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AUTHOR(S) Stirling A. Colgate, T-6

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Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

THE ENTROPY IN SUPERNOVA EXPLOSIONS

Stirling A. Colgate
Theoretical Division, MS B275
Los Alamos National Laboratory
Los Alamos, New Mexico 87545 USA

Abstract

The explosion of a supernova forms because of the collapse to a neutron star. In addition an explosion requires that a region of relatively high entropy (about 1000 in units of the Boltzmann constant) be in contact with the neutron star and persisting for a relatively protracted period of time, (1000s). The high entropy region ensures that the maximum temperature in contact with the neutron star and in hydrostatic equilibrium is less than some maximum. This temperature must be low enough (less than mc^2) such that neutrino emission cooling is small, otherwise the equilibrium atmosphere will collapse adding a large accretion mass to the neutron star.

A so-called normal explosion shock that must reverse the accretion flow corresponding to a typical stellar collapse must have sufficient strength or pressure to reverse this flow and eject the matter with 10^{51} ergs for a typical type II supernova. Surprisingly the matter behind such a shock wave has a relatively low entropy (s=10) low enough such that neutrino cooling (in hydrostatic equilibrium) would be orders of magnitude faster than the expansion rate. The resulting accretion flow would be inside the Bondi radius and result in free-fall accretion inside the expanding (at the speed of sound) rarefaction wave. The accreted mass or reimplosion mass unless stopped by a high entropy bubble could then exceed that of bound neutron star models. In addition the explosion shock would be overtaken by the rarefaction wave and either disappear or at least weaken. Hence, a hot, high entropy bubble is required to support an equilibrium atmosphere in contact with a relatively cold neutron star. Subsequently during the expansion of the high entropy bubble that drives or pushes on the shocked matter, mixing of the matter of the high entropy bubble and lower entropy shock-ejected matter is ensured. The mixing is driven by the negative entropy gradient between the high entropy bubble accelerating the shocked matter and the lower entropy of the matter behind the shock. Ultimately the shock propagates into the lower density matter of the presupernova envelope resulting in increasing entropy of the shocked matter. Mixing stops when the entropy of the interior bubble and that of the exterior shocked matter (near the hydrogen-helium boundary at $\rho = 1$ to $0.1\ \mathrm{g\ cm^{-3}}$) become equal as predicted for SN 1987A.

Introduction

Hans Bethe has taught us the advantages of considering the entropy of the various states of matter occurring during the collapse to a neutron star and the subsequent bounce shock (Bethe, 1991). Matter in the core of the presupernova star starts collapse in a state that is close to degenerate and hence low entropy, less than unity in units of the Boltzmann constant. The bounce of the core on the stiffening repulsive potential of the the nuclear equation of state involves less than a solar mass. The bounce of this homologous mass reflects a strong sound wave into the imploding additional matter. The strong sound wave steepens because of the converging velocity field and a shock is driven outwards.

the bounce shock. The bounce shock strengthens in the velocity gradient to an entropy of roughly 6 to 8 where most of this entropy resides in the degrees of freedom of the nuclei, that is, excited states and disassociated nuclei. It is primarily the energy of disassociation to free nucleons that soaks up the internal energy density of the shock and attenuates it to the point where the embryonic explosion is reversed. (Neutrino emission from the hot shocked matter also weakens the shock.) A continuing implosion or accretion of matter results and a new mechanism of explosion must take place, or else a black hole will become the inevitable final state. We see supernovae, and so black holes cannot be the final state.

