a}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{a}\right)\right\} + e^{\theta/L^2}\left\{\frac{a^2}{L+a} e^{-(r-a)/L}\left\{\operatorname{erf}\left(\frac{r-a}{2\sqrt{\theta}} - \frac{\sqrt{\theta}}{L}\right) + 1\right\}\right.\right.$$

$$\left.\left. + \frac{a^2}{L-a} e^{r-a/L}\left\{\operatorname{erf}\left(\frac{r-a}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{L}\right) - 1\right\}\right\}\right]. \qquad (17)$$

It may be verified that (17) is indeed a solution of (16). For an infinite sphere, the boundary condition at ∞ implies $B = 0$. If we take for our interior boundary condition $\psi(a) = 0$, we obtain

$$A = \frac{3L}{8\ell_t v \pi a^2}\left[\frac{2a^2L}{L^2 - a^2} e^{\theta/a^2}\left(\operatorname{erf}\left(\frac{\sqrt{\theta}}{a}\right) - 1\right)\right.$$

$$\left. + e^{\theta/L^2}\left\{\frac{a^2}{L+a}\left(\operatorname{erf}\left(\frac{\sqrt{\theta}}{L}\right) - 1\right) + \frac{a^2}{L-a}\left(1 - \operatorname{erf}\left(\frac{\sqrt{\theta}}{L}\right)\right)\right\}\right] \qquad (18)$$

The current into the cavity will be given by

$$4\pi a^2 \frac{\ell_t v}{3} \frac{\partial(\psi/r)}{\partial r}\Bigg)_{r=a} = 4\pi a \frac{\ell_t v}{3}\left[\frac{\partial \psi}{\partial r}\Bigg)_{r=a} - \frac{\psi(a)}{a}\right] \qquad (19)$$

However, the solution (17) is so chosen that if $A = B = 0$, there is zero current at $r = a$. Therefore, the current into the cavity is given by

[redacted]

[redacted]

$$\text{Current} = -4\pi a \frac{\ell_t v}{3} A\left[\frac{1}{L} + \frac{1}{a}\right]. \tag{20}$$

This current is exactly equal to the probability of capture by the gas, so that substituting (18) into (20), we find

$$P(a) = \frac{1}{L - a}\left[L\, e^{\theta/a^2}\left(1 - \operatorname{erf}\left(\frac{\sqrt{\theta}}{a}\right)\right) - a\, e^{\theta/L^2}\left(1 - \operatorname{erf}\left(\frac{\sqrt{\theta}}{L}\right)\right)\right] \tag{21}$$

This is the probability for a neutron which is born in a spherical cavity of radius a inside an infinite sphere to be thermalized and returned to the core. "a" is presumably an extrapolated radius. We have plotted $P(a)$ in Fig. 2 for the reflectors D_2O, Be, and C.

If, as before, we take the interior boundary condition to be a condition upon the logarithmic derivative of the neutron density, we obtain:

$$\left(\frac{\partial \psi}{\partial r} - \frac{\psi}{a}\right)_{r=a} = \Gamma \psi(a). \tag{22}$$

The left side is simply $-\frac{L+a}{aL} A$, while the right side may be written $\Gamma (A - A_o)$, where by A_o we denote the value of the particular solution of the inhomogenous equation at $r = a$, namely the value of A given by Eq. (18). When we solve Eq. (22) for A, we find

$$\left(1 + \frac{1}{\Gamma}\frac{L + a}{aL}\right) A = A_o.$$

Thus

$$P(\beta, a) = \frac{P(a)}{1 + \frac{1}{\Gamma}\frac{L+a}{aL}} \tag{23}$$

where $P(a)$ is given by Eq. (21), Γ by Eq. (5b), and $P(\beta, a)$ denotes the probability for a neutron which is born in a spherical cavity of radius a inside an infinite sphere to be thermalized and captured by

[redacted]

the core. In Figs. 3a - 3c, are plotted $P(\beta, a) \div P(a)$ vs. β for a number of values of a and for the reflectors D_2O, Be, and C. This ratio is

$$\frac{1}{1 + \frac{2}{3} \ell_t \frac{L+a}{La} \frac{1+\beta}{1-\beta}} .$$

I have assumed that capture of 48% of the neutrons is required for criticality and from Figs. 2 and 3 computed critical sizes. I have assumed a mean free path for capture (and fission) in the gas of 70 cm, for UF_6 at 70°C. For the albedo, I have used results from Section III, Fig. 4. The critical sizes are shown in Table III:

TABLE III

Material	Critical Radius (cm)	Oy Mass (kg)
D_2O	28.5	0.90
Be	63	9.7
C	79	19.2

Evidently deuterium bearing reflectors are the most efficient reflectors. One could replace outer portions of the D_2O reflector by graphite, without much increasing the critical radius.

B. REFLECTOR OF FINITE THICKNESS, SPHERICAL GEOMETRY

Let us now consider the cavity of the previous problem to be surrounded by a spherical shell of finite thickness. Assume that the shell is thick enough to thermalize and return a substantial fraction ($\gtrsim 0.5$) of the neutrons to the core. Two approaches to the age and

diffusion problems suggest themselves: (1) one might expand the slowing down density and thermal neutron density as Fourier series or (2) one might try adding hypothetical sources as in Section I, B to satisfy the boundary conditions. I have attempted the second method with the feeling that it is more appropriate for a thick sphere, where presumably many modes of any series expansion would be required.

We seek a solution for the age equation which satisfies the boundary conditions of zero current at $r = a$ and zero density at $r = D + a$ where D is the thickness of the spherical shell. Because of lack of symmetry in the $+ r$ and $- r$ directions, it does not appear possible to add two fictitious sources as in Section I, B and thereby satisfy both boundary conditions exactly. However, by adding one fictitious source at $r = a + 2D$, I have been able to obtain a solution to the age equation which satisfies both boundary conditions to good approximation for thick spherical shells. The solution in question is:

$$r\,\chi(r,\theta) = r\,\chi_o(r,\theta) - \frac{1}{4a\pi\sqrt{\pi\theta}}\left[e^{-\frac{(r-2D-a)^2}{4\theta}} - \frac{\sqrt{\pi\theta}}{a}\,e^{\theta/a^2}\,e^{-\frac{r-2D-a}{a}}\left(1 - \operatorname{erf}\left(\frac{2D+a-r}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{a}\right)\right)\right], \tag{24}$$

where χ_o is the solution for the infinitely thick shell which is given by Eq. (15). The additional terms may be written $(r-2D-a)\,\chi_o(2D+a-r,\theta)$ and correspond to a negative source at $r = 2D + a$. They are a solution of the age equation because said equation for $\theta \neq 0$ is

$$\frac{\partial^2 r\,\chi(r,\theta)}{\partial r^2} = \frac{\partial r\,\chi(r,\theta)}{\partial \theta}$$

which is also satisfied if the derivative on the left is taken with respect to 2D+a-r.

$\chi(r, \theta)$ as given by Eq. (24) also has the property $\chi(D+a, \theta) = 0$. The other boundary condition is not satisfied by Eq. (24), for the current at $r = a$ is not zero. This means that Eq. (24) is a solution of that problem in which instead of just a perfect reflector at $r = a$, we have, in addition, a source of neutrons of all ages at $r = a$. However, it can be shown that for thick spherical shells, the current into the shell (at $r = a$) which results from these erroneous sources is very small compared to the current through the outer boundary of the sphere at $r = D+a$. Hence the fraction of the neutrons contributed to the problem by the erroneous sources is very small. I, therefore, conclude that for thick shells, Eq. (24) is a good approximation to the exact solution of the age equation subject to the two boundary conditions.

In Appendix I, it is shown that, for thick shells, the current at $r = a$ is indeed negligible compared to that at $r = D+a$, and that in fact the former is of order $e^{-3D^2/4\theta}$ with respect to the latter.

