LAMS--894

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Reactor

LAMS-894

~~Series A~~

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24 May 1949 This document contains 16 pages.

FEASIBILITY OF SHADOW SHIELDING IN NUCLEAR POWERED SUBMARINES

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Feasibility of Shadow Shielding in Nuclear Powered Submarines

Abstract

Calculations concerning the shadow shielding of a nuclear reactor for submarines of the type proposed by Teller are given. A lower bound on the scattered radiation has been calculated by considering only the singly scattered γ rays. It is found that if the reactor operates at a power level of 10^5 kw, the magnitude of the singly scattered γ radiation that would reach the crew is far above lethal dosage.

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The purpose of this paper is to attempt to establish whether or not simple shadow shielding is adequate to protect personnel from γ radiation in a nuclear energy powered submarine.

In order to establish a lower bound for the scattered radiation which will reach the crew, only single scattering will be considered.

A typical reactor of the type proposed by Teller would have the following approximate specifications:

1. The reactor is in the form of a cube 1 meter on a side.

2. The reactor operates at a power level of 10^5 kw.

The reactor will be schematized in the submarine as shown in Fig. 1.

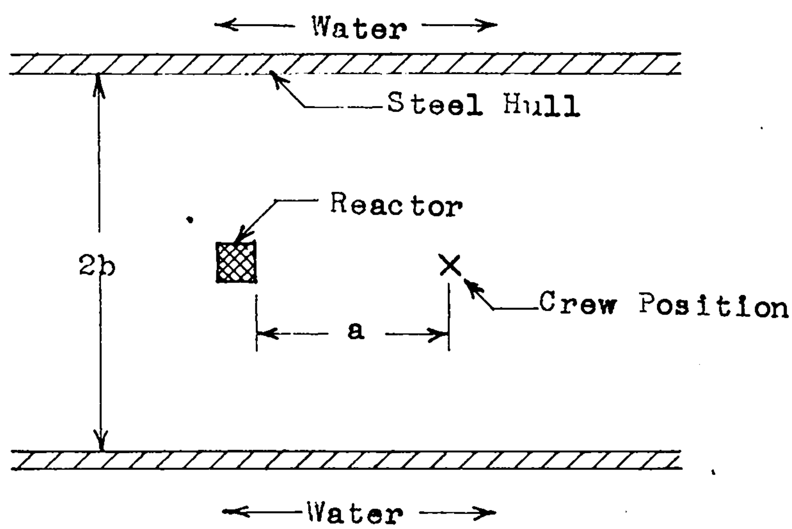

Fig. 1

What is desired to be calculated is the following: Assuming a simple shadow shield placed in front of the reactor

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(i.e. - no direct radiation from the reactor can reach the crew position), what is the gamma ray intensity at the crew position due to singly scattered radiation that has been reflected from the hull and the water outside the ship?

Assume that 2 prompt gamma rays, having a photon energy of 2.5 Mev each, are emitted per fission. For a power level of 10^5 kw, the rate of fission in the reactor is 3.68×10^{18} fissions/sec and thus there are 7.36×10^{18} gamma rays/sec produced in the reactor.

In order to know the gamma ray energy flux emerging from the reactor per second, it is necessary to know the self-absorption of the reactor. If the dimensions of the reactor are large compared to the mean-free-path of the gamma rays in the reactor, then the self-absorption may be obtained in the following manner.

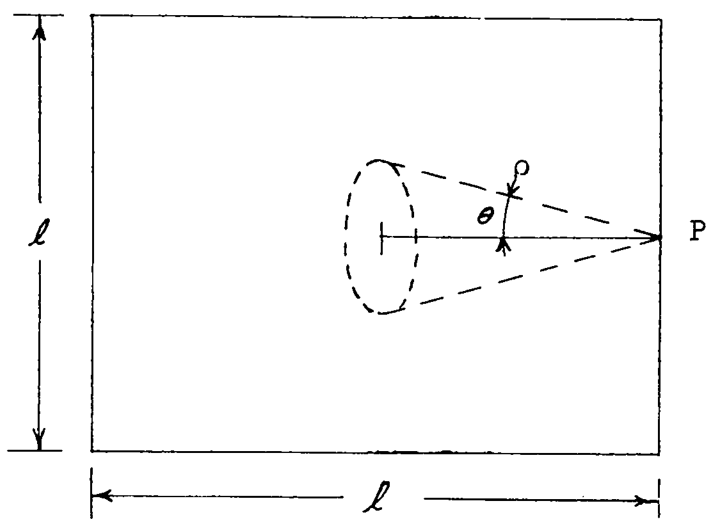

Fig. 2

Let:

W = power level of reactor; $\frac{\text{Mev}}{\text{sec}}$.
V = volume of reactor; cm^3.
E = energy flux at P; $\text{Mev/cm}^2\text{sec}$.
μ = gamma ray absorption coefficient in reactor; cm^{-1}.