The delayed neutrino emission and resulting heating from the interaction of these neutrinos with the imploding matter (Wilson and Mayle 1989, and Colgate and White 1966, Bruen 1985,1989a,b, Bludman, 1988) has long been considered the alternate mechanism for creating the explosion. However, the explosions created by neutrino deposition have been uncertain because small changes in neutrino transport lead to large changes in the kinetic energy of the ejected matter. This is because the energy emitted by neutrinos, the binding energy of the neutron star or approximately 3×10^{53} ergs is 300 times the ejected kinetic energy of approximately 10^{51} ergs. Hence, small changes in the coupling efficiency of these neutrinos to the energy of the ejected matter is bound to make a substantial effect on this very small residual fraction, the ejected kinetic energy, derived from the binding energy of the neutron star. The major conceptual problem is how to find a process or sequence of processes that lead to a robust and natural and physically logical explanation of the supernova mass ejection, yet independent of the initial stellar structure and independent of the numerous exotic mechanisms of neutrino transport. (We see a vast difference among various type II supernova and infer a similar difference in initial masses and structure.) In this paper it is hoped that some light may be shed on the state conditions associated with such a universal mechanism.

Reimplosion Mass

It has been pointed out before (Colgate 1971) that regardless of the details of the explosion mechanism itself, and the existence of a very strong shock wave for ejecting matter, nevertheless a significant fraction of the ejected matter could subsequently fall back onto the neutron star unless some rather extreme conditions took place that prevented it. This fall-back mass fraction appeared in the beginning to be so large and unavoidable that the early mechanism suggested for the explosion of supernova (Colgate and White 1966) of large neutrino emission seemed in jeopardy as was indicated by all subsequent calculations. The problem is that any matter initially on an ejection or escape trajectory would later find that the pressure support from the neutron star had disappeared because of neutrino energy loss. Subsequently a major fraction of the matter initially on an ejection trajectory would be overtaken by a rarefaction wave (Fig. 1) and reverse its velocity such as to fall back on the neutron star. Since the neutron star core forms in the initial explosion with a mass of the order of one $M_\odot$, any significant addition to this mass would not only violate the few cases of neutron star mass that are known, but would also lead to masses perilously close to collapse to a black hole. Consequently, it has long been a major problem to find a mechanism of explosion such that subsequent mass fall-back would be small.

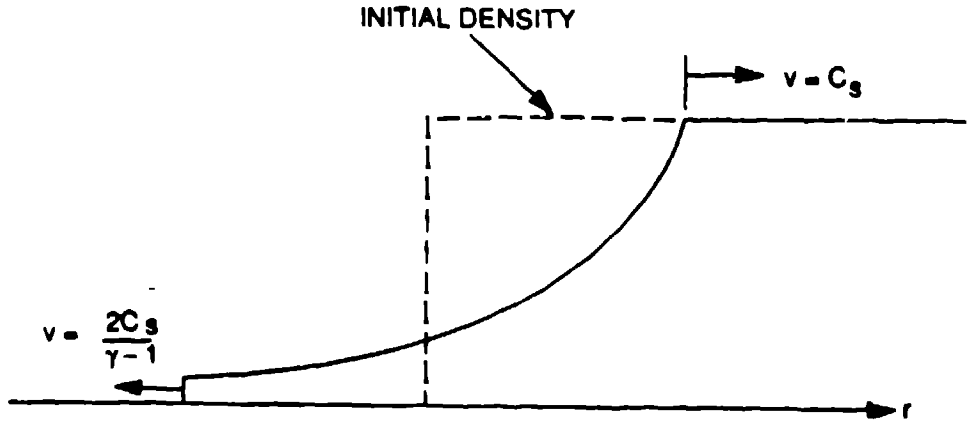


Figure 1 shows the classical rarefaction wave that progresses within the fluid at sound speed and reverses the flow. The maximum velocity of blow off from a rarefaction wave is $2C_S/(\gamma-1)$.

