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Effects & Instruments

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This document contains 5 pages
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The following is a memorandum from von Neumann to Oppenheimer dated 8/28/44.

TO: J. R. OPPENHEIMER

FROM: J. VON NEUMANN

SUBJECT: SURFACE WATER WAVES EXCITED BY AN UNDERWATER EXPLOSION

The Navy Bureau of Ordnance, Section Re 2c has made experiments on this subject, and further experiments are now in progress there, as well as at the Underwater Explosive Laboratory, Div. 2, NDRC, at Woods Hole.

The Bu. Ord. experiments were made with big charges: 3, 6, 21 and 46 (?) thousand pounds of TNT, in 40 feet of water, exploded at the bottom. The maximum bubble radii of these are about 38, 48, 73, 95 feet, respectively. Thus the two first ones nearly or just about break surface, while the two last ones ought to produce well defined holes, comparable to those which are produced by McMillan's procedure. The hole radii ought to be somewhat less than the computed bubble radii, since the bubble in the last stages of its expansion has less than atmospheric pressure.

Now one of McMillan's series of observations gave this:

Depth of Water (h)	Diameter of Hole (D)	Wave Velocity (V)	Wave Length (λ)	Wave Period ($t=\lambda/V$)	Time Scaling Factor $h^{-\frac{1}{2}}$	Scale Comparison
1 "	3 "	15.2 $\frac{\text{in}}{\text{sec}}$	4.2 "	.28 sec	1	.28
2½ "	8 "	24.7 $\frac{\text{in}}{\text{sec}}$	11.2 "	.45 sec	.61	.275
4 "	12 "	31.9 $\frac{\text{in}}{\text{sec}}$	18.2 "	.57 sec	.5	.285

Scaling this up to 40' = 480" depth gives:

480" (40')	1440" (120')	320 $\frac{\text{in}}{\text{sec}}$ (26.7 $\frac{\text{ft}}{\text{sec}}$)	2016" (168')	6.3 sec	.045	.283

The actual observations on the 46,000 pounds charge were as follows: The equivalent hole radius " is of the order of but less than 190' (see above), hence the scaled up value of 120' is acceptable. Then:

40'	~120'	~20 to 40 $\frac{ft}{sec}$ average 25 $\frac{ft}{sec}$	200 to 100'	8 to 4 sec

observed values.

So the 46,000 pounds charge scaled essentially correctly from McMillan's smallest scale experiments, which were below it by factors of 480 to 120 on the linear scale!

Theoretical efforts were also made to understand the surface wave excitation by an underwater explosion. The inital work is Kirkwoods, some modifications were suggested by Herring, what I describe in the following is a still later version, which seemed most satisfactory to me.

The calculations which follow describe the explosion by its gas bubble, which is assumed to be at depth R, in water of depth H, and the bubble is treated as a point. It is supposed, however, to displace a volume $V(t)$ of water which must be known as a function of time, i.e., the bubble is described as a point source, with the intensity $x(t) = (dV(t))/dt$ at the time t.

This is, of course, rather far from the "hole in the water" model, but it may give an idea of the orders of magnitude of the quantities involved.

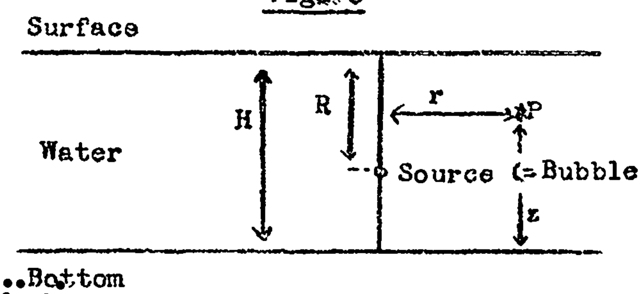
Using coordinated z, r (see Figure) for a general point P, let

$$\Phi = \Phi(P,t) = \Phi(z, r, t)$$

be the velocity potential. The water is treated as incompressible.

The effect of the source alone would be a velocity field given by the potential

Figure



$$\Phi_{\infty} = \frac{x(t)}{4\pi} \frac{1}{\sqrt{r^2 + (z + H - R)^2}}$$

The influence of the bottom is taken into account by adding an image point, i.e., by using the potential

$$\Phi_0 = \frac{x(t)}{4\pi}\left[\frac{1}{\sqrt{r^2 + (z - H - R)^2}} + \frac{1}{\sqrt{r^2 + (z - H + R)^2}}\right].$$

In order to account for the influence of the surface as well, a further potential Φ_1 must be added, which is regular everywhere in the water, i.e., the final potential is

$$\Phi = \Phi_0 + \Phi_1 .$$

Now the differential equations for Φ_1 become:

(1) $$\nabla^2 \Phi_1 = 0$$ (incompressibility),

(2) $$\left.\frac{\partial \Phi_1}{\partial z}\right|_{z=0} = 0$$ (bottom),

(3) $$\left.\left\{\frac{\partial^2 \Phi_1}{\partial t^2} + g\,\frac{\partial \Phi_1}{\partial z}\right\}\right|_{z=H} = -\left.\left\{\frac{\partial^2 \Phi_0}{\partial t^2} + g\,\frac{\partial \Phi_0}{\partial z}\right\}\right|_{z=H}$$ (surface).

