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# SMoDDimensional Cross-Section <br> minitiviti and Uncertainty Analysis :or Fusion Reactor Blankets 




#### Abstract

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# Two-Dimensional Cross-Section Sensitivity and Uncertainty Analysis For Fusion Reactor Blankets 

Mark Julien Embrechts



## CONTENTS

Nomenclature ..... viii
List of Figures ..... x
List of Tables ..... xi
Abstract ..... xiv

1. Introduction ..... 1
1.1 Motivation ..... 2
1.2 Literature Review ..... 4
2. Sensitivity Theory ..... 7
2.1 Definitions ..... 8
2.1.1 Cross-section sensitivity function, cross-section sensitivity profile and integral cross-section sen sitivity ..... 8
2.1.2 Vector cross section ..... 9
2.1.3 Geometry related terminology ..... 10
2.2 Cross-Section Sensitivity Profiles ..... 12
2.2.1 Introduction ..... 12
2.2.2 Analytical expression for the cross-section sen- sitivity profile ..... 14
2.2.3 Explicit expression for the cross-section sensi- tivity profile in discrete ordinates for a two- dimensional geometry representation ..... 20
2.3 Source and Detector Sensitivity Profiles ..... 34
2.4 Sensitivity Profiles for the Secondary Energy Distribution and the Secondary Angular Distribution ..... 38
2.4.1 Introduction ..... 38
2.4.2 Further theoretical development ..... 40
2.4.3 Secondary energy and secondary angular distribution sensitivity profile ..... 43
2.4.4 Integral sensitivities for SEDs and SADs ..... 44
2.4.5 Explicit expressions for integral SED sensitivity profiles in a two-dimensional geometry representation ..... 47
2.5 Design Sensitivity Analysis ..... 48
3. Application of Sensitivity Theory to Uncertainty Analysis ..... 52
3.1 Definitions ..... 52
3.2 Cross-Section Covariance Matrices ..... 54
3.3 Application of Cross-Section Sensitivity Profiles and Cross- Section Covariance Matrices to Predict Uncertainties ..... 55
3.4 Secondary Energy Distribution Uncertainty Analysis ..... 57
3.5 Overall Response Uncertainty ..... 62
4. SENSIT-2D: A Two-Dimensional Cross-Section Sensitivity and Uncertainty Analysis Code ..... 64
4.1 Introduction ..... 64
4.2 Computational Outline of a Sensitivity Study ..... 66
4.2.1 Cross-section preparation module ..... 70
4.2.2 The TRIDENT-CTR and TRDSEN block ..... 71
4.2.3 The SENSIT-2D module ..... 71
4.3 The SENSIT-2D Code ..... 73
4.3.1 Flow charts ..... 73
4.3.2 Subroutines used in SENSIT-2D ..... 78
5. Comparison of a Two-Dimensional Sensitivity Anlysis with a One-Dimensional Sensitivity Analysis ..... 83
5.1 Sample Problem \#1 ..... 84
5.1.1 TRIDENT-CTR and ONEDANT results ..... 92
5.1.2 SENSIT and SENSIT-2D results for a standard cross- section sensitivity analysis ..... 94
5.1.3 Comparison between a two-dimensional and a one- dimensional cross-section sensitivity and uncertainty analysis ..... 99
5.2 Sample Problem \#2 ..... 104
5.2.1 Influence of the quadrature order on the sensi- tivity profile ..... 104
5.2.2 Comparison between the two-dimensional and one- dimensional analysis of sample problem 非2 ..... 111
5.2.3 Comparison between the $X^{\prime}$ s calculated from the angular fluxes and the $\chi$ 's resulting from flux moments ..... 114
5.2.4 Evaluation of the loss term based on flux moments in the case of low c ..... 116
5.3 Conclusions ..... 117
6. Sensitivity and Uncertainty Analysis of the Heating in the TF-Coil for the FED ..... 118
6.1 Two-Dimensional Model for the FED ..... 118
6.2 Two-Dimensional Sensitivity and Uncertainty Analysis for the Heating in the TF-Coil due to SED and Cross-Section Uncertainties ..... 123
6.3 Comparison of the Two-Dimensional Model with a One- Dimensional representation ..... 129
7. Conclusions and Recommendations ..... 133
8. References ..... 138
Appendix A: SENSIT-2D Source and Code Listing ..... 146
Appendix B: TRIDSEN ..... 190

NOMENCLATURE
E reflects energy function
$\underline{\Omega} \quad$ reflects angular distribution
$\sigma \quad$ microscopic cross section
$\Sigma_{\mathrm{x}}$ macroscopic cross section, where x indicates the material and/or the type of cross section
$\Sigma_{x, s} \quad$ macroscopic scattering cross section for material $x$
$\Sigma_{x, T} \quad$ total cross section for material $x$
angular flus
angular flux in discrete ordinates representation spherical harmonics representation for the angular flux

Legendre polynomials
associated Legendre polynomials
transport operator
adjoint transport operator
source
response function
integral response
fractional uncertainty for the secondary angular distribution
part of the loss term in the sensitivity profile part of the gain term in the sensitivity profile

| $\mathrm{F}_{\Sigma}{ }_{x}$ | cross-section sensitivity function for cross section |
| :---: | :---: |
|  | $\Sigma_{x}$ |
| $\mathrm{p}_{\Sigma_{x}}^{\mathrm{g}}$ | cross-section sensitivity profile for a cross-section |
|  | $\Sigma_{x}$ for group g |
| $\Delta u^{g}$ | lethargy width for group g |
| w | quadrature weight |

## LIST OF FIGURES

| Figure 1 | Illustration of the terminology: blank region, source region and detector region |
| :---: | :---: |
| Figure 2a | Coordinates in $x-y$ geometry |
| Figure 2b | Coordinates in r-z geometry |
| Figure 3 | Definition of median energy and integral SED sensitivity |
| Figure 4 | Interpretation of the integral SED uncertainty as spectral shape perturbations and definitions of the spectral shape uncertainty parameter " $f$ " |
| Figure 5 | Computational outline for a two-dimensional sensitivity and uncertainty analysis with SENSIT-2D |
| Figure 6 | Data flow for the SENSIT-2D module |
| Figure 7 | Flow chart for SENSIT-2D |
| Figure 8 | ```Cylindrical geometry representation for sample problem 非``` |
| Figure 9 | Two-dimensional (TRIDENT-CTR) representation for sample problem \#1 |
| Figure 10 | One-dimensional (ONEDANT) representation for sample problem 非 |
| Figure 11 | Two-dimensional model for sample problem \#2 |
| Figure 12 | Two-dimensional model for the FED |
| Figure 13 | The TRIDENT-CTR band and triangle structure for the FED |

Table I Formulas for the sensitivity function
Table II Median Energies ( $E_{m}^{\prime}$, in MeV ) and fractional
uncertainties (F) for secondary energy distributions at incident neutron energies $E_{0}$

Table III
Table IV
Table V
Table VI
Table VIIa

Table VIIb
Table VIIIa

Table VIIIb

Table IX

Table Xa

Table Xb

Summary of the features of SENSIT-2D
List of subroutines used in SENSIT-2D
30-group energy structure
Atom densities of materials
Comparison of the heating in the copper region calculated by ONEDANT and TRIDENT-CTR

Computing times on a CDC-7600 machine
Comparison of the heating in the copper region calculated by ONEDANT and TRIDENT-CTR
two-dimensional analysis
Comparison between the $X$ 's calculated from angular fluxes and flux moments

Predicted response uncertainties due to estimated cross-section and SED uncertainties in a onedimensional analysis

Predicted response uncertainites due to estimated cross-section and SED uncertainties in a twodimensional analysis


Table XVII Predicted Uncertainties (standard deviation) due to estimated SED and cross-section uncertainties for the heating of the TF-coil

Table XVIII Predicted SED and cross-section uncertainties in the TF-coil due to uncertainties in the SS316 regions

Table XIX Comparison between the $X$ 's calculated from angular fluxes and the $X^{\prime}$ s resulting from flux moments for region 11 (SS316)

Table XX Predicted uncertainties (standard deviation) due to estimated SED and cross-section uncertainties in regions 1 and 3 for the heating in the TF-coil.

TWO-DIMENSIONAL CROSS-SECTION SENSITIVITY AND UNCERTAINTY ANALYSIS FOR FUSION REACTOR BLANKETS
by

## Mark Julien Embrechts

ABSTRACT
Sensitivity and uncertainty analysis implement the information obtained from a transport code by providing a reasonable estimate for the uncertainty for a particular response (e.g., tritium breeding), and by the ability to better understand the nucleonics involved. The doughnut shape of many fusion devices makes a two-dimensional calculation capability highly desirable. Based on first-order generalized perturbation theory, expressions for a two-dimensional SED (secondary energy distribution) and cross-section sensitivity and uncertainty analysis were developed for $x-y$ and r-z geometry. This theory was implemented by developing a two-dimensional sensitivity and uncertainty analysis code, SENSIT-2D. SENSIT-2D has a design capability and has the option to calculate sensitivities and uncertainties with respect to the response function itself. SENSIT-2D can only interact with the TRIDENT-CTR code.

A rigorous comparison between a one-dimensional and a two-dimensional analysis for a problem which is one-dimensional from the neutronics point of view, indicates that SENSIT-2D performs as intended.

A two-dimensional sensitivity and uncertainty analysis for the heating of the $T F$ coil for the FED (fusion engineering device) blanket was performed. The uncertainties calculated are of the same order of magnitude as those resulting from a one-dimensional analysis. The largest uncertainties were caused by the cross section uncertainties for chromium.

## 1. INTRODUCTION TO SENSITIVITY THEORY AND UNCERTAINTY ANALYSIS

In a time characterized by a continuously growing demand for sophisticated technology it should not be surprising that the production of fusion energy might materialize more rapidly than commonly predicted. With fusion devices going into a demonstration phase there is a need for sophisticated nucleonics methods, tailored to the fusion community. In a relatively short time frame fusion nucleonics has established itself as a more or less mature subfield. In this context sensitivity theory has become a widely applied concept which provides the reactor designer with a deeper understanding of the information obtained from transport calculations.

Under the term sensitivity theory usually algorithms based upon classical perturbation and variational theory are understood. The scope of this work will be limited to cross-section and design sensitivity analysis with respect to fusion reactors. Since fusion nucleonics do not involve eigenvalue calculations, the mathematical concepts utilized will be simpler than those required by the fission community.

Sensitivity theory determines how a design quantity changes when one or more of the design parameters are altered. Uncertainty analysis
provides the error range on a design quantity due to errors on the design parameters. Sensitivity information can easily be incorporated into an uncertainty analysis by introducing covariance matrices.

Cross-section sensitivity and uncertainty analysis will give error estimates of response functions (such as tritium breeding ratio, heating and material damage) due to uncertainties in the cross-section data. Such a study will reveal which partial cross sections and in what energy range contribute most to the error and will recommend refinements on cross-section evaluations in order to reduce that error. Although those results will depend on the particular response and the particular design, general conclusions can still be drawn for a class of similar designs. ${ }^{18}$ Sensitivity theory is a powerful design tool and is commonly applied to cross-section adjustment procedures. ${ }^{1-3}$ Design sensitivity analysis is frequently used to reduce the many and expensive computer runs required during the development of a new reactor concept.

### 1.1 Motivation

The purpose of this work is to assess the state of the art of sensitivity and uncertainty analysis with respect to fusion nucleonics, fill existing gaps in that field and suggest areas which deserve further attention.

At this moment the literature about sensitivity theory is scattered between various journal articles and technical reports. Therefore, the
author considered it as one of his responsibilities to provide a consistent monograph which explains, starting from the transport equation, how analytical and explicit expressions for various sensitivity profiles can be obtained. Current limitations with respect to the applicability of sensitivity theory are pointed out and the application of sensitivity theory to uncertainty analysis is explained. At the same time the scope has been kept limited to those algorithms which are presently used in calculation schemes.

Due to the particular geometry of fusion devices (toroidal geometry, non-symmetric plasma shape, etc.), a one-dimensional transport code (and therefore a one-dimensional sensitivity analysis) will generally be inadequate. In order to mock-up a fusion reactor more closely, a two-dimensional analysis is required. Although a two-dimensional sensitivity code - VIP ${ }^{4,5}$ - already exists, VIP was developed with a fission reactor in mind, and does not include an $r-z$ geometry option, nor a secondary energy distribution capability. To answer the needs of the fusion community, a two-dimensional sensitivity and uncertainty analysis code, SENSIT-2D, has been written.

A sensitivity code uses the regular and adjoint fluxes of a neutron transport code in order to construct sensitivity profiles. SENSIT-2D requires angular fluxes generated by TRIDENT-CTR. ${ }^{6,7}$ TRIDENT-CTR is a two-dimensional discrete-ordinates neutron transport code specially developed for the fusion community. Since SENSIT-2D incorporates the essential features of TRIDENT-CTR, i.e., triangular meshes and r-z geometry option, toroidal devices can be modeled quite accurately. SENSIT-

2D has the capability of group-dependent quadrature sets and includes the option of a secondary energy distribution (SED) sensitivity and uncertainty analysis. An option to calculate the loss term of the crosssection sensitivity profile based on either flux moments or angular fluxes is built into SENSIT-2D. The question whether a third-order spherical harmonics expansion of the angular flux will be adequate for a 2-D sensitivity analysis has not yet been adequately answered. ${ }^{8}$ The flux moment/angular flux option will help provide an answer to that question.

As an application of the SENSIT-2D code, a two-dimensional sensitivity and uncertaintly analysis of the inboard shield for the FED (fusion engineering device), currently in a preconceptual design stage by the General Atomic Company, was performed.

### 1.2 Literature Review

The roots of cross-section sensitivity theory can be traced to the work of Prezbindowski. ${ }^{9,10}$ The first widely used cross-section sensitivity code, SWANLAKE, ${ }^{11}$ was developed at ORNL (Oak Ridge National Laboratory). In order to include the evaluation of the sensitivity of the response to the response function, SWANLAKE was modified to SWANLAKE-UW by Wu and Maynard. ${ }^{77}$ Already early in its history, sensitivity theory was applied to fusion reactor studies. ${ }^{12-16}$ It has now become a common practice to include a sensitivity study in fusion neutronics. ${ }^{17-23,54}$

The mathematical concepts behind sensitivity theory are based on variational and perturbation theory. ${ }^{24-29}$ The application of sensitivity profiles to uncertaintly analysis was restricted not due to a lack of adequate mathematical formulations, but due to the lack of crosssection covariance data. An extensive effort to include standardized covariance data into ENDF/B files has recently been made. ${ }^{30-34}$

The theory of design sensitivity analysis can be traced to the work of Conn, Stacey, and Gerstl. $14,26,35,40$ The current limitation of design sensitivity analysis is related to the fact that the integral response is exact up to the second order with respect to the fluxes, but only exact to the first order with respect to design changes. Therefore, only relatively small design changes are allowed. The utilization of Padé approximants ${ }^{42}$ might prove to be a valuable alternative to higher-order perturbation theory, but has not yet been applied to design sensitivity analysis. ${ }^{63}$

The two-dimensional sensitivity code VIP ${ }^{4,5}$ was developed by Childs. VIP is oriented towards fission reactors and does not include a design sensitivity option, nor a secondary energy distribution capability.

The theory of secondary energy distribution (SED) and secondary angular distribution (SAD) sensitivity and uncertainty analysis was originated by Gerst1 ${ }^{43-45}$ and is incorporated into the SENSIT ${ }^{46}$ code. The FORSS ${ }^{47}$ code package has been applied mainly to fast reactor studies ${ }^{48,49}$ but can be applied to fusion reactor designs as well. Higherorder sensitivity theory ${ }^{42,50-51,78}$ still seems to be too impractical to
be readily applied. Recently however, the French developed a code system, SAMPO, ${ }^{52}$ which includes some higher-order sensitivity analysis capability.

## 2. SENSITIVITY THEORY

In this chapter the theory behind source and detector sensitivity, cross-section and secondary energy distribution (SED) sensitivity, and design sensitivity analysis will be explained. Starting from the transport equation, expressions for the corresponding sensitivity profiles will be derived. Those formulas will then be made more explicit and applied to a two-dimensional geometry. The theory presented in this and the following chapter is merely a consistent combination and reconstruction of several papers and reports. $3,13,16,17,18,43-46,53$

Since up to this time no single reference work about the various concepts used in sensitivity and uncertainty analysis has been published, the author uses the most commonly referred to terminology. In an attempt to present an overview with the emphasis on internal consistency, there might be some minor conflicts with the terminology used in earlier published papers.

### 2.1 Definitions

### 2.1.1 Cross-section sensitivity function, cross-section sensitivity profile and integral cross-section sensitivity

Let $I$ represent a design quantity (such as a reaction rate, e.g., the tritium breeding ratio), depending on a cross-section set and the angular fluxes. The cross-section sensitivity function for a particular cross section $\Sigma_{x}$ at energy $E, F_{\Sigma_{x}}(E)$, is defined as the fractional change of the design parameter of interest per unit fractional change of cross section $\Sigma_{x}$, or

$$
\begin{equation*}
F_{\Sigma_{x}}(E)=\frac{\partial I / I}{\partial \Sigma_{x} / \Sigma_{x}} \tag{1}
\end{equation*}
$$

In a multigroup formulation the usual preference is to work with a sensitivity profile $P_{\Sigma_{X}}^{g}$, which is defined by

$$
\begin{equation*}
\mathrm{P}_{\Sigma_{x}^{g}}^{g}=\frac{\partial I / I}{\partial \Sigma_{\mathrm{x}}^{\mathrm{g}} / \Sigma_{\mathrm{x}}^{g}} \cdot \frac{1}{\Delta u^{g}}, \tag{2}
\end{equation*}
$$

where $\Delta u^{g}$ is the lethargy width of group $g$ and $\Sigma_{x}^{g}$ is the multigroup cross section for group $g$. The sum over all the groups of the sensitivity profiles for a particular group cross section $\Sigma_{X}^{g}$, multiplied by
the corresponding lethargy widths, is called the integral cross-section sensitivity for cross section $\Sigma_{x}$, or

$$
\begin{align*}
S_{\Sigma_{x}} & =\sum_{g} P_{\Sigma_{x}}^{g} \cdot \Delta u^{g} \\
& =\int d E F_{\Sigma_{x}}(E) \tag{3}
\end{align*}
$$

The integral cross-section sensitivity can be interpreted as the percentage change of the design parameter of interest, I, resulting from a simultaneous one percent increase of the group cross sections $\Sigma_{X}^{8}$ in all energy groups $g$.

### 2.1.2 Vector cross section

The term "vector cross section". describes a multigroup partial cross-section set with one group-averaged reaction cross section for each group. Such a cross-section set can be described by a vector with GMAX elements, where GMAX is the number of energy groups. The term vector cross section was introduced by Gerstl to discriminate it from the matrix representation of a multigroup cross-section set. Differential scattering cross sections can obviously not be described in the form of a vector cross section.

### 2.1.3 Geometry related terminology

Under the term region we will understand a collection of one or more zones. A zone will always describe a homogeneous part of the reactor. We will make a distinction between source regions, detector regions and perturbed regions, and as a consequence between source, detector and perturbed zones. We will introduce the term blank region for a region that is neither a detector, source or perturbed region. A zone will further be divided into intervals.

The source region will describe that part of the reactor which contains a volumetric source. The detector region indicates the part of the reactor for which an integral response is desired. In the perturbed region changes in one or more cross sections can be made.

A source or a detector regions can contain more than one zone, and each zone can be made up of a different material. Due to the mathematical formulations a perturbed region can still contain more than one zone, but in this case all the zones have to contain identical materials. If there is more than one perturbed region, all those regions should contain the same materials.

The geometry-related terminology is illustrated in Fig. 1. In this case, there are six regions; a source region, two perturbed regions, one detector region and two blank regions. The source region contains three zones (identified by $\underline{a}, \underline{b}$, and $\underline{c}$ ). The first zone, $\underline{a}$, is a vacuum, while the other two zones are made up of iron. Note that both perturbed regions satisfy the requirement that the zones in these regions contain


MATERIALS


ZONES
a, $\underline{e}, \underline{f}$
$\underline{\underline{b}}, \underline{\bar{c}}, \underline{\bar{n}}, \underline{o}, \underline{p}, \underline{h}, \underline{i}, \underline{j}, \underline{k}, \underline{1}, \underline{m}$
g, $\underline{r}$
s,t
d, g

Figure 1. Illustration of the terminology: blank region, source region, perturbed region and detector region
identical materials. This requirement does not have to be met for source and detector regions.

### 2.2 Cross-Section Sensitivity Profiles

### 2.2.1 Introduction

Perturbation theory is most commonly applied in order to derive analytical expressions for the cross-section sensitivity profile. We therefore will follow in this work Oblow's approach. ${ }^{11,25}$ Based on the analytical expression, an explicit formula for the cross-section sensitivity profile in discrete ordinates form for a two-dimensional geometry will then be derived.

During the last few years there has been a trend towards using generalized perturbation theory for sensitivity studies. ${ }^{5,55,61}$ Generalized perturbation theory has the advantage that it can readily be applied to derive expressions for the ratio of bilinear functionals and that it can be used to study nonlinear systems. ${ }^{59,60}$ Also, higher-order expressions, based on generalized perturbation theory, have been derived. $57,58,61$

The differential approach is closely related to generalized perturbation theory and has been applied to cross-section sensitivity analysis by Oblow. ${ }^{28}$ A more rigorous formulation of the differential approach was made by Dubi and Dudziak. ${ }^{50,51}$ Although higher-order expressions
for cross-section sensitivity profiles can be derived, 50,51 the practicality of its application has not yet been proved. $50,51,78$

The evaluation of a sensitivity profile will generally require the solution of a direct and an adjoint problem. Such a system carries more information than the forward equation and it is therefore not surprising that this extra amount of information can be made explicit (e.g., through sensitivity profiles).

The higher-order expressions for the cross-section sensitivity profiles derived by Dubi and Dudziak involve the use of Green's functions. ${ }^{50,51}$ The Green's function - if properly integrated - allows one to gain all possible information for a particular transport problem. It therefore can be expected that higher-order sensitivity profiles can be calculated up to an arbitrary high order by evaluating one Green's function. For most cases, the derivation of the Green's function is extremely complicated, if not impossible. It therefore can be argued that the Green's function carries such a tremendous amount of information that it is not surprising that higher-order expressions for the sensitivity profile can be obtained, and that while the use of Green's functions can prove to be very valuable for gaining analytical and physical insight, they will not be practical as a basis for numerical evaluations.

From the study done by Wu and Maynard, ${ }^{78}$ it can be concluded that a first-order expression allows for a $40 \%$ perturbation in the cross sections (or rather the mean free path) and will still yield a reasonably accurate integral response (less than $10 \%$ error). Larger perturbations give rapidly increasing errors (the error increases roughly by a power
of three). Expressions exact up to the second order allow a $65 \%$ perturbation, and a sixth-order expression allows a $190 \%$ perturbation, both for an error less than $10 \%$. Also, for higher-order approximations, if was found that the error on the integral response will increase drastically once the error exceeds $10 \%$. It can be concluded therefore that the higher-order expressions do not bring a tremendous improvement over the first-order approximation (unless very high orders are used), while the computational effort increases drastically. Higher-order sensitivity analysis can only become practical when extremely simple expressions for the sensitivity profiles can be obtained, or when a suitable approximation for Green's functions can be found. ${ }^{79}$
2.2.2 Analytical expression for the cross-section sensitivity profile

Consider the regular and adjoint transport equations

$$
\begin{equation*}
\text { L. } \Phi=Q \text {, } \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{L}^{\stackrel{*}{*}} \cdot \Phi^{\frac{1}{*}}=\mathrm{R} \tag{5}
\end{equation*}
$$

where $\phi$ and $\phi^{*}$ represent the forward and the adjoint angular fluxes, $L$ and $L^{*}$ are the forward and adjoint transport operator, $Q$ is the source,
and $R$ is the detector response function. The integral response, $I$, can then be written as

$$
\begin{equation*}
I=\langle R, \phi\rangle \tag{6}
\end{equation*}
$$

or

$$
\begin{equation*}
I^{*}=\left\langle Q, \phi^{*}\right\rangle, \tag{7}
\end{equation*}
$$

where the symbol < , > means the inner product, i.e., the integral over the phase space. In a fully converged calculation $I^{\frac{1}{*}}$ will be equal to I. For the perturbed system, similar expressions can be obtained:

$$
\begin{equation*}
L_{p} \Phi_{p}=Q \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{L}_{\mathrm{p}}^{*} \phi_{\mathrm{p}}^{*}=\mathrm{R} \tag{9}
\end{equation*}
$$

$$
I_{p}=\left\langle R, \Phi_{p}\right\rangle
$$

and $I_{p}^{\stackrel{+}{\hat{m}}}=\left\langle Q, \phi_{\mathrm{p}}^{\stackrel{+}{\hat{*}}}\right\rangle$,
where

$$
\begin{equation*}
\Phi_{\mathrm{p}}=\phi+\delta \phi \tag{12}
\end{equation*}
$$

$$
\begin{equation*}
\phi_{\mathrm{p}}^{*}=\phi^{\star}+\delta \phi^{\dot{*}}, \tag{13}
\end{equation*}
$$

and $I_{p}=I+\delta I$.

From Eqs. (9), (13), and (5) we have

$$
\begin{equation*}
L_{p}^{*} \cdot \delta \Phi^{*}=\left(L^{*}-L_{p}^{*}\right) \cdot \phi^{*} . \tag{15}
\end{equation*}
$$

Further, we have from Eqs. (14), (11), (6), (12), and (9)

$$
\begin{align*}
\delta \mathrm{I} & =\mathrm{I}_{\mathrm{p}}-\mathrm{I}, \\
& =\left\langle\mathrm{R}, \Phi_{\mathrm{p}}-\phi\right\rangle, \\
& =\langle\mathrm{R}, \delta \phi\rangle, \\
\text { or } \delta \mathrm{I} & =\left\langle\mathrm{L}_{\mathrm{p}}^{*} \phi_{\mathrm{p}}^{*}, \delta \phi\right\rangle . \tag{16}
\end{align*}
$$

Using the definition of the adjoint transport operator and Eqs. (15) and (16) transforms to

$$
\delta I=\left\langle\Phi_{\mathrm{p}}, \mathrm{~L}_{\mathrm{p}}^{\dot{\hbar}} \delta \phi^{\star}\right\rangle
$$

or

$$
\begin{equation*}
\delta I=\left\langle\Phi_{p},\left(L^{*}-L_{p}^{*}\right) \Phi^{*}\right\rangle \tag{17}
\end{equation*}
$$

It is assumed that the perturbed differential scattering cross section can be expressed as a function of the unperturbed differential scattering cross section by

$$
\begin{equation*}
\Sigma_{\mathrm{sp}}\left(\underline{r}, \underline{\Omega}^{\underline{\Omega}} \underline{\Omega}^{\prime}, \mathrm{E} \rightarrow \mathrm{E}^{\prime}\right)=\mathrm{C} \cdot \Sigma_{\mathrm{p}}\left(\underline{r}, \underline{\Omega} \rightarrow \underline{\Omega}^{\prime}, \mathrm{E} \rightarrow \mathrm{E}^{\prime}\right), \tag{18}
\end{equation*}
$$

and similarly for the total cross section

$$
\begin{equation*}
\Sigma_{\mathrm{Tp}}(\underline{r}, \mathrm{E})=\mathrm{C} \cdot \Sigma_{\mathrm{T}}(\underline{r}, \mathrm{E}), \tag{19}
\end{equation*}
$$

where $C$ is a small quantity, which can be a function of $E$ and $\Omega$. Defining $\delta C=C-1$, we have

$$
\begin{equation*}
\delta C=\frac{\Sigma_{T p}(\underline{r}, E)-\Sigma_{T}(\underline{r}, E)}{\Sigma_{T}(\underline{r}, E)}=\frac{\Sigma_{s p}\left(\underline{r}, \underline{\Omega} \rightarrow \underline{\Omega}^{\prime}, E \rightarrow E\right)-\Sigma_{\mathrm{S}}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right)}{\Sigma_{\mathrm{s}}\left(\underline{\underline{r}}, \underline{\Omega} \rightarrow \underline{\Omega}^{\prime}, \mathrm{E} \rightarrow \mathrm{E}^{\prime}\right)} \tag{20}
\end{equation*}
$$

so that

$$
\begin{align*}
\delta I(E)= & \delta C \int d \underline{r} \int d \underline{\Omega} \cdot \phi_{p}\left\{-\Sigma_{T}(\underline{r}, E) \cdot \phi^{*}(\underline{r}, \underline{\Omega}, E)\right. \\
& \left.+\int d E^{\prime} \int d \underline{\Omega}^{\prime} \Sigma_{s}\left(\underline{r}, \underline{\Omega}^{\prime} \underline{\Omega}^{\prime}, E^{\prime} \rightarrow E^{\prime}\right) \cdot \phi^{*}\left(\underline{r}, \underline{\Omega}^{\prime}, E^{\prime}\right)\right\} \tag{21}
\end{align*}
$$

The cross-section sensitivity function $F_{\Sigma_{X}}(E)$ is defined by

$$
\begin{equation*}
F_{\Sigma_{x}}(E)=\frac{\partial I / I}{\partial \Sigma_{x} / \Sigma_{x}} \tag{22}
\end{equation*}
$$

and can be approximated by

$$
\begin{align*}
F_{\Sigma_{x}}(E) & \cong \frac{1}{\bar{I}} \int d \underline{r} \int d \underline{\Omega}\left\{-\phi(\underline{r}, \underline{\Omega}, E) \cdot \Sigma_{x, T}(\underline{r}, E) \cdot \Phi^{*}(\underline{r}, \underline{\Omega}, E)\right. \\
\quad & \left.+\int d \underline{\Omega}^{\prime} \int d E^{\prime} \phi(\underline{r}, \underline{\Omega}, E) \cdot \Sigma_{x, s}\left(\underline{r}, \underline{\Omega} \rightarrow \underline{\Omega}^{\prime}, E^{\prime} \rightarrow E^{\prime}\right) \cdot \phi^{*}\left(\underline{r}, \underline{\Omega}^{\prime}, E^{\prime}\right)\right\} \tag{23}
\end{align*}
$$

The sensitivity function $F_{\Sigma_{X}}(E)$ represents the dependence or sensitivity of a design parameter of interest to a particular cross section $\Sigma_{x}$ at energy $E$. The first term is usually referred to as the loss term and the second term is called the gain term. ${ }^{27}$

The cross-section sensitivity profile $P_{\Sigma_{X}}^{g}$ is then defined as

$$
\begin{equation*}
P_{\Sigma_{x}}^{g}=\frac{1}{\Delta u^{g}} \int_{E_{g}}^{E_{g-1}} d E F_{\Sigma_{X}}(E) \tag{24}
\end{equation*}
$$

The scaling factor $\Delta u^{g}$ is the lethargy width of group $g$ and is introduced as a normalization factor in order to remove the influence of the choice of the group structure.

## Remarks

1. In the previous section $\Sigma_{x}$ represents a partial cross section for a particular material. $\Sigma_{x}$ can be an absorption cross section, a total cross section, a differential scattering cross section, a reaction cross section, etc. Therefore $\Sigma_{x}$ has a surpressed index
which indicates the specific partial cross section. When evaluating the cross-section sensitivity profile for a partial cross section only the appropriate part, either the loss term or the gain term, will have to be considered in Eq. (23). When the partial cross section is not related to the production of secondary particles (e.g., a differential scattering cross section) the sensitivity profile in the multigroup form is referred to by Gerstl as a vector cross-section sensitivity profile. Obviously such cross sections contribute only to the loss term.
2. It is possible to define a net or a total sensitivity profile, which can be obtained by summing the loss and the gain terms for various partial reactions. The net sensitivity profile can be used to determine how important a particular element is with respect to a particular response.
3. Note that while deriving an expression for the cross-section sensitivity profile, we implicitly assumed that the response function was independent from the partial cross section for which a sensitivity profile is desired. If this assumption does not hold, an extra term has to be added to the previously obtained expressions. When the response function is also the cross section for which a sensitivity profile is sought, the sensitivity function will take the form

$$
\begin{equation*}
\frac{\partial \mathrm{I} / \mathrm{I}}{\partial \Sigma_{\mathrm{X}} / \Sigma_{\mathrm{x}}}=\frac{\langle\mathrm{R}, \phi\rangle}{\mathrm{I}}+\frac{\left\langle\phi, \mathrm{L}_{\Sigma} \dot{\phi}^{*}\right\rangle}{\mathrm{I}} \tag{25}
\end{equation*}
$$

where $L_{\Sigma_{X}}$ represents that portion of the transport operator that contains the cross-section set $\left\{\Sigma_{\mathrm{X}}\right\}$. In this expression the first term is a direct effect and the second term is an indirect effect. If the direct effect is present, the indirect effect will usually be negligible. A summary of the various possibilities is given in Table I.
4. The spatial integration in Eq. (23) has to be carried out over the perturbed regions only.