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The self-absorption will be derived by considering only the unscattered gamma radiation escaping from the surface of the reactor. This is a good approximation because the largest contribution to the energy flux at P comes from the volume immediately adjacent to P; and further, the scattered radiation reaching P is not only small in number but degraded in energy.

The contribution to the energy flux at P from a volume dV defined by

$$dV = 2\pi\rho^2 \sin\theta \, d\theta \, d\rho \text{ (Fig. 2)} \tag{1}$$

is

$$dE = \frac{\frac{W}{V} \cdot dV}{4\pi\rho^2} \quad e^{-\mu R} \tag{2a}$$

$$= \frac{W}{2V} \, e^{-\mu R} \sin\theta \, d\theta \, d\rho \tag{2b}$$

Integration over proper limits gives

$$E = \frac{W}{2V} \int_0^\infty e^{-\mu R} \, d\rho \int_0^{\pi/2} \sin\theta \, d\theta \tag{3a}$$

$$E = \frac{W}{2V\mu_R} \tag{3b}$$

If the self-absorption, η, is defined as $\eta = \frac{\text{emergent gamma rays}}{\text{produced gamma rays}}$

then $$\eta = \frac{6\ell^2 E}{W} \tag{4a}$$

$$\eta = \frac{3}{\mu_R \ell} \tag{4b}$$

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To determine the proper value of μ_R the composition of the reactor must be known. Assuming the reactor to have the volume ratio of U to H_2O of 1/3, the mean density of the reactor is

$$\bar{\rho} = 6.9 \text{ gm/cm}^3$$

and the partial densities are

$$\rho_U = 6.2 \text{ gm/cm}^3$$

$$\rho_{H_2O} = .67 \text{ gm/cm}^3$$

This gives an absorption coefficient

$$\mu_R = \underline{.314 \text{ cm}^{-1}}$$

Thus the self-absorption, is $\eta = 0.096$

The singly scattered gamma ray intensity at the crew position will now be calculated. Since the hull has an appreciable thickness in terms of its mean-free-path for the incident gamma radiation and yet is not sufficiently thick to be considered infinite for calculational purposes; calculations will be made for infinite thickness of steel and water, with the correct answer lying between the two extremes.

The expression for the singly scattered intensity is derived as follows:

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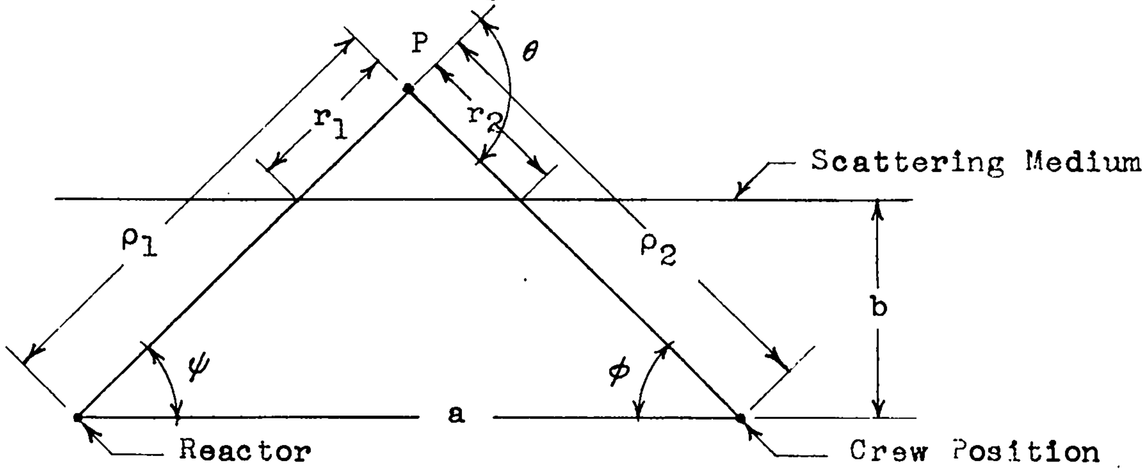
Fig. 3



Here we have idealized the reactor to a point source.