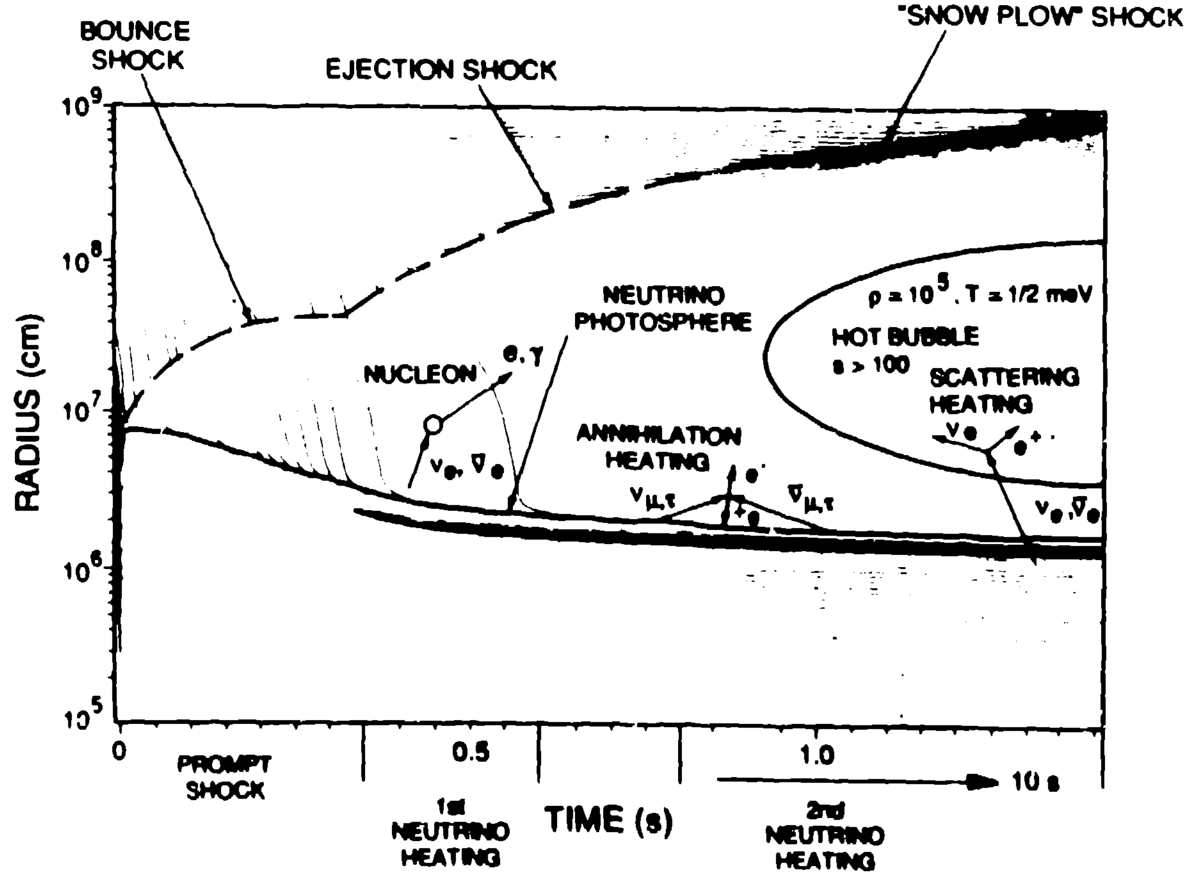


Figure 2. The sequence of processes occurring in a Type II supernova according to Wilson and Mayle (1989). First a star collapses, owing to exhaustion of the fuel and emits electron neutrinos as the compressing matter turns into neutron-rich matter. The forming neutron star bounces producing a strong shock that weakens owing both to the thermal decomposition of the infalling nuclear matter and the reversal of the neutron star bounce trajectory. Subsequently, a large burst of neutrinos of all flavors heats the infalling matter, starting an explosive shock. Yet later, mu and tau neutrino-antineutrino annihilations in nearly opposed collisions just above the neutrino photosphere generate heat without nucleonic matter, making a hot, high-entropy (10^4 Boltzmann-units) bubble, which pushes out the ejected matter in a thin "snow plow" shock. The pressure is maintained because the high-entropy ensures a large scale height adjacent to the neutron star.

