LA-10

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LA-10 Series [redacted]

THE EXPONENTIAL SHOCK AND RAREFACTION WAVES

Robert Davis, S. Frankel

The following curves give the results of the numerical solution of the equations of motion of the one dimensional shock and rarefaction waves derived by R. Serber.* These waves occur at the interface of two materials, one of which (the core) is heated by an energy release, the spatial distribution of which is constant per gram, and which increases exponentially with time. Each of the two materials is assumed to have zero initial temperature and to obey the equation of state of a perfect gas. The calculations for the shock wave have been carried thru for two values of the ratio of specific heats, $\gamma = 1.4$ and 1.67. The rarefaction wave has been treated only for $\gamma = 1.4$.

With increasing time the scale of linear dimensions increases exponentially, as do the scales of pressure and temperature. If the scales are so changed the graphs giving the spatial distribution of pressure, temperature, and displacement are invariant. The density as a function of the exponentially increasing scale of length is invariant. The scale of length which has been used is the co-moving coordinate, i.e. the initial distance from the interface of the material at the point considered, normalised to unity at the head of the wave. This length, denoted by x, has the range 0 to 1 for the shock wave and -1 to 0 for the rarefaction wave. For the rarefaction wave the letter η has been used in place of x.

The actual pressure or temperature at any stage of the expansion can be determined by making use of the fact that the head of the rarefaction wave moves with sound velocity, $[P/\rho]^{1/2}$. For a specified rate of increase of energy density, pressure, and temperature, $e^{2\lambda t}$, and of position and velocity, $e^{\lambda t}$, this condition determines the pressure and temperature at the head of the rarefaction wave in terms of its position and density. The pressure is continuous across the

* SEE A REPORT OF R. SERBER, TO APPEAR LATER IN THIS SERIES, FOR FURTHER CLARIFICATION.

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interface. This determines the pressure, hence also the temperature, in the shock wave.

The character of the shock wave is determined by γ_{tamper} alone. The character of the rarefaction wave is then determined by the interface boundary condition which equates the pressure and displacement on the two sides of the interface, hence also the invariant ratio, pressure over displacement squared. This condition depends on the two values of γ and on the ratio of initial densities, $\alpha = \rho_{tamper}/\rho_{core}$:

$$\frac{2\rho_t \; P(0)}{(\gamma_t+1)P(1)\;\varphi^2(+0)} = \frac{\rho_c \; P(0)}{\gamma_c \; P(-1)\;\varphi^2(-0)}$$

In the curves given here γ has been taken to be 1.4 in both materials.

The notation used in the graphs is as follows:

ρ_o = initial density of core or tamper

α = ρ_{ot}/ρ_{oc}

z = η = the initial position (measured from the initial position of the interface) of the material at the point considered, in units of the position of the head of the shock or rarefaction wave.

φ = actual position in units of the head position

χ = the displacement, $\varphi - z$

P = pressure in units of pressure at head of wave

ρ = density in units of ρ_o

T = P/ρ , temperature

It should be observed that the units in which z, φ, P, ρ, and T are measured are not neccessarily the same in the tamper as in the core. The relationship between the two sets of units is determined by the equality of pressure and position at the interface.

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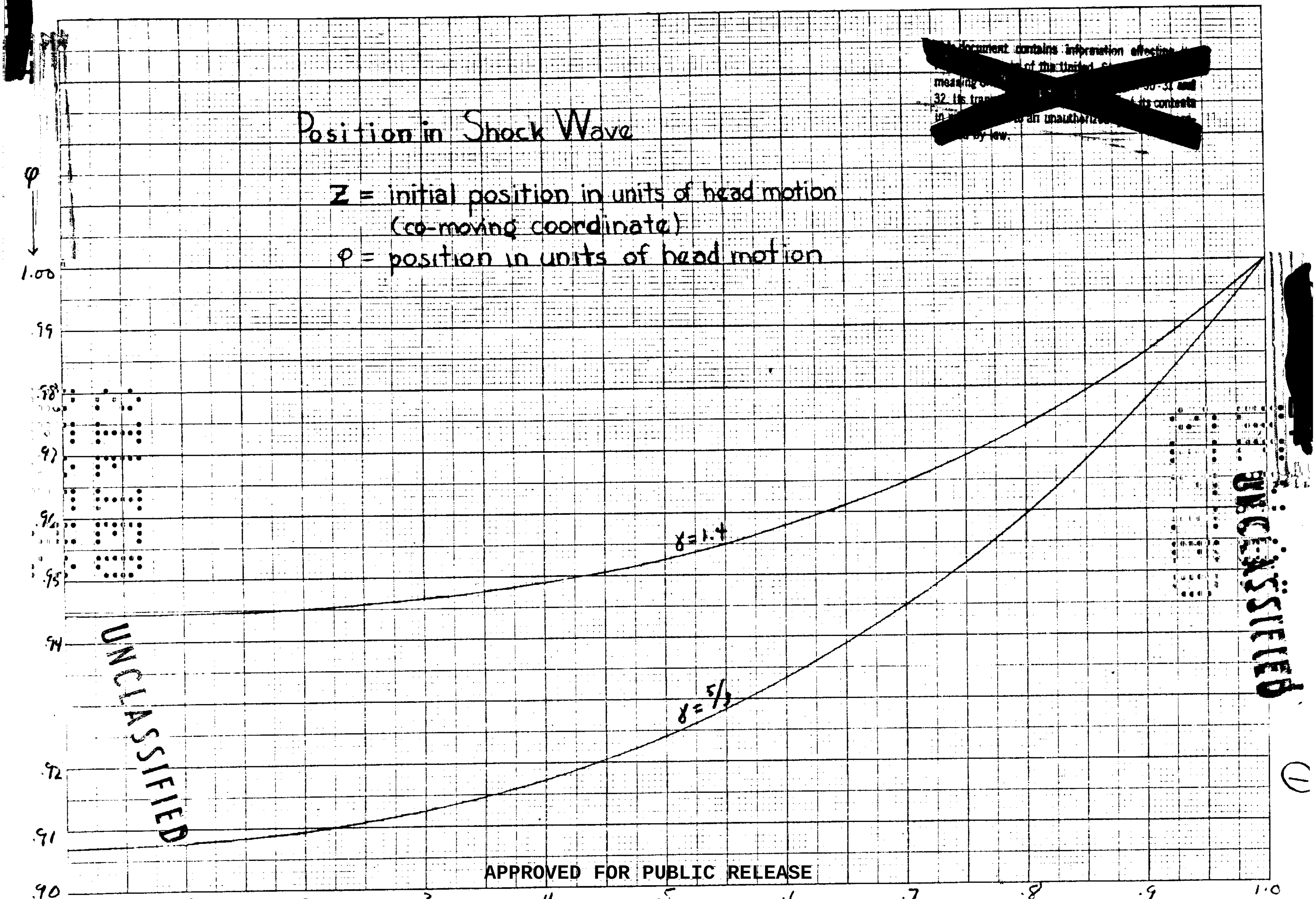
Position in Shock Wave
Z = initial position in units of head motion
(co-moving coordinate)
φ = position in units of head motion
φ
1.00
.99
.98
.97
.96
.95
.94
.92
.91
.90
γ = 1.4
γ = 5/3
.7
.8
.9
1.0