No scattering in the air inside the submarine will be considered.

Let

M = number of gamma rays emerging from reactor per second.

α_o = energy per gamma photon; mc^2.

μ = absorption coefficient in scattering medium; cm^{-1}.

The photon flux at P is

$$\frac{M\, e^{-\mu(\alpha_o)r_1}}{4\pi\rho_1^2}$$

Define a scattering volume, dV, by

$$dV = 2\pi\rho_2^2 \sin\phi \, d\phi \, d\rho_2 \tag{5}$$

The number of gamma rays, N, that will scatter in the volume dV through an angle θ and into a solid angle, $d\Omega$, is

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$$dN = \frac{M\, e^{-\mu(a_o) r_1}}{4\pi \rho_1^2} \cdot n dV \left(\frac{d\sigma}{d\Omega}\right)_{\theta, a_o} d\Omega \tag{6}$$

where

n = number of electrons per cm3 in the scattering medium.

$\left(\frac{d\sigma}{d\Omega}\right)_{\theta, a_o}$ = Klein-Nishina differential scattering cross section per electron.

If the receiver volume is taken as a small sphere of cross sectional area dA, then $d\Omega = \frac{dA}{\rho_2^2}$.

Consequently, the differential expression for the gamma ray flux at the crew position is $dI = \frac{dN}{dA}$ so that

$$dI = \frac{M\, e^{-\left[\mu(a_o) r_1 + \mu(a_1) r_2\right]}}{2\rho_1^2} \cdot n \left(\frac{d\sigma}{d\Omega}\right)_{\theta, a_o} \sin\phi \, d\phi \, d\rho_2 \tag{7}$$

If a_1, is the new energy of a singly scattered photon, where the relation between a_1, a_o and the scattering angle θ is given by

$$a_1 = \frac{a_o}{1 + a_o(1 - \cos\theta)} \tag{8}$$

then the expression for the energy flux (mc^2/cm^2 sec) at the crew position is

$$dE = a_1 \, dI \tag{9}$$

This above expression may be simplified and put into integrable (numerically) form by use of the following relations obtained from Fig. 3.

$$\rho_1 = \frac{a \sin\phi}{\sin\theta} \tag{10}$$

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which gives

$$\frac{\rho_2}{\sin(\theta - \phi)} = \frac{a}{\sin \theta}, \tag{11}$$

$$\rho_2 = a \cos \phi - a \sin \phi \cot \theta \tag{12}$$

If ϕ is held constant and (12) differentiated with respect to ρ_2 and θ

$$d\rho_2 = a \sin \phi \csc^2 \theta \, d\theta \tag{13a}$$

$$d\rho_2 = \frac{a \sin \phi}{\sin^2 \theta} \, d\theta \tag{13b}$$

Putting these expressions in the equation previously obtained for the energy flux gives

$$dE = \frac{Mn}{2a} \, a_1 \left(\frac{d\sigma}{d\Omega}\right)_{\theta, a_o} e^{-\left[\mu(a_o) r_1 + \mu(a_1) r_2\right]} \, d\phi \, d\theta \tag{14}$$

Also from Fig. 3 we find r_1 and r_2 to be

$$r_1 = \rho_1 - \frac{b}{\sin(\theta - \phi)} \tag{15a}$$

$$r_1 = \frac{a \sin \phi}{\sin \theta} - \frac{b}{\sin(\theta - \phi)} \tag{15b}$$

$$r_2 = \rho_2 - \frac{b}{\sin \phi} \tag{16a}$$

$$r_2 = \frac{a \sin(\theta - \phi)}{\sin \theta} - \frac{b}{\sin \phi} \tag{16b}$$

The integration limits on ϕ and θ are easily obtained from Fig. 3.