The most likely solution to this problem has been the formation of a high entropy bubble due to the annihilation by nearly opposed collisions of the high temperature mu and tau neutrinos and antineutrinos (Goodman, Dar, and Nussinov, 1987). Calculations using this process (Wilson and Mayle, 1989) are shown in Fig. 2 along with the formation of a high entropy bubble. It is the formation of this high entropy region that is so optimistic as a demonstration of a robust mechanism for a model independent explosion mechanism. However, we have already heard in this conference, H.T. Janka, and (in press 1990) that this approximation to the neutrino diffusion and annihilation may not be adequate. Thus it is important that we understand the need for a high entropy region next to the neutron star or rather the consequences of the lack of it.

Prior Calculations of Supernova Neutron Star Accretion

Chevalier (1989) has considered the fall-back mass due to the reflected shock wave in Sedov type power law density distributions (Sedov 1959). As a consequence not only is the Mach number of the explosion shock wave constant, but also the time of interest is 10^4 seconds and longer. The solutions have the advantage that they are then independent of the explosion mechanism. On the other hand Zel'dovich et al. (1972) considered the accretion onto a neutron star of a cloud of relatively small mass of 10^{-5} $M_\odot$ chosen as a power law distribution and initially static. The accretion takes place for roughly 24 seconds and the conditions close to the neutron star are calculated in detail. An extremely high neutrino luminosity and neutrino temperature are predicted but the relationship of the initial conditions to the explosion process is obscure. Hence, there is a need to understand the implication of this early accretion on the dynamics of the explosion. The later time phenomena is well considered in Chevalier's analysis and leads to less than 0.1 $M_\odot$ addition to the original neutron star limited by Bondi accretion and reduced (as erroneously discussed in Colgate, 1988) by the radioactive decay heating of the ^{56}Ni. Chevalier also makes the point that although this accretion is modest, nevertheless the accretion pressure is always large compared to that of a magnetic field of 10^{12} gauss. Hence, any discussion is moot of the emergence of a pulsar until this phase of the explosion is complete.

Accretion of High Entropy Matter on a Cold Neutron Star

The purpose of the high entropy matter is to insure that the ejected matter can be supported with high enough pressure, yet at a low enough density and temperature such that neutrino emission adjacent to the neutron star surface does not cool the interface so much that the pressure support disappears. Without pressure support, matter originally on an escape trajectory may fall back to the neutron star and negate the explosion. The question of fall-back or escape depends upon the local state conditions and local expansion velocity at the time that the rarefaction wave from the neutrino cooling over takes the mass element in question. Contrary to the previous treatment of this problem, an attempt will be made here to quantify the fall-back problem in terms of the entropy of the matter surrounding the neutron star. It has been customary to describe the collapse and ejection shock in terms of entropy, and so it is only reasonable to extend these concepts to a quantitative discussion of the necessary hot bubble and fall-back problem (see Bethe and Wilson 1985). The advantages of characterizing the fall-back matter in terms of entropy are several fold. First entropy is conserved in the initial expansion of the ejected matter, and second if a gradient in entropy exists in static equilibrium, then only a positive gradient is stable and as Bethe (1990) has pointed out the convective mixing especially close to the neutron will be reduced.

The plan of this paper will be to discuss first the structure of an explosion in a polytropic pre-supernova star in terms of entropy of the shock wave and second, to consider the cooling rate of an atmosphere by neutrino emission, both in hydrostatic equilibrium and in contact with a neutron star and third, to derive the reimplosion or accretion rate of matter onto the neutron star as a function of entropy, with the assumption that the pressure support has disappeared.

The Explosion

The explosion of a Type II supernova involves roughly 10^{51} ergs ejected kinetic energy and roughly equal energy in overcoming the original gravitational binding of the ejected matter. Since the major fraction of the mass of the ejected matter will have roughly the same velocity, namely 3×10^8 cm/s for a 10 $M_\odot$ star, the explosion shock wave that overtakes this matter must change the matter velocity from initially nearly at rest to something like this value following the explosion shock wave. This shock wave of course would have to be stronger in the interior where a smaller mass fraction is involved and a larger binding energy is to be overcome, and slightly weaker on the outside, but the general characteristic of a near constant velocity shock wave overtaking the matter of the star is a first crude approximation. This shock wave then traverses a density distribution starting from high density in the interior to low density in the envelope. The original density of the matter at the mass cut of ejection will be of the order of 10^8 g/cm^3 and in the outer mantle less than 10^{-8} g/cm^3 so that at least sixteen orders of magnitude of density change are involved. Similarly the original temperature of the matter decreases from the interior to the outer surface so that at near constant shock velocity the Mach number of the shock will steadily increase to relatively high values when it breaks through the surface.