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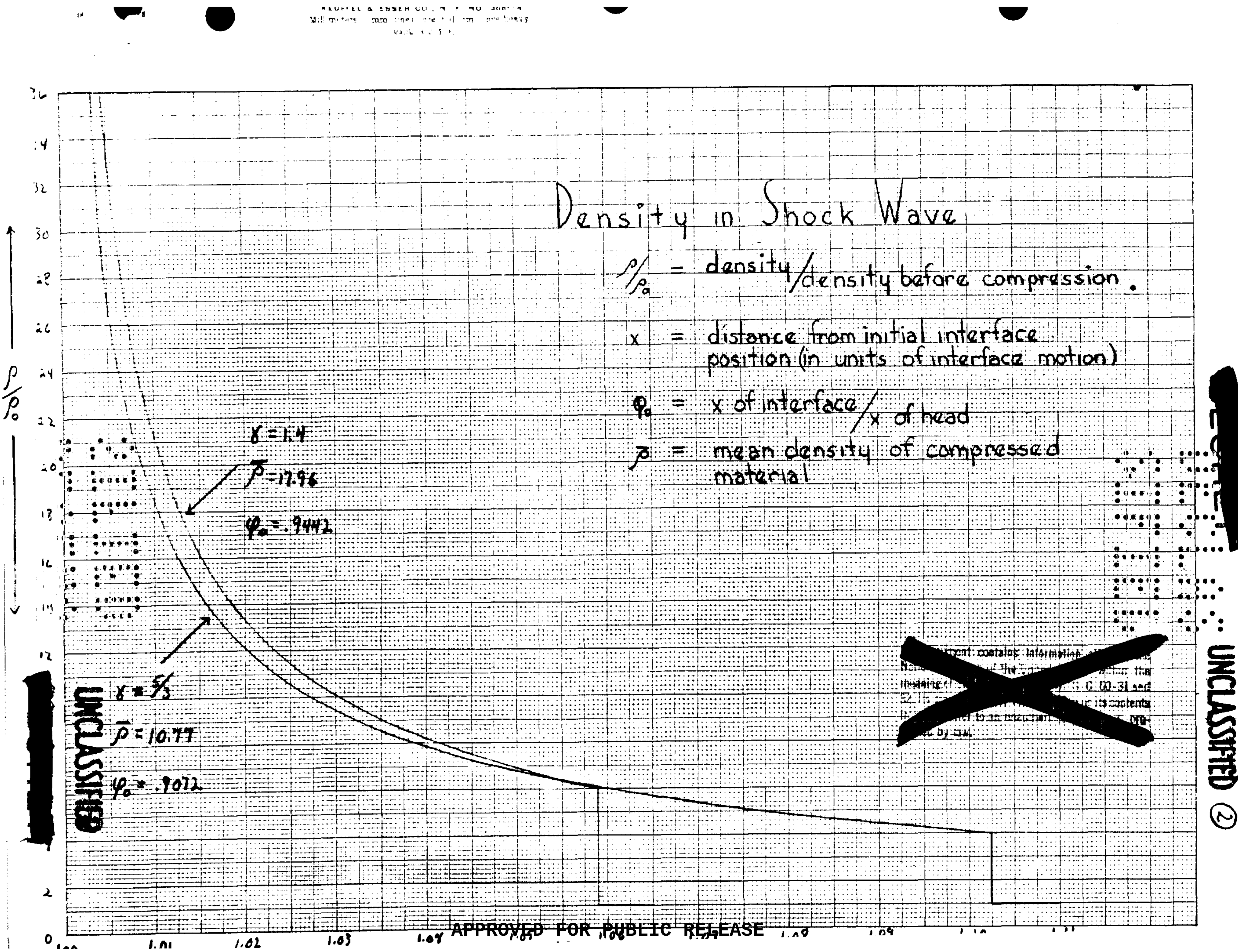
Density in Shock Wave
ρ/ρ₀ = density/density before compression
x = distance from initial interface position (in units of interface motion)
φ₀ = x of interface/x of head
ρ̄ = mean density of compressed material
γ = 1.4
ρ̄ = 17.96
φ₀ = .9442
γ = 5/3
ρ̄ = 10.77
φ₀ = .9072
ρ/ρ₀
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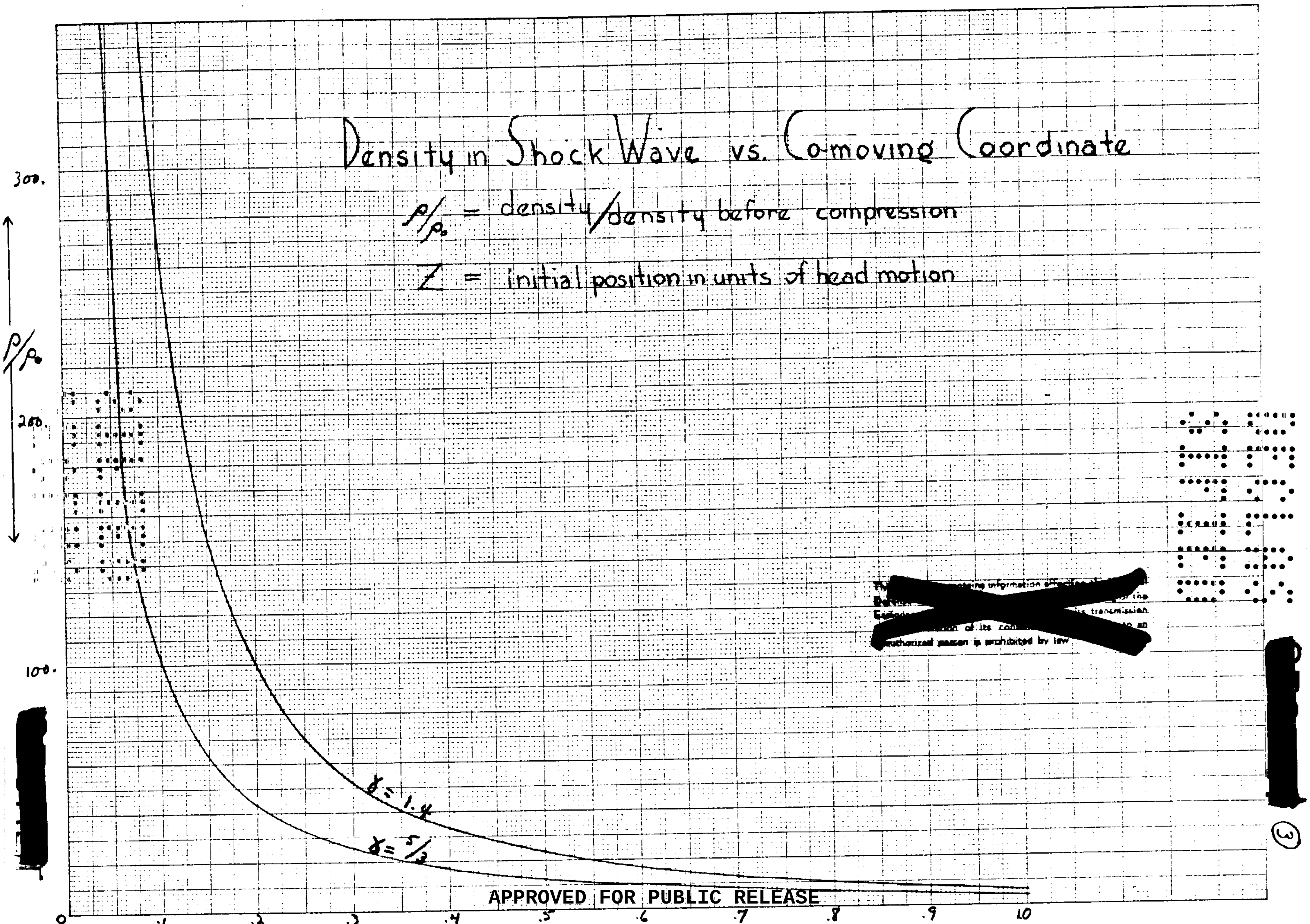
1.01
1.02
1.03
1.04