$$0 \leqslant \phi \leqslant \pi \tag{17}$$

For a fixed ϕ,

$$\phi + \psi \leqslant \theta \leqslant \pi \tag{18}$$

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ψ is seen to be

$$\psi = \tan^{-1} \left[\frac{b}{a - \frac{b}{\tan \phi}} \right] \tag{19}$$

Letting

$$\phi + \psi = \beta \tag{20}$$

the energy flux E is seen to be

$$E = \frac{Mn}{2a} \int_0^{\pi} d\phi \int_{\beta}^{\pi} \alpha_1 \quad \left(\frac{d\sigma}{d\Omega}\right)_{\alpha_0, \theta} e^{-\left[\mu(\alpha_0) r_1 + \mu(\alpha_1) r_2\right]} \cdot d\theta \tag{21}$$

Of special interest is the dependence of E on the angle ψ. This dependence may be obtained from equation (19).

Of more pertinent interest is the ionization level at the crew position rather than the energy flux. What is desired is the dosage rate, i.e. - roentgens/day, at the crew position. The dosage rate in r/day is obtained in the following manner: The differential contribution to the energy absorbed in air (mc^2/cm^3 sec) may be written as

$$dE = \mu_A(\alpha_1)\ dE \tag{22}$$

where

$\mu_A(\alpha_1)$ = coefficient of energy transfer to air; cm^{-1}

Since the roentgen is defined as an ionization level of 1 e.s.u./cm^3 of air at S.T.P., $\mu_A(\alpha_1)$ is taken at S.T.P. Further, the mean ionization potential for air is 66×10^{-6} mc^2 and the charge per ion is 4.80×10^{-10} e.s.u. Thus the

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differential contribution to the dosage rate in roentgens/sec is

$$dR_{\pi} = \frac{4.80 \times 10^{-10}}{66 \times 10^{-6}} \times \mu_A(\alpha_1)\, dE \tag{23}$$

The dosage rate, R_{π} in roentgens/day, due to single scattering may be written

$$R_{\pi} = .627 \times \frac{Mn}{2a} \int_0^{\pi} d\phi \int_{\beta}^{\pi} \alpha_1\, \mu_A(\alpha_1) \left(\frac{d\sigma}{d\Omega}\right)_{\theta,\alpha_o} e^{-\left[\mu(\alpha_o) r_1 + \mu(\alpha_1) r_2\right]} d\theta \tag{24}$$

For a typical submarine we shall assume that "a" and "b" are given by

$$a = 5 \times 10^2 \text{ cm}$$

$$b = 5 \times 10^2 \text{ cm}$$

Furthermore we are given that

$n = 3.34 \times 10^{23}$ electrons/cm^3 for H_2O

$n = 2.20 \times 10^{24}$ electrons/cm^3 for Fe

$\mu(\alpha_o) = 0.044$ cm^{-1} for H_2O

$\mu(\alpha_o) = 0.300$ cm^{-1} for Fe

and we have shown that

$M = 7.05 \times 10^{17}$ gamma rays/second

Putting these numbers into equation (24) we find

$R_{\pi}(H_2O) = 7 \times 10^6$ r/day

$R_{\pi}(Fe) = 2 \times 10^7$ r/day

Now let us see how, the interposition of a simple shadow shield between the reactor and the crew position effects the above

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dosage values R.

Let us define a simple shadow shield as one which restricts ψ to the range of integration

$$\psi_m \leqslant \psi \leqslant \pi \tag{25}$$

where

$$0 \leqslant \psi_m \leqslant \pi/2 \tag{26}$$

Then for a given ψ_m we obtain the dosage

$$R_m = R_\pi \frac{\int_{\psi_m}^{\pi} F(\psi)\, d\psi}{\int_0^{\pi} F(\psi)\, d\psi} \tag{27}$$

where $$F = \int_\beta^\pi G(\psi, \theta) d\theta \tag{28}$$

and $$G(\psi, \theta) = a_1 \mu_A(a_1) \left(\frac{d\sigma}{d\Omega}\right)_{\theta, a_o} e^{-\left[\mu(a_o)r_1 + \mu(a_1)r_2\right]} \tag{29}$$

Graphs 1 and 2 show that variation of $F(\psi)$ as a function of ψ.