### 2.2.3 Explicit expression for the cross-section sensitivity profile in discrete ordinates form for a two-dimensional geometry representation

## Coordinate system

The coordinate systems for $x-y$ and $r-z$ geometry are shown in Figs. 2 a and $2 \mathrm{~b} .{ }^{53}$ In both geometries $\phi$ was chosen to be the angle of rotation about the $\mu$-axis such that $d \Omega=d \mu . d \phi$, and since $\xi^{2}+\mu^{2}+v^{2}=1$, we have

TABLE I: FORMULAS FOR THE SENSITIVITY FUNCTION

| Case | Sensitivity Function |
| :---: | :---: |
| a. $I=\langle R, \phi\rangle$, where $\Sigma_{i} \neq R$ <br> b. $I=\langle R, \phi\rangle$, where $\Sigma_{i}=R$ <br> and $\quad \Sigma_{i} \not \subset L$ <br> c. $I=\langle R, \phi\rangle$, where $\Sigma_{i}=R$ <br> and $\quad \Sigma_{i} \subset L$ | $\begin{aligned} & \mathrm{F}_{\Sigma_{i}}=\left\langle\phi^{*}, \mathrm{~L}_{\Sigma} \phi\right\rangle / \mathrm{I} \\ & \mathrm{~F}_{\Sigma_{i}}=\langle\mathrm{R}, \phi\rangle / \mathrm{I} \\ & \mathrm{~F}_{\Sigma_{i}}=\langle\mathrm{R}, \phi\rangle / \mathrm{I}+\left\langle\phi^{*}, \mathrm{~L}_{\Sigma_{i}} \phi\right\rangle / \mathrm{I} \\ & \begin{array}{l} \text { direct } \\ \text { effect } \end{array} \\ & \begin{array}{c} \text { indirect } \\ \text { effect } \end{array} \end{aligned}$ <br> The direct effect is usually dominant |
| $<>$ indicates the inner produ <br> L stands for the transport <br> $L_{\Sigma_{i}}$ represents that portion contains cross-section $\{$ <br> $C$ means is included in <br> $\not \subset \quad$ means is not included in | over the phase space $\xi$ rator <br> he transport operator which |



Figure 2. a. Coordinates in $x-y$ geometry


Figure 2. b. Coordinates in r-z geometry

$$
\xi=\left(1-\mu^{2}\right)^{\frac{1}{2}} \cdot \sin \phi,
$$

and

$$
\eta=\left(1-\mu^{2}\right)^{\frac{1}{2}} \cdot \cos \phi
$$

Therefore both the $x-y$ and the $r-z$ geometry representation will lead to identical expressions for the sensitivity profile, with the understanding that in $x-y$ geometry the angular flux is represented by $\Phi(x, y, \mu, \phi)$, and by $\Phi(r, z, \mu, \phi)$ in the case of $r-z$ geometry.

We now will derive an expression for the sensitivity profile in an $x-y$ or in an $r-z$ geometry representation.

Method

Before deriving an expression in a discrete-ordinates formulation and a two-dimensional geometry for Eq. (23), a brief overview of the methods used is outlined.

Gain term:
I. order to represent the differential scattering cross section in a multigroup format, the common approach to expand the differential scattering cross section in Legendre polynomials is used. The number of terms in the expansion is a function of the order of anisotropic scattering. The Legendre polynomials are a function of the scattering angle $\mu_{0}$ (Fig. 2). Introducing spherical harmonics
functions and applying the addition theorem for spherical harmonics, the dependence on $\mu_{0}$ can be replaced by $\mu$ 's and $\phi$ 's. The angular fluxes are expanded in flux moments. The integrals are replaced by summations. Defining multigroup cross sections an expressions for the gain term can be obtained.

Loss term:
An explicit expression for the loss term can be derived based on angular fluxes or based on flux moments. In order to check the internal consistency in SENSIT-2D both methods will $b=$ applied. The derivation of an expression based on angular fluxes is straightforward: the integrations are replaced by summations and the appropriate multigroup cross sections are defined. An expression as a function of flux moments can be obtained by expanding the fluxes in flux moments, using spherical harmonics functions. The orthogonality relation of spherical harmonics is applied, the integrations are replaced by summations and appropriate multigroup cross sections are defined. Finally an expression for the loss term is the result.

## Analytical derivations

Expand the differential scattering cross section in Legendre polynomials according to

$$
\Sigma_{x, s}\left(\underline{\Omega}_{\underline{\Omega}} \underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right)=\Sigma_{x, s}\left(\mu_{0}, E \rightarrow E^{\prime}\right)=\sum_{\ell=0}^{\operatorname{LMAX}} \frac{2 \ell+1}{4 \pi} P_{\ell}\left(\mu_{0}\right) \Sigma_{s, \ell}\left(E \rightarrow E^{\prime}\right)
$$

where the $P_{\ell}\left(\mu_{0}\right)$ 's are the Legendre polynomials and LMAX the order of anisotropic scattering. Here, the scattering angle $\mu_{0}$ can be written as

$$
\mu_{0}=\underline{\Omega} \cdot \underline{\Omega}^{\prime}=\Omega_{x} \Omega_{x}^{\prime}+\Omega_{y} \Omega_{y}^{\prime}+\Omega_{z} \Omega_{z}^{\prime}
$$

or

$$
\begin{aligned}
\mu_{0} & =\mu \mu^{\prime}+\eta \eta^{\prime}+\xi \xi^{\prime} \\
& =\mu \mu^{\prime}+\left(1-\mu^{2}\right)^{\frac{1}{2}}\left(1-\mu^{\prime 2}\right)^{\frac{1}{2}} \cos \phi \cos \phi^{\prime}+\left(1-\mu^{2}\right)^{\frac{1}{2}}\left(1-\mu^{\prime 2}\right)^{\frac{1}{2}} \sin \phi \sin \phi
\end{aligned}
$$

or

$$
\mu_{0}=\mu \mu^{\prime}+\left(1-\mu^{2}\right)^{\frac{1}{2}}\left(1-\mu^{\prime} 2\right)^{\frac{1}{2}} \cos \left(\phi-\phi^{\prime}\right) .
$$

The spherical harmonics addition theorem states that (see e.g., Bell and Glasstone ${ }^{62}$ )

$$
\begin{equation*}
P_{\ell}\left(\mu_{0}\right)=P_{\ell}(\mu) P_{\ell}\left(\mu^{\prime}\right)+2 \sum_{k=1}^{\ell} \frac{(\ell-k)!}{(\ell+k)!} P_{\ell}^{k}(\mu) P_{\ell}^{k}\left(\mu^{\prime}\right) \cos \left[k\left(\phi-\phi^{\prime}\right)\right] \tag{27}
\end{equation*}
$$

where the $P_{\ell}^{k}(\mu)$ 's are the associated Legendre polynomials. The above expression can then be reformulated as

$$
\begin{align*}
P_{\ell}\left(\mu_{o}\right)= & \sum_{k=0}^{\ell} \frac{\left(2-\delta_{k o}\right)(\ell-k)!}{(\ell+k)!} P_{\ell}^{k}(\mu) P_{\ell}^{k}\left(\mu^{\prime}\right) \cos \left[k\left(\phi-\phi^{\prime}\right)\right] \\
= & \sum_{k=0}^{\ell}\left[\frac{\left(2-\delta_{k o}\right)(\ell-k)!}{(\ell+k)!}\right]^{\frac{3}{2}}\left[\frac{\left(2-\delta_{k o}\right)(\ell-k)!}{(\ell+k)!}\right]^{\frac{2}{2}} P_{\ell}^{k}(\mu) P_{\ell}^{k}\left(\mu^{\prime}\right) \\
& \times\left(\cos k \phi \cos k \phi^{\prime}+\sin k \phi \sin k \phi^{\prime}\right) . \tag{28}
\end{align*}
$$

We define

$$
\begin{equation*}
\mathrm{R}_{\ell}^{\mathrm{k}}(\mu, \phi)=\left[\frac{\left(2-\delta_{k o}\right)(\ell-k)!}{(\ell+k)!}\right]^{\frac{2}{2}} \mathrm{P}_{\ell}(\mu) \cos k \phi \tag{29}
\end{equation*}
$$

and

$$
\begin{equation*}
Q_{\ell}^{k}(\mu, \phi)=\left[\frac{\left(2-\delta_{k o}\right)(\ell-k)!}{(\ell+k)!}\right]^{\frac{1}{2}} P_{\ell}^{k}(\mu) \sin k \phi \tag{30}
\end{equation*}
$$

so that

$$
\begin{equation*}
P_{\ell}\left(\mu_{0}\right)=\sum_{k=0}^{\ell}\left\{R_{\ell}^{k}(\mu, \phi) R_{\ell}^{k}\left(\mu^{\prime}, \phi^{\prime}\right)+Q_{\ell}^{k}(\mu, \phi) Q_{\ell}^{k}\left(\mu^{\prime}, \phi^{\prime}\right)\right\} \tag{31}
\end{equation*}
$$

The $Q$ terms will generate odd moments which will vanish on integration, thus the $Q$ terms will be omitted in the following discussion. The $R_{\ell}^{k}$ terms are the spherical harmonics polynomials. Using the above expression for $P_{\ell}\left(\mu_{0}\right)$ in the expansion of the scattering cross section, we have

$$
\begin{equation*}
\Sigma_{x, s}\left({\underline{\Omega} \rightarrow \underline{\Omega}^{\prime}}, E \rightarrow E^{\prime}\right)=\sum_{\ell=0}^{L M A X} \frac{2 \ell+1}{4 \pi} \Sigma_{s, \ell}\left(E \rightarrow E^{\prime}\right) \sum_{k=0}^{\ell} R_{\ell}^{k}(\mu, \phi) R_{\ell}^{k}\left(\mu^{\prime}, \phi^{\prime}\right) \tag{32}
\end{equation*}
$$

where LMAX is the order of anisotropic scattering.
The second term of the sensitivity profile, Eq. (24), becomes

$$
\begin{align*}
& \frac{1}{I \Delta u^{g}} \int_{E_{g}}^{E_{g-1}} d E \int_{V} d \underline{r} \int_{0}^{\infty} d E^{\prime} \sum_{\ell=0}^{L M A X} \frac{2 \ell+1}{4 \pi} \Sigma_{s, \ell}\left(E \rightarrow E^{\prime}\right) \sum_{k=0}^{\ell} 2 \int_{-1}^{1} d \mu \int_{0}^{\pi} d \phi \\
& \quad . R_{\ell}^{k}(\mu, \phi) \Phi(\underline{r}, \underline{\Omega}, E) \cdot 2 \int_{-1}^{1} d \mu^{\prime} \int_{0}^{\pi} d \phi^{\prime} R_{\ell}^{k}\left(\mu^{\prime}, \phi^{\prime}\right) \phi^{*}\left(\underline{r}, \underline{\Omega}^{\prime}, E^{\prime}\right) \tag{33}
\end{align*}
$$

Note that

$$
\begin{equation*}
\int_{-1}^{1} d \mu \int_{0}^{\pi} d \phi R_{l}^{k}(\mu, \phi) R_{m}^{n}(\mu, \phi)=\frac{2 \pi}{2 \ell+1} \delta_{\ell m} \delta_{k n} \tag{34}
\end{equation*}
$$

and therefore the angular flux can be expanded according to

$$
\begin{equation*}
\phi(\underline{\Omega}, E)=\sum_{\ell=0}^{\infty}(2 \ell+1) \sum_{k=0}^{\ell} R_{\ell}^{k_{\phi}}{ }_{\ell}^{k}(E) \tag{35a}
\end{equation*}
$$

where $\phi_{\ell}^{\mathrm{k}}(E)=\int_{-1}^{1} \mathrm{~d} \mu \int_{\mathrm{p}}^{1} \mathrm{~d} \phi \mathrm{R}_{\ell}^{\mathrm{k}} \phi(\underline{\Omega}, \mathrm{E}) / 2 \pi$,
and similarly for the adjoint angular flux

$$
\begin{align*}
& \Phi(\underline{\Omega}, \mathrm{E})=\sum_{\ell=0}(2 \ell+1) \sum_{\mathrm{k}=0}^{\ell} \mathrm{R}_{\ell}^{\mathrm{k}^{\prime} \phi_{\ell}^{\prime} \mathrm{k}}(\mathrm{E}),  \tag{36a}\\
& \text { where } \phi_{l}^{\dot{k} k}(E)=\int_{-1}^{1} d \mu \int_{0}^{\pi} d \phi R_{l}^{k} \phi(\underline{\Omega}, E) / 2 \pi, \tag{36b}
\end{align*}
$$

Introducing these expansions in the sensitivity profile, the gain term becomes

$$
\begin{gather*}
P_{\sum_{x, g a i n}^{g}}=\frac{4 \pi}{I \Delta u^{g}} \int_{V} d \underline{r} \sum_{g^{\prime}=1}^{\text {GMAX }} \int_{E_{g^{\prime}}}^{E_{g^{\prime}}-1} d E^{\prime} \int_{E_{g}}^{E_{g-1}} d E \sum_{\ell=0}^{\text {LMAX }}(2 \ell+1) \Sigma_{s, \ell}\left(E \rightarrow E^{\prime}\right) \\
\cdot \sum_{k=0}^{\ell} \Phi_{\ell}^{k}(E) \Phi_{\ell}^{\prime \cdot k}\left(E^{\prime}\right), \tag{37}
\end{gather*}
$$

where GMAX is the number of energy groups. Defining
and discretizing over the spatial variable we have

$$
\begin{equation*}
\mathrm{P}_{\Sigma_{\mathrm{x}, \mathrm{gain}}^{\mathrm{g}}}=\frac{4 \pi}{\mathrm{I} \mathrm{\Delta u}^{g}} \sum_{\mathrm{g}^{\prime}=1}^{\text {GMAX }} \sum_{\ell=0}^{\text {LMAX }}(2 \ell+1) \Sigma_{\mathrm{s}, \ell}^{g g \mathrm{~g}^{\prime}} \sum_{\mathrm{k}=0}^{\ell} \sum_{\mathrm{i}=1}^{\text {IPERT }} \mathrm{V}_{\mathrm{i}} \phi_{\ell}^{\mathrm{kg}}(\mathrm{i}) \Phi_{\ell}^{* \mathrm{~kg}}{ }_{(\mathrm{i})}, \tag{39}
\end{equation*}
$$

where IPERT is the number of perturbed spatial intervals and indicates the spatial interval. If there is no upscattering, and introducing

$$
\begin{equation*}
\Psi_{\ell}^{g g^{\prime}}=4 \pi \sum_{k=0}^{\ell}(2 \ell+1) \sum_{i=1}^{\text {IPERT }} V_{i} \Phi_{\ell}^{\mathrm{kg}}(\mathrm{i}) \Phi_{\ell}^{*} \mathrm{~kg}^{\prime}(\mathrm{i}), \tag{40}
\end{equation*}
$$

we have

$$
\begin{equation*}
\mathrm{P}_{\sum_{x, g a i n}^{g}}=\frac{1}{I \Delta u^{g}} \sum_{\ell=0}^{\operatorname{LMAX}} \sum_{g^{\prime}=g}^{\mathrm{GMAX}} \sum_{\mathrm{s}, \ell}^{g^{g} g^{\prime}} \Psi_{\ell}^{\mathrm{gg}} . \tag{41}
\end{equation*}
$$

The loss term of the sensitivity profile is given by

$$
\begin{align*}
P_{\Sigma_{x, l o s s}^{g}} & =\frac{1}{I \Delta u^{g}} \int_{E_{g}}^{E_{g-1}} d E \int_{V} d \underline{r} \int \underline{\Omega}\left\{-\phi(\underline{r}, \underline{\Omega}, E) \Sigma_{x, T}(E) \phi^{*}(\underline{r}, \underline{\Omega}, E)\right\},  \tag{42}\\
& =\frac{1}{I \Delta u^{g}} \int_{E_{g}}^{E_{g}-1} d E \int_{V} d \underline{r} 2 \int_{-1}^{1} d \mu \int_{0}^{\pi} d \phi\left\{-\phi(\mu, \phi, E) \Sigma_{x, T}(E) \Phi^{*}(\mu, \phi, E)\right\},  \tag{43}\\
& =\frac{-4 \pi}{I \Delta u^{g}} \int_{E_{g}}^{E_{g}-1} d E \int_{V} d \underline{r} \Sigma_{x, T}(E) \sum_{m=1}^{M M} w_{m} \phi\left(\mu_{m}, \phi_{m}, E\right) \phi^{*}\left(\mu_{m}, \phi_{m}\right), \tag{44}
\end{align*}
$$

where

$$
\begin{align*}
& \phi_{m}=\tan ^{-1}\left(1-\mu_{m}^{2}-\eta_{m}^{2}\right)^{\frac{1}{2}} / \mu_{m} \quad \text { for } \mu_{m}>0,  \tag{45}\\
& \phi_{m}=\tan ^{-1}\left(1-\mu_{m}^{2}-\eta_{m}^{2}\right)^{\frac{1}{2}} / \mu_{m}+\pi \quad \text { for } \mu_{m}<0, \tag{46}
\end{align*}
$$

and $M M$ is the number of angular fluxes per quadrant.
Define

$$
\begin{equation*}
\sum_{m=1}^{M M} \int_{E_{g}}^{E_{g-1}} d E \Sigma_{x, T}(E) \cdot \phi\left(\mu_{m}, \phi_{m}, E\right) \cdot \phi^{*}\left(\mu_{m}, \phi_{m}, E\right)=\Sigma_{x, T}^{g} \sum_{m=1}^{M M} \phi_{m}^{g_{\phi}^{\prime-} g} \tag{47}
\end{equation*}
$$

so that

$$
\begin{equation*}
P_{\Sigma_{x, l o s s}^{g}}^{g}=\frac{-4 \pi}{I \Delta u^{g}} \Sigma_{x, T}^{g} \sum_{i=1}^{\text {IPERT }} V_{i} \sum_{m=1}^{M M} \psi_{m} \phi_{m}^{g}(i) \phi_{m}^{* g}(i) \tag{48}
\end{equation*}
$$

Introducing

$$
\begin{equation*}
x^{g}=4 \pi \sum_{i=1}^{\text {IPERT }} V_{i} \sum_{m=1}^{M M} \omega_{m} \phi_{m}^{g}(i) \phi_{m}^{*} g(i), \tag{49}
\end{equation*}
$$

we have

$$
\begin{equation*}
P_{\Sigma_{x, l o s s}^{g}}=\frac{-1}{I \Delta u^{g}} \Sigma_{x, T}^{g} x^{g} \tag{50}
\end{equation*}
$$

Note that the gain term was expressed as a function of flux moments, while the loss term was expressed in terms of angular fluxes. When the gain term is expressed as a function of flux moments, a very useful
relationship between the $\Psi^{\prime} s$ and the $X^{\prime} s$ will be obtained. For this case, substituting Eqs. (36) and (38) into Eq. (42), the loss term can be expanded as

$$
\begin{align*}
& P_{\Sigma_{x, l o s s}^{g}}=\frac{-2}{I \Delta u^{g}} \int_{E_{g}}^{E_{g-1}} \int_{V} d \underline{r} \Sigma_{x, T} \int_{-1}^{1} d \mu \int_{0}^{\pi} d \phi \sum_{\ell=0}^{\infty}(2 \ell+1) \sum_{k=0}^{\ell} R_{\ell}^{\left.k_{\phi}{ }^{k}{ }_{\ell \ell}\right\}} \\
& \sum_{\ell=0}^{\infty}\left\{(2 \ell+1) \sum_{k=0}^{\ell} R_{\ell}^{k_{\phi} \Phi^{\frac{\ell}{k}}} \ell\right\} . \tag{51}
\end{align*}
$$

Using the orthogonality relations Eq. (34) and defining the multigroup total cross section for group $g$ by

we have after discretizing the spatial variable, $\underline{r}$, and truncating the summation over \&,

$$
\begin{equation*}
P_{\Sigma_{x, l o s s}^{g}}=\frac{-4 \pi^{\Sigma_{x, T}^{g}}}{I \Delta u^{g}} \sum_{\ell=0}^{\text {LMAX }}(2 \ell+1) \sum_{i=1}^{\text {IPERT }} v_{i} \phi_{\ell}^{k g}(i) \phi_{\ell}^{k g}(i) \tag{53}
\end{equation*}
$$

Introducing

$$
\begin{equation*}
x^{g}=\sum_{\ell=0}^{\operatorname{LMAX}} \Psi_{\ell}^{g g}, \tag{54}
\end{equation*}
$$

the expression for the loss term reduces to Eq. (50) again.

## Summary

$$
\begin{equation*}
P_{\Sigma}^{g}=\frac{1}{I \cdot \Delta u^{g}}-\Sigma_{x, T}^{g} \chi^{g}+\sum_{\ell=0}^{\text {LMAX }} \sum_{g^{\prime}=g}^{\text {GMAX }} \sum_{s, \ell}^{g \rightarrow g^{\prime}} \Psi_{l}^{g g^{\prime}}, \tag{55}
\end{equation*}
$$

where

$$
\begin{align*}
& \Sigma_{x, T}^{g}=\text { total macroscopic cross section for reaction type } x \text {, } \\
& \Sigma_{s, \ell}^{g_{s} g^{\prime}}=\ell^{\prime} \text { th Legendre coefficient of the scattering matrix element for } \\
& \text { energy transfer from group } g \text { to group } g ' \text {, as derived from the } \\
& \text { differential scattering cross section for reaction type } x \text {, } \\
& \Psi_{\ell}^{g g^{\prime}}=4 \pi(2 \ell+1) \sum_{i=1}^{\text {IPERT }} \sum_{k=0}^{\ell} V_{i} \phi_{\ell}^{k g}(i) \phi_{\ell}^{\prime i} \mathrm{~kg}^{\prime}(\mathrm{i})  \tag{56}\\
& =\text { spatial integral of the product of the spherical harmonics } \\
& \text { expansions for the regular and adjoint angular fluxes, } \\
& x^{g}=4 \pi \sum_{i=1}^{\text {IPERT }} V_{i} \sum_{m=1}^{M M} \phi_{m}^{g}(i) \phi_{m}^{* g}(i) w_{m}  \tag{57}\\
& =\text { numerical integral of the product of forward and adjoint } \\
& \text { angular fluxes over all angles and all spatial intervals de- } \\
& \text { scribed by } i=1 \text {. . ., IPERT, } \\
& =\sum_{\ell=0}^{\text {LMAX }} \Psi_{\ell}^{g g} . \tag{58}
\end{align*}
$$

Note that expression (55) is identical with the expression for the cross-section sensitivity profile in a one-dimensional formulation. ${ }^{46}$ The flux moments can be expressed in terms of angular fluxes corresponding to

$$
\begin{equation*}
\Phi_{\ell}^{\mathrm{kg}}=\int_{-1}^{1} d \mu \int_{0}^{\pi} d \phi R_{\ell}^{k_{\phi} g}(\underline{\Omega}) / 2 \pi=\sum_{m=1}^{M M} \Phi_{m}^{g} R_{l}^{\mathrm{k}}\left(\mu_{m}, \phi_{m}\right) w_{m}, \tag{59}
\end{equation*}
$$

and

$$
\begin{aligned}
& \phi_{\ell}^{\dot{*} k g^{\prime}}=\int_{-1}^{1} d \mu \int_{0}^{\pi} d \phi R_{l}^{k^{\prime} \phi^{\prime} g^{\prime}}(\underline{\Omega}) / 2 \pi=\sum_{m=1}^{M M} \phi_{m}^{g^{\prime}} R_{l}^{k}\left(\mu_{m}, \phi_{m}\right) w_{m} . \\
& \mathrm{R}_{\ell}^{\mathrm{k}}(\Omega)=\text { spherical harmonics function } \\
& V_{i}=\text { volume of rotated triangles } \\
& \Delta u^{g}=\text { lethargy width of energy group } g \\
& =\ln \left(E^{g} / E^{g+1}\right) \text {, where } E^{g} \text { and } E^{g+1} \text { are upper and lower energy } \\
& \text { group boundaries } \\
& =\text { integral response as calculated from forward fluxes only } \\
& =\sum_{i=1}^{\text {IDET }} \sum_{g=1}^{\text {IGM }} V_{i} R_{i}^{g} \phi_{0}^{0 g}(i) \\
& R_{i}=\text { spatially and group-dependent detector response function. }
\end{aligned}
$$

2.3 Source and Detector Sensitivity Profiles ${ }^{46}$

Source and detector sensitivity profiles indicate how sensitive the
integral response $I$ or $I^{*}$ is to the energy distribution of the source, or to the detector response $R$. The integral response $I$ can be calculated from the forward flux, according to Eq. (63), or from the adjvint flux, according to Eq. (64). When the integral response is calculated from the adjoint flux it will be denoted as $I^{\frac{1}{\star}}$. Ideally, I will be equal to $I^{\star}$.

The sensitivity of the integral response to the energy distribution of the detector response function or the source can therefore be expressed by the sensitivity profiles

$$
\begin{equation*}
P_{R}^{g}=\int_{V_{d}} d \underline{r} \int_{E_{g}}^{E_{g-1}} d E \int d \underline{\Omega} R(\underline{r}, E) \cdot \underline{\Phi}(\underline{r}, \underline{\Omega}, E) / I \cdot \Delta u^{g} \tag{61}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{P}_{\mathrm{Q}}^{\mathrm{g}}=\int_{\mathrm{V}_{\mathrm{s}}} \mathrm{~d}_{\underline{\mathrm{r}}}^{\mathrm{E}_{\mathrm{g}}} \mathrm{E}^{\mathrm{E}-1} \mathrm{dE} \int \mathrm{~d} \underline{\Omega} \mathrm{Q}(\underline{r}, \underline{\Omega}, \mathrm{E}) \cdot \phi^{*}(\underline{r}, \underline{\Omega}, \mathrm{E}) / \mathrm{I}^{*} \cdot \Delta \mathrm{u}^{g} \tag{62}
\end{equation*}
$$

where $R(\underline{r}, E)$ is the detector response and $Q(\underline{r}, \underline{\Omega}, E)$ is the angular source, and $V_{d}$ and $V_{s}$ are the volumes of the detector and the source region. I was used in the denumerator of $P_{R}^{g}$ and $I^{\frac{1}{4}}$ was used in the denominator of $P_{Q}^{g}$ for internal consistency. It is obvious that the integral source and detector sensitivities, $S_{Q}$ and $S_{R}$, will be equal to one.

It is possible to derive an expression similar to Eq. (61) for the sensitivity of the integral response to the angular distribution of the source. The derivation of explicit expressions for $P_{R}^{g}$ and $P_{Q}^{g}$ is straightforward. The detector sensitivity profile as a function of the scalar fluxes becomes

$$
\begin{equation*}
P_{R}^{g}=\sum_{i=1}^{I D E T} V_{i} \cdot R_{i}^{g} \cdot \phi_{0}^{0 g}(i) / I \cdot \Delta u^{g} \tag{63}
\end{equation*}
$$

where the $\phi_{0}^{0 g}(i)$ are the scalar fluxes for group $g$ at interval $i$, IDET is the number of detector intervals, and $R_{i}^{g}$ is the detector response at interval i for group g.

For the source sensitivity profile in case of an isotropic source Eq. (62) transforms into

$$
\begin{equation*}
P_{Q}^{g}=\sum_{i=1}^{\text {ISRS }} V_{i} \cdot Q_{i}^{g} \cdot \Phi_{0}^{0 g}(i) / I^{\frac{1}{\star}} \cdot \Delta u^{g} \tag{64}
\end{equation*}
$$

where $Q_{i}^{g}$ is the voluminar source for group $g$ at source interval i.
In the case of an anisotropic source we defined $Q^{g}(\underline{r}, \underline{\Omega})$ by

$$
\begin{equation*}
Q^{g}(\underline{r}, \underline{\Omega}) \cdot \phi^{*} g(\underline{r}, \underline{\Omega})=\int_{E_{g}}^{E_{g-1}} d E Q(\underline{r}, \underline{\Omega}, E) \cdot \phi^{\dot{*}}(\underline{r}, \underline{\Omega}, E) \tag{65}
\end{equation*}
$$

and expand the angular source according to

$$
\begin{equation*}
Q^{g}(\underline{r}, \underline{\Omega})=Q^{g}(\underline{r}, \mu, \phi)=\sum_{\ell=0}^{I Q A N}(2 \ell+1) \sum_{k=0}^{\ell} R_{\ell}^{k}(\mu, \phi) \cdot Q_{\ell}^{\mathrm{kg}}(\underline{r}) / 2 \pi, \tag{66}
\end{equation*}
$$

where IQAN is the order of anisotropy of the source.
Substituting Eqs. (65) and (66) in Eq. (63), discretizing the spatial variable and using Eq. (36), the expression for the source sensitivity profile becomes

$$
\mathrm{P}_{\mathrm{Q}}^{\mathrm{g}}=2 . \sum_{\mathrm{i}=1}^{\text {ISRS }} \mathrm{V}_{\mathrm{i}} \sum_{\ell=0}^{\text {IQAN }}(2 \ell+1) \sum_{\mathrm{k}=0}^{\ell} \mathrm{Q}_{\ell}^{\mathrm{gk}}(\mathrm{i}) \phi_{\ell}^{\dot{2} \mathrm{gk}}(\mathrm{i}) / \mathrm{I}^{\frac{\lambda}{\star}} \cdot \Delta \mathrm{u}^{\mathrm{g}}
$$

As in Eq. (61) we can also define an angular source sensitivity function. The angular source sensitivity function indicates how sensitive the integral response $I^{\prime \prime}$ is to the angular distribution of the source, or

$$
\begin{equation*}
F_{Q}^{\Omega}=\frac{1}{I} \int_{V_{s}} d \underline{r} \int_{0}^{\infty} d E Q(\underline{r}, \underline{\Omega}, E) \cdot \Phi^{\hat{\prime}}(\underline{r}, \underline{\Omega}, E) / I^{*} \tag{68}
\end{equation*}
$$

### 2.4 Sensitivity Profiles for the Secondary Energy Distribution and the Secondary Angular Distribution

The theory of the secondary energy distribution (SED) and the secondary angular distribution (SAD) sensitivity analysis was originated by Gerstl. 43-46 Physically the only difference between a secondary energy distribution and a cross-section sensitivity profile is the way in which the integration over the energy variable is carried out. The "hot-cold" and the "forward-backward" concepts lead to a simple formulation of secondary sensitivity theory and can easily be incorporated in an uncertainty analysis. Even when both those concepts are a rather coarse approximation they have the advantage that they are simple and can be physically understood.

A more rigorous formulation might be possible, but its simple physical interpretation would be lost. ${ }^{63}$ The primary restriction on the application of secondary energy distribution and secondary angular distribution sensitivity profiles is the lack of cross-section uncertainty information in the proper format.