Density in Shock Wave vs. Co-moving Coordinate
ρ/ρ_0 = density/density before compression
Z = initial position in units of head motion
ρ/ρ_0
300.
200.
100.
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\gamma = 1.4$
$\gamma = 5/3$
(3)

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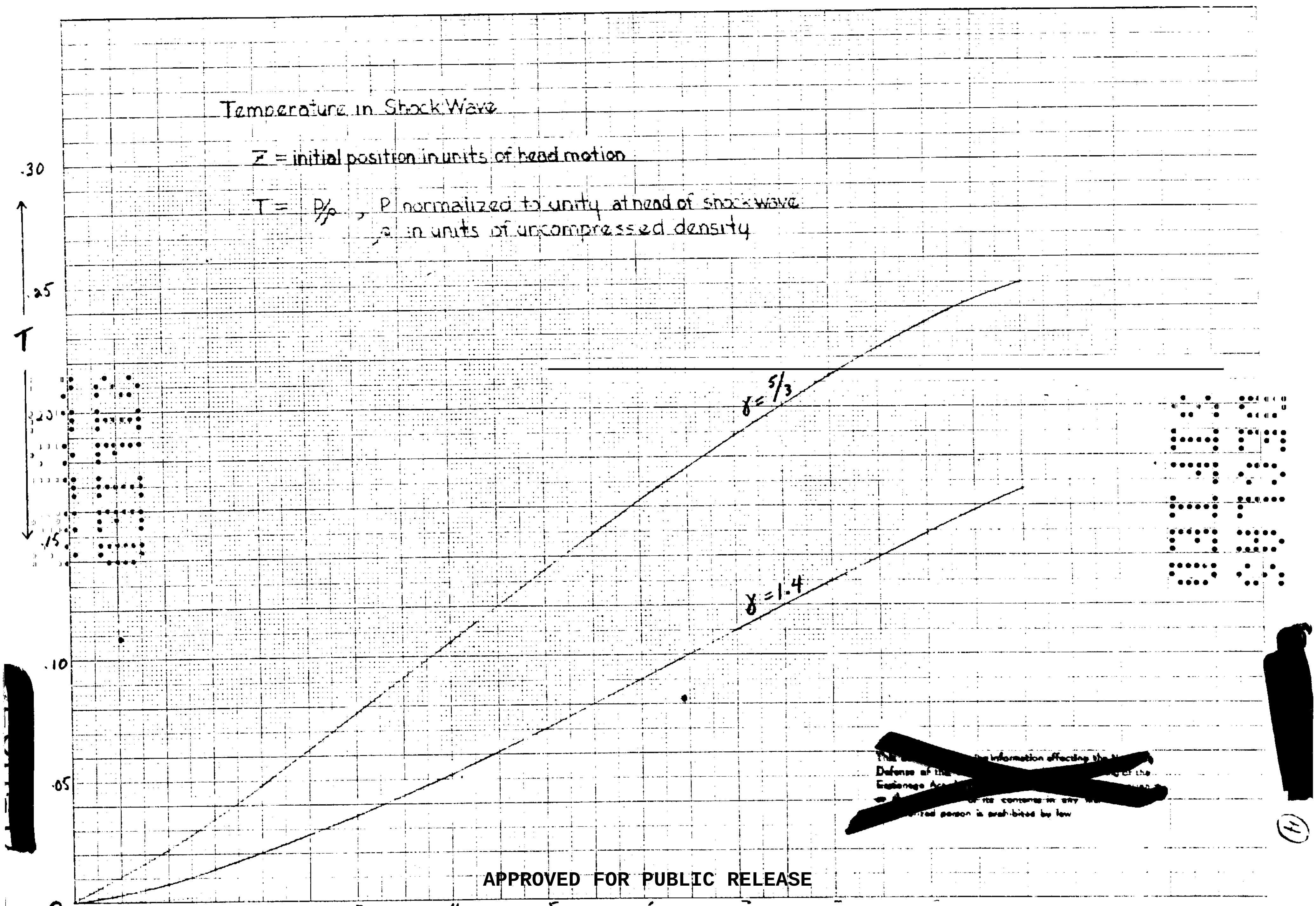