The following results are obtained:

For H_2O :

$$R_{\pi/4} = 6 \times 10^6 \text{ r/day}$$

$$R_{\pi/2} = 3 \times 10^6 \text{ r/day}$$

For Fe:

$$R_{\pi/4} = 2 \times 10^7 \text{ r/day}$$

$$R_{\pi/2} = 8 \times 10^6 \text{ r/day}$$

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Thus it is seen that only a small reduction is obtained by making ψ_m as great as $\pi/2$. Even the quasi shadow shielding with $\psi_m = \frac{3\pi}{4}$ would result in an intolerable dosage level.

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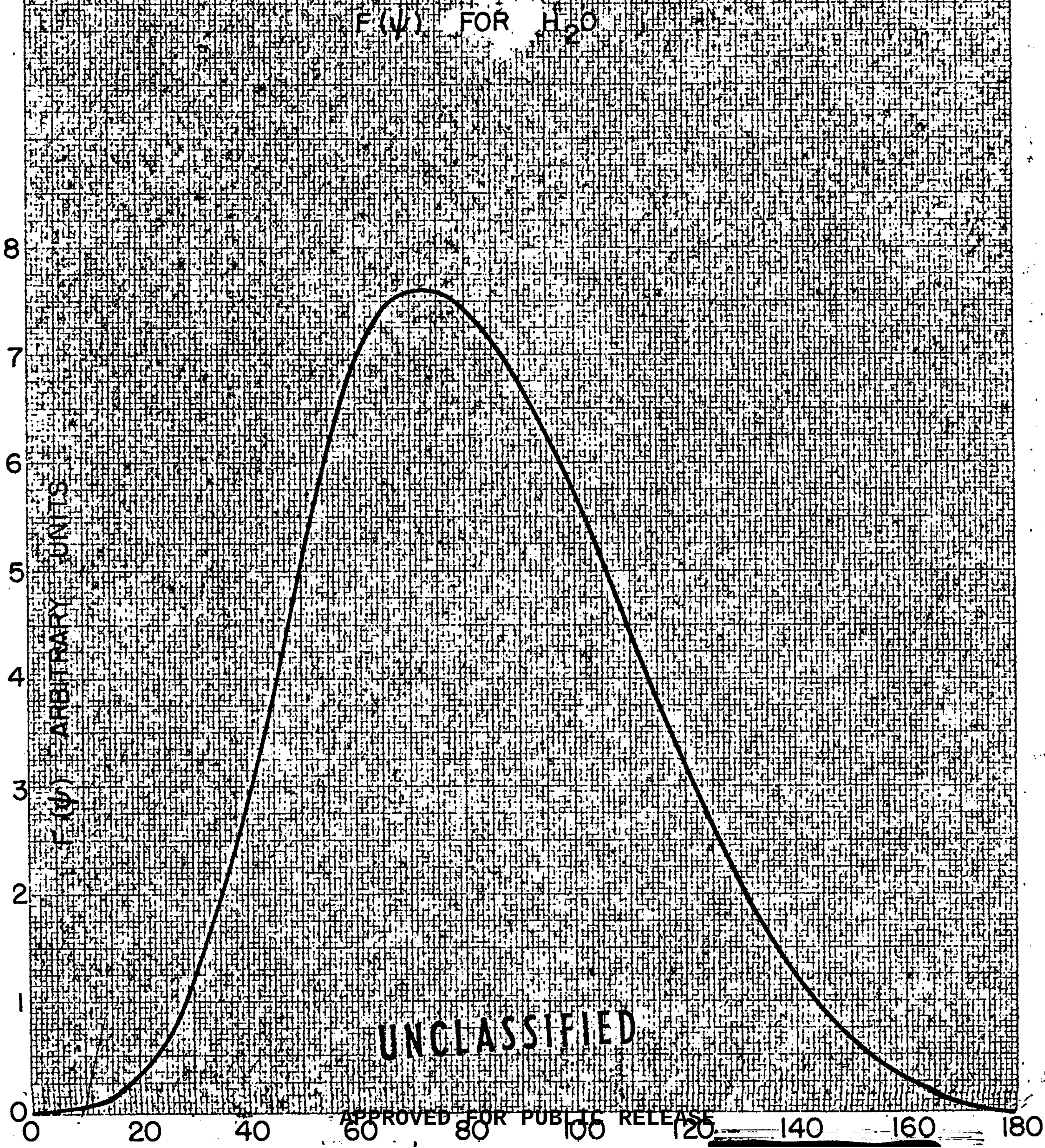
Graph 1