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The solution to the diffusion Eq. (16) with a source given by Eq. (24), follows from the discussion of Section II, A. If we again let $\rho = r\psi$, we obtain

$$\psi(r) = A\, e^{-r-a/L} + B\, e^{r-a/L} + \frac{3L}{8\ell_t v \pi a^2}\left[\frac{2a^2L}{L^2-a^2}\; e^{\theta/a^2}\; e^{r-a/a}\right.$$

$$\left(1 - \operatorname{erf}\left(\frac{r-a}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{a}\right)\right) + e^{\theta/L^2}\left\{\frac{a^2}{L+a} e^{-(r-a)/L}\left(1 + \operatorname{erf}\left(\frac{r-a}{2\sqrt{\theta}} - \frac{\sqrt{\theta}}{L}\right)\right)\right.$$

$$+ \frac{a^2}{L-a}\; e^{r-a/L}\left(\operatorname{erf}\left(\frac{r-a}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{L}\right) - 1\right)\Big\}$$

$$- \frac{2a^2L}{L^2-a^2}\; e^{\theta/a^2}\; e^{-\frac{r-2D-a}{a}}\left(1 - \operatorname{erf}\left(\frac{a+2D-r}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{a}\right)\right)$$

$$-e^{\theta/L^2}\left\{\frac{a^2}{L+a}\; e^{\frac{r-a-2D}{L}}\left(1 + \operatorname{erf}\left(\frac{2D+a-r}{2\sqrt{\theta}} - \frac{\sqrt{\theta}}{L}\right)\right)\right.$$

$$\left.\left. + \frac{a^2}{L-a}\; e^{\frac{2D+a-r}{L}}\left(\operatorname{erf}\left(\frac{2D+a-r}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{L}\right) - 1\right)\right\}\right] . \qquad (25)$$

The boundary condition at the outside of the spherical shell is

$$\psi(D+a) = A\, e^{-D/L} + B\, e^{D/L} = 0. \qquad (26)$$

The value of ψ at the interior boundary is

$$\Psi(a) = A + B + \frac{3L}{8\ell_t v \pi a^2}\left[\frac{2a^2L}{L^2-a^2}\; e^{\theta/a^2}\left\{1 - \operatorname{erf}\left(\frac{\sqrt{\theta}}{a}\right)\right.\right.$$

$$\left. - e^{2D/a}\left(1 - \operatorname{erf}\left(\frac{D}{\sqrt{\theta}} + \frac{\sqrt{\theta}}{a}\right)\right)\right\}$$

$$+ e^{\theta/L^2}\,\frac{a^2}{L+a}\left\{1 - \operatorname{erf}\left(\frac{\sqrt{\theta}}{L}\right) - e^{-2D/L}\left(1 + \operatorname{erf}\left(\frac{D}{\sqrt{\theta}} - \frac{\sqrt{\theta}}{L}\right)\right)\right\}$$

$$\left. + e^{\theta/L^2}\,\frac{a^2}{L-a}\left\{\operatorname{erf}\left(\frac{\sqrt{\theta}}{L}\right) - 1 + e^{2D/L}\left(1 - \operatorname{erf}\left(\frac{D}{\sqrt{\theta}} + \frac{\sqrt{\theta}}{L}\right)\right)\right\}\right] \qquad (27)$$

$$= A + B + T.$$

The current into the interior cavity is given by Eq. (19). We find that

$$\left.\frac{\partial \psi}{\partial r} - \frac{\psi}{r}\right)_{r=a} = -\frac{L+a}{aL} A + B \frac{a-L}{aL} + U \tag{28}$$

where U is given by:

$$U = \frac{3L}{8\ell_t v \pi a^2}\left[\frac{4aL}{L^2-a^2} e^{\theta/a^2} e^{2D/a}\left(1 - \operatorname{erf}\left(\frac{D}{\sqrt{\theta}} + \frac{\sqrt{\theta}}{a}\right)\right)\right.$$

$$+ \frac{a}{L}\frac{L-a}{L+a} e^{\theta/L^2} e^{-2D/L}\left(1 + \operatorname{erf}\left(\frac{D}{\sqrt{\theta}} - \frac{\sqrt{\theta}}{L}\right)\right)$$

$$\left. - \frac{a}{L}\frac{L+a}{L-a} e^{\theta/L^2} e^{2D/L}\left(1 - \operatorname{erf}\left(\frac{D}{\sqrt{\theta}} + \frac{\sqrt{\theta}}{L}\right)\right)\right]. \tag{29}$$

If for our interior boundary condition, we take $\psi(a) = 0$, it follows from Eqs. (26), (27) that

$$A = -\frac{T}{1 - e^{-2D/L}}.$$

Upon substitution in Eq. (28), we obtain:

$$\left.\frac{\partial \psi}{\partial r} - \frac{\psi}{r}\right)_{r=a} = T\left(\frac{1}{a} + \frac{1}{L}\frac{1 + e^{-2D/L}}{1 - e^{-2D/L}}\right) + U. \tag{30}$$

It is to be borne in mind that these results are valid only if $D \gtrsim 2\sqrt{\theta}$. Therefore, in the region of validity, we may expand the error functions of $D/\sqrt{\theta}$ and to good approximation equate them to unity. By this means we obtain

$$T \simeq \frac{3L}{4\ell_t v \pi a^2}\left[\frac{a^2 L}{L^2-a^2}\, e^{\theta/a^2}\left(1 - \operatorname{erf}\frac{\sqrt{\theta}}{a}\right) - \frac{a^3}{L^2+a^2}\left(1 - \operatorname{erf}\frac{\sqrt{\theta}}{L}\right) e^{\theta/L^2}\right.$$

$$\left. - e^{\theta/L^2}\,\frac{a^2}{L+a}\, e^{-2D/L}\right], \qquad (31)$$

and

$$U = \frac{3L}{4\ell_t v \pi a^2}\left[\frac{a}{L}\,\frac{L-a}{L+a}\, e^{\theta/L^2}\, e^{-2D/L}\right]. \qquad (32)$$

The current into the cavity (or return probability) follows from Eqs. (19) and (30)-(32):

$$P(D, a) \simeq \left[\frac{1 + e^{-2D/L}}{1 - e^{-2D/L}} + \frac{L}{a}\right]\frac{a}{L^2-a^2}\left\{L\, e^{\theta/a^2}\left(1 - \operatorname{erf}\frac{\sqrt{\theta}}{a}\right)\right.$$

$$\left. - a\, e^{\theta/L^2}\left(1 - \operatorname{erf}\frac{\sqrt{\theta}}{L}\right)\right\} - \frac{2a}{L+a}\,\frac{e^{-2D/L}}{1 - e^{-2D/L}}\, e^{\theta/L^2}. \qquad (33)$$

This is the probability for a neutron which is born in a cavity of radius a to be thermalized in a spherical shell of thickness D and returned to the core. Eq. (33) is plotted in Figs. 1a - 1c, as $P(D, a)$ vs. D for various values of a and the reflectors D_2O, Be, and C. A glance at Eqs. (13), (21), and (33) shows that $P(D, a) \xrightarrow[D \to \infty]{} P(a)$ and $P(D, a) \xrightarrow[a \to \infty]{} P(D)$.

Use of the more accurate boundary condition given by Eq. (22) leads to

$$-\frac{L+a}{aL} A + \frac{a-L}{aL} B + U = \Gamma(A + B + T). \qquad (34)$$

Upon solving for A with use of Eq. (26) we find

$$A = \frac{U - \Gamma T}{\left(\frac{1}{a} + \Gamma + \frac{1}{L}\,\frac{(1 + e^{-2D/L})}{(1 - e^{-2D/L})}\right)(1 - e^{-2D/L})} . \qquad (35)$$

Thus

$$\left(\frac{\partial \psi}{\partial r} - \frac{\psi}{r}\right)_{r=a} = \Gamma \left[\frac{U - \Gamma T}{\frac{1}{a} + \Gamma + \frac{1}{L}\,\frac{(1 + e^{-2D/L})}{(1 - e^{-2D/L})}} + T\right], \qquad (36)$$

and finally by comparison with Eq. (30), we see

$$P(\beta, D, a) = \frac{P(D, a)}{1 + \frac{1}{\Gamma}\left\{\frac{1}{a} + \frac{1}{L}\,\frac{(1 + e^{-2D/L})}{(1 - e^{-2D/L})}\right\}} . \qquad (37)$$

This expression represents the probability that a neutron, born within a cavity of radius a, is thermalized in the reflector (a spherical shell of thickness D) and then captured by the core. P(D, a) is given by Eq. (33), and Γ by Eq. (5b).