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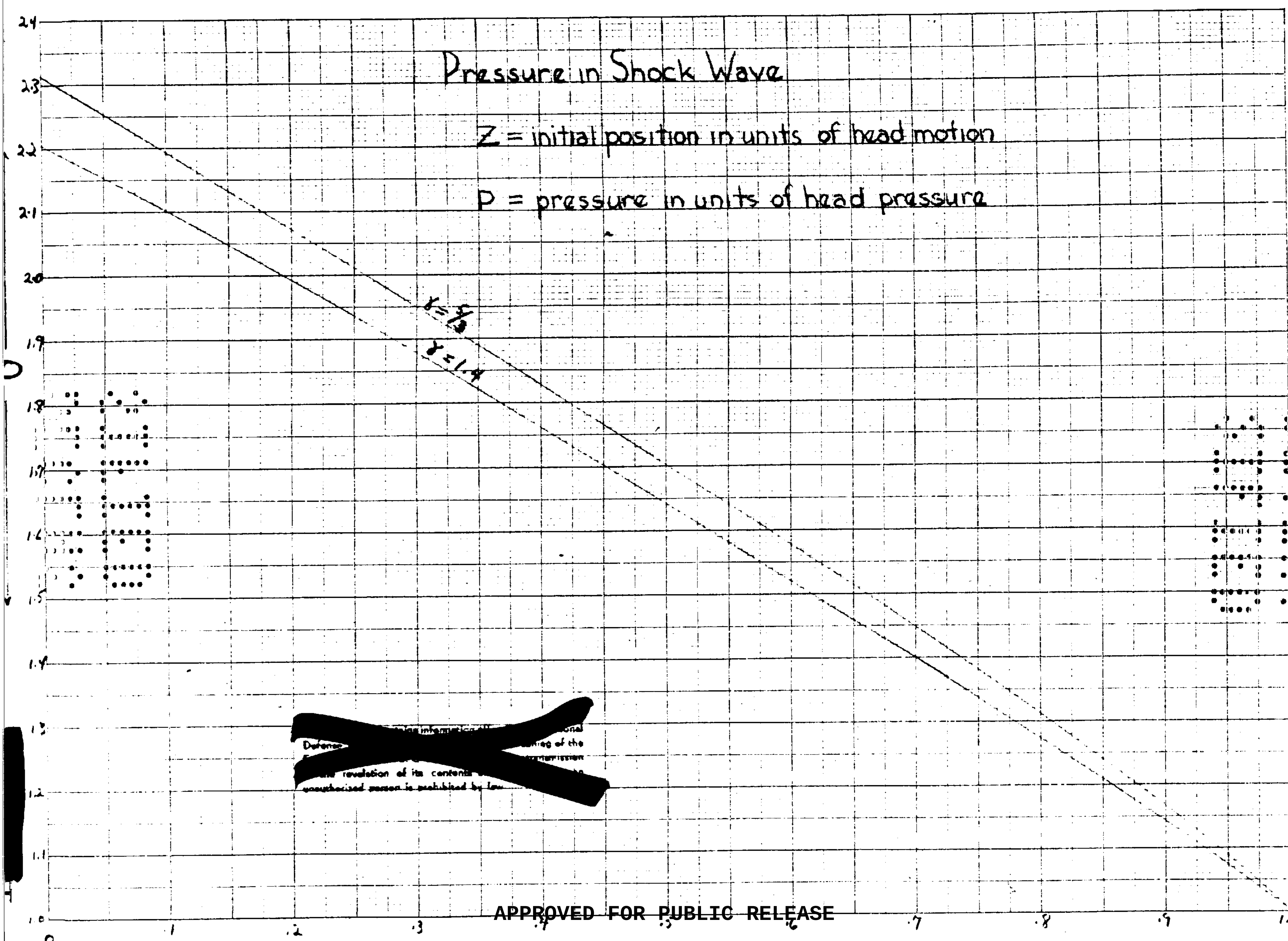
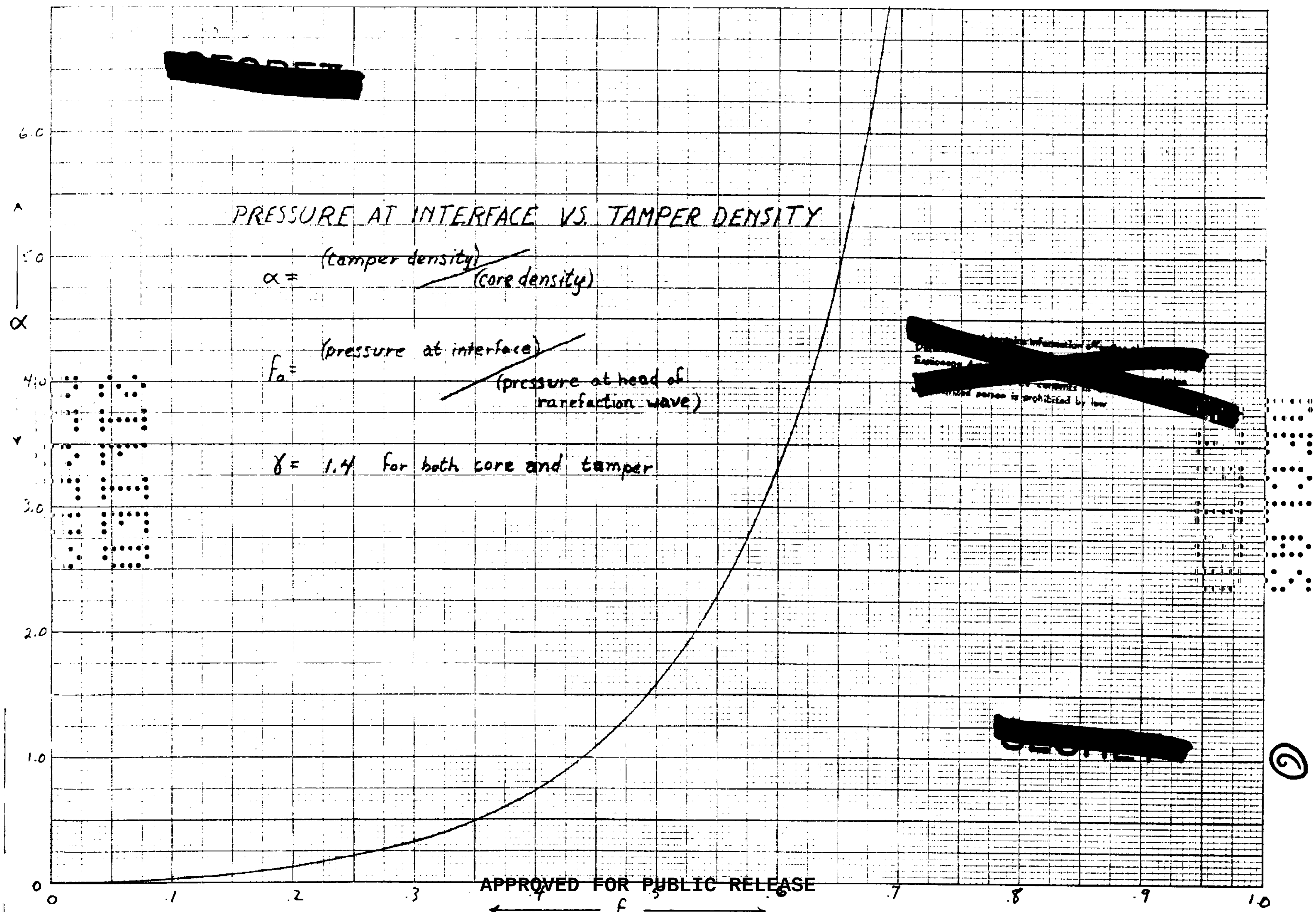