In addition, because of the large density change, the entropy behind the shock will increase to very high values on the exterior. As a consequence, we expect any explosion shock near the neutron star to produce matter of ever increasing entropy so that whatever high entropy bubble may originally be driving the shock wave, somewhere in the exterior this entropy will be exceeded by the shock ejected matter. This has consequences for subsequent convective mixing as well as restricts the use of Sedov solutions (Sedov 1959) which characteristically give a constant Mach number (Chevalier 1989). Consequently, we will start with the assumption of matter imploding onto the neutron star that is subsequently shocked and reversed in trajectory by a shock wave whose velocity in the core is at first dependent on radius and then becomes nearly constant.

A density distribution characteristic of nearly all models of collapse (Bethe and Wilson 1985) is characterized by $\rho=\rho_0\, r^{-3}$ where typically $\rho_0=10^{32}$ g. This density distribution has the characteristic that the mass increases as the logarithm of the radius which is a reasonable approximation for a stellar structure out to an envelope terminated by a surface. This value of ρ_0 results in a mass of 8 - 9 $M_\odot$ for a surface at 2×10^{12} cm typical of 1987A. When part of the mass distribution is in free fall, the density drops because of velocity divergence by roughly a factor of 10 to $\rho_0=10^{31}$ g. We next consider an explosion shock wave strong enough with shock velocity, v_s, to eject the matter from the gravitational potential or $v_s = (MG/r)^{1/2}$ where we have purposely left out the factor of 2 because of equal kinetic and internal energy behind the shock wave. We also characterize the "shock velocity" as approximately the change in fluid velocity across the shock transition. The large compression ratio, 7 for $\gamma=4/3$ means that the shock geometry is close to a snow-plow model and hence, the shock and fluid velocity are nearly the same.

The entropy of the matter, s, is expressed in terms of the Boltzmann constant k, but note that the pseudoentropy, $s_p=P/\rho^{4/3}$, is frequently more useful for calculational purposes. The relationship can be derived by noting that s=2 when the radiation energy content of a gas equals the particle energy content, or when $a\,T^4=3/2\,nkT$. The pseudoentropy representation is only valid for P(*radiation*) >> P (*particle*) because we have chosen $\gamma=4/3$, or equivalently for s >> 1. Then for this condition $s=9.1\times10^{-16}$ $(P/\rho^{4/3})$ for a mean atomic weight of unity, that is, electrons and ions of A=2.

The entropy behind the explosion shock can be derived assuming $\rho_o=10^{3}$ g noting the compression ratio is 7 and assuming $M_{neutron\ star} = 1.4\ M_\odot$. Then

$$s = 9.16\times10^{-16}\,\rho\, v_s^2/\rho^{4/3} = 3.8. \qquad (1)$$

The entropy behind the explosion shock in this region of the core is therefore approximately independent of radius and modest in value. It is large enough to ensure our approximation of a radiation dominated gas. It also could be a factor of several larger due to the reversal of the infall velocity. In deriving this entropy the entropy of nuclear disassociation of the nuclei has been neglected relative to the entropy of neutron star matter. This adds roughly 6 to the entropy of the condition described by equation (1), but it will be neglected here so that the relative entropies of the radiation dominated ejected matter can be compared easily. Neutrino emission will, of course, lower it. Beyond a radius of about 2×10^9 cm, the shock velocity should remain nearly constant at 3×10^8 cm/s (since half the mass is inside) and so the entropy should increase as $\rho^{-1/3}$ or proportional to radius. This results in an entropy of greater than 2000 in the outer envelope. This description of an explosion shock is shown in Fig.3.