From the above information it appears that a sphere of UF_6 at 70°C, of about 40 cm radius and surrounded by 40 cm of D_2O + 40 cm of graphite would be critical. An aluminum shell of thickness ~0.5 cm could be between the gas and reflector. The oralloy mass would be about 2.5 kg.

[redacted]

SECTION III. ANGULAR DISTRIBUTION

In this section are presented some trivial considerations regarding the angular distribution of thermal neutrons emerging into the gas region. Some information about the angular distribution is useful for estimating the probability that a thermal neutron will traverse the gas region without an interaction which removes it from the thermal group. This probability I call the albedo, β, which determines the interior boundary condition on the thermal neutron density.

In considering the angular distribution, let us assume for simplicity that the reflector-gas interface is a plane. This should be a good approximation for spherical cavities of radius large compared to the scattering mean free path for thermal neutrons in the reflector. Let us further assume that when a thermal neutron suffers an elastic collision in the reflector, it emerges from the collision with an angular distribution which is isotropic in the laboratory system. Finally, because in the reflector the capture mean free path is very large compared to the scattering mean free path, we ignore, in these considerations, the possibility of capture in the reflector.

Let the plane $z = 0$ represent the interface with a moderator extending in the $+z$ direction, and let the density of thermal neutrons be $\rho(z)$. The collision density is obviously proportional to $\rho(z)$ and the number of neutrons which have suffered a collision at z and are heading in solid angle $\overrightarrow{d\Omega}$ is proportional to $\rho(z)\,\overrightarrow{d\Omega}$. Consider the neutrons which have made their last collision as a source of neutrons

[redacted]

which may emerge from the medium. Measuring z in units of the scattering mean free path and calling μ the cosine of the angle between neutron direction and $\overrightarrow{-z}$ we see that the number of neutrons emerging with directions between μ and $\mu + d\mu$ is proportional to

$$\int_0^\infty \rho(z) e^{-z/\mu} \, dz \, d\mu. \tag{38}$$

Hence the fraction emerging in these directions is

$$F(\mu)d\mu = \frac{\int_0^\infty \rho(z) \, e^{-z/\mu} \, dz \, d\mu}{\int_0^1 \int_0^\infty \rho(z) \, e^{-z/\mu} dz \, d\mu} . \tag{39}$$

Hence a knowledge of the neutron density enables one, with our assumptions, to predict the angular distribution.

For example, if the density is constant, $\rho = \rho_0$, we obtain

$$F(\mu) = 2\mu. \tag{40}$$

For such an angular distribution, the probability of crossing a slab of thickness d, capture + fission mean free path λ, and zero scattering, is

$$\int_0^1 2\mu e^{-d/\lambda\mu} \, d\mu = 2E_3(d/\lambda). \tag{41}$$

For a spherical cavity of diameter $2a$, the corresponding probability is

$$\int_0^1 2\mu e^{-2a\mu/\lambda} \, d\mu = 2\left(\frac{\lambda}{2a}\right)^2 \left[1 - \left(\frac{2a}{\lambda} + 1\right) e^{-2a/\lambda}\right]. \tag{42}$$

Equations (41) and (42) represent the albedos for slab and sphere under the assumption of constant neutron density near the surface.

However, if the albedo of the gaseous region is not large (close to unity), the density will not be nearly constant near the surface. In our

diffusion approximation, $\rho(z)$, will be determined by β (by means of the interior boundary condition), but also β is determined by $\rho(z)$ and geometry (through Eq. (39)). Hence a self-consistent approach is indicated. To this end, I expand $\rho(z)$ about $z = 0$ as $\rho = \rho_o + \rho_1 z$, where since

$$\left.\frac{\partial \rho}{\partial z}\right)_{z=0} = \Gamma\, \rho(0),$$

$\rho_1 = \Gamma\, \rho_o$. It follows that the angular distribution is proportional to $\mu + \Gamma \ell_t \mu^2$ and

$$F(\mu) = (1 + \beta)\mu + \tfrac{3}{2}(1 - \beta)\,\mu^2. \tag{43}$$

I now use this angular distribution to compute the albedo (β) for a sphere, namely:

$$\beta = \int_o^1 F(\mu)\, e^{-2a/\lambda \mu}\, d\mu. \tag{44}$$

Solving for β in terms of $2a/\lambda = x$, I find

$$\beta = \frac{1 + \frac{3}{x} - e^{-x}\left(\frac{5}{2}x + 4 + \frac{3}{x}\right)}{x^2 - 1 + \frac{3}{x} - e^{-x}\left(\frac{x}{2} + 2 + \frac{3}{x}\right)}. \tag{45}$$

In Fig. 4 have been plotted β vs. x from Eqs. (42) and (45). Of course, the value from (45) is smaller than that for (42) for all $\beta<1$; however, for small x the difference is small.

[redacted]

SECTION IV. CAPTURE DURING SLOWING DOWN

In this section, I wish to estimate neutron reactions with the gas during the slowing down process. That the effect for spherical geometry is apparently neither negligible nor overriding can be seen from the following: the density of neutrons during slowing down will be roughly constant in the central cavity and an adjacent reflector region of usually comparable volume. However, the nuclear density of the solid reflector is of order 10^3 times that of the gas. Hence the epithermal captures are likely to be characteristic of a system with one oralloy atom per 10^3 moderating atoms. Thus of the order of 10% of the captures and fissions may be of epithermal neutrons.

It is not a priori evident whether the effect of epithermal reactions is to increase or decrease reactivity. The probability for a neutron to undergo either capture or fission with oralloy evidently increases. However, the higher capture to fission ratio for epithermal neutrons makes it unclear whether the probability of fission per se will increase or decrease.

Where the effect of epithermal reactions is small, a perturbation approach suggests itself, as follows: let $\chi(r, \theta)$ be the slowing down density computed with no epithermal reactions. The flux per logarithmic energy interval is then

$$\phi(r, u)\, du = \frac{\chi(r, \theta)}{\xi \sum_s}\, du, \qquad (46)$$

where $\xi \sum_s$ is the macroscopic slowing down cross section and $u = \ln \frac{E_o}{E}$,

[redacted]

with E the neutron energy and E_o its source energy. Let us assume that the flux, at any age above thermal, is constant within the gas volume and is equal to $\phi(o, u)$ for a slab or $\phi(a, u)$ for a spherical cavity. Let us restrict our attention to spherical geometry in what follows.

The probability of a reaction per cm^3 and per logarithmic energy interval is then

$$\frac{\Sigma_r}{\xi \Sigma_s} \chi(a, \theta)\, du$$

where Σ_r is the macroscopic reaction cross section in the gas; namely, $N_g(\sigma_c + \sigma_f)$ with N_g the density of reactive atoms and σ_c and σ_f the capture and fission cross sections, respectively. We ignore elastic scattering in the gas. Then the probability of reaction per logarithmic energy interval, $p_r(\theta)$ is,

$$p_r(\theta)\, du = \frac{4\pi a^3}{3} \frac{\Sigma_r}{\xi \Sigma_s} \chi(a, \theta)\, du. \tag{47}$$

The probabilities of capture and fission, $p_c(\theta)$ and $p_f(\theta)$ are:

$$\begin{aligned} p_c(\theta) &= \frac{\sigma_c}{\sigma_c + \sigma_f} p_r(\theta) \\ p_f(\theta) &= \frac{\sigma_f}{\sigma_c + \sigma_f} p_r(\theta). \end{aligned} \tag{48}$$

Hence the probabilities of capture or fission during slowing down to thermal age θ_o, $u = u_o$, are

$$p_c^e = \frac{4\pi a^3}{3} \int_o^{u_o} \frac{\Sigma_c}{\xi \Sigma_s} \chi(a, \theta)\, du \tag{49a}$$

[redacted]

$$P_f^e = \frac{4\pi a^3}{3} \int_o^{u_o} \frac{\Sigma_f}{\xi \Sigma_s} \chi(a, \theta)\, du \tag{49b}$$

$$P_r^e = P_c^e + P_f^e. \tag{49c}$$

In Eqs. (49), Σ_c and Σ_f denote the macroscopic capture and fission cross sections in the gas, while Σ_s is the macroscopic scattering cross section in the reflector.