Applying a Bessel transformation (in r)

$$\Phi_1 = \Phi_1(z, r, t) = \int_0^\infty f(z, k, t)\; J_0(kr)\, k\,dk,$$

(1), (2) are satisfied exactly by

$$f(z, k, t) = F(k, t)\cosh(zk),$$

and (3), after a Bessel transformation becomes

$$\frac{\partial^2 F}{\partial t^2}\cosh(Hk) + F\cdot gk\sinh(Hk) =$$

$$= -\int_0^\infty \left.\left\{\frac{\partial^2 \Phi_0}{\partial t^2} + g\,\frac{\partial \Phi_0}{\partial z}\right\}\right|_{z=H} \cdot J_0(kr)\, r\,dr =$$

$$= \left(gx - \frac{1}{k}\frac{d^2 x}{dt^2}\right)\frac{1}{2\pi}\; e^{-Hk}\cosh\big((H - R)k\big).$$

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i.e.,

$$\frac{\partial^2 F}{\partial t^2} + \omega^2 F = \left(gx - \frac{1}{k}\frac{d^2x}{dt^2}\right) \frac{1}{2\pi} e^{-Hk} \frac{\cosh\left((H-R)k\right)}{\cosh(Hk)},$$

where

$$\omega = \omega(k) = \sqrt{g\,k \tanh(Hk)}.$$

This gives

$$F = F(k,t) = \frac{1}{\omega}\frac{1}{2\pi} e^{-Hk} \frac{\cosh\left((H-R)k\right)}{\cosh(Hk)} \cdot \int_{t_o}^{t} \left(gx(s) - \frac{1}{k}\frac{d^2x(s)}{ds^2}\right) \sin\left(\omega(t-s)\right) ds +$$

$$+ A_{t_o}(k)\cos(\omega t) + B_{t_o}(k)\sin(\omega t).$$

Assume that the explosion occured at $t = 0$, i.e., $V(t) \equiv 0$ and $x(t) \equiv 0$ for $t < 0$. Then necessarily $\phi \equiv 0$ and with it $f \equiv 0$, $F \equiv 0$ for $t < 0$. Hence for a choice $t_o < 0$ necessarily $A_{t_o}(k) \equiv B_{t_o}(k) \equiv 0$.

Two partial integrations in the above expression for F allow to replace $\frac{d^2x(s)}{ds^2}$ by $x(s)$, and one more, to replace $x(s) = \frac{dV(s)}{ds}$ by $V(s)$. Remembering $t_o < 0$ and its implications, this gives

$$F = F(k,t) = \frac{1}{2\pi}\frac{\cosh\left((H-R)k\right)}{\cosh(Hk)} \cdot \left[-\frac{1}{k e^{Hk}}\frac{dV(t)}{dt} + \frac{g}{\cosh(Hk)} \int_{t_o}^{t} V(s)\cos\left(\omega(t-s)\right) ds \right].$$

Now one can put, by continuity $t_o = 0$. The main effects occur after the bubble has collapsed. Assume therefore

$V(s) \equiv 0$ for $s < 0$ as well as

for $s > T$,

the $T > 0$ being the significant bubble period. Assume further $t > T$. Then

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$$F = F(k,T) = \frac{g}{2\pi}\,\frac{\cosh\,(H-R)k}{\cosh^2\,(Hk)}\int_0^T V(s)\,\cos\big(\omega(t-s)\big)\,ds.$$

From this

$$\Phi_1 = \Phi_1(z,r,t) = \frac{g}{2\pi}\int_0^\infty \frac{\cosh\big((H-R)k\big)\,\cosh\,(zk)}{\cosh^2\,(Hk)}\,\cdot$$

$$\cdot\int_0^T V(s)\,\cos\big(\omega(t-s)\big)\,\cdot\,J_0\,(kr)\,r\,dr.$$

(Remember that $\omega = \omega(k) = \sqrt{gk\,\tanh\,(Hk)}$)

These formulae allow to compute the velocity, pressure, surface wave height everywhere. Numerical calculations are being carried out now in the Bu. Ord., Section Re 2c, Washington, and by the Applied Mathematical Panel Computing Project, New York.

Some preliminary results seemed to agree reasonably well with the big charge experiments.

(P.S. My formulae ought to be checked, although I think that they are correct.)

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