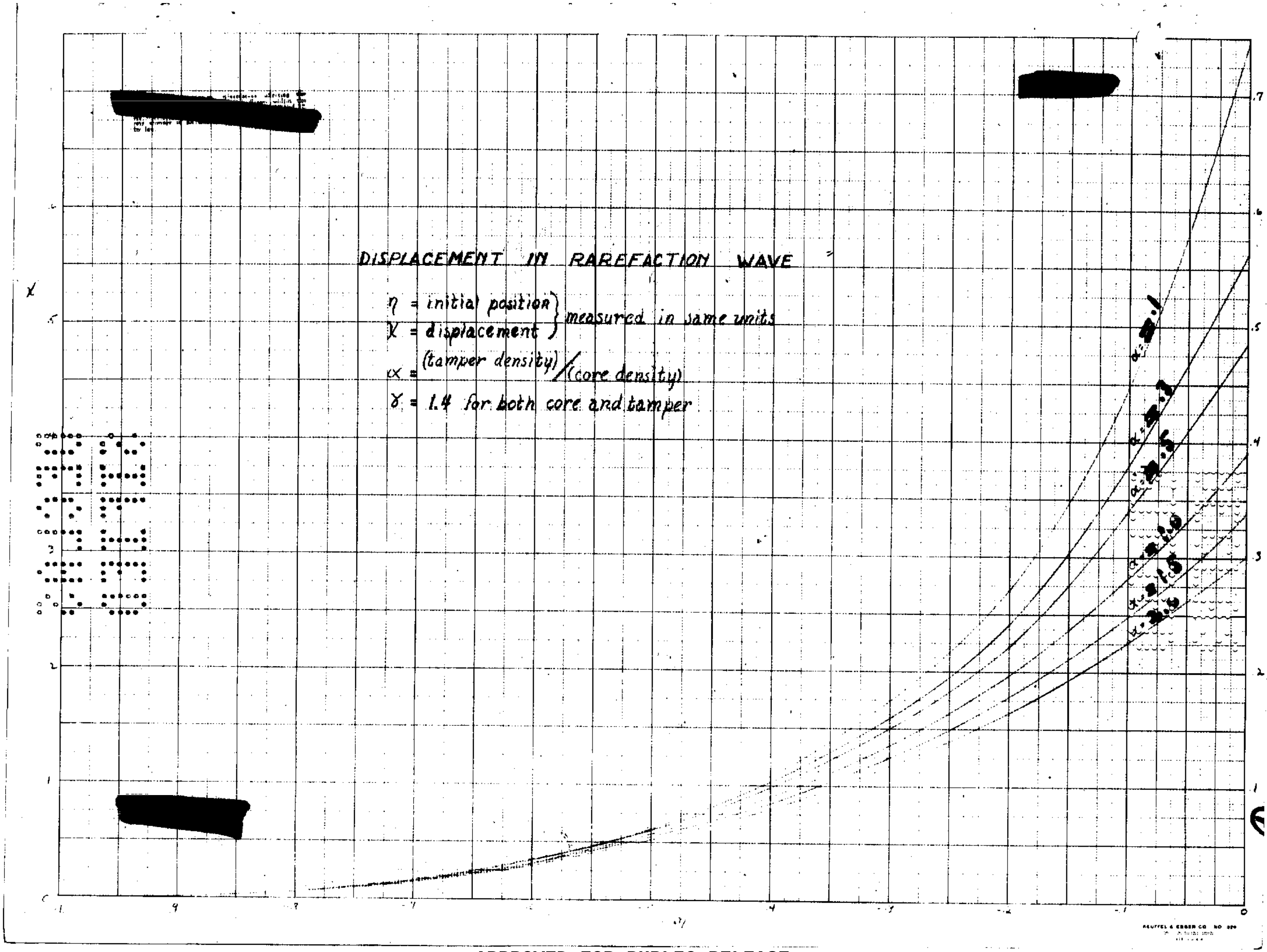
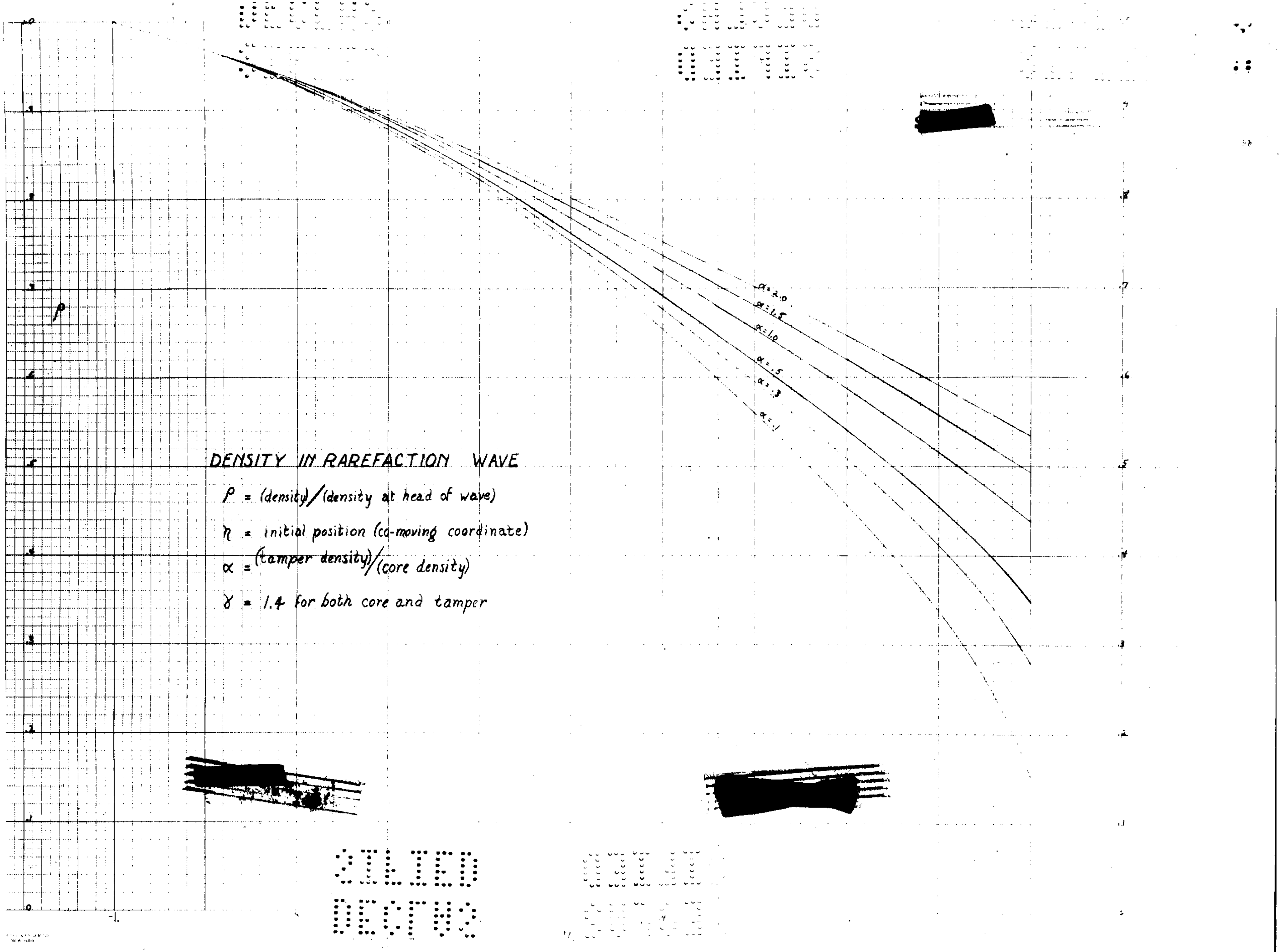