### 2.4.1 Introduction

The expression for the sensitivity profile for the differential scattering cross section is part of the gain term of the cross-section sensitivity profile and takes the form

$$
\begin{align*}
P_{\Sigma_{x}}\left(\underline{\Omega} \rightarrow \underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right) & =\frac{1}{\Delta u^{g} \cdot I} \int d \underline{r} \int d \underline{\Omega} \int_{E_{g}}^{E} g-1 \\
d E & \int_{0}^{\infty} d E^{\prime} \int d \underline{\Omega}^{\prime}  \tag{69}\\
& \times R_{\Sigma_{x, \text { gain }}}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right) \quad,
\end{align*}
$$

where $R_{X, \text { gain }}\left(\underline{x}, \underline{\Omega} \rightarrow \underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right)$ is a shorthand notation for

$$
\begin{equation*}
R_{\Sigma_{X, \operatorname{gain}}}\left(\underline{r}, \underline{\Omega} \rightarrow \underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right)=\Phi(\underline{r}, \underline{\Omega}, E) \Sigma_{x, s}\left(\underline{r}, \underline{\Omega}_{\rightarrow} \underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right) \Phi\left(\underline{r}, \underline{\Omega}^{\prime}, E^{\prime}\right) \tag{70}
\end{equation*}
$$

and similarly,

$$
\begin{equation*}
R_{\Sigma_{x, \text { gain }}}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}, E^{\prime} \rightarrow E\right)=\phi(\underline{r}, \underline{\Omega}, E) \Sigma_{x, s}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}, E \rightarrow E\right) \Phi^{\dot{*}}(\underline{r}, \underline{\Omega}, E) \tag{71}
\end{equation*}
$$

Equation (70) gives the contribution to the integral detector response, $I$, from the particles born at position $\underline{r}$ with energy $E^{\prime}$, traveling in direction $\underline{\Omega}^{\prime}$, since

$$
\begin{equation*}
\mathrm{I}=\left\langle\Phi, \mathrm{L}^{* \prime \prime} \phi^{*}\right\rangle=\left\langle\phi^{*}, \mathrm{~L} \phi\right\rangle . \tag{72}
\end{equation*}
$$

Similarly, $R_{\Sigma_{x, g a i n}}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}^{\prime}, E^{\prime} \rightarrow E\right)$ gives the contribution to the integral detector response from the particles born at position $\underline{r}$, with energy $E$, traveling in direction $\Omega$.

As it turns out, up to this point there is no difference in the physical interpretation of Eqs. (70) and (71). The way the integrations
are carried out will distinguish between the differential scattering cross-section sensitivity profile and the secondary energy distribution and secondary angular distribution sensitivity profile.

### 2.4.2 Further theoretical development

In this section we will elaborate on the physics behind the derivation of SEDs and SADs. Consider

$$
\begin{equation*}
F_{\Sigma}\left(E, E^{\prime}\right)=\frac{1}{\bar{I}} \int d \underline{r} \int d \underline{\Omega} \int d \underline{\Omega}^{\prime} R_{\Sigma, \operatorname{gain}}\left(\underline{r}, \underline{\Omega}-\underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right) \tag{73}
\end{equation*}
$$

In this expression $F_{\Sigma_{x, s}}$ represents the fractional change in the integral response per unit fractional change in the differential scattering cross section $\Sigma_{x, s}\left(E \rightarrow E^{\prime}\right)$; i.e., it is the fractional change in the integral response when the number of particles that scatter from $E$ into $E^{\prime}$ is increased by one percent. Obviously this will always be a positive effect and will therefore be included in the gain term.

Similar to Eq. (73),

$$
\begin{equation*}
\tilde{\mathrm{P}}_{\Sigma}^{g}{ }_{x, s}=\frac{1}{\bar{I}} \int_{E_{g}}^{E_{g-1}} \mathrm{dE} \int_{0}^{\infty} d E^{\prime} \int d \underline{\Omega} \int d \underline{\Omega}^{\prime} R_{\Sigma}{ }_{x, g \text { ain }}\left(\underline{r}, \underline{\Omega} \rightarrow \underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right) \tag{74}
\end{equation*}
$$

represents the fractional change in the integral response when the number of particles that scatter from group $g$ is increased by one percent. The tilda in Eq. (74) is introduced to distinguish from a lethargy normalized sensitivity profile.

In the adjoint formulation the equivalent of Eq. (73) will be

$$
\begin{equation*}
F_{\Sigma_{x, s}}\left(E^{\prime}, E\right)=F_{S E D}\left(E^{\prime}, E\right)=\frac{1}{\bar{I}} \int d \underline{r} \int d \underline{\Omega} \int d \underline{\Omega}^{\prime} R_{\Sigma}{ }_{x, \operatorname{gain}}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}, E^{\prime} \rightarrow E\right), \tag{75}
\end{equation*}
$$

which represents the fractional change in the integral response per unit fractional change in differential scattering cross section $\Sigma_{x, s^{\prime}}\left(E^{\prime} \rightarrow E\right)$, i.e., it is the fractional change in the integral response when the number of primary particles that scatter from $E^{\prime}$ to $E$ is increased by one percent, or for that matter that the number of secondary particles that were scattered from $E$ into $E$ were increased by one percent. Again, this will always have a positive effect and will therefore constitute a gain term in the sensitivity profile.

Define

$$
\begin{equation*}
\tilde{P}_{S E D}^{g}=\frac{1}{\bar{I}} \int_{E_{g}}^{E_{g-1}} d E \int_{0}^{\infty} d E^{\prime} \int d \underline{\Omega} \int d \underline{\Omega}^{\prime} R_{\Sigma}{ }_{x, g a i n}\left(\underline{r} \cdot \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}, E^{\prime} \rightarrow E\right) \tag{76}
\end{equation*}
$$

While there is no difference in the physical meaning of Eqs. (73) and (75), the formulations (74) and (76) are different. Equation (74)
represents the fractional change in the integral response when the number of secondary particles that were scattered into group $g$ have been increased by one percent.

It is clear from these examples that, depending on the way the integrations are done, several different sensitivity profiles can be constructed. In order to study the secondary angular distribution, we can introduce

$$
\begin{equation*}
F_{\Sigma_{x, \operatorname{gain}}}\left(\underline{\Omega}, E^{\prime}\right)=F_{S A D}\left(\underline{\Omega}, E^{\prime}\right)=\frac{1}{\bar{I}} \int d \underline{r} \int_{0}^{\infty} d E \int d \underline{\Omega}^{\prime} R_{\Sigma_{x, \text { gain }}}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}, E^{\prime} \rightarrow E\right) \tag{77}
\end{equation*}
$$

This expression gives the fractional change in the integral response when the number of secondary particles scattered from initial energy $E^{\prime}$ into final direction $\underline{\Omega}$ is increased by one percent. It will therefore be clear that

$$
\begin{equation*}
\tilde{\mathrm{P}}_{\mathrm{SAD}}(\underline{\Omega})=\int \mathrm{dE}^{\prime} \mathrm{F}_{\mathrm{SAD}}\left(\underline{\Omega}, \mathrm{E}^{\prime}\right) \tag{78}
\end{equation*}
$$

is the fractional change in the response function when the number of secondary particles which were scattered into direction $\underline{\Omega}$ was increased by one percent.

### 2.4.3 Secondary energy and secondary angular distribution sensitivity profiles

A double secondary energy distribution (SED) sensitivity profile is defined by

$$
\begin{equation*}
P_{S E D}^{g^{\prime} g}=\frac{1}{J \Delta u u^{g} \Delta u^{\prime}} \int_{E_{g}}^{E_{g-1}} d E \int_{E_{g^{\prime}}}^{E_{\prime^{\prime}-1}} d E^{\prime} \int d \underline{r} \int d \underline{\Omega} \int d \underline{\Omega}^{\prime} R_{\Sigma}{ }_{x, g a i n}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}, E^{\prime} \rightarrow E\right), \tag{79}
\end{equation*}
$$

The energy integrated SED sensitivity profile becomes

$$
\begin{equation*}
P_{S E D}^{g}=\frac{1}{I \Delta u^{g}} \int_{0}^{\infty} d E^{\prime} \int_{E_{g}}^{E_{g-1}} d E \int d \underline{r} \int d \underline{\Omega}^{\prime} R_{\Sigma}{ }_{x, \text { gain }}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}, E^{\prime} \rightarrow E\right) \tag{80}
\end{equation*}
$$

The differential sensitivity profile for the angular distribution of secondary particles scattered from initial energy $E^{\prime}$ is

$$
\begin{equation*}
P_{S A D}^{g^{\prime}}(\underline{\Omega})=\frac{1}{I \Delta u^{g}} \int_{E_{g}}^{g^{\prime}-1} d E^{\prime} \int_{0}^{\infty} d E \int d \underline{r} \int d \underline{\Omega}^{\prime} R_{\Sigma}{ }_{x, g a i n}\left(\underline{r}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}, E \rightarrow E^{\prime}\right) \tag{82}
\end{equation*}
$$

An energy integrated SED sensitivity profile can be defined by

$$
\begin{equation*}
\mathrm{P}_{\mathrm{SAD}}(\underline{\Omega})=\frac{1}{\mathrm{I}} \int_{0}^{\infty} \mathrm{dE} \cdot \int_{0}^{\infty} \mathrm{dE} \int \mathrm{~d} \underline{\underline{r}} \int \mathrm{~d} \underline{\Omega}^{\prime} \mathrm{R}_{\Sigma_{\mathrm{s}, \mathrm{gain}}}\left(\underline{\mathrm{r}}, \underline{\Omega}^{\prime} \rightarrow \underline{\Omega}, \mathrm{E} \rightarrow \mathrm{E}^{\prime}\right) . \tag{82}
\end{equation*}
$$

### 2.4.4 Integral sensitivities for SEDs and SADs

In order to make the sensitivity and uncertainty analysis for secondary energy distributions and secondary angular distributions less tedious, Gerstl introduced the concepts of the "hold-cold" SED and the "forward-backward" SAD integral sensitivity:

$$
\begin{equation*}
S_{S E D}^{g^{\prime}}=\int_{H O T} d E \tilde{P_{S E D} g^{\prime}}(E)-\int_{C O L D} d E \tilde{P_{S A D} g^{\prime}}(E), \tag{83}
\end{equation*}
$$

and

$$
\begin{equation*}
S_{S A D}=\int_{\substack{\text { forward } \\ \text { angles } \\(\mu>0)}} d \underline{\Omega} P_{S A D}(\underline{\Omega})-\int_{\substack{\text { backward } \\ \text { angles } \\(\mu<0)}} d \underline{\Omega} P_{S A D}(\underline{\Omega}) \cdot \tag{84}
\end{equation*}
$$

The forward-backward SAD integral sensitivity can be interpreted as the fractional change in the integral response when the number of secondaries which were scattered forward is increased by one percent, while the number of secondaries that were scattered backwards ( $\mu<0$ ) is decreased by one percent. The integral SAD sensitivity is a positive number which is labeled "forward" or "backward" depending whether the first or the second term in Eq. (84) is the larger one. Physically,
that positive number indicates how much more sensitive the response function is to forward scattered particles than to backward scattered particles, or vice versa.

For the hot-cold integral SED sensitivity, the concept of the median energy has to be introduced. In the multigroup formulation, the median energy defines the energy boundary which roughly divides the cross-section profile into two equal parts. The median energy and the integral SED sensitivity are illustrated in Fig. 3. ${ }^{43}$ Note that the median energy $g^{\prime}$ is a function of the primary energy group $g^{\prime}$. For that reason also the integral SED sensitivity will depend on $g^{\prime}$.

The hot-cold integral SED sensitivity expresses the fractional change in integral response when the number of secondaries which scatter in the "hot" part of the secondary energy distribution is increased by one percent while the number of secondaries scattered into the "cold" part is decreased by one percent. The integral hot-cold SED sensitivity is a positive number; labeled "hot" or "cold" depending on which term dominates in Eq. (83). That number indicates how much more sensitive the integral response is to particles scattered into the hot part of the secondary energy distribution than to particles scattered into the cold part, or vice versa.


$$
\left.\begin{array}{rl}
\sigma_{\text {hot }} & =\sum_{\mathrm{g}=1}^{g_{\mathrm{m}}} \sigma_{\mathrm{g}} \\
\sigma_{\text {cold }} & =\sum_{\mathrm{g}=\mathrm{E}_{\mathrm{m}}+1} \sigma_{\mathrm{g}}
\end{array}\right\} \quad \sigma_{\text {hot }}=\sigma_{\text {cold }}=1 / 2 \sigma
$$



$$
S_{S L D}^{g^{\prime}}=\sum_{g_{m}}^{g_{m}\left(g^{\prime}\right)} \hat{P}_{S E D}^{g}-\sum_{g_{m}\left(g^{\prime}\right)+1}^{G} \hat{P}_{S E D}^{g} \quad \begin{aligned}
& \text { "HOT" } \\
& \text { or } \\
& \text { or }
\end{aligned}
$$

Figure 3. Definition of median energy and integral SED sensitivity ${ }^{43}$

### 2.4.5 Explicit expressions for integral SED sensitivity profiles in a two-dimensional geometry representation

The expression for the double SED sensitivity profile, Eq. (79), is similar to the gain term of the cross-section sensitivity profile, Eq. (24). By comparing Eq. (79) with Eq. (24) and using Eq. (41), the explicit expression for the double SED sensitivity profile becomes

$$
\begin{equation*}
\mathrm{P}_{\mathrm{SED}}^{g^{\prime}, g}=\frac{1}{I \Delta u^{g} \Delta u^{g}} \sum_{\ell=0}^{\mathrm{LMAX}} \sum_{s, \ell}^{g^{\prime} \rightarrow g} \psi_{\ell}^{g^{\prime} g}, \tag{85}
\end{equation*}
$$

From Eqs. (85) and (80), it follows that the energy integrated SED sensitivity profile for the case of no upscattering can be represented by

$$
\begin{equation*}
\mathrm{P}_{\mathrm{SED}}^{g}=\frac{1}{I \Delta u^{g}} \quad \sum_{g^{\prime}=1}^{g} \sum_{\ell=0}^{\mathrm{LMAX}} \sum_{S, \ell}^{g^{\prime} \rightarrow g} \psi_{\ell}^{g^{\prime} g} \tag{86}
\end{equation*}
$$

Using the definition for the integral SED sensitivity (83), it becomes clear that

$$
\begin{equation*}
S_{S E D}^{g^{\prime}}=\sum_{g=g^{\prime}}^{g_{\mathrm{m}}\left(g^{\prime}\right)} \Delta u^{g} \cdot P_{S E D}^{g}-\sum_{g=g_{\mathrm{m}}\left(g^{\prime}\right)+1}^{\operatorname{GMAX}} \Delta u^{g} \cdot \mathrm{P}_{\mathrm{SED}}^{g} \tag{87}
\end{equation*}
$$

where $g_{m}\left(g^{\prime}\right)$ is defined in Fig. 1.

### 2.6 Design Sensitivity Analysis

Design sensitivity analysis provides a method to estimate changes in integral response for a slightly altered design. The results are exact up to the second order with respect to the corresponding flux changes, but only exact up to the first order with respect to design changes. The theory presented in this section is applicable only when the design changes can be expressed in terms of macroscopic crosssection changes. Methods based on generalized perturbation theory have been applied to design sensitivity analysis. 14,37

The integral response for the perturbed system can be expressed by Eq. (88) for the adjoint difference formulation, ${ }^{35}$

$$
\begin{equation*}
\mathrm{I}_{\mathrm{AD}}=\langle\mathrm{R}, \phi\rangle-\left\langle\Phi^{*}, \Delta \mathrm{~L} \phi\right\rangle=\mathrm{I}-\delta \mathrm{I}_{\mathrm{AD}}, \tag{88}
\end{equation*}
$$

and by Eq. (89) in the forward difference formulation

$$
\begin{equation*}
\mathrm{I}_{\mathrm{FD}}=\left\langle\mathrm{Q}, \mathrm{I}^{*}\right\rangle-\left\langle\phi, \Delta \mathrm{L}^{*} \phi^{*}\right\rangle=\mathrm{I}-\delta \mathrm{I}_{\mathrm{FD}} \tag{89}
\end{equation*}
$$

Proceeding in a manner similar to the derivation of the crosssection sensitivity profile, the second-order term in the right hand side of Eqs. (88) and (89) can be written as

$$
\begin{align*}
\delta I_{A D} & =\int_{0}^{\infty} d E \int_{V_{d}} d \underline{r} \int d \underline{\Omega}\left\{\Phi(\underline{r}, \underline{\Omega}, \mathrm{E}) \delta \Sigma_{\mathrm{x}, \mathrm{~T}}(\underline{\mathrm{r}}, \mathrm{E}) \phi^{*}(\underline{\mathrm{r}}, \underline{\Omega}, \mathrm{E})\right. \\
& \left.+\int_{0}^{\infty} \mathrm{dE} \int \mathrm{~d} \underline{\Omega}^{\prime} \Phi\left(\underline{\mathrm{r}}, \underline{\Omega}^{\prime}, \mathrm{E}^{\prime}\right) \delta \Sigma_{\mathrm{x}, \mathrm{~s}}\left(\underline{\mathrm{r}}, \underline{\Omega}^{\prime} \not \underline{\Omega}, \mathrm{E}^{\prime} \rightarrow \mathrm{E}\right) \Phi^{*}(\underline{\mathrm{r}}, \underline{\Omega}, \mathrm{E})\right\}, \tag{90}
\end{align*}
$$

and

$$
\begin{align*}
\delta I_{F D} & =\int_{0}^{\infty} d E \int_{V_{s}} d \underline{r} \int d \underline{\Omega}\left\{\phi(\underline{r}, \underline{\Omega}, E) \delta \Sigma_{x, T}(\underline{r}, E) \Phi^{*}(\underline{r}, \underline{\Omega}, E)\right. \\
& +\int_{0}^{\infty} d E^{\prime} \int d \underline{\Omega}^{\prime} \phi(\underline{r}, \underline{\Omega}, E) \delta \Sigma \Sigma_{x, s}\left(\underline{r}, \underline{\Omega} \rightarrow \underline{\Omega}^{\prime}, E \rightarrow E^{\prime}\right) \phi^{*}\left(\underline{r}, \underline{\Omega}^{\prime}, E^{\prime}\right) \tag{91}
\end{align*}
$$

In the above expressions we used

$$
\begin{equation*}
\delta \Sigma_{x, T}=\Sigma_{x, T}-\bar{\Sigma}_{x, T} \tag{92}
\end{equation*}
$$

and

$$
\begin{equation*}
\delta \Sigma_{x, s}=\Sigma_{x, s}-\bar{\Sigma}_{x, s} \tag{93}
\end{equation*}
$$

where $\Sigma$ refers to a perturbed cross section and $\bar{\Sigma}$ to a reference cross section.

A design sensitivity coefficient $X$ can be defined as the ratio of the integral response for the altered design over the integral response
for the original model. Depending whether the forward or the adjoint difference method are used, the design sensitivity coefficient equals

$$
\begin{equation*}
\mathrm{X}_{\mathrm{AD}}=\mathrm{I}_{\mathrm{AD}} / \mathrm{I}=1-\delta \mathrm{I}_{\mathrm{AD}} / \mathrm{I} \tag{94}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{X}_{\mathrm{FD}}=\mathrm{I}_{\mathrm{FD}} / \mathrm{I}^{*}=1-\delta \mathrm{I}_{\mathrm{FD}} / \mathrm{I}^{*} \tag{95}
\end{equation*}
$$

Note that respectively, $I$ and $I^{*}$ were used in the denominator of Eqs. (94) and (95) for internal consistency. Numerically $\delta \mathrm{I}_{\mathrm{AD}}$ and $\delta \mathrm{I}_{\mathrm{FD}}$ are identical; $I$ and $I^{*}$, however, can be different. Gerstl and Stacey ${ }^{35}$ indicate that the adjoint formulation is more accurate for perturbations closer to the detector, while the forward difference method gives better results for perturbations closer to the source. If both reference fluxes $\phi$ and $\phi^{*}$ are completely converged, Eqs. (94) and (95) will give identical results.

Explicit expressions for Eqs. (94) and (95) can be formulated. The procedure for the evaluations of $\delta \mathrm{I}_{\mathrm{AD}}$ and $\delta \mathrm{I}_{\mathrm{FD}}$ is similar to the derivation of the cross-section sensitivity profile and leads to the equations

$$
\begin{equation*}
\delta I_{A D}=\sum_{g=1}^{\text {IGM }}\left\{\delta \Sigma_{\mathrm{x}, \mathrm{~T}}^{\mathrm{g}} \chi^{g}-\sum_{\ell=0}^{\operatorname{LMAX}} \sum_{\mathrm{g}^{\prime}=1}^{g} \delta \Sigma_{\mathrm{s}, \ell}^{g^{\prime} \rightarrow \mathrm{g}} \Psi_{\mathrm{s}, \ell}^{g^{\prime} \rightarrow \mathrm{g}}\right\}, \tag{96}
\end{equation*}
$$

and

$$
\begin{equation*}
\delta I_{\mathrm{FD}}=\sum_{\mathrm{g}=1}^{\text {IGM }}\left\{\delta \sum_{\mathrm{x}, \mathrm{~T}}^{\mathrm{g}} \mathrm{X}^{\mathrm{g}}-\sum_{\ell=0}^{\mathrm{LMAX}} \underset{\mathrm{~g}^{\prime}=\mathrm{g}}{\mathrm{GMAX}} \underset{\mathrm{~s}, \ell}{\delta \Sigma_{\ell}^{g \rightarrow g^{\prime}}} \Psi_{\ell}^{\mathrm{gg}}{ }^{\prime}\right\} . \tag{97}
\end{equation*}
$$

## 3. APPLICATION OF SENSITIVITY THEORY TO UNCERTAINTY ANALYSIS

Sensitivity theory can be used to do an uncertainty analysis by introducing the concepts of cross-section covariance matrices and fractional uncertainties for SEDs. In this chapter we will explain how sensitivity profiles can be used in order to calculate the uncertainty of a reaction rate due to the uncertainties in the cross sections.

### 3.1 Definitions

Let I represent a design parameter depending on a multigroup crosssection set $\left\{\Sigma_{i}\right\}$, so that

$$
\begin{equation*}
I=I\left(\Sigma_{i}\right), \tag{98}
\end{equation*}
$$

where the index $i$ can reflect a group, a partial cross section or a material.

The variance of $I$ is defined as the expected value of the square of the difference between the actual value of $I$ and the expected value of I, or

$$
\begin{equation*}
\operatorname{Var}(I) \equiv E\left\{(\delta I)^{2}\right\}=E\left\{(I-E\{I\})^{2}\right\} . \tag{99}
\end{equation*}
$$

The standard deviation of $I$ is the square root of the variance,

$$
\begin{equation*}
\Delta \mathrm{I} \equiv[\operatorname{Var}(\mathrm{I})]^{\frac{3}{2}} \tag{100}
\end{equation*}
$$

The covariance of $a$ and $b$ is defined as

$$
\begin{equation*}
\operatorname{Cov}(a, b) \equiv E\{\delta a \cdot \delta b\} \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d a \cdot d b \cdot(a-E\{a\}) \cdot(b-E\{b\}) \cdot f(a, b), \tag{101}
\end{equation*}
$$

where $f(a, b)$ is $a$ joint probability density function. A nonzero covariance between the quantities $a$ and $b$ indicates $a$ mutual dependence on another quantity. Obviously we have

$$
\begin{equation*}
\operatorname{Cov}(a, a)=\operatorname{Var}(a), \tag{102}
\end{equation*}
$$

since $f(a, a)=1$.
A relative covariance element is defined by

$$
\begin{equation*}
R(a, b) \equiv \operatorname{Cov}(a, b) / a \cdot b \tag{103}
\end{equation*}
$$

### 3.2 Cross-Section Covariance Matrices

During the experimental evaluation of cross-section data, statistical errors arise from the fact that two similar experiments never agree completely. Also a systematic error reflects the fact that no equipment and no evaluation procedure is perfect, and that - among other factors - reference standards are used.

Cross-section covariance data describe the uncertainties in the multigroup cross sections and the correlation between those uncertainties. A nonzero nondiagonal covariance matrix element indicates that there was a common reason why an uncertainty in two different (e.g., partial cross sections or energy range) cross section was introduced. The evaluation procedure for covariance data is tedious and requires a sophisticated statistical analysis. $2,30,31$

Multigroup cross-section covariance data are ordered in covariance matrices. Such a covariance matrix contains GMAX rows and GMAX columns, where GMAX is the number of energy groups. A covariance matrix can contain covariance data of a particular partial cross section with itself over an energy range, with a different cross section for the same element, or with a partial cross section of a different element.

It has become a common practice to include formatted uncertainty data in the ENDF/B data files. Even though the uncertainty files are
still missing for many materials in ENDF/B-V, extensive work is underway. Based on these uncertainty data, covariance libraries can be constructed. ${ }^{32,33}$ A 30 -group covariance library based on ENDF/B-V which contains most of the elements commonly used in reactor shielding has been constructed by Muir and LaBauve. ${ }^{33}$ The covariance data in this library were processed into a 30 -group format by using the NJOY code. ${ }^{64,65}$ In this particular library, called COVFILS, the multigroup cross sections and the relative covariance matrices for ${ }^{1} H,{ }^{10} \mathrm{~B}, \mathrm{C},{ }^{16} 0$, $\mathrm{Cr}, \mathrm{Fe}, \mathrm{Ni}, \mathrm{Cu}$, and Pb are included. Another covariance library was set up by Drischler and Weisbin. ${ }^{32}$

### 3.3 Application of Cross-Section Sensitivity Profiles and Cross Section Covariance Matrices to Predict Uncertainties

Using first-order perturbation theory, the change in the integral response $I$, $\delta I$, as a consequence of small changes in $\Sigma_{i}$ can be approximated by

$$
\begin{equation*}
\delta I \cong \sum_{i} \frac{\partial I}{\partial \Sigma_{i}} \delta \Sigma_{i} \tag{104}
\end{equation*}
$$

We further have

$$
\begin{equation*}
\operatorname{Var}(I)=E\left\{\delta I^{2}\right\}=E\left\{\sum_{i, j} \frac{\partial I}{\partial \Sigma_{i}} \frac{\partial I}{\partial \Sigma_{j}} \delta \Sigma_{i} \delta \Sigma_{j}\right\} \tag{105}
\end{equation*}
$$

or

$$
\begin{equation*}
\operatorname{Var}(I)=\Sigma_{i, j} \frac{\partial I}{\partial \Sigma_{i}} \frac{\partial I}{\partial \Sigma_{j}} \operatorname{Cov}\left(\Sigma_{i}, \Sigma_{j}\right) \tag{106}
\end{equation*}
$$

From Eqs. (100) and (106) it now becomes obvious that

$$
\begin{equation*}
\left[\frac{\Delta I}{\bar{I}}\right]_{x s}^{2}=\sum_{i, j} \underbrace{P_{\Sigma_{i}} P_{\Sigma_{j}}}_{I} \frac{\operatorname{Cov}\left(\Sigma_{i}, \Sigma_{j}\right)}{\underbrace{\Sigma_{i}{ }_{j}{ }_{j}}_{I I}} \tag{107}
\end{equation*}
$$

where $P_{\Sigma_{i}}$ and $P_{\Sigma_{j}}$ are sensitivity profiles, and the subscript xs refers to reactor cross sections.

The concept of covariance data and sensitivity profiles leads to a simple way to evaluate the error in I. The first part in the summation requires sensitivity profiles and is highly problem dependent. The second part requires cross-section uncertainty information and is problem independent.

When trying to apply the theory presented here, very often covariance data will be missing for certain materials. One way of going around this problem would be to substitute the covariance file of the missing material by a covariance file for another material for which the cross sections are less well known. ${ }^{45}$ Other methods to eliminate this problem would be to make very conservative estimates. 16,17

The most conservative method would be to assume that the error in the cross section is the same for all groups and equal to the largest error for any one group. In that case it can be shown that ${ }^{16,17}$

$$
\begin{equation*}
\left[\frac{\Delta I}{I}\right]_{\max } \leq \frac{\Delta \Sigma_{i}}{\Sigma_{i}} \sum_{i}\left|P_{\Sigma_{i}}\right| \tag{108}
\end{equation*}
$$

### 3.4 Secondary Energy Distribution Uncertainty Analysis

For evaluating uncertainties in the integral response due to uncertainties in the secondary energy distribution we will follow Gerstl's approach 44,46 and introduce the spectral shape uncertainty parameter for the hot-cold concept.

When the total number of secondaries scattered from group $g^{\prime}$ are held constant, then necessarily

$$
\begin{equation*}
\frac{\delta \Sigma_{\text {HOT }}}{\Sigma_{\text {HOT }}}=-\frac{\delta \Sigma_{\text {COLD }}}{\Sigma_{\text {COLD }}} \equiv f_{g^{\prime}} . \tag{109}
\end{equation*}
$$

Therefore $f_{g}{ }^{\prime}$ quantifies the uncertainty in the shape of the SEDs and is called the spectral shape uncertainty parameter (Fig. 4) ${ }^{44}$.

It now becomes possible to express the relative change in integral response due to the uncertainty in the secondary energy distribution in a form similar to Eq. (107):

$$
\begin{equation*}
\left[\frac{\delta I}{I}\right]_{\text {SED }}=\sum_{g^{\prime}, g} P_{S E D}^{g^{\prime} g} \frac{\delta \Sigma g^{\prime} \rightarrow g}{\Sigma} g^{\prime} \rightarrow g . \tag{110}
\end{equation*}
$$



Figure 4. Interpretation of the integral SED uncertainty as spectrum shape perturbations and definition of the spectral shape uncertainty parameter "f" (ref. 44)

Substituting Eqs. (87) and (109) in Eq. (110), it follows that

$$
\begin{equation*}
\left[\frac{\delta I}{I}\right]_{S E D}=\sum_{g^{\prime}} \quad s_{S E D}^{g^{\prime}} f_{g^{\prime}} . \tag{111}
\end{equation*}
$$

Denote $f_{g^{\prime}}$ by $f_{j}$, where the index $j$ refers to a particular nuclear reaction, e.g., $(n, 2 n)$, at specific incident energy $g^{\prime}$, and let $f_{i}$ represent some different reaction/primary energy combination. Then the uncertainty in integral response corresponding to correlated uncertainties of all SEDs for a specific isotope is

$$
\begin{equation*}
\left[\frac{\Delta I}{I}\right]_{S E D}^{2} \equiv \frac{\operatorname{Var}(I)}{I^{2}}=E\left\{\frac{(\delta I)^{2}}{I^{2}}\right\}=E\left\{\sum_{i, j} S_{S E D}^{i} S_{S E D}^{j} f_{i} f_{j}\right\} \tag{112}
\end{equation*}
$$

or

$$
\begin{equation*}
\left[\frac{\Delta I}{I}\right]_{S E D}^{2}=\sum_{i, j} S_{S E D}^{i} S_{S E D}^{j} \operatorname{Cov}\left(f_{i}, f_{j}\right) \tag{113}
\end{equation*}
$$

If the spectral shape uncertainty parameters for a specific particle interaction, identified by the subscript $\ell$, are assumed to be fully correlated, it can be shown that ${ }^{67}$

$$
\begin{equation*}
\operatorname{Cov}\left(f_{i}, f_{j}\right) \operatorname{cor}(+1)=\left[\operatorname{Cov}\left(f_{i}, f_{i}\right)\right]^{\frac{1}{2}} \cdot\left[\operatorname{Cov}\left(f_{j}, f_{j}\right)\right]^{\frac{3}{2}}, \tag{114}
\end{equation*}
$$

so that

$$
\begin{equation*}
\left[\frac{\Delta I}{\mathrm{I}}\right]_{\ell}=\left|\sum_{g^{\prime}} \mathrm{s}_{\mathrm{SED}}^{\ell, \mathrm{g}^{\prime}}\left[\operatorname{Cov}\left(\mathrm{f}_{\ell g^{\prime}}, \mathrm{f}_{\ell g^{\prime}}\right)\right]^{\frac{1}{2}}\right| \tag{115}
\end{equation*}
$$

or,

$$
\begin{equation*}
\left[\frac{\Delta I}{\mathrm{I}}\right]_{\ell}=\sum_{g^{\prime}}\left|S_{S E D}^{\ell, g^{\prime}}\right|\left[\operatorname{Var}\left(\mathrm{f}_{\ell g^{\prime}}\right)\right]^{\frac{1}{2}} . \tag{116}
\end{equation*}
$$

If $N$ independent measurements of the same SED are available, the values for $\operatorname{Var}\left(f_{\ell g},\right)$ can easily be evaluated. For each cross-section evaluation, weights, $w_{n}$, are assigned, then

$$
\begin{equation*}
f_{g^{\prime}}^{\mathrm{n}}=\frac{\sigma_{\text {HOT }}^{\mathrm{n}}-\sigma_{\text {COLD }}^{\mathrm{n}}}{\mathrm{E}\{\sigma\}}, \quad \text { for } \mathrm{n}=1,2 \ldots \mathrm{~N} \tag{117}
\end{equation*}
$$

with

$$
\begin{equation*}
E\left\{f_{g^{\prime}}^{n}\right\}=\sum_{n=1}^{N} w_{n^{\prime}} f_{g^{\prime}}^{n}=0 \tag{118}
\end{equation*}
$$

The variance of $f_{g}$ will be

$$
\begin{equation*}
\operatorname{Var}\left(f_{g^{\prime}}\right)=E\left\{f_{g^{\prime}}^{2}\right\}=\sum_{n=1}^{N} w_{n} \frac{\left(\sigma_{\text {HOT }}^{2}-\sigma_{\text {COLD }}^{2}\right)}{[E\{\sigma\}]^{2}} \tag{119}
\end{equation*}
$$

$\operatorname{Var}\left(\mathrm{f}_{\mathrm{g}^{\prime}}\right)$ is called the fractional uncertainty for the secondary energy distribution and is identified by the symbol $F$. A short program which evaluates the values of $F$ has been written by Muir; ${ }^{66}$ the results for the 30 -group neutron structure ${ }^{45}$ is shown in Table II.