P_f^e is a probability of fission before thermalization. However, the source of thermal neutrons has been diminished by these reactions and is no longer $\chi(r, \theta_o)$ but

$$\chi(r, \theta_o) - \int_o^{u_o} p_r(\theta)\, \chi(r, \theta_o - \theta)\, du. \tag{50}$$

Furthermore the probability of capture or fission after thermalization is no longer $P(D, a, \theta_o)$ but

$$P(D, a, \theta_o) - \int_o^{u_o} p_r(\theta)\, P(D, a, \theta_o - \theta)\, du. \tag{51}$$

Thus the total probability of fission may be written

$$\begin{aligned} P_f &= \frac{\Sigma_f(th)}{\Sigma_r(th)} P(D, a, \theta_o) + \\ &+ \frac{4\pi a^3}{3} \int_o^{u_o} \chi(a, \theta) \frac{\Sigma_f}{\xi \Sigma_s} \left[1 - \frac{\Sigma_r}{\Sigma_f} \frac{\Sigma_f(th)}{\Sigma_r(th)} P(D, a, \theta_o - \theta) \right] du \\ &= \frac{P(D, a, \theta_o)}{1 + \alpha(th)} + \frac{4\pi a^3}{3} \int_o^{u_o} \chi(a, \theta) \frac{\Sigma_f(u)}{\xi \Sigma_s(u)} E\, du \end{aligned} \tag{52}$$

where

$$E = 1 - \frac{1 + \alpha(u)}{1 + \alpha(th)} P(D, a, \theta_o - \theta). \tag{53}$$

[redacted]

[redacted]

In the above equations, θ, Σ_f, Σ_r, and Σ_s will in general be functions of u. α is the capture to fission ratio, namely Σ_c/Σ_f.

Whether or not epithermal reactions make the system more or less critical depends on the sign of the integral in (52). The weighted average of the function E determines this sign. Note that the first term in E (unity) represents the increase in fissions due to direct epithermal fission while the second represents the loss in thermal fissions due to the disappearance of neutrons in epithermal reactions.

E is evidently a complicated function. However, on the average it will be small compared to unity. The term $1 + \alpha(u)/1 + \alpha(th)$ is likely to average around 1.20 and values of $P(D, a, \theta_o - \theta)$ about 0.85 are reasonable so that it is likely that $|\bar{E}| \sim 0.05$. By $\bar{E}$ I here mean the average weighted as in the integral in Eq. (52).

Let us now see how these results look for the case of infinite spherical reflectors. From Eq. (15) we find

$$\chi(a, \theta) = \frac{1}{4\pi a^2 \sqrt{\pi\theta}} \left[1 - \frac{\sqrt{\pi\theta}}{a} e^{\theta/a^2} \left(1 - \operatorname{erf} \left(\frac{\sqrt{\theta}}{a} \right) \right) \right]. \qquad (54)$$

Remembering that the original source is normalized to one neutron per sec, we see that the probability of fission in the epithermal region is

$$P_f^e = \frac{1}{3} \int_o^{u_o} \left(\frac{a}{\sqrt{\pi\theta}} - e^{\theta/a^2} \left(1 - \operatorname{erf} \left(\frac{\sqrt{\theta}}{a} \right) \right) \right) \frac{\Sigma_f(u)}{\xi \Sigma_s(u)} \, du. \qquad (55)$$

For most of the interesting energy range and many elements Σ_s is a constant θ proportional to u while $\Sigma_f(u)$ is a complicated function

[redacted]

with resonances. Thus the integral P_f^e must be computed numerically for an accurate evaluation. However, for at least a large energy interval, roughly 10 kev $\gtrsim E \gtrsim$ 10 ev, $\Sigma_f(u)$ on the average behaves as $E^{-1/2}$, while for some lower energy intervals it may be possible to consider Σ_f as a constant.

At any rate, if we assume $\theta = \dfrac{u}{3\xi\Sigma_s\Sigma_t}$ and that $\xi\Sigma_s$, Σ_t, and Σ_f are constant, we find (for the interval $\theta_1 \leq \theta \leq \theta_2$):

$$P_f = \Sigma_f \Sigma_t \, a^2 \left[e^{\theta_1/a^2} \left(1 - \operatorname{erf} \frac{\sqrt{\theta_1}}{a} \right) - e^{\theta_2/a^2} \left(1 - \operatorname{erf} \frac{\sqrt{\theta_2}}{a} \right) \right] \quad (56)$$

If we assume that Σ_f behaves as $E^{-1/2}$ and the above conditions on other parameters,

$$P_f^e = \frac{\Sigma_f(\text{final})}{3\xi\Sigma_s} \, e^{-u_o/2} \int_o^u e^{u/2} \left\{ \frac{a}{\sqrt{\pi\theta}} - e^{\theta/a^2} \left(1 - \operatorname{erf} \frac{\sqrt{\theta}}{a} \right) \right\} du$$

which expression I have not evaluated analytically. However, in general $e^{u/2}$ will be a rapidly varying function of the integrand compared to the other factor. Hence we may approximate the latter factor by its value at $u = u_o$, obtaining

$$P_f^e \simeq \frac{2\Sigma_f(\text{final})}{3\xi\Sigma_s} \left(\frac{a}{\sqrt{\pi\theta}} - e^{\theta/a^2} \left(1 - \operatorname{erf} \frac{\sqrt{\theta}}{a} \right) \right). \quad (57)$$

I have used these expressions plus approximations for the various cross sections to estimate that with a gas of UF_6 at 70°C and one atmosphere pressure, (a) for a 40 cm sphere in D_2O or Be, approximately

10% of the neutrons will undergo epithermal fission and (b) for a 70 cm sphere in graphite about 20% of the neutrons will undergo epithermal fission. In both cases, a majority of the epithermal fissions will occur for neutron energies below 1 ev, for which both α and θ are not much different from thermal values.

When I combine these results with an average value for E (Eq. (53)) of the order of 0.05, I conclude that the effect of epithermal reactions is not large for spheres of approximately minimal radii. More specifically, it appears unlikely to affect the reactivity by more than about 1%.

APPENDIX I. CURRENTS FROM FICTITIOUS SOURCES FOR THE FINITE SPHERE

In this section I consider Eq. (24), the approximate solution to the age equation satisfying boundary conditions $\chi(a + D, \theta) = 0$ and $\partial\chi/\partial r)_{r=a} = 0$. In particular I investigate the currents at r=a and r = D + a. The current density is proportional to:

$$\frac{\partial \chi(r, \theta)}{\partial r} = J(r, \theta). \qquad (A, 1)$$

It is evident that

$$J(a, \theta) = \frac{\partial r\chi_1(r, \theta)}{\partial r}\Bigg)_{r=a} - \chi_1(a, \theta) \qquad (A, 2a)$$

$$J(a + D, \theta) = \frac{\partial r\chi(r, \theta)}{\partial r}\Bigg)_{r=a+D} \qquad (A, 2b)$$

where the notation $\chi_1 = \chi - \chi_o$ has been employed. Substituting Eq. (15) for χ_o and Eq. (24) for χ_1, we find:

$$J(a, \theta) = \frac{1}{4a\pi\sqrt{\pi\theta}}\left[\left(\frac{2}{a} - \frac{D}{\theta}\right)e^{-D^2/\theta} - \frac{2\sqrt{\pi\theta}}{a^2}\, e^{\theta/a^2}\, e^{2D/a}\left\{1 - \mathrm{erf}\left(\frac{D}{\sqrt{\theta}} + \frac{\sqrt{\theta}}{a}\right)\right\}\right]$$

$$J(a + D, \theta) = \frac{1}{4a\pi\sqrt{\pi\theta}}\left[\left(\frac{1}{a} - \frac{D}{2\theta}\right)e^{-D^2/4\theta} - \frac{\sqrt{\pi\theta}}{a^2}\, e^{D/a}\, e^{\theta/a^2}\left\{1 - \mathrm{erf}\left(\frac{D}{2\sqrt{\theta}} + \frac{\sqrt{\theta}}{a}\right)\right\}\right].$$