F(ψ) FOR H_2O
F(ψ) ARBITRARY UNITS
0
1
2
3
4
5
6
7
8
0
20
40
60
80
100
120
140
160
180

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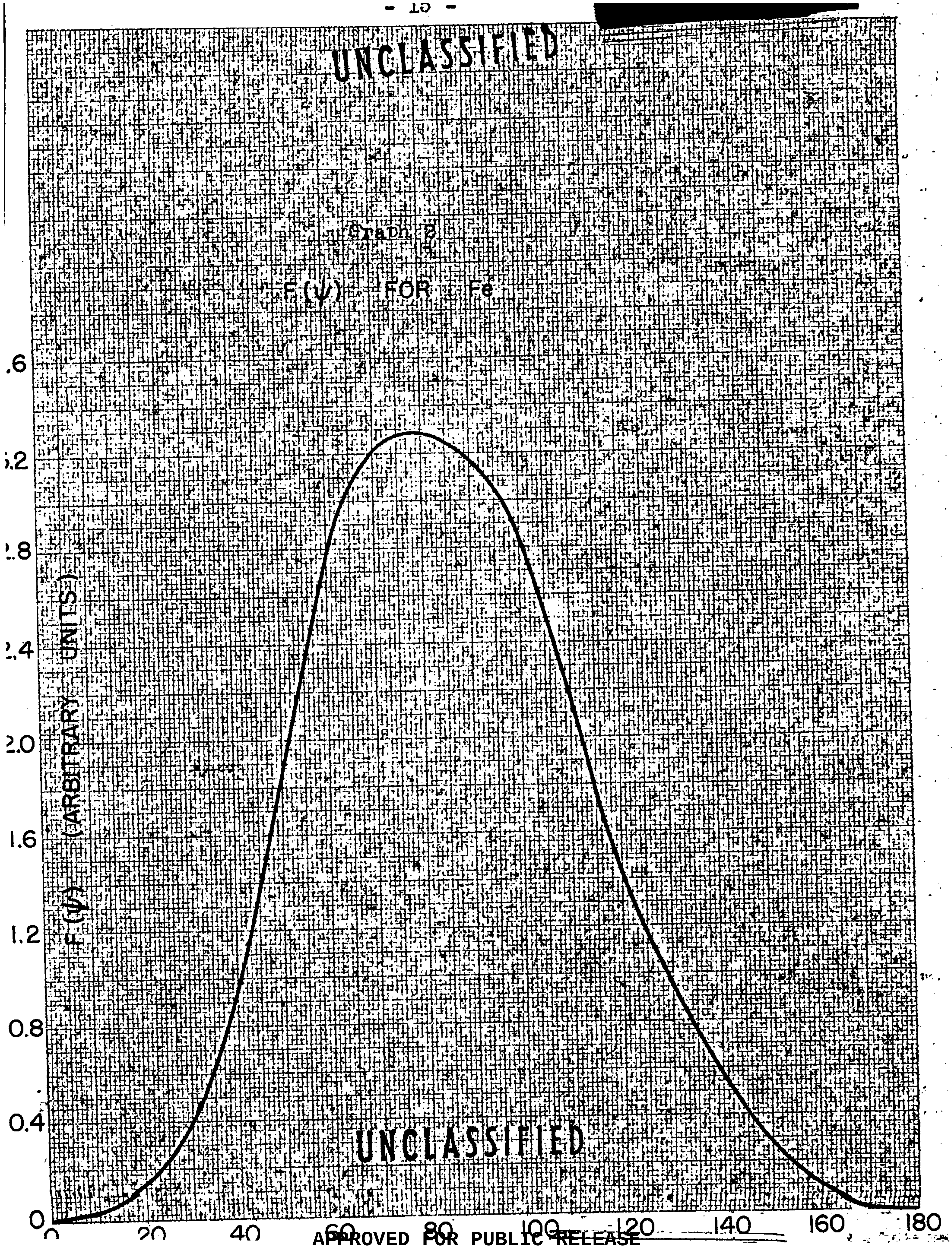
Graph 2
F(ψ) FOR Fe
F(ψ) (ARBITRARY UNITS)
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
0
20
40
60
80
100
120
140
160
180

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Acknowledgement

We are indebted to Miss Barbara Bissel of the RAND Corporation for the numerical computations in this report.

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