TABLE II
MEDIAN ENERGIES (E ${ }_{m}^{\prime}$, IN MEV) AND FRACTIONAL UNCERTAINTIES (F) FOR SECONDARY energy distributions at incident neutron energies e ${ }_{0}$
(Ref. 45)


### 3.5 Overall Response Uncertainty

The overall response uncertainty will be of the form

$$
\begin{equation*}
\left[\frac{\Delta I}{\mathrm{I}}\right]=\sqrt{\left[\frac{\Delta I}{\mathrm{I}}\right]_{\mathrm{SED}}^{2}+\left[\frac{\Delta I}{\mathrm{I}}\right]_{\mathrm{XS}}^{2}} \tag{120}
\end{equation*}
$$

where

$$
\begin{equation*}
\left[\frac{\Delta I}{I}\right]_{S E D}^{2}=\sum_{i}\left[\frac{\Delta I}{I}\right]_{S E D, i}^{2} \tag{121}
\end{equation*}
$$

and

$$
\begin{equation*}
\left[\frac{\Delta I}{I}\right]_{X S}^{2}=\sum_{i, k}\left[\frac{\Delta I}{I}\right]_{X S, i, k}^{2} \tag{122}
\end{equation*}
$$

The index $i$ reflects the unzertainties in the various materials. It was assumed that the effects from SED uncertainties for all possible reactions which generate secondaries are uncorrelated. It is also assumed that the uncertainties due to the SEDs are uncorrelated with other uncertainties due to reaction cross sections (XS), and that the uncertainties between the reaction cross sections themselves are uncorrelated.

1. To be absolutely correct, a term reflecting the uncertainty in the secondary angular distribution should be included. Due to the difficulty in generating uncertainty data from ENDF/B-V in the proper format, we do not include that term.
2. In order to evaluate the sensitivity profiles, we should keep in mind that the form of the sensitivity profile will depend on the particular reaction cross section for which a response is desired (Table I).
3. SENSIT-2D: A TWO-DIMENSIONAL CROSS-SECTION AND DESIGN SENSITIVITY AND UNCERTAINTY ANALYSIS CODE

### 4.1 Introduction

The theory explained in the previous chapters has been incorporated in a two-dimensional cross-section and design sensitivity and uncertannty analysis code, SENSIT-2D. This code is written for a CDC-7600 machine and is accessible via the NMFECC-network (National Magnetic Fusion Energy Computer Center) at Livermore. SENSIT-2D has the capability to perform a standard cross-section and a vector cross-section sensitivity and uncertainty analysis, a seconda־y energy distribution sensitivity and uncertainty analysis, a design sensitivity analysis and an integral response (e.g., dose rate) sensitivity and uncertainty analysis. As a special feature in the SENSIT-2D code, the loss term of the sensitivity profile can be evaluated based on angular fluxes and/or flux moments.

SENSIT-2D is developed with the purpose of interacting with the TRIDENT-CTR ${ }^{6}$ code, a two-dimensional discrete-ordinates code with triangular meshes and an r-z geometry capability, tailored to the needs of the fusion community. Angular fluxes generated by other 2-D codes, such as DOT, TWODANT, TRIDENT, etc., cannot be accepted by SENSIT-2D due to the different format. The unique features of TRIDENT-CTR (group dependent quadrature sets, r-z geometry description, triangular meshes) are reflected in SENSIT-2D. Coupled neutrun/gamma-ray studies can be performed. In contrast with TRIDENT-CTR however, SENSIT-2D is restricted to the use of equal weight ( $E Q_{n}$ ) quadrature sets, ${ }^{68}$ symmetrical with respect to the four quadrants. Upscattering is not allowed.

Many subroutines used in SENSIT-2D are taken from SENSIT ${ }^{46}$ or TRIDENT-CTR. SENSIT-2D is similar in its structure to SENSIT, but is an entirely different code. Unlike SENSIT, SENSIT-2D does not use the BPOINTR ${ }^{69}$ package for dynamical data storage allocation, but rather uses a sophisticated pointer scheme in order to allow variably dimensioned arrays. As soon as an array is not used any more, its memory space becomes immediately available for other data. SENSIT-2D does not include a source sensitivity analysis capability and cannot calculate integral responses based on the adjoint formulation. This has the disadvantage that no check for internal consistency can be made. Therefore, other ways have to be found in order to determine whether the fluxes are fully converged. One way for doing so would be to calculate the integral response based on the adjoint formulation while performing
the adjoint TRIDENT-CTR or the adjoint TRDSEN run, and compare with the integral response based on the forward calculation.

SENSIT-2D requires input files which contain the angular fluxes at the triangle midpoints multipled by the corresponding volumes, and the adjoint angular fluxes at the triangle midpoints. A modified version of TRIDENT-CTR, TRDSEN, was written by T. J. Seed ${ }^{70}$ to generate these flux files. A summary of these modifications was provided by T. J. Seed and is included as Appendix B. After a TRIDENT-CTR run, the TRDSEN code will use the dump files generated by TRIDENT-CTR, go through an extra iteration, and write out the angular fluxes in a form compatible with SENSIT-2D. Both SENSIT-2D and TRDSEN use little computing time compared with the time required by TRIDENT-CTR.

The features of SENSIT-2D are summarized in Table III. The SENSIT2D source code is generously provided with comment cards and is included as Appendix A.

### 4.2 Computational Outline of a Sensitivity Study

A flow chart (Fig. 5) illustrates the outline for a two-dimensional sensitivity and uncertainty analysis. From this figure it becomes immediately apparent that a sensitivity analysis requires elaborate data management. The data flow can be divided into three major parts: a cross-section preparation module, in which the cross sections required by TRIDENT-CTR and SENSIT-2D are prepared, a TRIDENT-CTR/TRDSEN block,

TABLE III: SUMMARY OF THE FEATURES OF SENSIT-2D (PART I)

## SENSIT-2D: A Two-Dimensional Cross-Section and Design Sensitivity and Uncertainty Analysis Code

## Code Information:

* written for the CDC-7600
* typical storage, 20K (SCM), 80K (LCM)
$\therefore \quad$ number of program lines, 3400
* used with the TRIDENT-CTR transport code
* typical ruir times, $10-100 \mathrm{sec}$


## Capabilities:

* computes sensitivity and uncertainty of a calculated integral response (e.g., dose rate) due to input cross sections and their uncertainties
$\star$ cross-section sensitivity
* vector cross-section sensitivity and uncertainty analysis
* design sensitivity analysis
$\star \quad$ secondary energy distribution (SED) sensitivity and uncertainty analysis

TABLE III: SUMMARY OF THE FEATURES OF SENSIT-2D (PART 2)

## SENSIT-2D

## TRIDENT-CTR Features Carried Over into SENSIT-2D:

```
* x-yor r-z geometry
* group-dependent }\mp@subsup{S}{n}{}\mathrm{ order
* triangular spatial mesh
```


## Unique Features:

* developed primarily for fusion problems
* group dependent quadrature order and triangular mesh
* can evaluate loss-term of sensitivity profile based on angular fluxes and/or flux moments


## Current Limitations:

* can only interact with TRIDENT-CTR transport code
* not yet implemented on other than CDC computers
* based on first-order perturbation theory
* upscattering not allowed


Figure 5. Computational outline for a two-dimensional sensitivity analysis with SENSIT-2D
where the angular fluxes in a form compatible with SENSIT-2D are generated, and a SENSIT-2D module, which performs the calculations and manipulations necessary for a sensitivity and uncertainty analysis.

### 4.2.1 Cross-section preparation module

There are many possible ways to generate the multigroup crosssection tables required by SENSIT-2D and TRIDENT-CTR. The flow chart of Fig. 5 illustrates just one of these possibilities. All the codes mentioned here are accessible via the MFE machine. Basically, three codes are required: NJOY, TRANSX, and MIXIT. Starting from the ENDF/B-V data file, the NJOY code system ${ }^{64}$ generates a multigroup cross-section library (MATXS5) and a vector cross-section and covariance library (TAPE10). A covariance library can be constructed by using the ERROR module in the NJOY code system. ${ }^{33}$

From the multigroup cross-section library (MATXS5), the desired isotopes can be extracted by the TRANSX code ${ }^{72}$ and will be written on a file with the name XSLIBF5. The MIXIT code ${ }^{73}$ can make up new materials by mixing isotopes from the XSLIBF5 library. The cross sections used in SENSIT-2D have to be written on a file called TAPE4. The cross sections used in TRIDENT-CTR and TRDSEN will be on file GEODXS. SENSIT2D and TRIDENT-CTR include the option to feed in cross sections directly from cards.

### 4.2.2 The TRIDENT-CTR and TRDSENS block

SENSIT-2D requires regular angular fluxes at the triangle centerpoints, multipled by the corresponding volumes, and adjoint angular fluxes at the triangle centerpoints. TRIDENT-CTR does not write out angular fluxes. Therefore the TRDSEN version of TRIDENT-CTR was written by SEFD. TRDSEN makes use of the flux moment dump files, generated by TRIDENT-CTR. These dump files will be the starting flux guesses for TRDSEN. TRDSEN will perform one more iteration and write out the angular fluxes. In this discussion we will represent the dump file families by DUMP1 for the regular flux moments, and DUMP2 for the angular flux moments. Except for a different starting guess option, TRDSEN requires the same input as TRIDENT-CTR.

### 4.2.3 The SENSIT-2D module

The SENSIT-2D code performs a sensitivity and uncertainty analysis. When vector cross sections and their covariances are required, they have to be present on a file with the name TAPE10. If the cross section data are read from tape, they have to be written on a file called TAPE4. The regular angular fluxes at the triangle centerpoints multiplied by the corresponding volumes (TAPE11, TAPE12,...) and the adjoint angular fluxes at the triangle centerpoints (TAPE15, TAPE16,...) can be quite voluminous. Writing out large files can be troublesome on the MFE
machine when there is a temporary lack of continuous disk space. Therefore TRIDENT-CTR and SENSIT-2D have the built-in option to specify the maximum number of words to be written on one file. This limit has to be set high enough to ensure that all the flux data related to une group can be written on one file. 1000000 words per file is usually a practical size and is the default in TRIDENT-CTR.

SENSIT-2D can generate four more file families:

1. TAPEl, which contains the regular scalar fluxes at the triangle centerpoints.
2. TAPE20, TAPE21,..., which are random access files and contain the adjoint angular fluxes at the triangle centerpoints,
3. TAPE 25, TAPE26,..., containing the regular flux moments at the triangle centerpoints, multipled by the corresponding volumes, 4. TAPE30, TAPE31,..., which contain the adjoint angular fluxes at the triangle midpoints.

SENSIT-2D has the option of not generating those file families, but using those created by a former run. The flux moments are constructed from the angular fluxes according to the formula

$$
\phi_{\ell}^{k}\binom{x, y}{r, z}=\sum_{m=1}^{M N} w_{m} R_{\ell}^{k}\left(\mu_{m}, \phi_{m}\right) \phi_{m}\binom{x, y}{r, z},
$$

where the $w_{m}^{\prime \prime s}$ are the quadrature weights, the $R_{l}^{k_{1}} s$ the sphericai harmonics functions, and $\mathbb{M N}$ the total number of angular fluxes.

### 4.3 The SENSIT-2D Code

In this section the structure of the SENSIT-2D code, its options and capabilities will be explained in more detail. SENSIT-2D is a powerful sensitivity and uncertainty analysis code. The description of this code from the user's point of view is given in the user's manual. ${ }^{71}$

### 4.3.1 Flow charts

The overall data flow within the SENSIT-2D module is repeated in Fig. 6. A simplified flow chart is illustrated in Fig. 7. The main parts of the flow chart include these steps:

* The control parameters and the geometry related information are read in.
* The quadrature sets and the spherical harmonics functions required to generate the flux moments are constructed.
$\therefore$ The adjoint angular fluxes at the triangle centerpoints are written on random access files, flux moments are generated and scalar fluxes are extracted.
* A detector sensitivity analysis is performed; if desired an uncertainty analysis is done.
$\pm \quad$ The $X^{\prime} s$ and $\psi^{\prime}$ s which form the essential parts of the cross-section and secondary energy distribution sensitivity profiles are calculated for each perturbed zone and for the sum over all perturbed zones.


Figure 6. Data flow for the SENSIT-2D module


Figure 7. Flow chart for SENSIT-2D (part 1)


Figure 7. Flow chart for SENSIT-2D (part 2)


Figure 7. Flow chart for SENSIT-2D (part 3)

Up to this point, all the subroutines used are different from those used in the SENSIT code. The remaining calculations are done with SENSIT subroutines.

* Cross sections are read in.
$\star \quad$ Vector cross sections are extracted.
$\therefore \quad$ Sensitivity profiles are calculated used in the appropriate $\psi^{\prime} s$ and $X^{\prime} \mathrm{s}$.
* If desired to do so, an uncertainty analysis is performed.
$\%$ A vector cross-section sensitivity and uncertainty analysis can be performed and partial sums of individual response variances can be made.


### 4.3.2 Subroutines used in SENSIT-2D

Table IV summarizes the subroutines used in SENSIT-2D and indicates their origin in case they were taken over or adapted from another code. The essential difference between SENSIT and SENSIT-2D is the way that the geometry is described and how the $\psi^{\prime} s$ and the $X^{\prime} s$ are calculated. Basically, all the subroutines are called from the main program with a few exemptions when subroutines are called from other subroutines. The subroutines for SENSIT-2D which were not taken over from other codes will now be described. For the SENSIT subroutines we refer to the user's manual. 46

TABLE IV: LIST OF SUBROUTINES USED IN SENSIT-2D

| Name Subroutine | Origin | If Taken From Another <br> Code, Were Changes Made? |
| :--- | :--- | :---: |
| EBND |  |  |
| GEOM | SENSIT-2D | - |
| SNCON | SENSIT-2D | - |
| TAPAS | TRIDENT-CTR | yes |
| PNGEN | SENSIT-2D | - |
| FLUXMOM | TRIDENT-CTR | yes |
| DETSEN | SENSIT-2D | - |
| CHIS | SENSIT-2D | - |
| POINT4B | SENSIT-2D | - |
| PSIS | SENSIT-2D | - |
| POINT8 | SENSIT-2D | - |
| SUB5 | SENSIT-2D | - |
| SUB6 | SENSIT | yes |
| TEXT | SENSIT | no |
| TESTA | SENSIT | no |
| SUB8 | SENSIT | no |
| SUB11 | SENSIT | yes |
| SUB8V | SENSIT | SENSIT |
| SUB9 | SENSIT | yes |
| SUB9V | SENSIT | no |
| SUB5V | SENSIT | no |
| COVARD | SENSIT | no |
| SETID | SENSIT | no |

1. Subroutine EDNB. Neutron and gamma-ray energy group structures are read in from cards and the lethargy widths for each group are calculated.
2. Subroutine GEOM. Geometry related information is read in and edited.
3. Subroutine SNCON. This routine was taken and adapted from the TRIDENT-CTR code. The $E Q_{n}$ cosines and weights are calcualted. The quadrature information is edited whenever IOPT is 1 or 3.
4. Subroutine TAPAS. Files are assigned to the various flux data. The filenames for the angular fluxes are read from the input file. Those filenames will have to be of the form TAPEXY, where XY will be the input information. Filenames in the same format will then be assigned to the adjoint angular fluxes (on sequential files in this case), and the flux moments. The maximum number of words to be written on each file is controlled by the input parameter MAXWRD. Groups will never be broken up between different files. 5. Subroutine PNGEN. This subroutine originates from the TRIDENTCTR code. Spherical harmonics functions, used for constructing flux moments, are calculated. For the adjoint flux moment calculation the arrays related to the spherical harmonics will be rearranged to take into account the fact that the numbering of the angular directions was not symmetric with respect to the four quadrants in TRIDENT-CTR.
5. Subroutine FLUXMOM. The adjoint angular fluxes will be rewritten on a random access file. The direct and adjoint flux
moments are constructed and written on sequential files. In the case that the input parameter IPREPl, it is assumed that those manipulations are already performed in an earlier SENSTT-2D run. In this case one has to make sure that the parameter MAXWRD was not changed. While creating the regular flux moments, the scalar fluxes will be extracted and written on a file named TAPE1.
6. Subroutine DETSEN. From the scalar fluxes, the integral response for each detector zone is read from input cards. The detector sensitivity profile is calculated and edited. In the case that the input parameter DETCOV equals one, a covariance matrix has to be provided, subroutine $S U B 9$ will be called and a detector response uncertainty analysis is performed.
7. Subroutine CHIS. The $X^{\prime}$ s are calculated for each perturbed zone and for the sum over all perturbed zones based on angular fluxes. In the case that the parameter ICHIMOM equals one, this subroutine will be skipped and the $X$ 's will be calculated based on flux moments via the $\psi^{\prime} s$.
8. Subroutine POINT4B. This subroutine sets LCM pointers for the flux moments which will be used in SUB4B.
9. Subroutine PSIS. The $\psi^{1} s$ are calculated for each of the perturbed zones and for the sum over all perturbed zones based on flux moments. In the case that ICHIMOM is not equal to zero also the $X$ 's will be calculated from flux moments. In the case that parameter IPREP equals one, the $\psi^{\prime} s$ will be read in from file TAPE3.
10. Subroutine POINT8. This subroutine sets pointers for the appropriate $\chi$ 's and $\psi^{\prime} s$, used in subroutine SUB8.
11. COMPARISON OF A TWO-DIMENSIONAL SENSITIVITY ANALYSIS WITH A ONE DIMENSIONAL SENSITIVITY ANALYSIS

Before applying SENSIT-2D to the FED (fusion engineering device) inboard shield design, currently in development at the General Atomic Company, it was necessary to make sure that SENSIT-2D will provide the correct answers. One way for checking on the performance of SENSIT-2D is to analyze a two-dimensional sample problem, which is one-dimensional from the neutronics point of view, and then to compare the results with a one-dimensional analysis. In this case ONEDANT ${ }^{74}$ and SENSIT $^{46}$ are used for the one-dimensional study, while TRIDENT-CTR, TRDSEN, and SENSIT-2D are used for the two-dimensional analysis.

Two sample problems will be studied. The first sample problem uses real cross-section data, while the second sample problem utilizes artificial cross sections. Computing times, the influence of the quadrature set order, and the performance of the angular fluxes versus the flux moments option for the calculation of the chi's will be discussed.
5.1 Sample Problem \#1

The first sample problem is a mock-up of a cylindrical geometry (Fig. 8). There are four zones present: a source zone (vacuum), a perturbed zone (iron), a zone made up of $40 \%$ iron and $40 \%$ water, and a detector zone (copper). The reaction rate of interest is the heat generated in the copper region. The source was assumed isotropic and had a neutron density of one neutron per cubic centimeter ( 1 neutron $/ \mathrm{cm}^{3}$ ). The source neutrons are emitted at 14.1 MeV (group 2). The left boundary is reflecting, and on the right there is a vacuum boundary condition. Thirty neutron groups were used with a third order of anisotropic scattering. The cross sections were generated using the TRANSX ${ }^{72}$ code. The energy group boundaries are reproduced in Table $V$.

In the two-dimensional model (TRIDENT-CTR) two bands--each $0.5-\mathrm{cm}$ wide--are present. In order to be consistent with the one-dimensional analysis the upper and the lower boundaries were made reflective (Fig. 9). Each band is divided into 35 triangles (5 triangles for the source zone, 10 triangles for each of the other three zones). The automatic mesh generator in TRIDENT-CTR was used. The convergence precision was set to $10^{-3}$. A convergence precision of $10^{-3}$ means here that the average scalar flux for any triangle changes by less than $0.1 \%$ between two consecutive iterations. A similar criterion is used in ONEDANT. The calculation is performed with the built-in $E Q_{n}-8$ (equal weight) quadrature set. The mixture densities are given in Table VI. For the adjoint calculation the source is in zone IV and consists of the copper


Figure 8. Cylindrical geometry representation for sample



REFLECTING BOUNDARY

30 neutron groups
neutron source: 1 neutron $/ \mathrm{cm}^{3}$ in group2 (14.1 MeV)
$P-3, E Q_{n}-8 \quad$ : third-order of anisotropic scattering
8 th-order equal weight quadrature set
response function:copper kerma factor in zone \#IV
convergence precision : $10^{-3}$

Figure 9. Two-dimensional (TRIDENT-CTR) representation for sample problem \#1

TABLE VI. ATOM DENSITIES OF MATERIALS

|  |  | Atoms $/ \mathrm{m}^{3}$ |
| :--- | :--- | :---: |
| ZONE \#1 | Vacuum | -- |
| ZONE \#II | Fe | $8.490+28^{\mathrm{a}}$ |
| ZONE \#III |  |  |
|  | Fe | $3.396+28$ |
|  | H | $4.020+28$ |
|  | 0 | $1.900+28$ |
| ZONE \#IV | Cu | $8.490+28$ |

a $8.490+28=8.49 \times 10^{28}$
b 40 vol \% Fe and 40 vol \% water.
kerma factors. The response is calculated in that case in zone I. It was found that the adjoint calculation required more iterations and time in order to reach convergence. Originally the forward calculation was done using 20 triangles per band. The adjoint problem, however, did not converge. In the evaluation process of the kerma factors, the kermas for some groups are made negative in order to satisfy energy balance. Making those negative sources zero in the TRIDENT-CTR run did not lead to any improvement. Subsequently, 35 triangles per band were used. When the negative sources were set to zero convergence was reached. Ignoring the negative kerma factors leads to a $20 \%$ increase in the total heating. The forward calculation required about 11 minutes cpu time (central processor unit time on a CDC-7600), while the adjoint calculation required about 13.5 minutes. Generating the angular fluxes using the TRDSEN code required about 20 seconds of cpu time for each case.

TRDSEN does on extra iteration in order to generate the angular fluxes. The convergence criterion in TRIDENT-CTR is based on the scalar fluxes, and therefore one extra iteration in TRDSEN should be adequate. However, restarting TRIDENT-CTR with the flux moments as starting guesses, revealed that for some groups two extra iterations were necessary to reach a convergence precision of $10^{-3}$. No explanation for this could be found.

The one-dimensional model (ONEDANT) contains 35 intervals (5 for the source zone, 10 intervals for each of the remaining zones). The one-dimensional description for the forward problem is summarized in
REFLECTING BOUNDARY



30 neutron groups
neutron source: 1 neutron $/ \mathrm{cm}^{3}$ in group 2 (14.1Mevo
P-3, S-8 : third-order of anisotropic scattering
8th-order Gaussian quadrature set
detector response: copper kerma factor in zone $\# \mathrm{IV}$
convergence precision: $10^{-4}$

Figure 10. One-dimensional (ONEDANT) apresentation for sample problem \#l

Fig. 10. Again it was found that the use of negative sources in the adjoint calculation caused difficulties with respect to the convergence. In that case, groups 18 and 19 triggered the message "TRANSPORT FLUXES BAD"; groups 4, 5, 6, 7, and 19 did not converge (max. number of inner iterations $300 /$ group). However, the overall heating in the copper region was within $0.1 \%$ of the heating calculated by the forward run. A coupled neturon/gamma-ray calculation ( 30 neutrons groups and 12 gammaray groups) in the adjoint mode led to some improvement. In that case, only group 2 did not converge. The required convergence precision in the ONEDANT runs was set to $10^{-4}$. The built-in S-8 Gaussian quadrature sets were used. In order to be consistent with the TRIDENT-CTR calculations, the negative sources in the adjoint case were set to zero, even though this did not seem to be necessary. Each run required about six seconds of cpu time.

A standard cross-section sensitivity analysis (the cross sections in zone II are perturbed) was performed using the SENSIT code and the SENSIT-2D code. A comparison between the SENSIT and the SENSIT-2D results revealed that SENSIT $^{76}$ does not rearrange the angular fluxes correctly (in cylindrical geometry). To correct this error, a shuffling routine which takes case of this deficiency was then built into SENSIT. The SENSIT results are in good agreement with those obtained from SENSIT-2D. The flux moments versus the angular flux option was tested out for the calculation of the loss term. Again there is good agreement. Finally, an uncertainty analysis was performed for the heating in the copper zone. The SENSIT-2D analysis matches the SENSIT analysis.

### 5.1.1 TRIDENT-CTR and ONEDANT results

A comparison of the heating in the copper region (zone IV) between TRIDENT-CTR (and SENSIT-2D) and ONEDANT (and SENSIT) is summarized in Table VII a. The adjoint calculations yield a $20 \%$ higher heating rate due to the fact that the negative kerma factors were set equal to zero. The one-dimensional and the two-dimensional analysis are in agreement. The computing times for those various runs are given in Table VII b.

Each ONEDANT run requires about 8 seconds of total computing time (LTSS time), whereas it takes about 12 minutes to do the TRIDENT-CTR runs. The TRIDENT-CTR runs were done with a convergence precision of $10^{-3}$, whereas for the ONEDANT runs a convergence precision of $10^{-4}$ was specified. In order to obtain the same convergence precision in TRIDENT-CTR about eight additional minutes of cpu time are required. It was found that a forward coupled neutron/gamma-ray calculation (30 neutron groups and 12 gamma-ray groups) required only 8 minutes of computing time with TRIDENT-CTR (convergence precision $10^{-3}$ ). An explanation for this paradoxial behavior is related to the fact that $\sigma_{s} / \sigma_{T}$ has a different (smaller) value in a coupled neutron/gamma-ray calculation. The flux moments generated by TRIDENT-CTR and ONEDANT were compared. In the ONEDANT geometry the angular fluxes are assumed to be symmetrical with respect to the $z$-axis, ${ }^{75}$ so that the odd flux moments ( $\phi_{1}^{0}, \phi_{2}^{1}, \phi_{3}^{0}$, and $\phi_{3}^{2}$ ) vanish. Since TRIDENT-CTR performs a real twodimensional calculation the odd moments will not be zero in that case. In our sample problems there is still symmetry with respect to the

TABLE VIIa. COMPARISON OF THE HEATING IN THE COPPER REGION CALCULATED BY ONEDANT AND TRIDENT-CTR

|  | FORWARD | ADJOINT |
| :--- | :--- | :---: |
| ONEDANT $^{\text {a }}$ | $2.37382+7$ | $2.40541+7$ |
| ONEDANT | $2.01189+7$ | $2.01882+7$ |
| TRIDENT-CTR | $2.01175+7$ | $2.39263+7$ |
| SENSIT | $2.01011+7$ | $2.40541+7$ |
| SENSIT-2D | $2.01098+7$ |  |

a negative KERMA factors set to zero

TABLE VIIb. COMPUTING TIMES ON A CDC-7600 MACHINE

|  | CPU-TIME $^{\text {a }}$ | I/0 TIME | LTSS TIME $^{\mathrm{c}}$ |
| :--- | :--- | :--- | :---: |
| ONEDANT FORWARD | 5.80 sec. | 1.87 sec. | 7.65 sec. |
| ONEDANT ADJOINT | 6.09 sec. | 1.82 sec. | 7.97 sec. |
| TRIDENT-CTR FORWARD |  |  | 13.5 minutes |
| TRIDENT-CTR ADJOINT |  |  | 11.1 minutes |
| SENSIT | 4.92 sec. | 0.55 sec. | 6.08 sec. |
| SENSIT-2D | 8.50 sec. | 9.02 sec. | 17.84 sec. |

[^0]z-axis. For that reason, the odd moments in TRIDENT-CTR will have opposite signs in band one and band two. For some zones and some groups this was not completely the case. There was about $30 \%$ difference in the absolute values of some flux moments in band one and band two, which indicates that the problem was not in a sense truly converged. The convergence criteria in ONEDANT and TRIDENT-CTR test only for the scalar fluxes between two consecutive iterations. Even when the convergence criteria are satisfied in both codes, a true convergence of the angular flux is not guaranteed. The even moments in band one are exactly the same as those for band two. Because the contribution of the odd moments is small compared to the contribution of the even moments (about one thousandth), the problem can be considered fully converged.

The scalar flux moments calculated by TRIDENT-CTR and ONEDANT are in very good agreement. The higher-order moments are different. Since TRIDENT-CTR and ONEDANT do not use the same coordinate system, they do not calculate the same physical quantity for the higher-order flux moments. As long as TRIDENT-CTR is consistent with SENSIT-2D, and ONEDANT consistent with SENSIT, the results from the one-dimensional sensitivity analysis should match those obtained from a two-dimensional sensitivity analysis.

### 5.1.2 SENSIT and SENSIT-2D results for a standard cross-section sensitivity analysis

A standard cross-section sensitivity analysis was performed using

SENSIT and SENSIT-2D. The sensitivity of the heating in zone IV to the cross sections in zone II was studied. SENSIT-2D requires about three times more computing time than SENSIT in this case (Table VII b). The main part of the calculation involves the evaluation of the $\psi^{\prime}$ s (gain term). A complete sensitivity and uncertainty analysis may involve several SENSIT (or SENSIT-2D) runs. Thus an option which allows one to save the $\psi^{\prime}$ s has been built into SENSIT-2D. It is obvious that the computing time required in SENSIT-2D is negligible compared to the computing time required for the forward and adjoint TRIDENT-CTR calculations.

The partial and the net sensitivity profiles calculated by SENSIT and SENSIT-2D are reproduced in TABLES VIII a and VIII b. It can be concluded that the SENSIT-2D results are in good agreement with those obtained by SENSIT. Note that the absorption cross section is negative for groups 2 and 3. A negative absorption cross section does not necessarily indicate that errors were made during the cross section processing. There are various ways to define an absorption cross section, and a controversy about a commonly agreed on definition is currently in progress. What is called an absorption cross section in a transport code is not truly an absorption cross section but the difference between the transport cross section and the outscattering ( $\sigma_{a}^{g}=\sigma_{t r}^{g}-\sum_{g^{\prime}} \sigma^{g \rightarrow g^{\prime}}$ ). Note that groups 2 and 3 are the main contributors to the integral sensitivity.