For the spherical shells with which we are concerned $\frac{D}{\sqrt{\theta}} \gtrsim 2$ so that we may expand the error function $J(a, \theta)$, using:

$$1 - \mathrm{erf}(x) \sim \frac{e^{-x^2}}{\sqrt{\pi x}}.$$

We find,

$$J(a, \theta) \simeq \frac{1}{4a\pi\sqrt{\pi\theta}} \; e^{-D^2/\theta}\left[\frac{D^2a/\theta - D}{aD + \theta}\right]. \qquad (A, 4)$$

If $D \gtrsim 4\sqrt{\theta}$, we may also expand the error function for $J(a + D, \theta)$, obtaining:

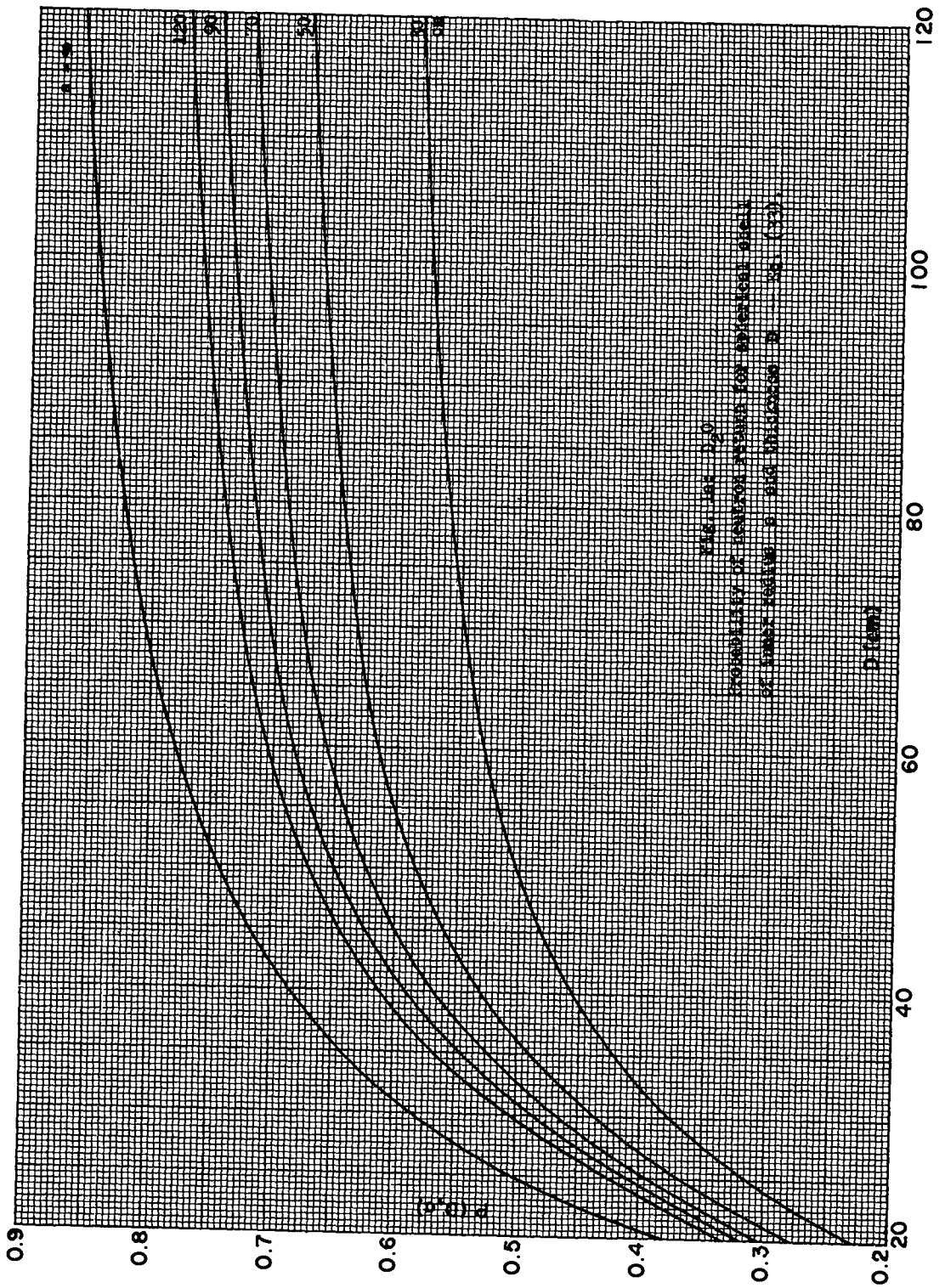
$$J(a + D, \theta) \simeq \frac{-1}{4a\pi\sqrt{\pi\theta}} \; e^{-D^2/4\theta} \; \frac{D^2a/\theta}{2(aD + 2\theta)} \cdot \qquad (A, 5)$$

From these two equations it follows that

$$\frac{J(a, \theta)}{J(a + D, \theta)} \sim O(e^{-3D^2/4\theta}). \qquad (A, 6)$$

For thick spheres this will be a small number. For some interesting values of D, it may not be permissible to use the expansion (A, 5) for $J(a + D, \theta)$. However, it can still be shown that $J(a, \theta)$ will be small compared to $J(a + D, \theta)$.

From our work with plane geometry, Section I, B and Fig. 1, we know that even plane reflectors having $D < 2\sqrt{\theta}$ (with θ the age to thermal) cannot return half the neutrons to a core. It follows that efficient reflectors are always thick in the sense that $D \gtrsim 2\sqrt{\theta}$; that (A, 4) is a good approximation; and that $J(a, \theta)$ is essentially negligible.


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
20
40
60
80
100
120
D (km)

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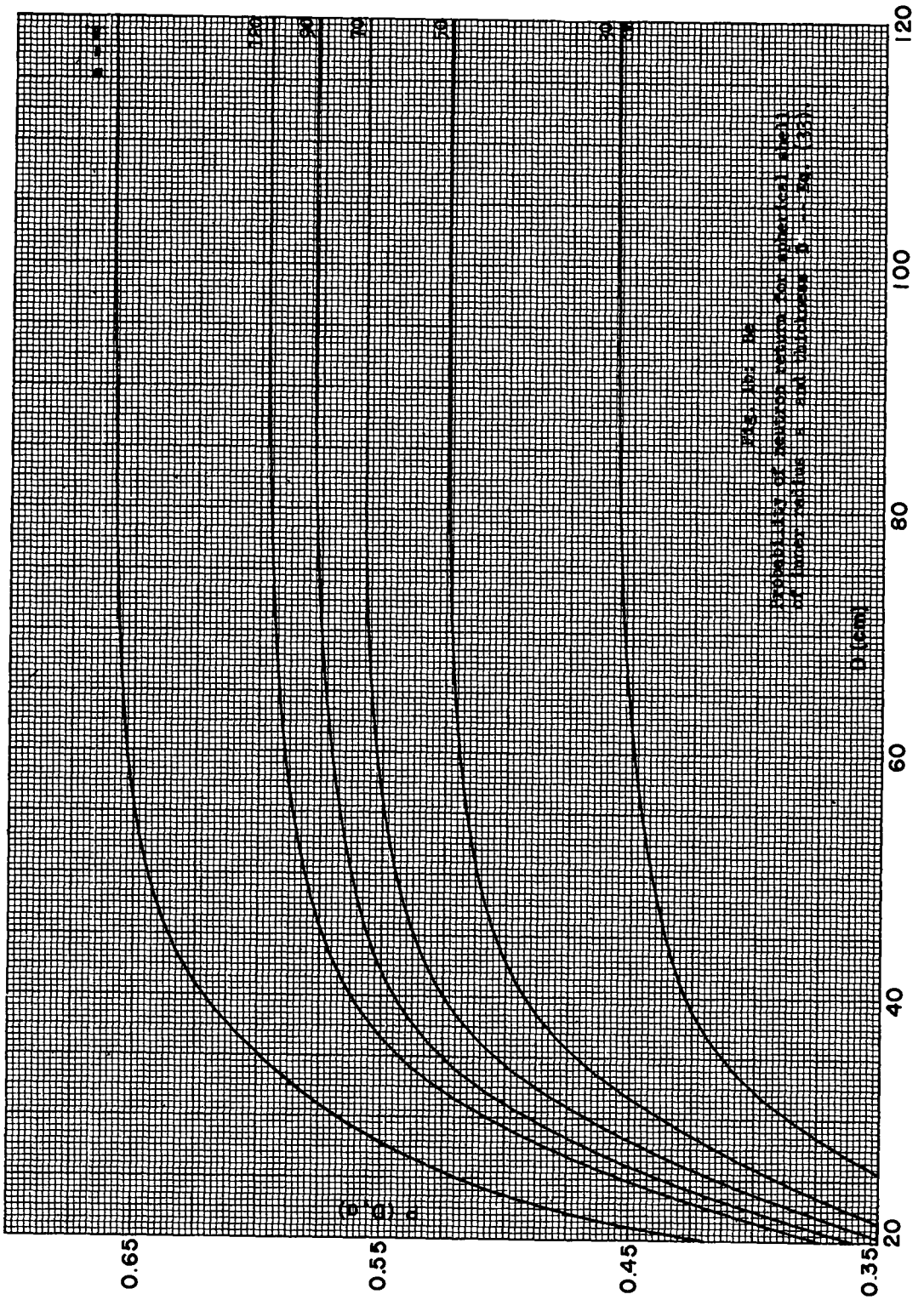