Pressure in Shock Wave
Z = initial position in units of head motion
P = pressure in units of head pressure
γ = 5/3
γ = 1.4
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.6
1.4
1.2
1.1
1.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0

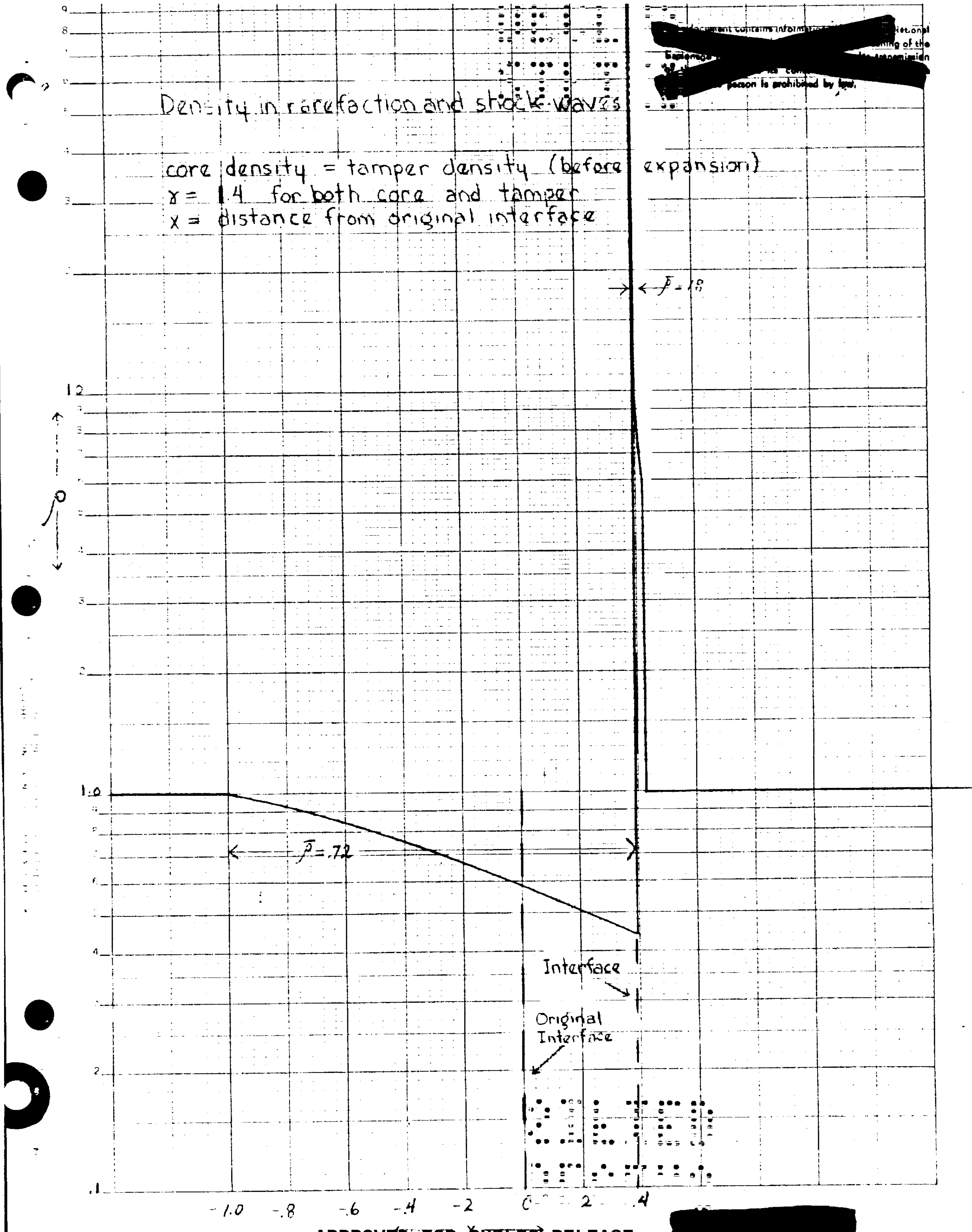
[redacted]