It was mentioned earlier that the $X$ 's can be calculated based on flux moments or based on angular fluxes according to

# TABLE VIIIa: PARTIAL AND NET SENSITIVITY PROFILES FOR THE ONE-DIMENSIONAL ANALYSIS <br> (Part 1) 

## dCFinitions df sengit simsitivity profile notenclature

| Qaxs | - SEIIGITIVITY PROFILE PCR DELTR-U FOR TIE ADSORPIIUN CROSS-SECTION ©TFIKEN FROII POSITIOK 1H: 1 N INPUT CKUSS-SECTIUN TRULES). PURE LUSS TERM |
| :---: | :---: |
| HU-F 155 |  LHICH IS USUALLY IUG-TIRES THE FISSION CRUSS SCCTIUN. PUAE LOSS TERIT |
| 5×5 | - partial sensitivity picofale per vel tinu for tie scittelihg cruje-section scoifuted for each ENERGY GROUP AS A DIAẼOHAL SUM FROM INPUU XS-TADLES). LOSS JENUY ONLY |
| TXS | - SEnSITIVITY PRORILE PCR DELTA-U FOR TIE TOTAL CI:OSS SECTION GAS GIVEN IN POSITION IHT IN HHPUT CROSS-SEETTIOH TACLES). PURE LOSS TERM |
| H-CAIN |  <br>  |
|  | ENERGY GROUPS. COMPUTED FROM FORLMAD DIFFERENCE FORHULATION. |
| c-cain |  FUL SENSITIVITY GFIHIS DUL TU BCNTIG:IIIG UITT UF GHIIR ENERGY GISUUP G INIO RLL LOLER GAFIA ENERGY COFPUTED FRDM FOFIKARD DIFFERENLE FURMELATIOK. |
| N-CAIn(Séd) |  <br>  <br>  <br> CORSESPGHDS IU SIHGLE-DIFFLILLIIIIKL SEL REIESITIVITY PKUFILE, PSEDGG-OUT) PER UELU-DUT. INTEGRATED OVER ALL IACIDLNT LINEIGY G!:UUUS. |
| H6-CAIN |  <br>  groul g into rel geith grourij. |
| SEN | - het Sensitivity profile plk lelta-u fur tik scatterihg cruje-sjcction (sellosichicain) |
| SENT | - net seissitivity profile per viltanu for tie tutal crioss-section (SClit-tesingalis |
| SEER | - SENSITIVITY PRDFILE PER DCLTA-U FOR THE detccion: mecipolise futictioit rig) |
| SEMO | - SEnsitivity prdfille Per dilta-u for the sourle disiriuution fuictioil oig) |



TABLE VIIIb：PARTIAL AND NET SENSITIVITY PROFILES FOR THE TWO－DIMENSIONAL ANALYSIS




| GLOUP | UPPER－EXEV | BELTA－U | $\text { ANE } P U$ | RE LOS | S S TERM |  | Mo：PU $N-C A 1 N$ | E GAIN TERTS H－GOIH（SED） |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1． diacier | 1．2SE－81 | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 2 | 1．SOSE＋07 | 1．03E－as | 2．4C4E +00 | 6. | －2．215E＋01 | －1．9CCE＋01 | 3．452E +00 | 4．053E 48 | 0. |
| 3 | 1．3SOE +67 | 1．18E－81 | 1．797E－02 | 0. | －1．791E430 | －1．773E＋U0 | 7．523E－01 | 1． $51 . \overline{J E}+C 0$ | 0. |
| 4 | $1.2005+07$ | 1．82E－81 | －1．977E－82 | $B$. | －2．959E－61 | －3．152E－21 | 1．3CEE－01 | 2．01E5－01 | 0. |
| 5 | 1．0こ0E＋67 | 2．50E－81 | －1．3EDE－02 | 0. | －3．39JE－01 | －3．53＇JE－01 | 1．002［－01 | 3．129E－01 | 0. |
| 6 | 7．7919E406 | 2．48E－81 | －S．6STE－03 | 0. | －2．6G2E－81 | －2．71EE－61 | 1．618E－81 | 2．470：－81 | 0. |
| 7 | $6.376[+86$ | S．805－01 | －1．77UE－03 | 0. | －2．427¢－01 | －2．4A5EE－01 | $1.719 \mathrm{E}-01$ | 2．3035－81 | 0. |
| 8 | 3．602E＋06 | 2．56E－91 | －9．SaCL－04 | 0. | －2．8G4巨－41 | －2．974ㄷ－01 | $2.32115-61$ | 2．CSEE－81 | 0. |
| 9 | 2．CGSE＋06 | 2．STEE－01 | －5．809E－04 | 6. | －3．594E－81 | －3．6日ĖE－01 | 3．111E－01 | 3.68 EE －01 | 0. |
| 10 | 2．233E＋E6 | 2．5GE－01 | －3．073E－04 | 0. | －3．4U1E－81 | －3．4以N［－01 | $3.15^{\prime \prime}$ 次－81 | 3．CGEE－D1 | 0. |
| 11 | 1．73CE＋36 | 2．595－01 | －2．624E－04 | 0. | －3．711E－81 | －3．71＜15－01 | 3．511E－41 | 3．SGCE－C1 | 0. |
| 12 | $1.3535: 05$ | 4．97E－01 | －5．92－1E－U4 | 0. | －4．SCBE－01 | －4．59．1E゙－01 | 4．301E－01 |  | 0. |
| 13 | 8．23ıE．205 | 4．98E－51 | －1．S09E－03 | 0. | －6．5：5¢［－81 | －6．9yEE－01 | 6.33 ＇tre01 | 7．563［－01 | 0. |
| 14 | S．firne：－0S | S．01E－31 | －1．032E－03 | 0. | －7．60＾E－01 | －7．61c：ㅌ－01 | 7．52：E－01 | 0．040゙家－も1 | 0. |
| 15 | 3．LE「E＋GS | 4．92E－81 | －3．72〕E－0．4 | 0. | －1．94UE－U1 | －1．9S＂E－ti | 1．9UJE－21 | 2．015E－01 | 0. |
| 16 | 1．ECJE＋OS | 1．8EE＋03 | －6．0＇j1E－04 | 0. | －2．914E－01 | －2．92EE－01 | 2．0ire－8i | 3.02 CE － 1 | 0. |
| 17 | 6．7605＋04 | 1．0－E＋ 00 | －4．20UE－U4 | 0. | －3．10GE－U1 | －3．20CE－01 | 3．191玉－01 | 3.217 ¢－b1 | 0. |
| 10 | 2．ACAE O 01 | 1． $595+03$ | －5．399E゙－65 | 0. | －2．655 -02 | －2．E0こ | 1．9．15E－6？ | 2．0心̇E－42 | 0. |
| 19 | $9.125 E \div 05$ | 1．01，E + CS | －7．097E－05 | 0. |  | －4．CJ1E－02 | 4．65：－022 | $4.695-02$ | 0. |
| 23 | 3． 5 CnE +33 | 9．93E－01 | －1．022E－0＇3 | 0. | －2．394E－62 | －2．こリコE－u2 |  | 2．J゙SE－02 | 0. |
| 21 | 1．235［＾的 | 1．06E＋33 | －2．503E－64 | 0. | －1．4i＇CE－62 | －1．502E－02 |  | 1．42こE－02 | 0. |
| 22 | 4．SN4E＋JJ？ | 1．0CE＋03 | －3．72CE－U5 | 0. | －1．EAEt－02 | －1．6c5e－02 | 1．CJTE－02 | 1．64JE－02 | 0. |
| 23 | 1．674Г．932 | 1．65¢\％ | －4．GLOE－05 | 0. | －1．35tE－02 | －1．3C2E－42 | 1．J51k002 | 1．5C1E－02 | 0. |
| 24 | 6．14GE．${ }^{\text {d }}$ | 9．92E－01 | －S．610¢－4＇ | U． | －1．0125－112 | －1．01EE－U2 | 1．60－E－02 | 1．01CE－ひ2 | 0. |
| 25 | 2．2CCE． 01 | 9．9ne－ 1 | －6．727E－05 | 0. | －6．927E－L3 | －6．954E－03 | $6.0 .35:-63$ | 6．リニ゙ニ－03 | 0. |
| 26 | 8． 3 － 0 － 403 | 1．CLETSJ | －7．023E－US | 0. | －4．3－75．－43 | －4．41CLi－U5 | 4．3－11E－63 | 4．412E－03 | 0. |
| 27 | 3．0GUE＋EJ | 9．2CE－01 | －6．6AGE－05 | 0. | －2．1）GE－U3 | －2．5C2E－03 | 2．509E－03 | 2． $25 C E-03$ | 0. |
| 20 | 1．130E +00 | $1.0 \leq 500$ | －S．720E－05 | 0. | －1．30．5－03 | －1．351E－03 | 1．ごロE－¢3 | 1．35リE－03 | 0. |
| 29 | 4．149E－01 | 1．CCE＋33 | －4．4E．JE－05 | 0. | －6．11LE－84 | －6．5CI［－ย4 | 6．3035－0．4 | 6．SCEE－04 | 0. |
| 30 | 1．520E－D1 | 1．11E＋00 | －2．02टE－04 | 0. | －9．99\％E－04 | －1．20＜E－93 | 1．16．JE－03 | 1．181E－03 | 0. |
| INTEGR |  |  | 2．479E－01 | 0. | －S．041ENO | $-4.793 E+00$ | $3.275 E+40$ | $3.2755+05$ | 0. |


| EROLP | UPPER－E（EV） | DELT | －IIET SEN | ILES SENT |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1．700E407 | 1．25E－01 | B． | 0． |
| 2 | 1．50nctat | 1．05E－81 | －1．3cscto | －1．123E461 |
| 3 | 1．35．E．${ }^{\text {P }}$ | 1．2以゙ニータ1 | －1．050ctoo | －1．021E＋ 0 |
| 4 | 1．25：c．t07 | 1．62i：－91 | －1．Suve－u1 | －1．70TL－ 11 |
| 5 | 1．001E＋67 | 2． 5 fiE－ 01 | －1．5UGE－B1 | －1．73－1E－01 |
| 6 | 7．77cE＋36 | 2．4FE－01 | －1．04．1E－U1 | －1．101E－U1 |
| 7 | $6.8712+46$ | S．OCE－01 | －7．601E－42 | －\％．isjoE－ 22 |
| 0 | 3．60！icti6 | 2．5EE－01 |  | －S．うJIE－82 |
| 9 | 2． $2 ¢ 5 E+06$ | 2．58E－81 | －4．032E－62 | －4．09IE－02 |
| 18 | $2.252 E+05$ | 2．53t－91 | －3．22LE－02 | －3．25iE－02 |
| 11 | 1．73．2C＋8G | 2．55E－01 | －2．Un7E－02 | －2．034E－U2 |
| 12 | 1．353E406 | $4.97 E-01$ | －2．CCCE－02 | －2．92\％E－02 |
| 13 | 0．236［405 | 4．9nE－81 | －1．417E－02 | －1．${ }^{\text {cicuco }}$ |
| 14 | 5．0！ric＋35 | S．U1E－01 | －8．19－E－${ }^{\text {－}}$ | －9．22AE－03 |
| 15 | 3．030C＋05 | 4．92E－01 | －4．5725－53 | －4．944－03 |
| 16 | 1．0－？E：\％ | 1．ORE＋D | －2．63ijc－03 | －3． $31 \times 5$ |
| 17 | 6．7CTLE194 | 1．0uE + E0 | －5．41515－14 | －9．719 -04 |
| 18 | 2．03C15＋04 | 1．0うit＋00 | －S．67EE－24 | －6．21－1E－04 |
| 19 | 9．120E：03 | 1．E1，E\％ 33 | －2．613E－64 | －3．3こॅ［－04 |
| 20 | 3． $5.65+33$ |  | －3．U．：0¢－04 | －3．272E－U4 |
| 21 | 1．2．ct＋U3 | 1．01－EM | S．${ }^{\text {corcter }}$ | －2．0．65c－04 |
| 22 | 4．5 5 ¢ $5+32$ | 1．8．5\％103 | －7．725－65 | －1．1小゙J－04 |
| 23 | 1．C：OE4U2 | $1.0 \times 5.103$ | －7．UULE－US | －1．16．E－H．4 |
| 24 | E． $140 \mathrm{~F}+01$ | 9．9JE－G |  | －1．1765－04 |
| 25 | 2． 2 CnE +01 | 9．91゙く－01 | －3．11：L－U5 | －9．11－ |
| 26 | 8． 3 そre +04 | 1.01 ¢く可 | －6．52\％${ }^{\circ} \mathrm{C}$－60 | －7．CM＇jE－fis |
| 27 | 3．85：E409 | 9．9CE－01 | 1．30゙ローU5 | －S．3c．e－us |
| 20 | 1．12ヶ¢ちJ8 | 1．0ciesob | 2．4：－15－以う | －3．27EL－115 |
| 29 |  | 1．05ctob | 2．4＇」lに－u＇j |  |
| 30 | 1．S2CE－01 | 1．1IE＋63 | 1．69LL゙－64 |  |
| INTECRAL |  |  | －1．7CCE | －1．510E480 |

$$
x^{g}=\sum_{\ell=0}^{\operatorname{LMAX}} \sum_{k=0}^{\ell} \psi_{\ell}^{\mathrm{kgg}}=\sum_{\ell=0}^{\operatorname{LMAX}} \psi_{\ell}^{\mathrm{gg}}
$$

Table IX provides a comparison between the $X$ 's calculated from angular fluxes and flux moments. There is a very good agreement. It was found that this relationship is also true in the one-dimensional analysis. For $\ell=0$ and $\ell=1$, the $\Psi_{\ell}^{\mathrm{k}^{\prime}} \mathrm{s}$ calculated in SENSIT and SENSIT-2D are different. However, the $\Psi_{\ell}{ }^{\prime} s$ defined by

$$
\begin{equation*}
\psi_{\ell}^{g g}=\sum_{\underline{k}=0}^{\ell} \psi_{\ell}^{\mathrm{kgg}} \tag{123}
\end{equation*}
$$

are in agreement.
5.1.3 Comparison between a two-dimensional and a one-dimensional crosssection sensitivity and uncertainty analysis

A cross-section sensitivity and uncertainty analysis was done for the heating in the copper region, using SENSIT and SENSIT-2D. In this analysis the effects of the uncertainties in the secondary energy distribution were included. Six separate SENSIT (or SENSIT-2D) runs were required:

TABLE IX: COMPAPISON BETWEEN THE CHI'S CALCULATED FROM ANGULAR FLUYES AND FPOM: FLUX MO:IENTS

| 5-50 | SFMSIT-PD |  | SE:SIT |  |
| :---: | :---: | :---: | :---: | :---: |
|  | chs (anp. Tuxes) | chi (flux mome:ts) | chi (ang. fluxss) | cr: ( ${ }^{\text {(fiax monents) }}$ |
| 1 | 0.0 | 0.0 | 0.0 |  |
| 2 | $2.5058+8$ | 2.5053+9 | $2.4933+8$ | $2.5003+9$ |
| 3 | $2.4390 \cdot 7$ | 2.4362+7 | $2.1493+7$ |  |
| 4 | $6.1304 \cdot 6$ | $6.1264 \times 6$ | $6.1829 \cdot 6$ |  |
| 5 | 8.3365-6 | $8.3329 \times 6$ | 8. 4135 -6 |  |
| 6 | $5.6969 \times 6$ | 5.6354-6 | $5.7424 \cdot 6$ |  |
| 7 | 9.3953*5 | 9.3944*6 | 9.4775 6 |  |
| 8 | $5.6893 \times 6$ | 5.6931*6 | $5.7332 \times 6$ |  |
| 9 | 7.154306 | $7.1543+6$ | 7.2079 .6 |  |
| 10 | $7.3349 \times 6$ | 7.3349 .6 | 7.3894 .6 |  |
| 11 | $7.5019+6$ | $7.9619+6$ | $7.9321+6$ |  |
| 12 | $2.1571+7$ | $2.1571+7$ | 2.1793*6 |  |
| 13 | 2.9571*7 | $2.9571+7$ | 2.9022*6 |  |
| 2 | 2.5499-7 | 2.5429+7 | $2.6026 \cdot 6$ |  |
| 15 | 6.3491 .6 | $6.3491+6$ | $6.5028 \cdot 6$ |  |
| 16 | 1.57 93*7 | $1.5783+7$ | $1.6345 \times 7$ |  |
| 17 | $6.5269 \times 6$ | 6.526906 | $7.1384 \times 6$ |  |
| 18 | $1.7636 \cdot 6$ | 1.7536 .6 | $1.8060 \cdot 6$ |  |
| 13 | 1.246306 | $1.2463 \times 6$ | $1.2862 \times 6$ |  |
| 20 | $7.6860 \times 5$ | $7.6860 \times 5$ | $7.7452 \times 5$ |  |
| 21 | $3.7764 \times 5$ | 3. $7764 \times 5$ | 3.54400 5 |  |
| 27 | 3.7425+5 | $3.7415+5$ | $3.9205+5$ |  |
| 23 | $2.9673 \times 5$ | $2.9673+5$ | $3.0745+5$ |  |
| 2i | $2.2041 \times 5$ | $2.2041+5$ | $2.2105 \times 5$ |  |
| 25 | $1.5057 \times 5$ | $2.5057 \times 5$ | $1.5857 \times 5$ |  |
| 26 | 9.4634 .4 | $9.4634 \cdot 4$ | $9.5405 \cdot 4$ |  |
| 27 | 5.409004 | 5.4091* | $5.4300 \% 4$ |  |
| 28 | $2.3476+4$ | 2.8476+4 | $2.8499+4$ |  |
| 29 | 1.3324*4 | 1. $3325+4$ | $1.3318+4$ |  |
| 30 | 2.39514 | 2.3849 .4 | $2.4510+4$ |  |

- three runs for the vector cross-section sensitivity and uncertainty analysis (one for the cross sections of zone II, one for the cross sections of zone III, and one for the cross sections of zone IV), - three runs for the SED sensitivity and uncertainty analysis.

Oxygen was not included in the vector cross-section sensitivity and uncertainty analysis, and hydrogen was ignored in the SED sensitivity and uncertainty analysis.

The procedure for an uncertainty analysis has been discussed by Gerstl. ${ }^{45}$ The results from the one-dimensional analysis are reproduced in Table $X a$, while those from the two-dimensional study are given in Table Xb . The studies are in good agreement. Sensit required a total of 89 seconds of computing time, while SENSIT-2D required 90 seconds on a CDC-7600 machine. The uncertainty of the heating rate due to all cross-section uncertainties is $30 \%$. The iron in zone II is the largest contributor to that uncertainty. The contribution of the SED uncertainty is smaller than that from the vector cross sections. Gerstl points out that the results obtained from the SED analysis might have been underestimated due to the simplicity of the "hot-cold" concept and due to the fact that the partial cross sections which contribute to the secondary energy distribution were not separated into individual partial cross sections. ${ }^{45}$

TABLE Xa. PREDICTED RESPONSE UNCERTAINTIES DUE TO ESTIMATED CROSS SECTION AND SED UNCERTAINTIES IN A ONE-DIMENSIONAL ANALYSIS

| CROSS SECTION | ZONE | RESPONSE UNCERTAINTIES DUE TO SED UNCERTAINTIES, IN \% |  | RESPONSE UNCERTAINTIES DUE TO CROSS-SECTION UNCERTAINTIES, IN \% |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\left[\frac{\Delta R}{R}\right]_{\text {zone }}$ | $\left[\frac{\Delta R}{R}\right]_{\text {zone }}^{\circ}$ | $\left[\frac{\Delta R}{R}\right]_{\substack{\text { zonect }}}$ | $\left[\frac{\Delta R}{R}\right]_{z o n e}$ |
| Fe | II | 8.18 | 8.18 | 23.80 | 23.80 |
| Fe | III | 2.50 |  | 10.33 |  |
| 0 | III | 0.78 | 2.61 | - | 10.52 |
| H | III | - |  | 1.96 |  |
| Cu | IV | 4.02 | 4.02 | 11.72 | 11.72 |
| All ${ }^{*}$ |  |  | 9.48 |  | 28.54 |

Overall uncertainty $=\left(9.48^{2}+28.54^{2}\right)^{\frac{1}{2}}=30.0 \%$

* quadratic sums

TABLE Xb. PREDICTED RESPONSE UNCERTAINTIES DUE TO ESTIMATED CROSS SECTION AND SED UNCERTAINTIES IN A TWO-DIMENSIONAL ANALYSIS

| $\begin{aligned} & \text { CROSS } \\ & \text { SECTION } \end{aligned}$ | ZONE | RESPONSE UNCERTAINTIES DUE TO SED UNCERTAINTIES, IN \% |  | RESPONSE UNCERTAINTIES DUE TO CROSS-SECTION UNCERTAINTIES, IN \% |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\left[\frac{\Delta R}{R}\right]_{\substack{\text { element } \\ \text { zone }}}$ | $\left[\frac{\Delta R}{R}\right]_{\text {zone }}^{*}$ | $\left[\frac{\Delta R}{R}\right]_{\text {zone }}$ | $\left[\frac{\Delta R}{R}\right]_{\text {zone }}^{:}$ |
| Fe | II | 8.17 | 8.17 | 23.88 | 23.88 |
| Fe | III | 2.50 |  | 10.27 |  |
| 0 |  | 0.79 | 2.62 | - | 10.46 |
| H | III | - |  | 1.96 |  |
| Cu | IV | 4.02 | 4.02 | 11.68 | 11.68 |
| All* |  |  | 9.47 |  | 28.57 |

Overall uncertainty $=\left(9.47^{2}+28.57^{2}\right)^{\frac{3}{2}}=30.1 \%$

[^1]
### 5.2 Sample Problem \#2

A simple one-band problem will be analyzed to study the influence of the mesh spacing, quadrature order, convergence precision, and the c-factor (mean number of secondaries per collision) on the sensitivity profile. The band is $1-\mathrm{cm}$ high and $20-\mathrm{cm}$ wide. There are ten distinct zones, each $1-\mathrm{cm}$ wide (Fig. 1l), and all zones are made of the same material. A three-group artificial cross-section set with a third-order anisotropic scattering is used (Table XI). The $\mathrm{P}_{1}, \mathrm{P}_{2}$, and $\mathrm{P}_{3}$ components of the scattering cross-section tables were chosen to be identical with the $P_{0}$ component. A volumetric source with a source density of 1 neutron/ $\mathrm{cm}^{3}$ in group 1 is present in the first zone. A standard cross-section sensitivity analysis will be performed, in which the cross sections in zone IV are perturbed, and the detector response is calculated in zones $I X$ and $X$ for a response function of $100 \mathrm{~cm}^{-1}$ in each group.
5.2.1 Influence of the quadrature order on the sensitivity profile

The detector response calculated by TRIDENT-CTR using $E Q_{6}, E Q_{12}$, and $E Q_{16}$ quadrature sets are compared in Table XII. For the first three cases, the pointwise convergence precision was set to $10^{-3}$ and each zone contained four triangles (using automatic meshes). Five additional cases are included in Table XII:

RETLf.CTIVs BOMYMAPY

$\mathrm{r}-\mathrm{z}$ geometry
All zones contain identical materials
3 neutron groups
Neutron source: 1 neutron $/ \mathrm{cm}^{3}$ in zone I and group 1
Response function: $100 \mathrm{~cm}^{-1}$ (all groups)

Figure 11. Two-dimensional model for sample problem \#2

TABLE XI: CROSS SECTION TABLE USED IN SAMPLE PROBLEM \#2 (THE $P_{0}, P_{1}, P_{2}$, AND $P_{3}$ TABLES ARE IDENTICAL)

| $\substack{\text { Cross } \\ \text { Section }}$ | $g=1$ | $g=2$ | $g=3$ |
| :--- | :--- | :--- | :--- |
| $\Sigma_{\text {edit }}^{g}$ |  |  |  |
| $\Sigma_{a}^{g}$ | - | - | - |
| $\Sigma_{f}^{g}$ | 0.02 | 0.05 | 0.1 |
| $\Sigma_{T}^{g}$ | 0.0 | 0.0 | 0.0 |
| $\Sigma_{S}^{g \rightarrow g}$ | 0.1 | 0.2 | 0.3 |
| $\Sigma_{S}^{g-1 \rightarrow g}$ | 0.05 | 0.1 | 0.2 |
| $\Sigma_{S}^{g-2 \rightarrow g}$ | 0.0 | 0.0 | 0.01 |

TABLE XII: INTEGRAL RESPONSE FOR SAMPLE PROBLEM \#2

| Transport | Quadrature | Convergence <br> Precision | \# Triangles |
| :--- | :---: | :---: | :--- | :--- |


|  |  |  | $10^{-3}$ | 40 | 593.968 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| TRIDENT-CTR | EQ-6 | 592.256 |  |  |  |
| TRIDENT-CTR | EQ-12 | $10^{-3}$ | 40 | 592.826 | 591.659 |
| TRIDENT-CTR | EQ-16 | $10^{-3}$ | 40 | 593.659 | 592.476 |
| TRIDENT-CTR | EQ-12 | $10^{-4}$ | 40 | 593.659 | 593.148 |
| TRIDENT-CTR | Eq-12 | $10^{-4}$ | 80 | 593.688 | 593.208 |
| ONEDANT | $\mathrm{S}-12^{3}$ | $10^{-4}$ | 40 | 593.855 | 590.370 |
| ONEDANT | $\mathrm{S}-32$ | $10^{-4}$ | 40 | 591.814 | 590.883 |
| ONEDANT | $\mathrm{S}-32$ | $10^{-4}$ | 80 | 592.055 | 590.900 |

[^2]1. integral response using an $E Q_{12}$ quadrature set with convergence precision $10^{-4}$;
2. integral response using an $\mathrm{EQ}_{12}$ quadrature set with convergence precision $10^{-4}$ and eight triangles per zone;
3. integral response calculated by ONEDANT using an $\mathrm{S}_{12}$ quadrature set, four intervals per zone and a $10^{-4}$ convergence precision;,
4. integral response calculated by ONEDANT, using an $S_{32}$ quadrature set, four intervals per zone and a $10^{-4}$ convergence precision;
5. integral response calculated by ONEDANT, using an $S_{32}$ quadrature set, eight intervals per zone and a $10^{-4}$ convergence precision.

The response functions in Table XII are in good agreement (maximum difference $0.6 \%$ ). The standard cross-section sensitivity profiles for the $E Q_{6}, \mathrm{EQ}_{12}$, and $E Q_{16}$ calculations are reproduced in Tables XIIIa, XIIIb, and XIIIc. The integral sensitivity for the $E Q_{6}$ case is $5 \%$ different from the $E Q_{12}$ case for $A X S$ (absorption cross-section sensitivity profile) and $5 \%$ different for N -GAIN (outscattering cross-section sensitivity profile). The results obtained from the $E Q_{12}$ calculation are in good agreement with those obtained from the $E Q_{16}$ calculation. The sensitivity profiles for the $E Q_{12}$ case ( $10^{-4}$ convergence precision) and the $E Q_{12}$ case ( $10^{-4}$ convergence precision, eight triangles per zone) are not shown. They are nearly identical with Table XIIIb.

TABLE IIIa: STANDARD CROSS-SECTION SENSITIVITY PROFILES CALCULATED BY SENSIT-2D FOR THE EQ-6 CASE (CONVERGENCE PRECISION 0.001, 4 TRIANGLES PER ZONE) FOR SAMPLE PROBLEM \#2

| GROUP | UPPER-E (EV) | DELTA-U | $\operatorname{maxratos}_{A X S} P$ | RELLOS | $\begin{aligned} & \text { TE R M } \\ & \text { SXS } \end{aligned}$ | $\begin{gathered} 5 \text { TXS } \end{gathered}$ | N-GA1H | RE GAIN TERTE H-GAIN(SED) |  NG-T.A1N |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.000E+01 | $6.93 \mathrm{E}-81$ | -4.632E-02 | 0. | -1.853E-81 | -2.316E-01 | 1.549E-01 | N-GAIN(SED) | NG-riAIN |
| 2 | S.008E+e日 | $1.61 \mathrm{E}+0 \mathrm{0}$ | -4. Si2E-03 | 0. | -1.359E-82 | -1.EIIEE-0? | 1. 186 E -82 | 2.19eE-02 | ${ }^{\text {日. }}$ |
| 3 | $1.680 \mathrm{E}+8 \mathrm{O}$ | 6.93E-81 | -1.142E-82 | 0. | -2.284E-82 | -3.426E-02 | 2.234E-82 | 3.677E-02 | 8. |
| INTEG |  |  | -4.732E-82 | 0. | -1.661E-01 | -2.135E-81 | 1.423E-81 | 1.423E-01 | 8. |
| GROUP | UPPER-E (EV) | DELTA-U | *aw NET PR EEN | $\begin{aligned} & \text { OF ILES } \\ & \text { SENT } \end{aligned}$ |  |  |  |  |  |
| 1 | 1.080Etal | 6.93E-81 | -3.039E-82 | -7.671E-02 |  |  |  |  |  |
| 2 | S.b8beteb | 1.61E+00 | -1.73IE-83 | -6.263E-83 |  |  |  |  |  |
| 3 | $1.000 E+80$ | $6.93 \mathrm{E}-81$ | 3.2S2E-86 | -1.14IE-82 |  |  |  |  |  |
| INTEGR |  |  | -2.385E-72 | -7.116E-82 |  |  |  |  |  |

TABLE XIIIb: STANDARD CROSS-SECTION SENSITIVITY PROFILES CALCULATED BY SENSIT-2d FOR THE EQ-12 CASE (CONVERGENCE PRECISION $0.001,4$ TRIANGLES PER•ZONE) FOR SAMPLE PROBLEM \#2

| GiROUP | UPPER-E (EV) | DELTA-U | AXS P UR | $\begin{aligned} & \text { EURLOS } \\ & \mathrm{NU} \text { LISS } \end{aligned}$ | TERM 5XS | 5 TXS | $\begin{gathered} N-G A I N \end{gathered}$ | pure gain terms N-GAINISED | NG-GAIN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1-0beetol | 6-93E-81 | -4.329E-03 | 0. | -1.731E-81 | -2.164E-81 | $1.458 \mathrm{E}-81$ | 1 1-87SE-01 |  |
| 2 | S. $000 \mathrm{C}=+8 \mathrm{~B}$ | $1.61 \mathrm{E}+8 \mathrm{~B}$ | -4.366E-03 | 0. | -1.310E-82 | -1.74EE-82 | 1.145E-82 | 2 2.125E-02 | 0. |
| 3 | 1.000E +80 | $6.93 \mathrm{E}-81$ | -1-103E-02 | 8. | -2.206E-82 | -3.310E-02 | 2-207E-82 | $2.761 E-32$ | 8. |
| INTEGR |  |  | -4.468E-02 | 0. | -1.564E-81 | -2.011E-81 | $1.348 \mathrm{E}-81$ | $11.348 \mathrm{E}-81$ | 0. |
| GRDUP | UPPEP-E (EV) | DELTA-U | nowow HET PROF SEN | $\begin{aligned} & \text { JF ILES } \\ & \text { SENT } \end{aligned}$ |  |  |  |  |  |
| 1 | $1.000 E+01$ | $6.93 \mathrm{E}-81$ | -2.731E-82 | -5.868E-02 |  |  |  |  |  |
| 2 | S.004? +80 | $1.61 \mathrm{E}+8 \mathrm{~B}$ | -1.668E-03 | -6.034E-83 |  |  |  |  |  |
| 3 | $1.800 \overline{c o}^{+80}$ | 6.93E-81 | 0.040E-07 | -1.103E-02 |  |  |  |  |  |
| INTEGR |  |  | -2.161E-82 | -6-629E-82 |  |  |  |  |  |

# TABLE XIIIc: STANDARD CROSS-SECTION SENSITIVITY PROFILES CALCULATED BY SENSIT-2D FOR THE EQ-16 CASE (CONVERGENCE PRECISION 0.001, 4 TRIANGLES PER ZONE) FOR SAMPLE PROBLEM \#2 

| GROUP | UPPER-E (EV) | DELTA-U |  | REMLOSS | $\begin{aligned} & \text { TE R M } \\ & \text { SXS } \end{aligned}$ |  | H-GAIN | JRE GAIN TERTS N-GAIN(SED) | NG-GAIN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.000=+01 | $6.93 \mathrm{E}-18$ | -4.293E-02 | 0. | -1.716E-81 | -2.145E-01 | 1.443E-01 | $1.062 \mathrm{E}-81$ |  |
| 2 | S.888E+08 | $1.61 \mathrm{E}+8 \mathrm{~B}$ | -4.330E-83 | 8. | -1.299E-82 | $-1.73 \div E-02$ | 1. 135E-82 | 2.189E-82 | 8. |
| 3 | 1.808E+C0 | 6.93E-81 | -1.094E-02 | 0. | -2.188E-82 | -3.28IE-02 | 2.187E-8? | 3.732E-62 | 0. |
| 1NTEGRAL |  |  | -4.4295-02 | 0. | -1.5S0E-81 | -1.993E-81 | 1.334E-H1 | 1.334E-81 | 0. |
|  |  |  | Mown NET PROFILES |  |  |  |  |  |  |
| GROUP | UPPEP-E (EV) | DELTA-U | SEM | SENT |  |  |  |  |  |
|  | $1.080 \mathrm{E}+01$ | $6.93 \mathrm{E}-81$ | -2.734E-82 | -7.024E-02 |  |  |  |  |  |
| 2 | 5.880E +80 | $1.61 \mathrm{E}+0 \mathrm{~B}$ | -1.ES3E-03 | -5.90\%E-03 |  |  |  |  |  |
| 3 | $1.080 \mathrm{E}+80$ | $6.93 \mathrm{E}-81$ | -1.484E-86 | -1.094E-02 |  |  |  |  |  |
| INTEGR |  |  | -2.162E-02 | -6.59IE-02 |  |  |  |  |  |

Note that the net sensitivity profiles SEN (= SXS + N-GAIN) for group 3 are respectively $3.252 \times 10^{-6}, 8.040 \times 10^{-7}$, and $1.484 \times 10^{-6}$ for the $E Q_{6}$, the $E Q_{12}$, and the $E Q_{16}$ case. The large discrepancies here can be attributed to the fact that those quantities result from subtracting two numbers that are nearly equal in magnitude.