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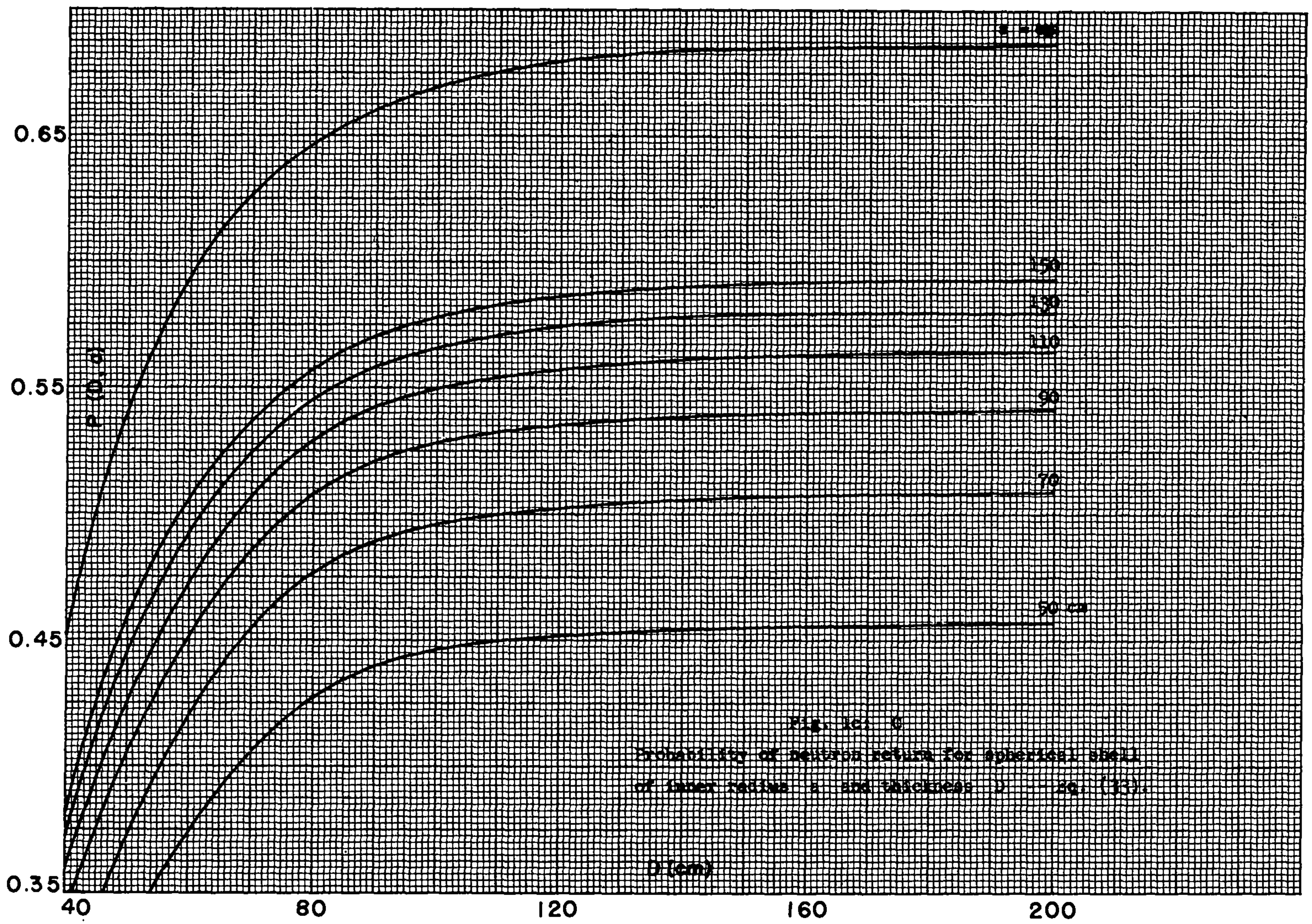

Fig. [illegible] 6
Probability of neutron return for spherical shell of inner radius a and thickness D — Eq. (13).

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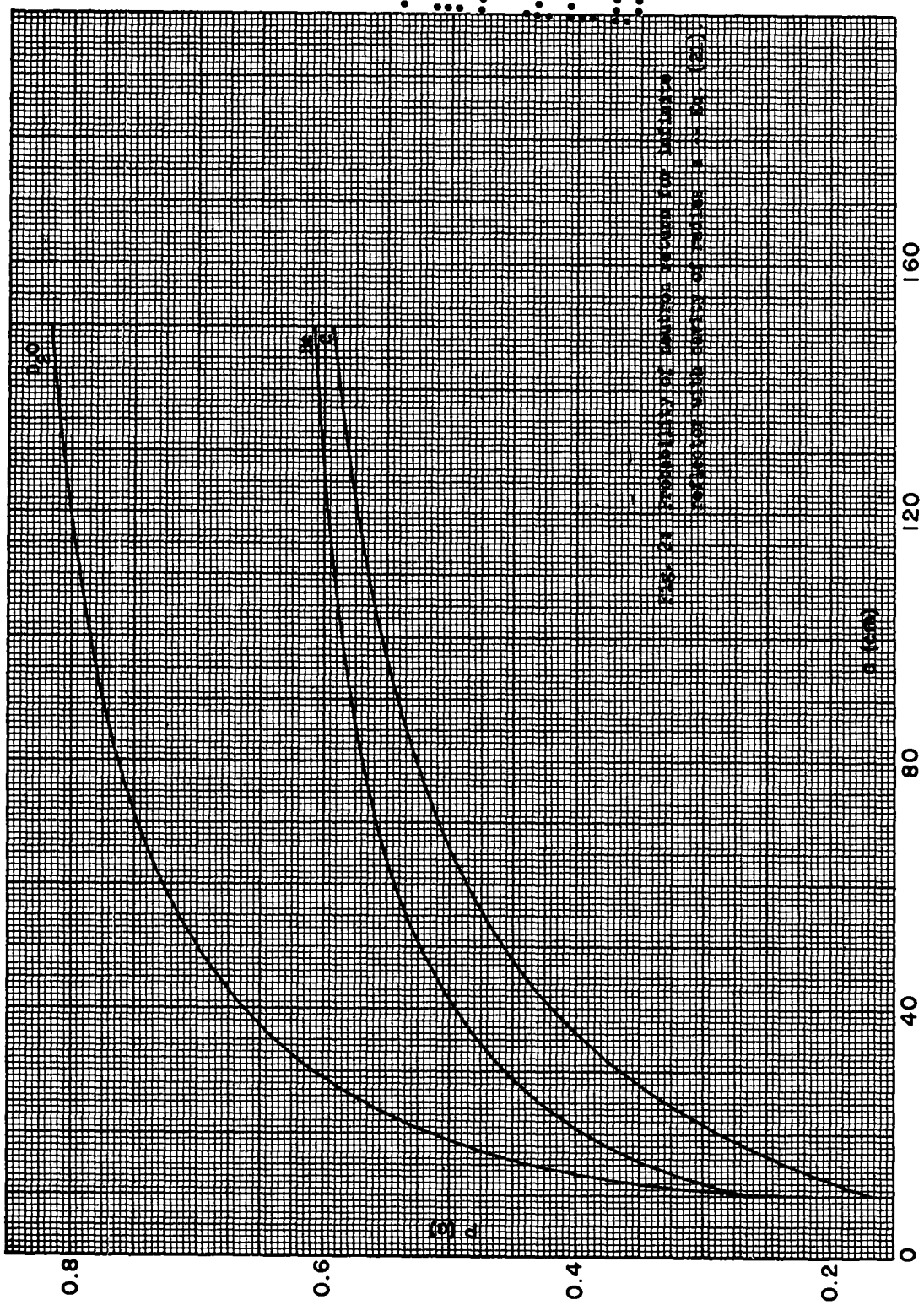