The Static Post-Shock Atmosphere

Since the flow behind the shock is subsonic, the density distribution will be hydrostatic. The entropy next to the neutron for a uniform distribution in radius, i.e., a constant, s_o is related to the temperature. For a low mass atmosphere

$$dP/dr = -\rho g = -MG\rho/r^2 . \qquad (2)$$

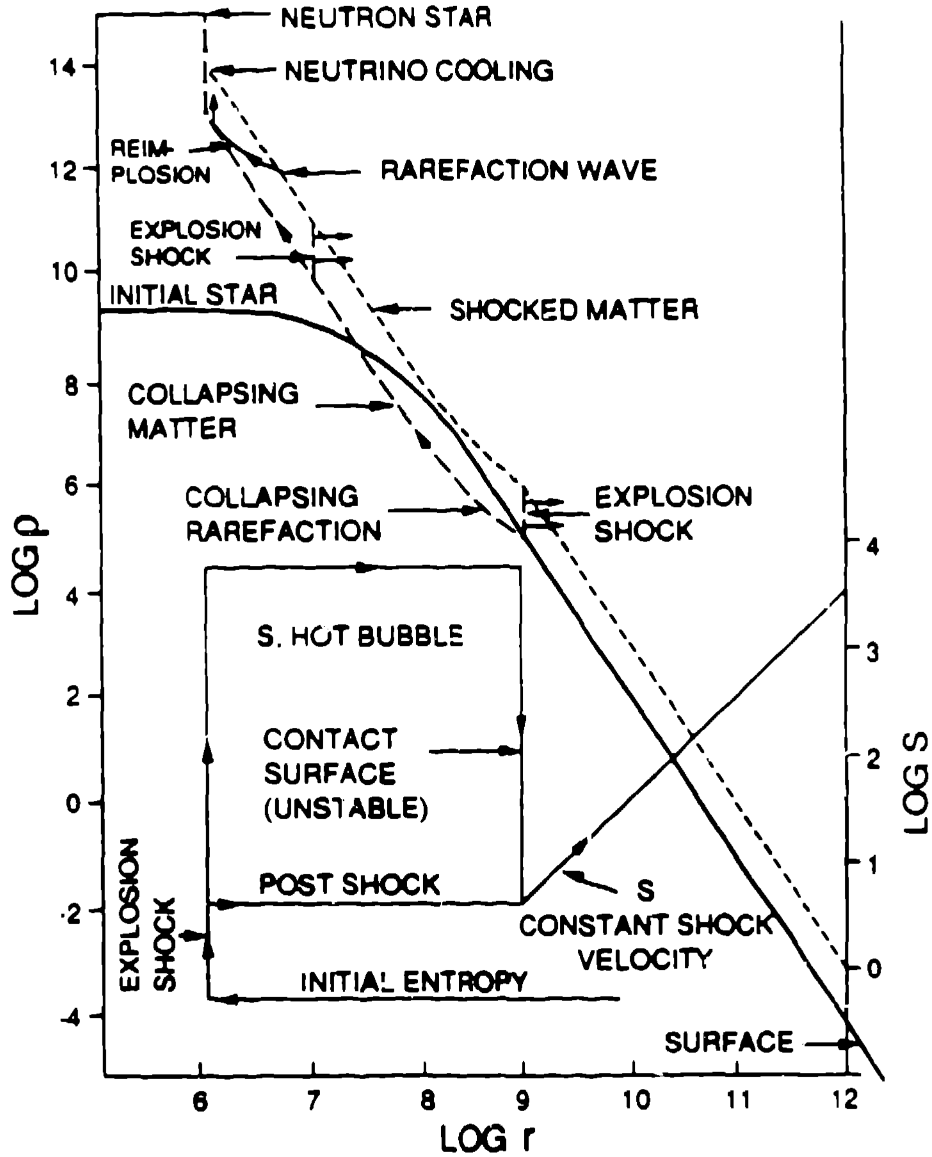
However, using the definition entropy in terms of P and ρ where $s=9.1\times10^{-16}\,P/\rho^{4/3}$, s_{ns} at the neutron star surface where $r_{ns}=10^6$ cm after several seconds, Fig.(2), becomes