It can be concluded from Tables XII and XIII that even when the integral responses differ by less than $0.4 \%$, the sensitivity profiles can differ by as much as $5 \%$ between an $\mathrm{EQ}_{6}$ and an $\mathrm{EQ}_{12}$ calculation. The close agreement between the results from the $E Q_{12}$ and the $E Q_{16}$ calculation suggest that this difference is probably due to the fact that the angular fluxes in the $\mathrm{EQ}_{6}$ calculation are not yet fully converged. ${ }^{81}$ Indeed, choosing the higher-order anisotropic scattering cross sections equal to the isotropic components is unphysical. The convergence criteria used in ONEDANT and TRIDENT-CTR do guarantee convergence for the scalar fluxes, but not for the higher-order flux moments.

### 5.2.2 Comparison between the two-dimensional and one-dimensional analysys of sample problem \#2

The cross-section sensitivity profiles resulting from a one-dimensional analysis ( $S_{12}$ quadrature set, $10^{-4}$ convergence precision and four intervals per zone; $S_{32}$ quadrature set, $10^{-4}$ convergence precision and eight intervals per zone) are compared with those obtained from a twodimensional analysis $\left(E Q_{12}\right.$ quadrature set, $10^{-4}$ convergence precision and eight triangles per zone in Tables XIVa, .IVb, and XIVc.

TABLE XIVa：STANDARD CROSS－SECTION SENSITIVITY PROFILES CALCULATED BY SENSIT FOR THE S－12 CASE（CON－ VERGENCE PRECISION 0．0001， 4 INTERVALS PER ZONL）FOR SAMPLE PROBLEM $\# 2$


TABLE XIVb：STANDARD CROSS－SECTION SENSITIVITY PROFILES CALCULATED BY SENSIT FOR THE S－32 CASE（CON－ VERGENCE PRECISION 0．0001， 8 INTERVALS PER ZONE）POR SAMPLE PROBLEM \＃2

| GROUP | UPPER－E（EV） | DELTA－U | $\underset{\text { MxS }}{\operatorname{man}} \mathbf{P}$ UR | $\begin{aligned} & \text { EU-FILO } \\ & \text { Nu-F } \end{aligned}$ | $\begin{aligned} & \text { TEERM } \\ & \text { SXS } \end{aligned}$ | 5 Tract $7 \times 5$ |  | PURE GAIN TERTS N－GAIM（SED） | $\text { NG-GA } 11$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $1.680{ }^{\text {c }}+81$ | $6.93 \mathrm{E}-81$ | －4．215E－0？ | 0. | －1．68GE－21 | －2．107E－81 | $1.2995-01$ | 9．517E－02 | 0. |
| 2 | S＿800ご＋00 | 1．61E＋80 | －4．277E－03 | 日． | －1．283E－82 | －1．711E－82 | 1．128E－82 | 1．991E－02 | 0. |
| 3 | 1．880E＋80 | $6.93 \mathrm{E}-81$ |  | 6. | －2．161E－02 | －3．242E－02 | 2．221E－02 | 3．689E－02 | B． |
| INTEGRAL |  |  | －4．359E－02 | 0. | －1．525E－01 | －1．961E－81 | 1．236F－81 | $1.236 \mathrm{E}-81$ | 0. |
| croup | UPPFR－E（EV） | DELTA－U | ＊＊0＊NET PROF ILES＊＊＊＊ |  |  |  |  |  |  |
| 1 | 1．000Etel | 6．93E－81 | －3．07IE－02 | －0．08бE－02 |  |  |  |  |  |
| 2 | 5．830 5 ＋ 08 | $1.61 \mathrm{E}+8 \mathrm{~B}$ | －1．5SIE－03 | －5． $829 \mathrm{E} \cdot 83$ |  |  |  |  |  |
| 3 | $1.800 E+80$ | 6．93E－81 | 5．989E－04 | －1．021E－82 |  |  |  |  |  |
| Integral |  |  | －2．891E－02 | －7．250E－82 |  |  |  |  |  |

# TABLE XIVc: STANDARD CROSS-SECTION SENSITIVITY PROFILES CALCU CALCULATED BY SENSIT-2D FOR THE EQ-12 CASE (CONVERGENCE PRECISION 0.0001, 8 TRIANGLES PER ZONE) FOR SAMPLE PROBLEM $\# 2$ 

| GROUP | UFPER-E (EV) | DELTA-U | $\underset{A \times S}{\operatorname{mactics}} P \cup R$ | $\underset{\mathrm{NU}-\mathrm{F} 15 \mathrm{~S}}{\mathrm{~L}}$ | $\underset{S \times S}{T E} \text { R M }$ |  | ${ }_{N-G A I N} P$ | URE GAIN TERTS H-GAIN(SED) | $N \in-\Gamma: \cap 101$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.e日uEtol | 6.93E-81 | -4.324E-02 | 0. | -1.730E-01 | -2.162E-01 | 1.457E-01 | $1.074 E-81$ | 0. |
| 2 | S.800E+80 | $1.61 \mathrm{E}+80$ | -4.362E-03 | 8. | -1.309E-02 | -1.i45E-82 | 1.142E-82 | 2.123E-62 | 4. |
| 3 | 1.00 j ¢ +00 | 6.93E-01 | -1.162E-02 | 6. | -2.203E-02 | -3.305E-82 | 2.235E-02 | 3.75SE-32 | 0. |
| INTEG |  |  | -4.463E-02 | e. | -1.562E-01 | -2.009E-81 | 1.346E-01 | 1.34uE-01 | 0. |
|  |  |  | - NECT PROF | OF ILES ***** |  |  |  |  |  |
| GROUP | UPPER-E(EV) | DELTA-U | SEN | SENT |  |  |  |  |  |
|  | 1. OugE+al | 6.93E-01 | -2.72TE-02 | -7.0515-02 |  |  |  |  |  |
| 2 | S_80JE +80 | 1.61E+80 | -1-66Ė̇-03 | -c.03dE-03 |  |  |  |  |  |
| 3 | $1.0685+80$ | 6.93E-01 | 7.918E-87 | -1.1825-82 |  |  |  |  |  |
| INTEG |  |  | -2-158E-02 | -6.621E-02 |  |  |  |  |  |

Note that the N-GAIN integral sensitivity differs by about $6 \%$ between Table XIV $b$ and XIV c. The integral net sensitivity shows a $35 \%$ difference for SEN (= SXS + N-GAIN) and a $10 \%$ difference for SENT (= TXS + N-GAIN) between the one-dimensional and the two-dimensional analysis. The bulk part of this large difference for the integral net sensitivity results from the subtraction of two numbers that are nearly equal in magnitude. A comparison of N-GAIN (integral) in Tables XIV a, XIV b, and XIV $c$ suggests that - even with an $S_{32}$ quadrature set - the one-dimensional calculation is not yet fully converged.

### 5.3.2 Comparison between the $X$ 's calculated from angular fluxes and the $X^{\prime}$ s resulting from flux moments

The $X$ 's (or the loss term of the cross-section sensitivity profile) can be evaluated based on flux moments (Eq. 58) or based on angular fluxes (Eq. 57). A calculation based on flux moments requires less computing time, less computer memory, and less data transfer. To have an idea of the order of expansion of the angular fluxes in flux moments necessary to reach a reasonable accuracy, the $\chi$ 's resulting from angular fluxes are compared with those obtained from a P-0, P-1, P-2, . . ., P-17 spherical harmonics expansion of the angular fluxes (Table XV). It is found that for any expansion of order greater than $\mathrm{P}-0$, there is good agreement (less than $1 \%$ difference for $\Sigma \chi^{g}$ ). For very high spherical $g$ harmonics expansions ( $\mathrm{P}-15$ and higher) there is divergence. This divergence can be avoided by doing the computations in quadruple precision.

TABLE XV: COMPARISON BETWEEN THE CHI'S CALCULATED FROM ANGULAR FLUXES AND THE CHI'S CALCULATED FROM FLUX MOMENTS

Order of Expansion of the Angular Flux
$x^{1^{a}} \quad x^{2} \quad x^{3} \quad \left\lvert\, \begin{aligned} & \Sigma\left(x_{m o m}^{g}-x_{a n g}^{g}\right)\end{aligned}\right.$

| angular fluxes | 889.88 | 83.382 | 45.352 | - |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 796.63 | 76.811 | 41.898 | 103.275 |
| 1 | 880.43 | 83.755 | 45.524 | 8.905 |
| 2 | 868.78 | 83.054 | 45.326 | 21.454 |
| 3 | 884.11 | 83.374 | 45.353 | 5.777 |
| 4 | 884.58 | 83.388 | 45.364 | 5.282 |
| 5 | 886.82 | 83.385 | 45.357 | 2.052 |
| 6 | 888.82 | 83.389 | 45.354 | 1.051 |
| 7 | 889.16 | 83.381 | 45.353 | 0.720 |
| 8 | 889.78 | 83.381 | 45.352 | 0.101 |
| 10 | 889.74 | 83.382 | 45.352 | 0.140 |
| 11 | 889.91 | 83.383 | 45.352 | 0.031 |
| 12 | 890.01 | 83.385 | 45.351 | 0.133 |
| 14 | 889.89 | 83.382 | 45.351 | 0.009 |
| 16 | 898.27 | 83.573 | 45.431 | 8.610 |
| 17 | 938.59 | 83.100 | 46.203 | 51.279 |
| 1.85 | 85.617 | 46.416 | 65.269 |  |

${ }^{a} \chi^{1}$ mean $\chi$ for group 1

The small differences in Table XV indicate that the loss term of the sensitivity profile can indeed be calculated based on a low-order spherical harmonics expansion of the angular fluxes.

### 5.2.4 Evaluation of the loss term based on flux moments in the case of low c

The question whether the $X$ 's can be computed with adequate accuracy from Eq. (58) in the case of low c (mean number of secondaries per collision) was raised. ${ }^{8}$ Based on an analytical one-dimensional analysis of the half-space problem (one group) with a mono-directional boundary source, it was found that for $c$ less than 0.8 , a low-order spherical harmonics expansion of the angular flux would lead to erroneous results in the $X^{\prime} s$.

In order to confirm the analytical study, sample problem \#2 was reexamined with a different cross-section table. The corresponding c's were 0.5 for the high-energy group, 0.4 for the second group, and 0.33 for the low-energy group. The $X$ 's calculated based on flux moments were still in agreement with those obtained from the angular fluxes (even for a P-l expansion). An explanation for this paradoxical behavior is probably related to the use of a distributed volumetric source in sample problem 非2, whereas the conclusions drawn in the analytical evaluation were based on the presence of a mono-directional boundary source.

### 5.3 Conclusions

The rigorous study of the two sample problems indicates that there is good agreement between the one- and two-dimensional analysis. Wherever discrepancies appear, a plausible explanation can be provided. Ultimately, the comparison between a one- and two-dimensional study proves to be a sound debugging procedure for SENSIT-2D as well as for the SENSIT code.

For the flux moments versus the angular fluxes comparison for the evaluation of the $X^{\prime} s$, there is a strong indication that the loss term can be calculated from lower-order flux moments ( $\mathrm{P}-1$ ) as well as from angular fluxes. By the same token. a $\mathrm{P}-1$ sensitivity and uncertainty analysis seems to provide sufficient accuracy.

The study of the influence of the quadrature sets on the sensitivity profiles reveals the importance of the angular-flux convergence in ONEDANT and TRIDENT-CTR. Furthermore, some doubts about the meaningfulness and practicality of the net sensitivity profile (SEN) can be raised.

## 6. SENSITIVITY AND UNCERTAINTY ANALYSIS OF THE HEATING IN THE TF COIL FOR THE FED

In this part a secondary energy distribution and a vector crosssection sensitivity and uncertainty analysis will be performed for the heating of the $T F$ coil in the inner shield of the FED. The results obtained from the two-dimensional analysis will be compared with selected results from a one-dimensional model. The blanket design for the FED is currently in development at the General Atomic Company. 82,83

### 6.1 Two-Dimensional Model for the FED

The two-dimensional model for the FED in r-z geometry is illustrated in Fig. 12, and is documented in more detail in reference 84. The material composition is shown in Table XVI. In the forward TRIDENTCTR model, which was set up by W. T. Urban, ${ }^{84}$ the standard Los Alamos 42 coupled neutron/gamma-ray group structure was used. ${ }^{85}$ There are 30 neutron groups and 12 gamma-ray groups. The TRIDENT-CTR model ${ }^{84}$ (Fig.
table Xvi. atom densities for the isotopes used in the materials (atom/b cm)

|  | Isotope | MATERIAL |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | SS316 | TFCOIL | SS304 | CNAT | IHDLC | IHDLB | IHDLA | SS312 |
|  | H-1 |  | 3.79E-3 |  |  | 5.03E-2 | 1.34E-2 | 1.68E-3 |  |
|  | He-4 |  | 6.67E-3 |  |  |  |  |  |  |
|  | B-10 |  | 2.98E-5 |  |  |  |  |  |  |
|  | B-11 |  | $1.20 \mathrm{E}-4$ |  |  |  |  |  |  |
|  | C |  | $1.90 \mathrm{E}-3$ |  | 8.03E-2 |  |  |  |  |
|  | 0-16 |  | 2.17E-3 |  |  | 2.51E-2 | 6.70E-3 | 8.38E-4 |  |
| ■ | A1-27 |  | $1.81 \mathrm{E}-4$ |  |  |  |  |  |  |
| $\stackrel{\square}{6}$ | Si |  | 5.59E-4 |  |  |  |  |  |  |
|  | Ca |  | 2.42E-4 |  |  |  |  |  |  |
|  | Cr | 1.67E-2 | 5.97E-3 | 1.77E-2 |  | 4.18E-3 | 1.34E-2 | 1.63E-2 | 3.34E-3 |
|  | Mn-55 | $1.75 \mathrm{E}-3$ | 6.27E-4 | $1.67 \mathrm{E}-3$ |  | 4.38E-4 | $1.40 \mathrm{E}-3$ | $1.71 \mathrm{E}-3$ | 3.50E-4 |
|  | Fe | $5.44 \mathrm{E}-2$ | 1.95E-2 | 6.06E-2 |  | $1.36 \mathrm{E}-2$ | $4.35 \mathrm{E}-2$ | 5.30E-2 | 1.09E-2 |
|  | Ni | 1.15E-2 | 4.12E-3 | $7.40 \mathrm{E}-3$ |  | $2.88 \mathrm{E}-3$ | 9.20E-3 | 1.12E-2 | 2.30E-3 |
|  | Cu |  | 2.11E-2 |  |  |  |  |  |  |
|  | $\mathrm{Nb}-93$ |  | $2.44 \mathrm{E}-4$ |  |  |  |  |  |  |
|  | Mo | 1.51E-3 | $5.41 \mathrm{E}-4$ |  |  | $3.78 \mathrm{E}-4$ | 1.21E-3 | 1.47E-3 | 3.02E-4 |



Figure 12. Two-dimensional model for the $F E D^{84}$
The numbers within each zone indicate the zone number.


Figure 13. The TRIDENT-CTR band and triangle structure for the FED
13) utilizes 2062 triangles, divided over 27 bands. The response functions for calculating the heating in the $T F$ coil, were prepared by the TRANSXX code. 72 Those response functions will be the sources for the adjoint calculation. It was noted earlier that negative sources can introduce instabilities in the sweeping algorithm for the adjoint TRIDENT-CTR calculation. The negative kerma factors are therefore set to zero. This will have a minor effect on the total heating calculated in the TF -coil (less than $1 \%$ ).

EQ-2 and EQ-8 quadrature sets are used for groups 1 and 2 respectively, EQ-3 is used for groups 3, 4, and 5, while an EQ-4 quadrature set is utilized for the remaining groups. The convergence precision is specified to be $10^{-3}$. The gamma-ray groups contribute most to the heating in the TF coil (93\%). The total heating in the TF coil is $823 \times$ $10^{-6} \mathrm{MW}$.

The heating calculated by the adjoint TRIDENT-CTR calculation is found to be $3 \%$ smaller than the heating resulting from the forward run. The forward calculation required about one hour of c.p.u. time on a CDC-7600 computer, while the adjoint run took about four hours. Groups 11 to 23 required significantly more inner iterations in the adjoint mode than the other groups. No explanation of this behavior could be found. Experience with other neutronics codes indicates that the adjoint mode for this type of calculation requires usually no more than $30 \%$ extra calculation time.
6.2 Two-Dimensional Sensitivity and Uncertainty Analysis for the Heating
in the TF Coil due to SED and Cross-Section Uncertainties

A secondary energy distribution and vector cross-section uncertainty analysis was performed with SENSIT-2D using the forward and adjoint angular flux files created by TRDSEN. A separate SENSIT-2D run is required for each zone. Because separate runs are necessary for a crosssection and a SED analysis, a total of 22 SENSIT-2D cases were analyzed. A total of 15 minutes c.p.u. time was used by SENSIT-2D. The bulk of this time is consumed during input/output manipulations.

The median energies and fractional uncertainties for the SED uncertainty calculations were taken from Table II. 45 A special cross-section table was created - using TRANSX - for the SED analysis. COVFILS ${ }^{33}$ data were used for generating the covariance matrices utilized in the crosssection uncertainty evaluation. Only $0-16, \mathrm{C}, \mathrm{Fe}, \mathrm{Ni}, \mathrm{Cr}$, and Cu were considered for the $S E D$ uncertainties, while $\mathrm{H}, \mathrm{Fe}, \mathrm{Cr}, \mathrm{Ni}, \mathrm{B}-10, \mathrm{C}$, and Cu were included for the cross-section uncertainties. With the exception of oxygen, no important materials were left out. It was found in an earlier study that the cross-section uncertainties for oxygen caused an $8 \%$ uncertainty in the heating. 45 The current version of SENSIT-2D does not include the option to extract the covariance data for oxygen from COVFILS.

The gamma-ray cross sections are generally better known than the neutron cross sections. Therefore, only the uncertainties resulting
from uncertainties in neutron cross sections are calculated. Throughout this analysis a third order of anisotropic scattering is used.

The predicted uncertainties in the heating of the TF-coil are summarized in Table XVII. It was assumed that the uncertainties for a particular element in the various $\operatorname{SS} 316$ zones (1, 3, 7, 11, and 12 in Fig. 12) are fully correlated, while all other uncertainties were assumed to be noncorrelated. This implies that the uncertainties for a particular element can be added over all SS313 zones, while all the other uncertainties are added quadratically. The approach of either assuming full correlation or assuming noncorrelation is rather simplistic. Translating the physics of this particular problem into a more sophisticated correlation scheme would be a major study by itself. The uncertainties resulting from the uncertainties in the cross sections for $\mathrm{Cr}, \mathrm{Fe}$, and Ni in the SS316 zones are reproduced in Table XVIII.

From Table XVII it can be concluded that the cross-section uncertainties (predicted to be $113 \%$ ) tend to be more important than the SED uncertainties ( $20 \%$ ) . Even when the overal uncertainty seems to be relatively large ( $115 \%$ ), the blanket designer is able to set an upper bound for the heating in the $T F$ coil. The largest uncertainties are due to uncertainties in the Cr cross sections. A more detailed look at the computer listings generated by this analysis reveals that the largest uncertainties are produced by uncertainties in the total Cr and the elastic Cr scattering cross sections. The heating is less sensitive to Cr than to Fe. This indicates that the calculated uncertainty is largely due to the fact that Cr has very large covariances. A re-evaluation

TABLE XVII: PREDICTED UNCERTAINTIES (STANDARD DEVIATION) DUE TO ESTIMATED SED AND CROSS-SECTION UNCERTAINTIES FOR THE HEATING IN THE TF COIL (part 1)

|  | ction <br> Zone | SED Uncertainti $\left[\frac{\Delta R}{R}\right]_{\text {Mat ,region }}$ | $\begin{aligned} & \text { ies in } \% \\ & {\left[\frac{\Lambda R}{R}\right]_{\text {Mat }}^{*}} \end{aligned}$ | XS Uncertainti $\left[\frac{\Delta R}{R}\right]_{\text {Mat, region }}$ | $\begin{aligned} & \text { es in } \% \\ & {\left[\frac{\Delta R}{R}\right]_{\text {Mat }}^{*}} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cr | SS316 | 3.8 | 4.9 | 60.0 | 96.7 |
|  | TFCOIL | 0.2 |  | 34.5 |  |
|  | SS304 | 0.1 |  | 4.5 |  |
|  | SS312 | 0.0 |  | 1.1 |  |
|  | ISDLC | 0.2 |  | 2.2 |  |
|  | ISDLB | 0.8 |  | 33.3 |  |
|  | ISDLA | 3.0 |  | 58.5 |  |
| Fe | SS316 | 14.8 | 18.4 | 18.9 | 47.3 |
|  | TFCOIL | 0.1 |  | 10.4 |  |
|  | SS304 | 0.0 |  | 2.2 |  |
|  | SS312 | 0.2 |  | 0.7 |  |
|  | ISDLC | 0.5 |  | 4.4 |  |
|  | ISDLB | 2.7 |  | 23.6 |  |
|  | ISDLA | 10.8 |  | 34.5 |  |
| Ni | SS316 | 1.5 | 4.3 | 18.6 | 31.4 |
|  | TFCOIL | 0.7 |  | 11.8 |  |
|  | SS304 | 0.0 |  | 0.9 |  |
|  | SS312 | 0.0 |  | 0.4 |  |
|  | ISDLC | 0.0 |  | 1.3 |  |
|  | ISDLB | 0.4 |  | 13.4 |  |
|  | ISDLA | 1.2 |  | 18.0 |  |

TABLE XVII: PREDICTED UNCERTAINTIES (STANDARD DEVIATION) DUE TO ESTIMATED SED AND CROSS-SECTION UNCERTAINTIES FOR THE HEATING IN THE TF-COIL (part 2)


Total uncertainty due to cross-section uncertainties and SEDs $=114.6 \%$ *

* Quadratic Sums

TABLE XVIII: PREDICTED SED AND CROSS-SECTION UNCERTAINTIES IN THE TF COIL DUE TO UNCERTAINTIES IN THE SS316 ZONES

| Cross Section Material Zone | SED Uncertainti $\left[\frac{\Delta R}{R}\right]_{\text {Mat, region }}$ | $\begin{aligned} & \text { ies in \% } \\ & {\left[\frac{\Delta R}{R}\right]_{\text {Mat }}^{\frac{t}{n}}} \end{aligned}$ | XS Uncertainti $\left[\frac{\Delta R}{R}\right]_{\text {Mat, region }}$ | $\begin{aligned} & \text { es in \% } \\ & {\left[\frac{\Delta R}{R}\right]_{\text {Mat }}^{=-\%}} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Cr} \quad 1$ | 0.1 | 3.8 | 12.0 | 60.0 |
| 3 | 0.7 |  | 45.5 |  |
| 7 | 0.0 |  | 0.8 |  |
| 11 | 3.0 |  | 4.3 |  |
| 12 | 0.0 |  | 0.4 |  |
| Fe | 0.5 | 14.8 | 2.3 | 18.9 |
|  | 3.1 |  | 11.3 |  |
|  | 0.1 |  | 0.7 |  |
|  | 11.0 |  | 4.3 |  |
|  | 0.1 |  | 0.3 |  |
| Ni $\begin{array}{rr} \\ & 1 \\ & 3 \\ & 7 \\ & 11 \\ & 12\end{array}$ | 0.0 | 1.5 | 3.6 | 18.6 |
|  | 0.3 |  | 12.8 |  |
|  | 0.0 |  | 0.3 |  |
|  | 1.2 |  | 1.8 |  |
|  | 0.0 |  | 1.1 |  |

of the covariance data for Cr is highly recommended. If new covariance data would not reduce the predicted uncertainty, new experiments for measuring the Cr cross sections are suggested. The conclusions drawn here are consistent with an earlier study of a similar design. 45

The SED uncertainties, although less relevant to overall predicted uncertainty, tend to become more important in the outboard shield (region 11 in Table XVIII). An explanation for this behavior is related with the fact that the heating in the TF coil will be very sensitive to backscattering in this region. An SAD (secondary angular distribution) sensitivity and uncertainty analysis might lead to very interesting results.

The $X$ 's for the region near to the plasma in the outboard shield are calculated for each group based on angular fluxes and based on flux moments (Table XIX). Both methods lead generally to the same $X$ 's. The difference for the upper neutron groups might indicate that a thirdorder spherical harmonics expansion of the angular flux tends to become inadequate, due to the peaked shape of the angular flux close to the source region. In this particular study no serious error in the calculation of the uncertainties would have been introduced if the loss term of the sensitivity profile would have been calculated from flux moments. For a situation where the angular flux would have a pronounced peaked behavior, it would be highly desirable to evaluate the $X$ 's based on angular fluxes.

It is obvious from Table XIX that some fluxes in the lower gammaray groups (groups 41 and 42 ) are negative. Since only neutron sensitívity profiles are utilized to calculate uncertainties, this will not affect the results.

### 6.3 Comparison of the Two-Dimensional Model with a One-Dimensional Representation

The results obtained from the two-dimensional sensitivity and uncertainty analysis will be compared with those of a one-dimensional analysis in selected regions (Table XX). The uncertainties in the heating in the TF coil due to the uncertainties in the $\mathrm{Cr}, \mathrm{Ni}$, and Fe crosssections and secondary energy distributions will be calculated with ONEDANT and SENSIT in zone 1 and zone 3 (Fig. 12). The one-dimensional model for ONEDANT is straightforward. The total heating calculated in the TF-coil is $1043 \times 10^{-6} \mathrm{MW}$ (compared to $823 \times 10^{-6} \mathrm{MW}$ for the twodimensional model). In this comparison the uncertainties calculated by SENSIT will be normalized to the response calculated in the two-dimensional model.

It can be concluded from Table XII that the calculated uncertainties agree reasonably well for zone 3. There are substantial differences for the results in zone l. The reason for those differences is probably related with the fact that the one-dimensional model is not adequate for calculating the overall heating in the TF coil (especially

TABLE XIX：COMPARISON BETNEEN THE $\mathcal{X}$＇s CALCULATED FROM ANGULAR FLUXES（UPPER PART）AND THE $X$＇s RESULTING FROM FLUX MOMENTS（LOWER PART）FOR PEGION 11 （SS316）${ }^{\text {a }}$

```
* * tESt PMRINTOUT FOR thE Chi's * * *
*+&K= 1++*
    0. .13195E-05 .17957E-06 .67647E-07 .86778E-07 .79271E-07
    .21326E-06 .19166E-06 .32175E-06
    .22361E-05 .14093E-05
    .8Є888E-08 .68189E-08
    .4943ЗE-09 .26407E-09
    .24385E-10 .71906E-10
    .¿ぞ\53E-16 - 51897E-23 -. 26794E-48
```

    \(\rightarrow+\) CHI'S GENEMATED FMOM FLUX MOMENTS \(\rightarrow \omega \rightarrow\)
    \(\bullet \bullet\) TEST PMINTOUT FOR THE CHI'S \(\bullet \bullet \bullet\)
    $\rightarrow+\infty x=1+\infty+$
0.0
.21318E-06
. 223E6E-05
- 86885E-08
$.49424 E-09$
.26401E-09
$.24438 \mathrm{E}-10$. $71918 \mathrm{E}-10$
. $13135 \mathrm{E}-10$.29048E-11 .18631E-12

| 10555E－05 | ． 17003 E －U6 | ．59837E－07 | ．81854E－07 | －78798E－07 |
| :---: | :---: | :---: | :---: | :---: |
| 06 | ．32167E－U6 | ． 40669 E －06 | ．48798E－06 | 15こ9こe－05 |
| 14096E－05 | ． $18630 \mathrm{E}-\mathrm{UG}$ | ． 37797 E －06 | ． 107888 －06 | －17605E－07 |
| 8187E－08 | ．28273E－U8 | ．17886E－08 | ．12638E－08 | ．54232e－09 |
| 01E－09 | ． 12118 E －U9 | ．48553E－10 | ． $16304 \mathrm{E}-10$ | 4－171E－11 |
| 8E－10 | ． 12068 E －US | ．66705e－10 | 49849E－10 | 26 |
| 048E－11 | ． $18631 \mathrm{E}-12$ | ． 2624 OE |  |  |

${ }^{\text {a }}$ The $X$＇s are ordered by group（high neutron energy to low neutron energy；high gamma－ray energy to low gamma－ray energy）

TABLE XX: PREDICTED UNCERTAINTIES (STANDARD LEVIATION) DUE TO ESTIMATED SED AND CROSS-SECTION UNCERTAINTIES IN ZONES 1 AND 3 FOR THE HEATING IN THE TF-COIL

| Cross Section Material Zone |  | SED Uncertainties in \%$\left[\frac{\Delta R}{R}\right]_{\text {Mat }} \underset{\text { l-D }}{ } \quad\left[\frac{\Delta R}{R}\right]_{\text {Mat }}^{ \pm}$ |  | XS Uncertainties in \%$\left.\left[\frac{\Delta R}{R}\right]_{\text {Mat, zone }}^{1-D}<\frac{\Delta R}{R}\right]_{\text {Mat }}^{2-D}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cr | 1 | 0.1 | 0.1 | 29.3 | 12.0 |
|  | 2 | 0.6 | 0.7 | 44.8 | 42.5 |
| Fe | 1 | 0.8 | 0.5 | 4.5 | 2.3 |
|  | 3 | 2.6 | 3.1 | 9.6 | 11.3 |
| Ni | 1 | 0.0 | 0.0 | 8.3 | 3.6 |
|  | 3 | 0.2 | 0.3 | 13.1 | 12.8 |

the source region is poorly simulated in the one-dimensional representation). A more relevant sensitivity analysis would be to consider the heating calculated at the hottest spot in the TF coil. The hottest spot is in the center plane of the toroid. We would expect that the onedimensional model would be an adequate representation in this case.

## 7. CONCLUSIONS AND RECOMMENDATIONS

Expressions for a two-dimensional SED (secondary energy distribution) and cross-section sensitivity and uncertainty analysis were developed. This izzeury was implemented by developing a two-dimensional sensitivity and uncertainty analysis code SENSIT-2D. SENSIT-2D has a design capability and has the option to calculate sensitivities and uncertainties with respect to the response function itself. A rigorous comparison between a one-dimensional and a two-dimensional analysis for a problem which is one-dimensional from the neutronics point of view, indicates that SENSIT-2D performs as intended. Algorithms for calculating the angular source distribution sensitivity and secondary angular distribution sensitivity and uncertainty are explained.

The analysis of the FED (fusion engineering device) inboard shield indicates that, although the calculated uncertainties in the 2-D model are of the same order of magnitude as those resulting from the $1-\mathrm{D}$ model, there might be severe differences. This does not necessarily imply that the overall conclusions from a $1-\mathrm{D}$ study would not be valuable. The more complex the geometry, the more compulsory a 2-D analysis becomes.