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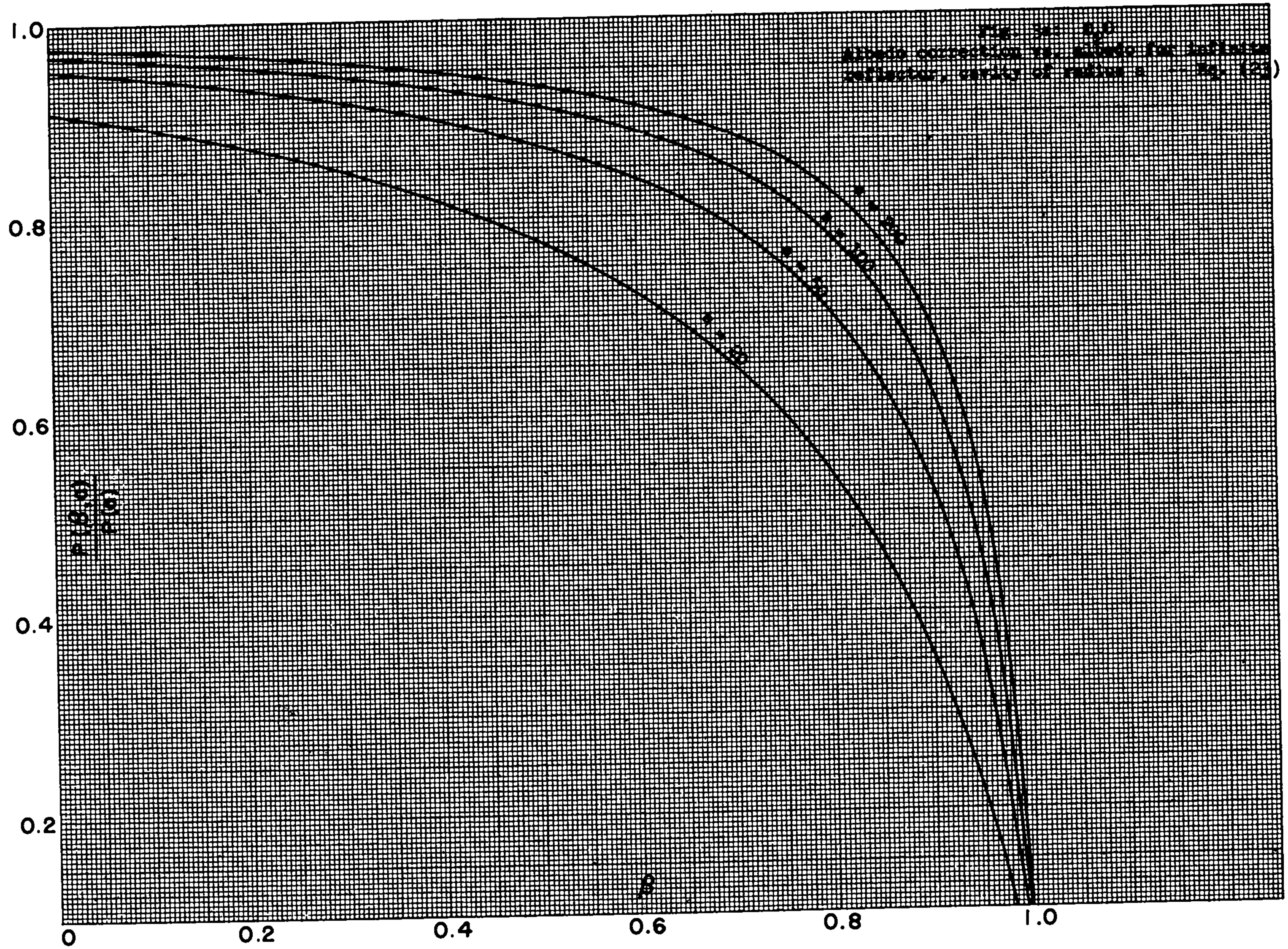

Albedo correction vs. albedo for infinite reflector, cavity of radius a -- Eq. (2)

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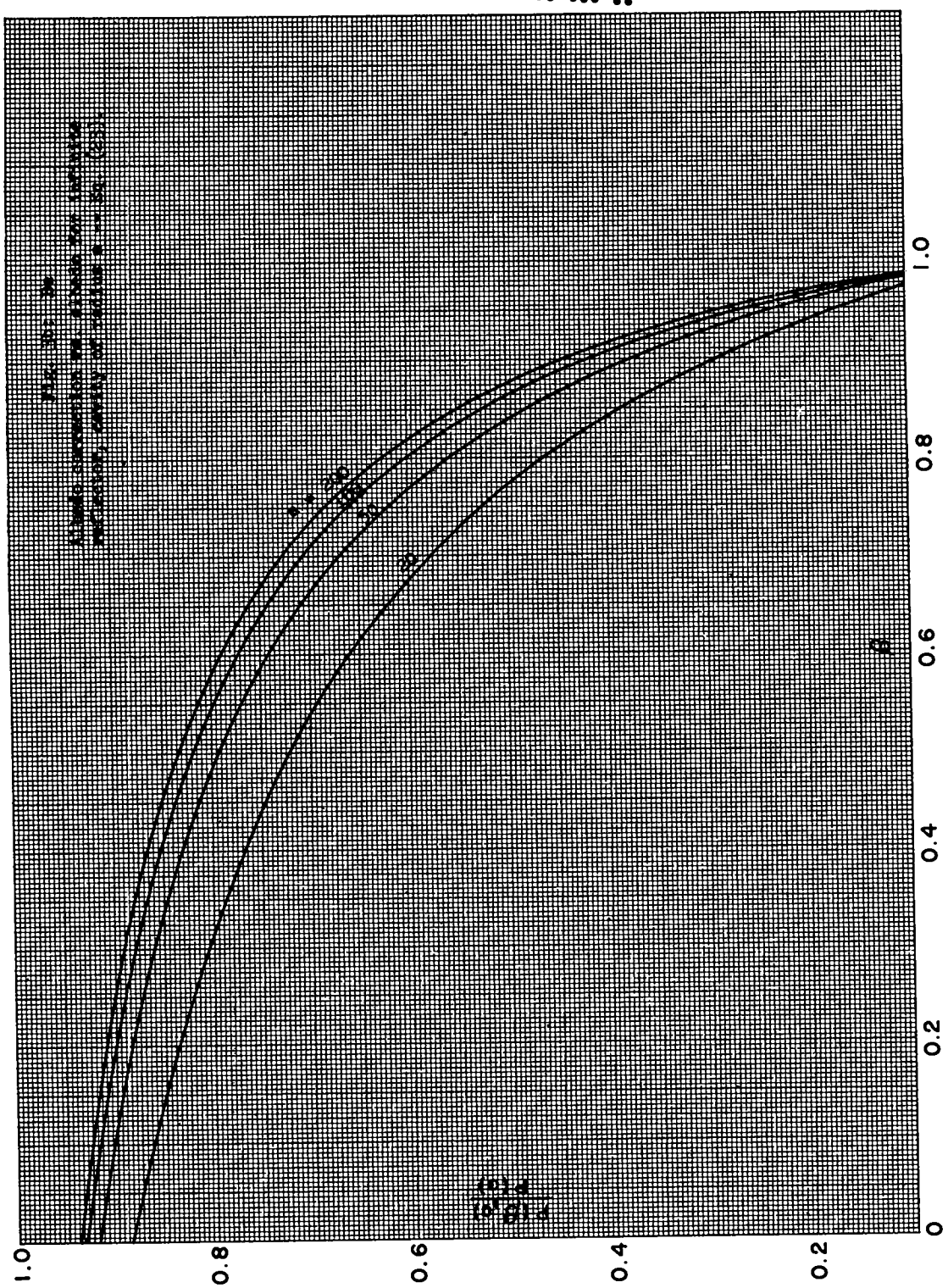