(9)


PRESSURE AT INTERFACE VS. TAMPER DENSITY
α = (tamper density) / (core density)
f_o = (pressure at interface) / (pressure at head of rarefaction wave)
γ = 1.4 for both core and tamper
α
6.0
5.0
4.0
3.0
2.0
1.0
0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
f


DISPLACEMENT IN RAREFACTION WAVE
η = initial position
χ = displacement
measured in same units
α = (tamper density)/(core density)
γ = 1.4 for both core and tamper
χ
η
[redacted]


DENSITY IN RAREFACTION WAVE
ρ = (density)/(density at head of wave)
η = initial position (co-moving coordinate)
α = (tamper density)/(core density)
γ = 1.4 for both core and tamper
α = 2.0
α = 1.5
α = 1.0
α = .5
α = .3
α = .1
ρ
-1.
0



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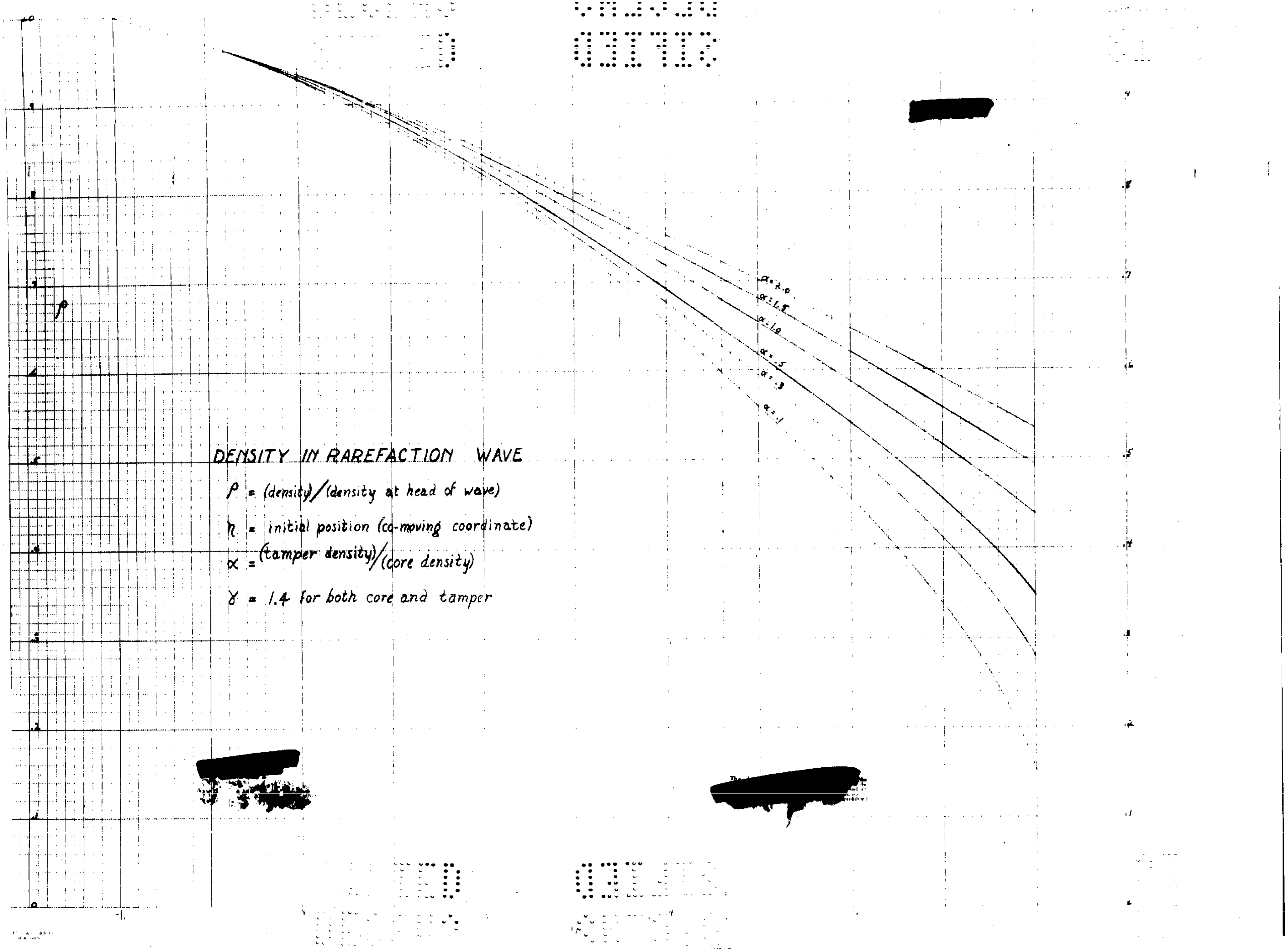
DENSITY IN RAREFACTION WAVE
ρ = (density)/(density at head of wave)
η = initial position (co-moving coordinate)
α = (tamper density)/(core density)
γ = 1.4 for both core and tamper
ρ
α = 2.0
α = 1.5
α = 1.0
α = .5
α = .3
α = .1
-1.

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