The most serious source of discrepancies between a l-D and a 2-D study are related to the difficulty of describing a complex geometry adequately in a one-dimensional model. However, several neutronics related aspects might introduce differences. The use of different quadrature sets - especially when streaming might be involved - could lead to different results. When the angular fluxes have a pronounced peaked behavior, the angular flux option for calculating the loss term of the sensitivity profile will provide a better answer than the flux moment option. The different sweeping algorithms and code characteristics used by the $1-D$ and $2-D$ transport codes might be another cause of discrepancies in the results. Needless to say, a meaningful transport calculation is compulsory in order to obtain reliable results from a sensitivity and uncertainty analysis.

The results from the FED study suggest that the SED uncertainties tend to be smaller than those generated by cross-section uncertainties. It has been pointed out ${ }^{45}$ that, because all secondary particle production processes for a particular element are presently treated as one single process, the simplicity of the hot-cold concept for SED sensitivity might mask several causes of a larger uncertainty than calculated by SENSIT or SENSIT-2D. A more elaborate algorithm for a SED analysis, as an alternative to the hot-cold concept, a separate treatment for the various particle production processes involved, or a combination of both, would eliminate this deficiency. Even with the hot-cold model, which might underestimate SED uncertainties, the SEDs might become the dominant cause of the calculated uncertainty in the case that the
response function is a threshold reaction or in the case that backscattering becomes important. In this latter situation, an SAD (secondary angular distribution) analysis might also contribute significantly to the overall uncertainty estimate. At present, the required crosssection data are not arranged in the proper format to do this type of study.

Sensitivity and uncertainty analysis estimates the uncertainty to a calculated response. It would be more meaningful to be able to implement those uncertainties with a confidence level. In order to do this, we have to know how reliable the covariance data are, what the effects of errors resulting from the transport calculations will be, and what the limits of first-order perturbation theory are. It was assumed in this study that the uncertainties, resulting from uncertainties in different regions, were either fully correlated or not correlated at all, depending on whether these regions have the same or a different material constituency. The evaluation of reliable correlation coefficients would be a major effort by itself.

The validity of an uncertainty analysis is often limited more due to the lack of the proper cross-section covariance data, than due to the lack of representative mathematical formalisms. Covariance data for several materials are still missing, or just guesstimates (e.g., Cu) ${ }^{33}$. The fractional uncertainties required for an SED analysis are evaluated for just a few materials and are not available for the various individuai particle production processes.

The current version of SENSIT-2D cannot yet access all the covariance data available in COVFILS, ${ }^{33}$ but will be able to do so in the future. Even when SENSIT-2D does not require a lot of computing time, the extra amount of c.p.u. time required by the adjoint TRIDENT-CTR run makes a two-dimensional sensitivity and uncertainty analysis demanding when it comes to computer resources. The development and implementation of acceleration methods for TRIDENT-CTR are therefore desirable. A sensitivity analysis involves a tremendous amount of data management. A mechanization of the various steps required, by the development of an interactive systems code, would provide a more elegant procedure for sensitivity and uncertainty analysis.

The algorithms to perform a higher order sensitivity analysis have been developed, but are still too complicated to be built into a computer program for general applicability. The increasing number of transport equations to be solved prohibits the incorporation of present higher order sensitivity schemes in a two-dimensional code. An effort to develop simple algorithms for higher order sensitivity can certainly be justified, however.

It becomes obvious that several flaws can be found in the state of the art of sensitivity and uncertainty analysis. Removing any one of them would require a major commitment.

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## APPENDIX A <br> SENSIT-2D SOURCE CODE LISTING

In this appendix a source listing of the SENSIT-2D code is reproduced. The source listing is documented by many comments.

A source listing of the SENSIT-2D code can also be obtained from the NMFECC by typing the command

FILEM\$READ 5043 .SENS2D SSSS\$END

```
Los Alamos Identification No. LP-1390.
```



```
c
C TMIS is THE MAIN PANT OF THE PmOEmAM (SENSIT-ZD), NOV. 1 UEOSIDN
    level 2.le
    common ac(22000)
    COMMON /⿴LOT/ TITLE(B)
    COMMON/ITE/ITEST;ITYP
    COmman/COUANB/JCOUAM
    COmmaN/XSFOm/*K5,IMT,3m
    COmmON/UNS/LMAXD
    COMMON /LLC/ LC(40000)
    INTEGEREOEO
    CALL ECZEmO(LC)
C +OQ STANT MEALING COMTMOL mAMAMETENS
    NEAD(5,1010) (T1TLE(3),1=1,8)
1010 Fammat(8n10)
    HNITE(6,1U20) (TITLE(1),:1=1,8)
1020 Fammat(1m1,8a10)
        MEAE (5,1030) zTYF, MAXNAD, MNPE, MNEL,IO#EDIST, JTMAX,
    1
                                    IGH,NCOURLILMAX, ITEST:SIMAX
1030 FOmmat(12:6)
```



```
    1 DETCOUNNSEDIIDUTPUT,NSUMCON
        mEAD(5,1030) 1Cmimam, 10लT, 15TOP, 16EO, 2m-3
        WNITE (6,1(40) & TYPIMA,MWMDIMNPDIMNEL,IPNEN, IT, JTMAX,
    1
```



```
    O,2-vECTHA-xS,3-sEDE, 16x,1m=,84,1
        * MAXNND = MAXIMUM NUMAEN OF MOMDS DN NFILE }\times10000
        35x,1N=184,1
```



```
        * MNEL = MA%. MUMBEN OF ETA LEVELS*,50x,1ME,14,/
```



```
        1005,84%/
```



```
        - JTmax = maximum mumgen of TMINNGLES IN P,
        -aNY CNE SNWDO, डEX, 1mx,84,/
```



```
        * NCOURL = NUMDEN OF NEUTMON GMOURS IN CFL. CALC.' 2ENOQ,
        - Fom NEUTmONS ONLT*,1Sm,1m#,14,1
```



```
        - ITEST = TEST PNINTOUT FLNGI O-NDNE,I-XSIS-NONE@,
        \bullet,3-vECTOD-xs*,ご5x,1M=,14,"
```



```
c
        H#ITE(6,1050) 2XSTHPE,NPEAXS,IDES,KDZIKEZ,KXS,IMT,IMM,
        1 DETCONONSEV,IDUTPUTPNSUNCON
c
1050 FOWMAT(IM |OIXTAPE = EDUNCE OF INPUT COOSS-SECTIONS: 0-CAmDSIOP
```



```
        * NFEARS = NUMDEN OF SUCCESSIVE CASESD ALSO ND. OF iNPUTQ,
        * x5-sETS TO SE mEADQ,11\x,1m=,84,/
        * IDES = ASSUMED 1 PEN CENT DENSITY INCNEASE IN PERT.*,
        * 25.FDN DES.-SEN., OノI*NO/YES*,O1X,1m#,14/
        * KDz E NUMDEN OF DETECTDN zONESt,51x,1*NE,14,/
```




```
        3Ex,1N=.14,
        * INT = POITIDN DF TOTAL CNOSS SECTIDN IN XS-TABLES*,
        31x,1m=,14,
```



```
        *TABLES*,26x,1Hm,84%/
        * DETCOL = 0/1 = DD MOT/DD mEAD COUNmiANCE mATRIx FD**:
        * -(6)*, さ8x,1
        * NSED F 0/1 = DO NOT/DD mEAE INTEGOAL SEDEUWCEGTMIN也,
        *TES - E7x,1m=,84,/
        * IOUTPUT = OUT#UT #NINT DETAILE 0-SUM OUE# PENT. zONESPO
        - ONLYM 1- OLSD INDIV. PEPT. 25.*,01x,1m#,84!/
        - NSUNCOV N NO. OF EESP,-vANINCES SUMED FON ITYPE2,*,
```



```
    c
```



```
C
```



```
81 % 0
- NOCYES GNEI 0.2EX:1mE:24,'
```



```
        31x,1m=,84%,
```



```
        -0/1 NOCYES*IE2x,1mEOI4:/
```



```
        - IAR3 = 0,1 USE EXISTING SEO. MNG. FLUX FILET NOVYESQ,
    29^:1M*:14:ノ
C
C SET MOINTEMS FOD SUDMOUTIME EIND
C
    LE=1
    LDELU#LE*IGN+2
C
    CALL EBNE (AC (LDELU), AC (LE), IEM,NCOUNL)
C SET mOINTENS FON SUNmOUTINE GEOM
    LITZ = LDELU + IEM
    LIIT = LITZ + JT
    LNTEZ = LIIT + JT
    LNFZ = LNTRZ + JT*JZMAX
    LNDZ = LNPZ + KmZ
    LI\SigmaZ = LNEZ + KDZ
    LNFIDZ =.LIDZ + IT&JZMAX
    LNEIEZ = LNFIDZ + JT*JZmAX
    LIPELC = LNEIDZ * JT@JZMAX
    L&DELE\hat{E LIPELE * KOz}
    LPT = LIDELE + KDZ
    LまT = LeT + JTOK#z
    LKTP = LDT * JTGKDZ
    LKTD = LKTP + JTOKMz
    LKELEI = LKTD + JT*KDZ
    LKEL#2 = LKELO1 * sT*KEZ
    LKELDI = LKEL#2 + JT4KMz
    LNELDD = LKELDI * JT4KDZ
    LCOVN = L*ELEE + JTHnDZ
    LAST = LCOUM + IEN+IEm
    CALL EEOM(AC(LITZ),AC(LIIT),AC(LNTOZ),AC(LNPZ),AC(LNEZ), NC(LIDZ),
    1 ac(LN-1DD),AC(LNDIEZ),AC(LIDELE゙),AC(LIDELÉ),AC(LOT),
                                    AC(LDT):AC(L&TP),AC(LHTD:,AC(LMELP1),AC(LKELPE゙),
                                    AC(LKELE1), AC (LKELEL),JTOKPZ,KまZ,ITSUM)
c
C CALCULATE AUXzLANY UANIABLES
c
    KPz#= <*2%1
    Lmaxf = Lmax + 1
    LMAXE= Lmax t
    wax =0
    Da 110 1=1.LMMAX
    NM = NM - 2
    110 CONTIMUE
c
C SET DOINTEPS FOD sUPmOUTIME SNCON ANE SUDEOUTINE TMPAS
c
C
    MA=NRE=mAKHND*1000
    LKTAPELAST
    LMT = LNTMP + 16M*5 + 1
    LMME LHT + IEm
    LISN=LMM + IGM
    LNPD = LISN + 16M
    LNUP = LNHE + 4
    LAST = LNUP + MNEL$A
    3CE=1
    3CN=3CE + 44MMPIOPEM
    3LAST = ICM 44MNPOCIGM
    CALL SNCDM(LC(ICE),LC(zCM), AC (LHT),AC(LNUP), AC (LNEB),
    1
                AC (LMM), WNPO, WNEL,AC (LISN),IGM,10PT)
    CALL TADAS (AC (LKTMO), AC (LMM) ; NMO ITSUN: IGMPMAXMmD,
    1 AC (LKTO),AC(LKELP1),AC(LKEL-2),K#Z,JT)
c
C SET DOINTEOS FDN SUMMOUTIMES #NGEN OND FLUXMOM
```

| 161 162 | c | L＊－LNPE |
| :---: | :---: | :---: |
| 163 |  | Le＝L－＋MNPDQLmAxPtLmAx |
| 164 |  | LPMNI＝Ln＋4tMNPOCNM |
| 165 |  | LT＝LPHW 3 ＋MNPD |
| 166 |  | LAST＝LT＋ÉCLMAX＋ 1 |
| 167 |  |  |
| 168 |  | 1FFLUX＝3LAST |
| 169 |  | IFLUX＝8FFLUX＊JTMAXAMPE |
| 170 |  |  |
| 171 |  | IFMOM＝IFUX＋NH®JT®JTmAx |
| 172 |  | ILASTEIFMOM＋JT®JTMAX |
| 173 |  |  |
| 174 |  | IF（3PMEP．EE．1） 60 T0 140 |
| 175 |  | Do 130151.2 |
| 176 |  | 1F（1．ER．2）KAd＝1 |
| 177 |  | 30 120 cemi．1em |
| 178 |  | $c=c$ |
| 179 |  |  |
| 180 |  |  |
| 181 |  |  |
| 162 |  |  |
| 183 |  |  |
| 184 |  |  |
| 185 |  |  |
| 186 |  |  |
| 187 |  |  |
| 186 | 120 | CONT 3 NUE |
| 189 | 130 | CONT INUE |
| 190 | 140 | CONT 2 NUE |
| 191 c |  |  |
| 192 | C SET | －IINTEES FDE SEUNDOUTIME DETSEW |
| 193 C |  |  |
| 194 |  | LFSUME＝Leme |
| 195 |  | LPMIV＝LFSUM＊ICm |
| 196 |  | Lの＝LPmiv＊Jtmax |
| 197 |  |  |
| 198 |  | LSENP＝LZON＋KDZ |
| 159 |  | LSSENA＝LSENM＋ 26 M |
| 200 |  |  |
| 201 |  | LAST＝LSIGMn＋KDz＊16m |
| 202 |  |  |
| 203 |  |  |
| 204 | 1 |  |
| 205 | 2 |  |
| 206 |  | 3 AC（LEELU），AC（LE），NCOUPL，1 CEG） |
| 207 c |  |  |
|  | C set | －Dintems Fon cmi＇s and msi＇s |
| 209 C |  |  |
| 210 |  | L1Psi＝Late |
| 211 |  | LCMI＝LiPsi＋1Gmoxpzo |
| 212 |  |  |
| 213 |  | LAST1＝LCCMz＋ 8 cm |
| 214 |  |  |
| 215 |  | IF（iCmimam．ED．1）©0 To 145 |
| E16 | C set | EDINTEFS FOE sunmautime CMIs |
| 217 （1） |  |  |
| 218 |  | iflux＝ 1 |
| 219 |  | IAFFLUX $=1$ iFLUX + maresstanx |
| 220 |  | zafluy $=$ imflux + mNPDestmax |
| 221 |  | ILAST＝IAFLUX＋MnPotitmax |
| こど |  |  |
| 2ES C CALCULATE TME CMI＇s |  |  |
| 224 |  |  |
| 225 |  |  |
| 226 | 1 |  |
| 227 | $E$ |  |
| 208 c |  |  |
| EE9 | C SET | POINTENS FOM susmoutime Paintus |
| 230 | c |  |
| 231 | 145 | LImmm Lasti |
| 232 |  |  |
| 233 |  | L－si＝LiLP＋ЈT＊＊－z |
| 234 |  | LFPsimLPsi＋Lmaxpeismempzo |
| 235 |  | LAST＝LPPSI＋LMAx＠＋${ }^{\text {cem }}$ |
| 236 |  |  |
| 237 | c sey | Maintems Fom susmoutine msis |
| 236 |  |  |
| 239 |  |  |
| 240 |  | ifflux＝ 2 sum +1 |


| 241 242 | IFAFLUE E IFFLUE + ITSUAANM <br> ILAST＝IFAFLUX + ITSUM＊＊ |  |
| :---: | :---: | :---: |
| 243 |  |  |
| 244 | C CALCULATE TWE OSI＇s mod stome im LCM |  |
| 245 |  |  |
| 246 | 2 |  |
| 247 |  |  |
| 248 | 3 |  |
| 249 |  |  |
| 250 | IF（15TOP．EE，1）stam |  |
| 251 |  |  |
| 252 | C CLEAO MPPmopeinte scm mid LCM space |  |
| 253 | INELEI＝ 1 Sum＋1 |  |
| 254 | NELFCEILAST |  |
| ごち |  |  |
| 256 | 155 | LC（1HEL的）$=0.0$ |
| 257 |  | IMEL－1 $=$ LAST1 |
| 258 | 1 MELP2xLAst |  |
| 259 |  |  |
| 260 | 157 | AC（IMEL－）$=0.0$ |
| 261 |  |  |
| 262 | $c \quad c$ | CALL SUEMOUTIMES TO EEAD IN ANL／OR CALCULATE UALUES OF Cmoss |
| 263 | $c$ | SECTIONS |
| 264 |  | NXS $=0$ |
| E05 | c $+\infty$ |  THE CODE DOMNCMES WEDE IWTO TMO DIFFEDENT EXECUTION MODES |
| 26E | c $\rightarrow$＋ |  |
| E¢7 |  | 1F（iTYP．EE，2）¢0 T0 290 |
| 268 | c |  |
| Ees |  |  |
| ごフ | $c+\infty$ | An\＃AYS GMED MWt FSED FOM MLL NEUTODN EMOUPS |
| e71 | 150 |  |
| 272 | 1F（NSEと，EA，O）60 To 170 |  |
| 273 |  |  |
| 274 |  | IF（NCOUPL，NE． 0 ）1 Emi＝NCOUPL |
| ご5 | C SET | －DIMTEPS FOE GMED MWE FSED |
| C76 |  | LGMEL－LAST1 |
| 277 |  | LFSED＝LGAED＋ 8 Eml |
| 278 |  | LAST1 LFSED＋86m1 |
| 279 |  |  |
| 280 |  |  |
| 281 | $10 \pm 0$ | Fommat（ 1216 ） |
| 282 |  |  |
| 283 | 1070 | FOmmAT（6E12．5） |
| 284 |  | Weite（6，1080） |
| こ8き | 1080 |  |
| 286 |  | 1 －UNCEFTAINTIES（FSEE）INPUT FOA SED UNCE日T，MNALTSIS＊ |
| 287 |  |  |
| 288 |  | D0 160 1 21.26 ml |
| 289 |  | W－1TE（6，1090）1，AC（LGMED－1＋1），AC（LFSED－1＋1） |
| こ90 | 1090 |  |
| 291 | $1 \in 0$ | CONTINUE |
| こ92 | $c \rightarrow \infty$ | END OF SED－UNCEFTAINTY INPUT MND ONIMT |
| 293 | c |  |
| 294 | 170 | COnTINUE |
| こ5． | c calc | CULATE AUxILAmy umminales |
| 296 |  | 1TL $=16 \mathrm{H}+1 \mathrm{mT}$ |
| 297 |  | NWS＝1TL＊IEAMLmAxP |
| 296 |  | NHL＝16M＋1TL |
| こ99 |  |  |
| 300 | C SET | POINTEFS FOM DSLED NWD DSL（LCM）IISLFD |
| 301 |  |  |
| 302 |  | $\begin{aligned} & \text { IISLFD IISUM \& } 1 \\ & \text { IथSL }=\text { IDSLFD \& NaNK } \end{aligned}$ |
| 303 | C sET LCM－mointems Fom CROSS sECTIDNS |  |
| 304 |  |  |
| 305 |  |  |
| 306 |  | 1F（Wxs．EE． 2 ） $1 \times 51$ \＃ $1 \times 5+0 \mathrm{~ms}$ |
| 307 |  | ILAST＝ixsi＋m |
| 308 |  |  |
| 309 |  | W－ITE（6，1100）NXSI，NPE日xs |
| 310 | 1100 |  |
| 311 |  | 1 －CAsEs＊） |
| 312 |  |  |
| 313 | C stit | POINTEOS FOO SECDWป CAOSS SECTIDN SET |
| 314 |  | 2xs3an $=3 \times 5$ |
| 315 |  | 2F（3TYP．NE．1）EOTO 180 |
| 316 |  | 1F（1DES．EE．1） 60 TO 180 |
| 317 |  | 2xsen＝ $2 \times 5$－mms |
| 318 |  | 2xs |
| 319 |  |  |
| 3E0 |  | ILAST $=2 \times 51$＋ |



```
482
483
485
4E6
4 8 7 ,
4E6 C
489C
4 9 1 \mathrm { C }
4 9 2 ~ C
4 9 3
4 9 4 C
445
4 9 8
499 c
501
502
503 TIMENSION DELU(1),E (1)
```



```
506 IF(NCOUPL,NE,O) IGMO1=NCOUPL+
508
509
5111
512
$13
516
518
519
$520
522
EE3
525
E34 150 CONTINLE 
```



```
E36 4E0 FOMmAT(1m,14,
```



```
    E3S 4SO FOMmAT(Im:10(ix,IFEIO.3))
    450 FOmmAT(1m, 1O(1x, 1PE\ (0.3))
    450 FOMmAT(1m.10(1x,1PEIO.3))
```



```
    #ETUNO
    END
C
C
C EEDOM MEADS MNE EDITS TME GEDMETGY FOO PEOTUMDED NWD DETECTOM zONES
GE{
S52
553 c
110 CONTINUE
```



```
    IF (NCDUPL.EE.O) WNEUTFIEM
    IF (NCOUPL. NE. U) WNEUTENCOUPL
    z0 1こ0 E=1%mmEUT
    EDUCz* E(E)/E(E*1)
    DELU(G) = MLDG(EDUAZ)
    120 cONTINUE
    iF(NCOUNL.EE, O) E0 TO 150
    DO 130 E= z6m-1,Iem
    EDUOZ=E(E+1)/E(E+で)
    上ELU(E)=ALOG (E|UOZ)
130 CONTIMUE
    NHITE (6:460)
    20 140 6=1.2 cm
    MmITE(6,470) 6,DELU(6)
    140 camtinue
    150 CONTINUE
```




```
    l ILELE,FT,DTOWTPHん
c
```





```
\ 2 DT(JT,1),KTO(JT,1),WTD(JT,1),KELOI(JT,1),KELPE゙(JT,1),
5E C
55S C * * quTPUT COMNENTS * * *
560 c
EINL WEALS IN WEUTBON MWD EAMOA BAY STMUCTUPE AWD CALCULATES LETOANGT
    WIDTMS PE希 EmOUO
    SUBGOUTIME EDNE(DELUPE, IGMPNCOUPL)
C * | INPUT CDMNENTS * * 
```



```
* * DUTOUT COMNENTS * * *
            DELU(コ) - LETMAmEY MzDTMS
        INTEGEG 6
```



```
    W@ITE (6,420) 1 0mpl
    umitE (6,410) (E (1), i=1, 1 cmp 1)
    IFINCOUOL.EE.OS EO TO 110
    13लF2 = 1%M + 2
    NCP2 = NCDUPL + 2
    NGAMP1 = 1GM - NCGUNL + 1
    mEND (5,430) (E,I), %*NCPE, zEMPC)
    WOITE(6,440) NEANOI
```



```
    410 FOmmAT(1m 1 10(1x,1PE10.3))
    OETUN
```



```
561
C NDIDZ(J,K) - IDENTIFIES LETECTOD ZONE E FON IOND I
5 6 2
563 c
5 6 4
$65 c
566 c
567 c
568 c
509 C
570
571
572 c
575 c
O
**
```



```
    1PELE(N) - FEFT. 2ONE K SMONS UP IN IDELE(N) INWDS
```



```
    DT(JJOW) - DET. ZONE SMOMS UP IN THE INWD DT(N,1) . - -
    MTR(J,K) - IS FEFT. ZONE MOESENT IN ENND J T 0/1 MOMYES
    *TE(J,K) - IS EET. ZONE N POESENT IM DNOL J ? U/I MOCYES
```



```
    *ELP2(J,*) - PERT. zONE K IN BANL JENES MITM TGI. KELPE
```



```
    WELD己(J,*) - DET, ZONE * IM BMND JENDS MITM TOI. WELEE
* & INPUT COMNENTS * * * 
```




```
    IDZ(J,IZ) - ZQNE IDENTIFICATIONFON TOE IZ'TM ZONE IN INNE J
    ITz(g) - : zONES IN INND J
```



```
    KDZ - & DETECTOD ZONES
    NPZ(NPZ) - FERTU#DEZ ZONE IDENTIFICATIGNFODKPZ'TM #E#T. ZONE
    NZZ(以DZ) - LETECTON ZONE IDENTIFICNTIGNFOD KとZ'TM DET. ZONE
    ITEST - DETAILED DUTPUT EESIMED? 0/ET.O NOUYES
    JT - DEYARLEANDS
OEAD IN E ZONES FOD EACM ENWD ITZ, E TGIANELES FOM EACM EANV IIT
```



```
WEAS IN IONE IDENTIFICATIDWS IDZ
    ITsum=0
    DO 110 j=1.jT
    GEAD(5,40き) 2TZ(J):2IT(J)
    &TSUM=\SigmaTSUN+82T(J)
    I2F1T2(J)
    mEAz(5,403)(NTPZ(s,z),1=1,12)
```



```
    110 COMTINUE
```



```
#EAL &ETECTOD zONE IDENTIFICNTIDNE NDZ
    GEAD(5,403)(NDz(1z):12*1, <-z)
    mEAg(5.403)(Nzz(1z),1z=10*zz)
```



```
    y0 120 w=1.*-z
    z\mp@code{#LE(*)=0}0
    Do 120 j=1.sT
    *TP(J.*)=0
    120 cONTINUS
    DO 125 W=1.*Dz
    2DELS (m)=0
    125 CDNTINUE
        DO 210 J=1.jT
        JJ=1TZ(د)
        20 130 12=1%ココ
    NF12Z(J,12)#0
    130 ND:Dz(J.:z)=0
    m0 160 12=1,J.3
    20 150 k=1.**Z
    NOENOZ (m)
    IF(zDz(J,&z).NE.NW) EO T0 1SO
    1F(NTP(J,*).NE.0) E0 To 14U
    1FELE (x)=2 =ELE゙}(x)*
```



```
    140 NPIDz(J, &2) ##N
    KT# (J,K)=1
    150 CONTINUE
    160 contiance
        DO 170 k:1.<D2
        <TD(J,K)=0
    170 CONTINUK
        D0 200 :2F1:3.3
        DO 190 k=1.*DZ
        ND#NDZ (*)
        IF(1DZ(3,12).NE.ND) E0 TO 1YO
        8F(KTD(J,K).NE.O) 60 T0 180
        1DELE゙(k)=1DELE(k)+1
    DT (IDELE(m):以)=3
    180 NDIDz(3,IZ)EW
        *TD(J,*)=1
    190 EDNTINUE
    %00 COMTImul
    210 cOMT HONE
C
```

```
41
&i\hat{C}
*S Do 280 JE1,دT
```



```
645 1F(MTP(I,12).EE.O) 60 TO <4O
646 *ELF1(J,12)=1
S47 KELP2(1,12)=0
648
648
649
650
651
GE3
S
##Nam
M&1TE(6,501) WELP1(J,12),KELPC(I,12),1,12
C #mWO END mEMOVING
    E40 CONT&NGE
    Da 270 12=1,kDz
    8F(NTD(J,12).EQ.O) GO TO E7O
    1F(NTD(J,12).E
        MELD1(J,12)=1
        MELDE(1,12)=0
        DO 2*O z=1,MEL#4
        MELP3*NTEz(S,I)
        IF(NDIDZ(I,I).EE.IZ) GO Ta EbU
        KELDI(S,12)=KELDI(J,12)+ WELPS
    ESO CONTINU
```



```
    C mmN= mEMGUE NEXT CAND IN PmOEmON
        WmiTE(6,50E) KELDI(s,12),WFLDE(土,12),,1,12
C ENZ EEMOUZNS
    ci
    2GO CONTIMUE
c
C EDITING
    MW ITE (6,410)
    WWITE(6.410)
    DO C90 J=1,3T
    z2=itz(J)
    WM1TEN(6,409) J
    WWITE(6,408)
    NEITE(6,405)(1D2(1,1),1:1,12)
    W#ITE (6.408)
    WW1TE (6,4(14) (NTMZ(S,1),1:1,12)
    WBITE (6,4(16) (NPIDZ(S,1),1=1,12)
    WMITE(G:4(17)(NDIDZ(N:1):1=1:12)
    WEITE,6.410)
    290 cONTINUE
    HWITE (6,410
    WEITE(6,411)
    DO 310 k=1,x-z
    IPEL=IPEL2(K)
    HEITE(6,413) (PT(J,K),g=1,10EL)
    310 CONTINUE
        WNITE (6.410)
        WOITE(6,415)
        DO 320 N=1.kDz
        1שEL=IDELS(*)
        WNITE(6,414) K,IDEL
        NAITE(6,413)(DT(J,K),J=1,8DEL)
    3EO cONTzMUE
C
mamm memove FOLLDNING COMDS IN ACTUNL Pmogmaw
    H0ITE(6,410)
    HEITE(6,500)
    D 330 j=1:3T
```



```
    330 MWITE (6,504)(NTD(J;K),N=1;KD2)
    GUO FOMMAT(Im ,4ON* * TOIMNGLE INFD FDN PENT ZONES * * *)
```





```
    SO4 Fgmmat(im ,\tilde{Cux, SmmTD,Ex,1210)}
    ##N- END MEMOUZNG
    401 Fammat (16)
    40゙̇ Fammat(2:6
    40S Fommat (1E:6)
    404 Fammt(1m, 8x,11N= T#ImNGLES,15(16,1x))
```

```
801
802
803
804
805
805
806
808
810
811
811
813
814
815
817
618
819
820
821
822
823 c
824 C EDIT SENSITIVITY PMOFILES SUNMEV OUEW MLL DET. ZDNES
825
825
827
829
830
831
832
833
834
835
836
837
838
840 C EDIT SENSITIUITT mmOFILES FDW INDIVILUML zONES
840
841
843
844
845
8 4 7
848
849
850
8E1
852
853
e54
855
860 410 FOmmAT (6E12.5)
```



```
    SSIGNA(N,G) I GPOUP FOD EETECTON 2ONE E © {6)
    430 Fammat(5m = i3. 3x,1-E1c.S)
    440 Fammat(1m 0,0Bn10.<)
```



```
    SE FUNCTION (E) *, 25(1~*))
```








```
    510 Fgmmar (1m, ड̇x,\bullet----------*)
```



```
    Gご FOMmATI4Gm * * WESPONSE D, GEOUP ANE zETECTDP 2ONE * * *)
```



```
    1ご復12.5)
    360 WETUMN
    MGO NET
80 C
```

```
881 c
&EE C sUDPDUTINE TAPAS ASSIGNS LDEICAL UNITS TO THE ANEULAN FLUXES MOL TME
EE二 C FLUY MOMENTS
854C
685
8&6
88:
88G
8EG
850 c
```



```
B9icc ITSUM NM FOTML NUMEENOFT T
```



```
894
895 c * - DUTPuT cOmAENTS * * *
896 
896 C M C MEE.
898
899 C
900 c
901 c
903
904
905
906
906
908
9 0 9
910
911
912
913
913
914
915
916
917
918
9189
920
921
9E2
923
9こ4
9E4
926
928
928

```

    LलsT = <TAN (ミ, I*M) +1
    I sum=0
    DO 230 6= 1.8%M
    EG=15m-8+1
    ANGMON=ITSUM@MM(GE)
    IF (ANGMOD.GT.MAMMMD) 50 TO E40
    I&UN = ISUN+MNGMOD
    IF(isu*.LT.FANK*RD) 60 Ta 21U
    I5UN工のNGMOR
    *TAP(3.GE) =LAST+1
    60 T0 E20
    210 *TA0(j.66)=LAET
    220 LAsT##TA0(3,E\epsilon)
    230 cOMTINUE
    *a Ta 250
    240 WもITE(6,4た0)
    STO*
    ぞ0 LAST*@TMP(3,1) + 1
    Da 290 1*1.E
    iF(z.EE.2) LAST = MNT(4,icm)+1
    IP=34%
    15UM=0
    1PLDF=0
    DO 255 j=1.jT
    的 255 j=1,JT
    IF(NTO(J,N).EE,0) &0 T0 EFS
    ```

```

    2#5 cOMTIMUE
    MDMNOCEIPLDF ENM
    1F (~ONNDO.ET.mAxMmD) Ea Ta z4U
    Dロ こ80 cG*1.1%M
    6##G
    IF(I.EE.2) क# IEm-s*+1
    Isum = isum + MCNWON
    IF(18UM.LT.MAXGMD) so T0 <60
    IsUM=MDN**OM
    *TAP(10,5)=L@sT+1
    6a T0 270
    #60 NTAN(IP,G) =LAST
    270 LASTwWTAP(ID,छ)
    280 CONTINUE
    290 CDNTINUE
    -1 FaNmAT(1き16)
    ```