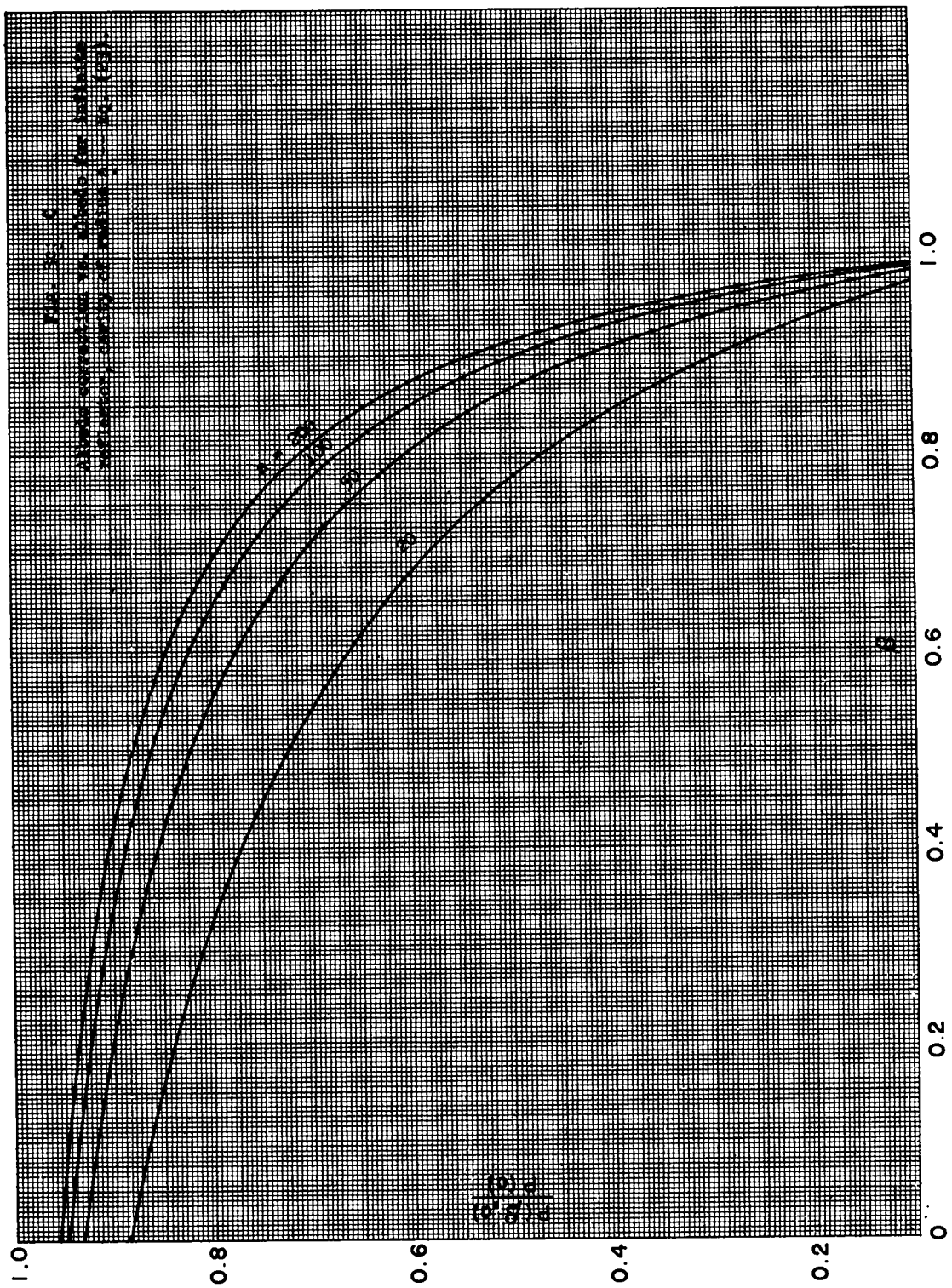

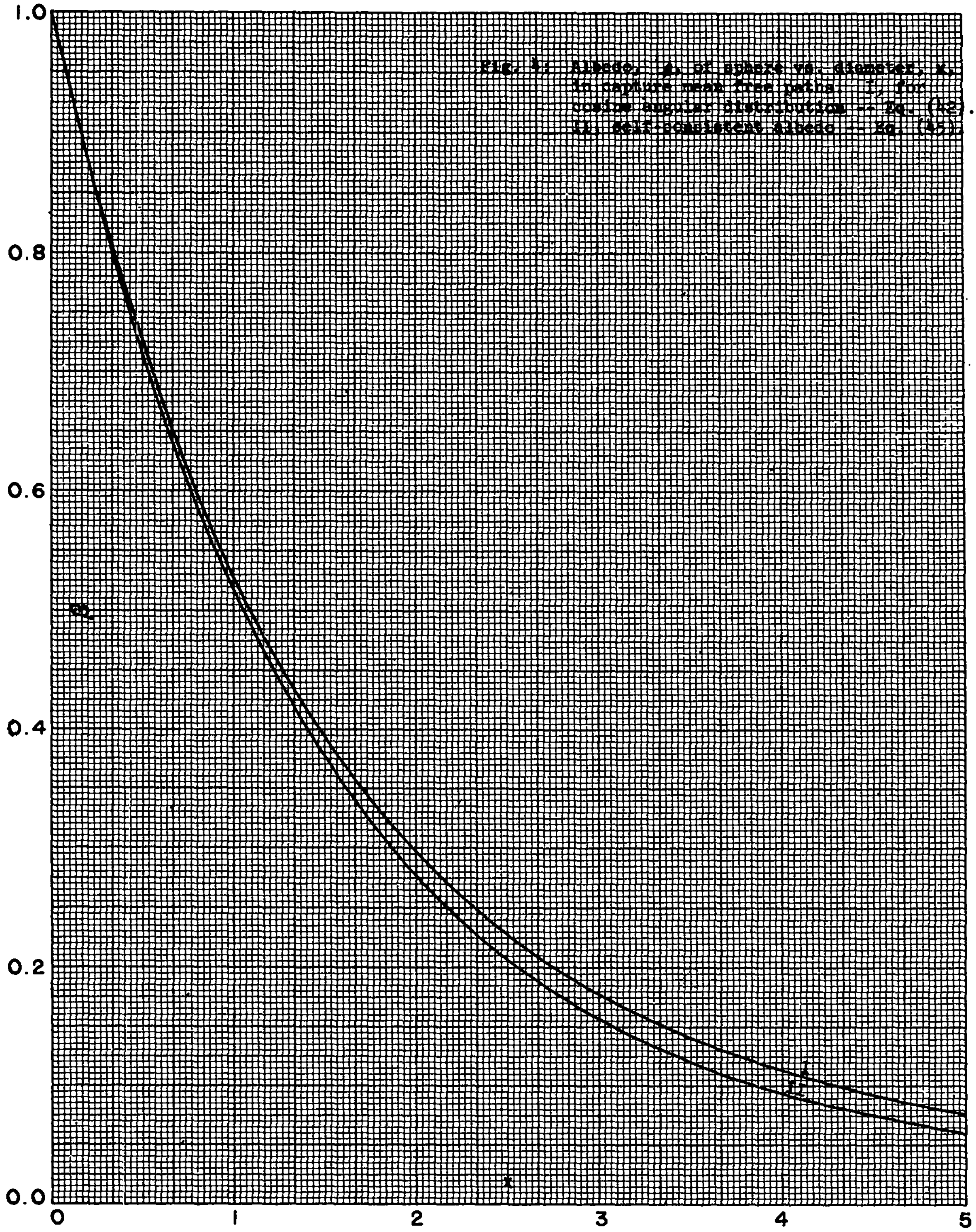

Fig. 4: Albedo, A, of sphere vs. diameter, x, in capture mean free paths. I, for cosine angular distribution -- Eq. (42). II, self-consistent albedo -- Eq. (45).

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