$$s_{ns} = 1.34\times10^{11}\, r_6^{-4/3}\ P_{ns}^{-1/3}. \qquad (3)$$

If this is expressed in terms of a temperature in MeV, T_{mev}, at the neutron star surface, then

$$s_{ns} = 189\ T_{mev}^{-4/3}\, r_6^{-4/3}. \qquad (4)$$

Figure 3 shows the initial density distribution in the star precollapse, during collapse, and with a hypothetical explosion shock wave strong enough to deposit the energy necessary for ejecting the matter external to the neutron star. Neutrino cooling gives rise to a rarefaction wave that reverses the explosion. The entropy distribution before and later behind the explosion shock is shown along with the entropy of a hot bubble necessary to support the explosion matter despite neutrino cooling.



Thus at an Mev temperature the entropy for an equilibrium atmosphere must be almost 100 times larger than that created by a typical explosion shock wave. However, the neutrino cooling of such an atmosphere in contact with the neutron star surface at this temperature is given by Schinder *et al.* (1987). This results in a cooling rate of $9\times10^{24}\ T_{Mev}^{9}$ ergs $cm^{-3}\ s^{-1}$ and therefore a cooling time of

$$\tau_{cool} = 42\ T_{mev}^{5}\ \text{sec, or}\ 42\ (s_{ns}/189)^{-15/4}\ \text{sec.} \qquad (5)$$

If the entropy of the matter is roughly 6 in these units or closer to 12 when the nuclei are included, the cooling time becomes 10^{-4} seconds, or the free-fall time of the atmosphere onto the neutron star. The temperature of such a collapsing atmosphere becomes 13.3 MeV without including the nuclear emission The more exact calculations of Zel'dovich et.al. (1972) give a temperature closer to 11 MeV, but from a low density cloud.

Explosion Mechanism

These temperatures are sufficiently high such that a collapsing or accreting atmosphere is a logical explanation of the late time (10 s) high energy neutrinos observed from 1987A. The neutron star itself should cool to an interior temperature of an Mev in several seconds (Wilson and Mayle 1990, Van Riper 1991, and Nomoto and Tsuruta, 1987). The implication is that this high temperature collapsing atmosphere is emitting neutrinos at high enough energy (3 kT=40 MeV) such their enhanced cross section for deposition of their energy in the imploding matter at larger radius and lower density creates the high entropy bubble that causes the explosion. This was exactly the mechanism mocked up in Colgate and White, (1966) where the newly formed neutron star was treated as a ridged boundary after a small fraction of a second and the energy of subsequently accreted matter was emitted as neutrinos from a shocked gas with a temperature of 12 to 15 MeV. The appropriate fraction of the neutrino flux was deposited in the in-falling matter, heated it, and caused the explosion.

Mixing Behind the Shock

The consequence of the hot bubble is that it will mix at the contact surface of the expanding shocked matter. This shocked matter has considerably lower entropy than the hot bubble. To investigate the degree of this mixing there are several surprising simplifications to the problem. These are (1) the expected mixing is close to the thickness of matter behind the shock and (2) the shock entropy increases due to the decreasing density of the envelope and the two entropies: that of the expanding hot bubble and that of the material behind the shock will become equal somewhere near the boundary between helium and hydrogen or when $\rho=0.1$ to 1 g cm^{-3}. Thus the outer mass fraction of ejected hydrogen will be unmixed, and that inside everything will be mixed. Thus we see that the requirement for a hot bubble to make the supernova explosion is already manifested in the signature of mixing in x-rays and gamma rays of 1987A. The mixing created by the small molecular weight difference in the supernova structure is probably too small to give rise to the necessary mixing. This is because of the stabilizing entropy gradient created by the density in combination with the near constant shock velocity. Also, t[illegible] mixing should be nearly complete by the time of ^{56}Ni decay, so that no further entropy gradients would be created.

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