```

        NETUMM
        END
        C
    957 c
gef
95E c
959 c
900
<TAD(5,1GN) = LOGICNL UNITS FOD FLUXES
NTAD(1,巨) = LDGICAL UNITS FG\# NNGULAN FLUXES

```

```

        MTAP(3,6) L LO&CML UNITS FOM AI,JOINT ANGULAD FLUMES-OMNzDM ACCESS
    ```

```

        <TAP(5,@) = LOGICAL UNIT FOP ALJDIMT FLUX MOMEMTS
    ```

```

    INTEEEO 6,#E
    z0 200 %*1.2
    3エUN=0
    c

```

    LEUEL こっCE,CN
    DIMENSIDN CM(MNPE,4,1), CE (MNPD,4,1),NT(1), WUP(MNEL,1),NEL (4),
    12ELS(8), SN(4):MPO(1), 15N(1),MM(1)
    DIMENSIDN U4(3), U6(6), U&(1u), U1U(15), v12(き1), ul4(28), u16(36)
    INTEGER
    Dnta v2/.5773503/
    dата U4/.868R903..3500212..3SUUE12/
```



```
    дATA U8,.GE.035NE,.606E570..8U65570..EE12958..5773503..5512958,
    1.1971380..E133981,.2133981..19%1580/
```



```
    1.4567576..4&97749:.4897745..456/576..1631468..1755273..1755č73.
    2.1755こ73..1631408/
    zATA U12ノ.9E10344..908uSĖ..Yu&u522..7827706..8030727..7827706,
    1.6(140252..6400755,.6400755..6U4UES2..3511744..4\overline{13515.,4249785,}
    2.4c113515,.3911744,.1370611%.14y1456..1497456..1497456..1457456,
    3.1370611/
    DATA U14/.9855865.,9314035..9514035..8362916..8521252,.8362916,
    1.7010923..7324250..7324そ50..7u1UYغ3..5326134.,5691823..5773503,
```



```
    3.3359i38..1196230%.1301510..1541510%.1301510..1301510..1301510,
    4.1196230 /
    gnta u16/.9589102,.5464165,.4464163,.8727534,.8e55877,.8727534,
    1.7657351,.7925089,.7925089,.7657351,.6327389,.6666774,.6752671,
```



```
    z.666t.774,.65c7369,.4743525,.S1U\519,. S21,431,.5215431,.51 07319,
```



```
    5. 1158880%.1050159/
    z^Tの SN/1.0.-1.0.-1.0.1.0/
C
C
    #EME(5,440) (2SN(0), E=1, % EM)
    Do 350 *=1, iGm
    120 MEL=85m(6)/?
```



```
    130 DO 140 L=1,4
        MPE(L)=1
        CE(1,L,E)=Sm(L)*N2
        NT(ङ)=0.2S
    140 cONTINUE
    #0 Ta 290
150 DO 160L=1.4
    NPD(L)=3
    Do 160m*1,3
    CE(m,L,G) =SN(L)*U4(m)
    WT(\sigma) =0.06333333
    160 CDNTIMNK
    C口 Ta 290
    170 DO 180 L*1.4
    NPD(L)=6
    20 180 m=1.6.
    CE(M,L,G)=SN(L)*U6 (m)
    WT(E) =0.04166667
```



```
    *0 T0 #90
    190 DO 200 L=1.4
    MFO(L)=10
    DO 200 m=1.10
    CE(NOL,E)=SN(L)*UB (m)
    由T(ब)=.025
    200 CONTIANL
    60 T0 290
    210 Do 220L*1.4
        NOO(L)=15
        DO 220 H=1.15
        CE(MOL,G) =SN(L)OU10(M)
        4T(6)=0.01666667
    220 cowTINuT
    & T0 290
    230 DO E40 L=1.4
        NPF(L)=21
        ma c40 m=1,21
        CE(MPL,G)=sm(L)ON12(M)
        \omegaT(E)=0.01190476
```

```
240 continu{星
    250 DC E60 L=1.4
    NPD(L)=ç8
    DO 260 m=1.28
    CE(m,L,G)=sm(L)*U14(m)
    uT(\sigma)=0.008928571
    260 cONT IMUE
    co To 290
    2TO DO 280 L=1.4
        NPD(L)=36
        D0 c'80 *E1.36
        CE(MOLOG)ESN(L)OU16(M)
        wr(\sigma)=0.006944444
    280 CONTINUE
    290 Mm(E):=40NeC(1)
        DO 310 %L=1.MEL
        DO 300 LE1.4
    NUP(ILOL)=IL
    300 CONTINUE
        zELS(IL)=(1L-(IL-1))/2
    310 CONTINUE
        00 320 L=1:4
        NEL(L)EmEL
    320 CONTIMUR
        DO 330 iL=1,MEL
        ILI=mEL-8L
    DO 330 mp|l:IL
    MEIELS(IL)+MO
    H1=IELS(ILI +*O) ++\infty
    CM(m,4,*1E-CE(m1,*4,5)
    CM(m; 5, 6, =-CE(m1,4,5)
    M己=1ELS(IL)+IL-N-+1
    CM'mट,d,*)ECE(M1,4,5)
```



```
    330 contimue
```



```
    3F(1OPT.NE.1.AWD.IOPT.NE.3) E0 T0 350
    W#ITE (6,410) 6, ISN(E)
    DO 340LE 1. 4
    WOITE (6,420) L
    MPD=NE(L)
    D0 340m=10 mpe
    MOITE (6,430) M,CM(M,L,E),CE(WOLOE),MT(6)
    340 cONTINUE
    3*0 continue
```





```
    4ड0 FOmmAT (5x, 13. ЗЕ20.8)
    440 Fommer(12:6)
C
    GETUAM
    END
1095
1096 c
1097 C SUDPDUTINE FNEEN MAS SEEN COPIED MND MODIFIED FMOM TME TGIDENTOCTM
1096 C CODE.
109G C TWIS SUDMOUTINE EENEONTES SPWEPICAL MAMNONICS POLYMOMIMLS AND
```



```
1101 C NEJOImT FLUX-NOMENT CONSTMUCTION.
1102 C
1104
1104
1105
1106
1107
1108
1109
1210
1110
1111
1112 Dnracer/3.1415926/
1113
1115 C CMIMNPE:4,IGM) BuADmATURE MU*g
115 C CMIMMPOP4,IGM, LUADMATUNE MU*g
1116 C CE (MNPO,4,1GM) EUADOATUAE ETA-s
1117C (G) C OUNIDATUME DImmECTIDNS FDO GmOU E
111BC LMAX OMEEE OF SCATTEGING
1119C LINAXP ELEMAX + 1
1120c NM TOTAL MUNSEG DF MOMENTS
```



| 1201 |  | $\cdots(m, M-4 m)=P(m, N, J)$ |
| :---: | :---: | :---: |
| 1 102 | 190 | CONT 3 MuE |
| 1203 |  | $\cdots \times m+1$ |
| 1204 | 200 | cantinue |
| 1205 |  | ¢a ro 210 |
| 1206 | 220 | IF（LQ．E日．1）MD＝0 |
| 1207 |  | 3F（Le．E日．2）Mm＝2mpor |
| $1 \dot{128}$ |  |  |
| 1205 |  | 3F（LE．E日－4）mpmeme |
| 1210 |  | $x=1$ |
| 1211 |  | 20 240 n＝1，Lmax |
| $121 \bar{c}$ |  | Do 240 JFI 1 N |
| 1213 |  | MEVENSE COUNT ON EACM ETA LEUEL FOE AEJOINT MOMENTS |
| 1214 |  | $\cdots \pm 1$ |
| 1215 |  | 3NEEXI $=1$ |
| 1216 |  | DOĖ35 8J＝1，NEL |
| 1217 |  | 1NEE』1＝ 3 NDEx $1+15-1$ |
| 1218 |  | Do $230 \mathrm{ml}=1.13$ |
| 1219 |  |  |
| 12 ¢0 |  |  |
| 1221 |  |  |
| 1222 |  | $\cdots \times \mathrm{m+1}$ |
| 1223 | 230 | CDMTIMES |
| 1224 | 235 | cont inue |
| 1225 |  | $\cdots=*+1$ |
| 1 12．26 | 240 | contimul |
| 12 27 | 210 | cont $i$ muk |
| 1228 |  | D0 $500 \quad 1=1.10$ |
| 1229 | c |  |
| 1230 | 500 | CONT I NuT |
| 1231 | 420 | Fomat（6E12．5） |
| 1232 |  | GETUMOM |
| 1233 |  | END |
| 1234 | c |  |
| 1235 | c suzm | OUTINE FLUYMOM GENEMATES TME FLUX MDMENTS |
| 1236 |  |  |
| 1237 | c |  |
| 1238 |  |  |
| 1239 | 1 |  |
| 1240 | 2 |  |
| 1241 |  |  |
| 1242 | c | INPUT COMmENTS＊＊ |
| 1243 | － |  |
| 1244 | c | FLUX（itsumemm（6），4）maUlam FLUXES 1N LCM |
| 1245 | c | W（IGM）DIJALEATUME MEICMTS |
| 1246 | c | IIT（JT）EiTMIAMILESJPAND |
| 1247 | c |  |
| 1248 | c |  |
| 1249 | c | ¢ Emour incex |
| 1250 | c | 16m tatal ticadues |
| 12゙51 | c | NM NUMEEP OF MOMENTS |
| $125 \pm$ | c | $3 T$ Number of sands |
| 1253 | c | KAD IEENTIFIEA ADSOINT／DIEECT FLUXES |
| 1254 | c |  |
| 1255 | c | FHom SCALAR FLUXES IN DETECTOM MEGzDN |
| 1256 |  |  |
| 1 ごち7 | c＊ | －ourput camments－－ |
| 1ごく8 |  |  |
| 1255 | c | FLU＊MOMENTS maE CmCULATED AnI Mistten ． |
| 1260 | c |  |
| 1261 | c | WEITTEN ON TAEEI |
| 1 こ62 | c |  |
| 1263 | c |  |
| 1264 |  |  |
| 1265 |  | LEVEL 2，FFluxifluxifuxifmom |
| 1266 |  |  |
| 1267 |  |  |
| 1268 |  |  |
| 1269 |  |  |
| 1270 |  | DIMENSION MTD（JTIJ），KELDI（JT，1）OKELDE（ST，1）PFMOM（ST，1） |
| 1271 |  |  |
| 1272 |  |  |
| 1273 |  |  |
| 1274 |  |  |
| 1275 | C INIT | 1AL 32 E |
| 1276 |  |  |
| 1277 |  |  |
| 1278 |  | IF（KAD．ME．O）ED TO 130 |
| 1279 |  | MmE $=$ M（6） |
| 1280 |  | Und＝＜tat（36， |


| 1こと1 |  | UNĖ＝Tap（86＋3．E） |
| :---: | :---: | :---: |
| 12゙を |  |  |
| 1̇๕3 |  | 60 T0 110 |
| 1284 | 100 | CALL Assicn（uml $0,-3$ ） |
| 1285 |  |  |
| 1 こ®6 | c |  |
| $1{ }^{\text {c }} 8$ | 1000 | Fammat（23n4） |
| 1288 |  | DEAD（UN1）（SPEAM（ $\alpha \sim$ ），$\alpha \sim=1,10$ ） |
| 1289 | c |  |
| 1290 | 1010 | Fammat（12：6） |
| $1 E 91$ | 110 |  |
| 1292 |  | 60 T0 190 |
| 1293 | 120 | CALL ASSIGN（UNZ，O，－3） |
| 1294 |  | CALL FAmstz（unċimaxmmb） |
| 1295 |  | co To 190 |
| 1296 |  |  |
| 1 こ97 | C INIT | TALI2E FON TME NDJOIMT CASE |
| 1298 | 130 | c＝1 $6 \times-6+1$ |
| 1299 |  | Mmट $=$ man（ 6 ）／2 |
| 1300 |  | UNI $=$ ¢ TMP（IG， 6 ） |
| 1301 |  |  |
| 1302 |  |  |
| 1303 |  |  |
| 1304 |  | ca TO 150 |
| 1305 | 140 | CALL Assicw（uwl，0，－5） |
| 1316 |  |  |
| 1307 |  |  |
| 1308 | 150 |  |
| 1309 |  | 60 T0 170 |
| 1310 | 160 | CALL Assicw（uner 0，－3） |
| 1311 |  | CNLL FAMS 22 （UNẼ，MAXMmD） |
| 1312 | 170 |  |
| 1313 |  | 60 T0 190 |
| 1314 | 180 |  |
| 1315 |  |  |
| 1316 |  | k⿴囗十介¹ |
| 1317 | 190 | CDMTIMUE |
| 1318 |  |  |
| 1319 | C INIT | 3HLI2E FLUX MOMENTS |
| 1 ちき 0 |  |  |
| 1321 |  | 20 205 j＊1．JT |
| 13 ぞ2 |  |  |
| 1323 |  | とロ 205 1T＊1．1Tコ |
| 13 ç 4 |  | DO EOO $2 \mathrm{~N}=1.0 \mathrm{~m}$ |
| 1 こご | 200 | Fux（zN：J，IT）$=0$. |
| 1こご6 | 205 | FMom（J－1T）$=0$ |
| 1 こ̇て |  | JコTEセ＊」T |
| 1328 |  |  |
| 13 ¢9 |  | 20 $605 \mathrm{~s}=1.35$ |
| 1350 |  |  |
| 1351 | c mend | のNBULAO FLUXES |
| 133 F |  |  |
| 1335 |  | 1COUN 1＝ 1 |
| 1334 |  | JJ＊ |
| 1335 |  | 1F（g．6T．JT）JJF2•JT－s＋1 |
| 1336 |  |  |
| 1337 |  | 17コニ887（コ） |
| 1338 |  | ーツx＝－mmzノて |
| 1339 |  | Do El0 mmis 1 ，mmx |
| 1340 |  | 1COUNべあ＝ |
| 1341 |  | －EAD（UN1）ik |
| 1342 |  | WEAL（UN1）（FFLUx（IEDUN），ICOUN＝ICOUN1，ICOUN2） |
| 1343 |  |  |
| 1244 | E10 | CONTIMUE |
| 1345 | $c$ |  |
| 1346 |  |  |
| 1347 | c | Wezte（10，430）Kezt |
| 1348 |  | CALL WD：sm（UN3，FFLUX，İTJ＊KGIT） |
| 1349 |  | ＊＊ITE＊ロIT＊IITJ |
| 1350 | ころ0 | 3 couwd $=1$ |
| 1351 |  | Da ė20 mixalimmy |
| 135 こ |  | 1 Counでar icaun $1+1$ TJ－1 |
| 1353 |  | HEAE（UN1）IM |
| 1354 |  |  |
| 1355 |  | 1 COUw $1=1$ counciol |
| 1356 | EこO | CONTIMUE |
| 1357 | c |  |
| 1356 |  |  |
| 1359 | c | HeITE（10．430）K⿴IT |
| 1360 |  | CALL Wzism（un3，FLuxtiztjokmit） |

```
1361
1362
15Є3
1304 2SO 1F(J.ET.JT) EO T0 3ÉO
l304 ESO iF(J.ET.JT) 60
l366 13F(NTO(JJ+k),EE,
1367
1368
1369
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1371
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14ट1
1422
1422
14ट゙
14ご 360 COWTIMUE
14ट6
14\hat{C7 390 %F (MAD.ME.O) 50 T0 605}
1428 c
1429 c
1430 C IFS1 (J.ET.JT) E0 TO 560
1432 Da 550 M=1.*Dz
1433 IF(NTD(JJ,K).EE.0) 60 T0 ESO
1433 (%)
1435 1TE=WELEE(JJOK)
1436 Mm1 =mmごノご
1437 COUNT=0
1438 ICOUN=2T1
1439 DO S20 Im=1.mml
1440 20 510 &TF&T1.1T2
```

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1441
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145?
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1459
14*0
4E0
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1462
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1465
1466
1407
1488
1469
1470
1470
1471
1412
1473
1474
147%
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1476
1477
1478
1478
148!
14E1
14EE
14E%
1484
1485
1486
1486
14E7
1488
1485
1490c
1491 c
1492 c
1493
1494
1495
1496
1497
1496
1499
1500
1501
1502
1502
1503
1504
1505
1506
1507
1507
1508
1509
1510c
2E11 c
151% C
1513
1514
1514
1516
1517
1517
1518
1519
15E0
```

```
    Fmam(JJ,IT) = FMOM(JJ:IT) * N(G)*FFLUx(ICOUN)
```

    Fmam(JJ,IT) = FMOM(JJ:IT) * N(G)*FFLUx(ICOUN)
        ICOUN=1COUN+1
        ICOUN=1COUN+1
    510 CONTINUE
    510 CONTINUE
    ICOUNT=ICOUNT+ITI
    ICOUNT=ICOUNT+ITI
    ICOUN= ICOUNT+IT1
    ICOUN= ICOUNT+IT1
    S20 CONTINUE
S20 CONTINUE
ICOUNT=0
ICOUNT=0
ICOUN=IT1
ICOUN=IT1
Mm1工mm2/2*1

```
    Mm1工mm2/2*1
```




```
    z0 530 iT#iT1,it?
```

```
    z0 530 iT#iT1,it?
```




```
    ICOUN=1COUN+1
```

    ICOUN=1COUN+1
    S30 CONTIMUE
S30 CONTIMUE
ICOUNT= \& COUNT+ITJ
ICOUNT= \& COUNT+ITJ
ICOUNEICOUNT+1T1
ICOUNEICOUNT+1T1
540 CONTINUE
540 CONTINUE
S50 CDNTINUK
S50 CDNTINUK
6a ra 605
6a ra 605
560 Da 600 k=1, wDz

```
    560 Da 600 k=1, wDz
```




```
    1T|=mELD!(JJ,K)
```

    1T|=mELD!(JJ,K)
    1T2=mELDで(ココ, K)
    1T2=mELDで(ココ, K)
    1COunT=0
    1COunT=0
    1COUW=1T1
    ```
    1COUW=1T1
```




```
    *m4=3****2/2
```

```
    *m4=3****2/2
```




```
    065 1T=1T1,1TE
```

    065 1T=1T1,1TE
    FMON(JJ, IT) =FMOM(JJ, IT) tw(G)*FFLUX(ICDUN)
    FMON(JJ, IT) =FMOM(JJ, IT) tw(G)*FFLUX(ICDUN)
    ICDun=zCOUN+1
    ICDun=zCOUN+1
    565 cONTINUE
565 cONTINUE
ICOUNT=ICDUNT+ITS
ICOUNT=ICDUNT+ITS
ICOUN=ICOUNT+IT1
ICOUN=ICOUNT+IT1
570 CgNTINUE
570 CgNTINUE
ICRUNT:O
ICRUNT:O
COQUN=IT1
COQUN=IT1
Mm4=m*44+1
Mm4=m*44+1
MM5 = men(E)
MM5 = men(E)
20 590 Im=mom4, am55
20 590 Im=mom4, am55
y0 5\&0 zTEITI|IT2

```
    y0 5&0 zTEITI|IT2
```




```
    zCOUN=:COUN+1
```

    zCOUN=:COUN+1
    5BO CONTINUE
5BO CONTINUE
COUNT:ICDUNT+ITJ
COUNT:ICDUNT+ITJ
COUN=ICOUNT+ITI
COUN=ICOUNT+ITI
590 CONTIONUE
590 CONTIONUE
GOO CONTINUE
GOO CONTINUE
605 CONTINUE
605 CONTINUE
MO1TE FLUY MOMENTS GND SCALAN FLUXES
MO1TE FLUY MOMENTS GND SCALAN FLUXES
Da 6こ0 y=1.コT
Da 6こ0 y=1.コT
Da 610**1,x-z
Da 610**1,x-z
IF(NTP(3,N).EE.U) Sa T0 610
IF(NTP(3,N).EE.U) Sa T0 610
121**FLP1(Jom)

```
    121**FLP1(Jom)
```




```
    N=ITE(UNE)((FU)(IN:J,IT), IN=1,NW), IT=121, Iz2)
```

    N=ITE(UNE)((FU)(IN:J,IT), IN=1,NW), IT=121, Iz2)
    610 CONTINUE
    610 CONTINUE
    6EO CONTINUE
    6EO CONTINUE
        IF(MAD.ME.0) 60 T0 650
        IF(MAD.ME.0) 60 T0 650
    DO 640 y=1.jT
    DO 640 y=1.jT
    D0 650 k=1.*2z
    D0 650 k=1.*2z
    IF(KTD(J,*).EEO.0) ©0 T0 630
    IF(KTD(J,*).EEO.0) ©0 T0 630
    121=wELD1(J,m)
    121=wELD1(J,m)
    1ZきシMELDき(J.K)
    1ZきシMELDき(J.K)
    HWITE(1) (FMOM(J. {2), I2=121,:I2&)
    HWITE(1) (FMOM(J. {2), I2=121,:I2&)
    650 CONT % NuE
    650 CONT % NuE
    640 CONTINUE
    640 CONTINUE
    c
C CLOSE TAOES IF MECESSEOR
C CLOSE TAOES IF MECESSEOR
650 1F(NAE.ME.0) 60 T0 400

```
650 1F(NAE.ME.0) 60 T0 400
```






```
    @0 T0 410
```

```
    @0 T0 410
```








```
410 CONTINUE
```

```
410 CONTINUE
```

| 1521 | 4 cio Fgmat（6E12．5） |  |
| :---: | :---: | :---: |
| $15 \dot{2}$ | $430.1$ | Famat（1m，6monit＝，16） |
| 1523 | －ETUMOM |  |
| $15 ⿰ 氵$ 4 | END |  |
| 1525 | c |  |
| 15 E6 | c |  |
| 15.27 | sunadutime cmis calculates the cmiss |  |
| 1528 | c |  |
| 1529 |  |  |
| 1530 |  |  |
| 1531 |  |  |
| 15.32 |  |  |
| 1533 | c＊＊input comments |  |
| 1534 |  |  |
| 1535 |  |  |
| 1536 | OFLUX（itJimper）－menmmanced nejoint meulan fluxes（LCM， |  |
| 1537 | KTAF（5，icm）－Disk index |  |
| 1538 |  |  |
| 1539 |  |  |
| 1540 |  |  |
| 1541 |  |  |
| $15.4{ }^{\text {c }}$ | mm（s）－Tatal ti manents fof emout |  |
| 1543 |  |  |
| 1544 | W（E）－ED．－OUndimatume heicmis Fon gmour |  |
| 1545 | KPz－：FEPTUNEED 2DNES |  |
| 1546 | 15 m －－amours |  |
| 154？ | JT－－smans |  |
| 1548 | StMAX－MAx．：Tezanglesfannd |  |
| 1549 | ITSUM－Tatal NumBee DF TRIMNELES |  |
| 1550 |  |  |
| 15E1 | C－－DUTPLT COMENTE • • • |  |
| 1552 | CMI（3GN1KP2F）－CMI＇s |  |
| 15E3 |  |  |
| 15.4 | LEVEL 2，FLUX，AFFLUX，Aflux |  |
| 1555 |  |  |
| 1556 |  |  |
| $15 ¢ 7$ |  |  |
| 1558 |  |  |
| 1559 |  |  |
| 1560 | 2 |  |
| 1561 |  |  |
| 1562 | DATA CP1／6．283185307／ |  |
| 1563 | 1F（zGEO．E日．1）Cers 1.0 |  |
| 1564 |  |  |
| 1565 | MEITE（6，420） |  |
| $15 \in 6$ |  | Do $190 \quad 6=1.16 m$ |
| 1567 | c ${ }^{\text {c }}$ |  |
| 1568 | INITIALI2E |  |
| 1569 |  |  |
| 1570 |  |  |
| 1571 | 1＊ 8 ¢m＋6－12 |  |
| 1572 | N－ITENGIT＋itsumant（ 2 ） |  |
| 1573 | 100 |  |
| 1574 |  | NELEISN（6）／2 |
| 1575 |  | ココTEE¢JT |
| 1576 |  | mpormin（ 6 ） 14 |
| 1577 |  | UN1＝ntan（1，6） |
| 1578 |  |  |
| 1579 | C DEEN FILES IF NECESSOMY |  |
| 15E0 |  |  |
| 1581 |  |  |
| 1582 1583 |  50 TO 90 |  |
| 1564 | 80 | CALL ASSIEN（UN1． $0,-3$ ） |
| 1585 |  |  |
| 1586 |  |  |
| 1557 | 902 |  |
| 1588 |  |  |
| 1569 |  |  |
| 1590 1591 |  |  |  |
| 1592 |  |  |
| 1593 |  |  |
| 1594 |  |  |
| 1595 |  |  |
| 1596 |  | Do $1701=1,2$ |
| 1597 |  | ＊Witwonit－izus ya 95 2melime |
| 1598 |  |  |
| 1599 |  | vo 95 1m＝1．me NEAD（UN1）IK |
| 1600 | 95 mend（UN1）（FLUX（iT，3M），iTE1，1TJ） |  |






| 1921 |  | END |
| :---: | :---: | :---: |
| 1922 | c |  |
| 1923 | c |  |
| 1924 | c |  |
| 1925 | c |  |
| 1926 | c |  |
| 1927 |  |  |
| 1928 | c ma |  |
| 1929 | C T |  |
| 1930 |  | LEUEL 2，XME゙，XW1 |
| 1931 |  |  |
| 1932 |  |  |
| 1933 |  | －ITITLE（11），TITLEx（ぐU） |
| 1934 |  |  |
| 1935 |  | COMmgmritelitesteitro |
| 1936 |  | COMmONRCOUAE：／JCOUAD |
| 1937 |  | COmmanrxsFommrkxs＋ImT，Imm |
| 1938 |  | COMMAN／DENS／MUMDEN |
| 1939 |  | －Em M M MLEN |
| 1940 |  | INTEEER MLCOM |
| 1941 |  |  |
| 1942 |  | NE＝5 |
| 1943 |  | NET＝6 |
| 1944 |  | GOP＝000m |
| 1945 |  | NC $=0$ |
| 1946 |  | NC1 $=0$ |
| 1947 |  | NT1＝ 32767 |
| 1948 |  | LLMAE＝ 1 LMAX＊ 1 |
| 1949 |  |  |
| 1950 |  | 1F（KXS．E日．2）60 T0 40 |
| 1951 | c |  |
| $19 \pm$ | c $\rightarrow$＋ | －mend Laslofammar cmoss sections Fmom zmout fle（om Cmmds） |
| 1953 |  | 1 continue |
| 1954 |  |  |
| 1955 |  | E Fammer（20nt） |
| 1956 |  | － |
| 1957 |  | 3 Faman（12x，（12．6．11x．11） |
| 1958 |  | Weitce（6，305） |
| 1959 | 303 |  |
| 1960 |  |  |
| 1961 |  |  |
| 1962 |  | 4 Fnomat（ 1 － 2 ， $20 n 4$ ） |
| 1963 |  | Heite（ 6,5 ）Numden |
| 1964 |  |  |
| 1965 |  |  |
| 1966 |  | 6 D0 900 LLE10LLmax |
| 1967 |  |  |
| 1968 |  |  |
| 1969 |  |  |
| 1970 | 301 | 1 Fammat（6E12．5） |
| 1971 | 10 | 0 20 $3001=1$ ，wce |
| 1972 |  |  |
| 1973 |  |  |
| 1974 | 300 | 0 CONTINUE |
| 1975 |  | IF（ITEST．EE．1）ED T0 304 |
| 1976 |  | N－ITE（6，305） |
| 1977 | 305 |  |
| 1978 |  | 60 T0 910 ． |
| 1979 | 304 |  |
| 1980 | 302 | （ Famme（1m，6（2x，1－E12．5）） |
| 1981 | 910 | 0 continue |
| 1982 | 900 | 0 contimut |
| 1983 | 999 | 9 netumm |
| 1984 | c |  |
| 1985 | 500 | 0 cantimut |
| 1986 |  | MENIND 4 |
| 1987 | coos |  |
| 198.5 | c＋＊＊e |  |
| 1989 |  | M1 $=$（nCsomCTL）／6 |
| 1990 | c＊ | IF M1 25 MOT A MULTIPLE DF 6 ，THEN MDD 1 mome mecond |
| 1991 |  |  |
| 1992 | c＊ | のยz ONE FOめ TITLE mecomd |
| 1993 |  | Mर＝ $\cos _{1}+1$ |
| 1994 | c＊ |  |
| 1995 |  |  |
| 1956 |  |  |
| 1997 | 5000 | 0 Fcmat（ $16,6 \times 1$［ $12.5,2 x, N 10$ ） |
| 1998 | c＊＊ |  |
| 1999 |  | 1SC1P＝（1D－1） |
| 2000 | c＊ |  |


| 2001 2002 |  | 1F（ID．EE．1）EO TO 5007 Do 510 1＝1，iscip |
| :---: | :---: | :---: |
| 2003 | 510 | －EEAD（4，5001）（TITLEX（N），NEI．EU） |
| 20104 | 5007 | COMTINUE |
| 2005 |  | DO SOE LLE 10 Llmax |
| 2006 | $c * *$ | DEAD TITLE OF MATE円IM DOM－L COMPONENT DESIMED F\＃OM TAEE |
| 2007 |  |  |
| 2008 | 5001 | FOmmt（20n4） |
| 2009 |  |  |
| 2010 | 5002 | Fomme（1m，1x，20n4） |
| 2011 |  | WHITE（W6．5003）NUNEENP 5 SNAME |
| 2012 | 5003 |  |
| 2013 | c＊＊ | DEAL COOSS SECTIONS OF MATEOIM DESIMED |
| 2014 |  |  |
| 2015 | 5004 | Fomat（6E12．5） |
| 2016 | c＊＊ | IF ITEST FLAEE0 DD NDT MGINT MICmox＇s |
| 2017 |  | IF（ITEST．ME．1）©0 TO 507 |
| 2018 |  | LELL－1 |
| 2019 |  | H－ITE（6，5005）L |
| E020 | 5005 |  |
| 2021 |  | $1=0$ ， 3 ） |
| 2022 |  |  |
| 2023 | 509 | FOmmt（1m．6（2x，1PE12．5） |
| E0E4 | c＋${ }^{+}$ | MAME THE MACmoscanic Cmoss sectiows |
| 2025 | 507 |  |
| 2026 |  | ED $505 \mathrm{~J}=1$ NCTL |
| 2027 | 505 |  |
| 2028 | 502 | COMTIMUE |
| 2029 |  | ©0 T0 499 |
| 2030 | c |  |
| 2031 | c ${ }^{+\infty}$ |  |
| 2032 | c |  |
| 2033 | 40 | Do 9999 LN＊1，LLMAX |
| 2034 | 50 | 1F（NC）1E1．121．31 |
| 2035 | 121 | －EAI．（NS，11）NCC，PLCOMP M NMEI |
| 2036 | 11 | Fommat（2i6，10n4） |
| 2037 |  | NCID＝mCOM +1 |
| 2036 |  | NCI＝NCID |
| 2039 |  | 1F（nce）22，22，21 |
| 2040 | 21 | 1F（nCC－E゙）24，22，24 |
| 2041 | 2 2̇ | $\boldsymbol{j}=0$ |
| 2042 |  | NCOUNTENCE＊WCTL |
| 2043 | 6ざ |  |
| 2044 | 6 |  |
| 2045 |  | 20 635 $2=1.6$ |
| 2046 |  |  |
| 2047 | c | Ma mepents |
| 2048 | 810 |  |
| 2049 |  | $j=s+1$ |
| 2050 |  | XN1（JPLN／EV（1） |
| 2051 |  | co To EOO |
| 2052 | c | WEmeat |
| 2053 | 700 | L＝IN（1） |
| 2054 |  | Do $809 \mathrm{~m}=1 . \mathrm{L}$ |
| 2055 |  | $j=j+1$ |
| 2056 | 809 | \＃N1（J＊LN）\＃V（1） |
| 2057 | 800 | 1F（s－mCaUNT）635，24，24 |
| 2058 | 635 | CONTINUE |
| 2059 |  | 60 \％0 622 |
| 20¢0 | 24 | NC＝ 1 |
| 2061 |  | IF（nce－7）31，25，31 |
| 2062 | 25 | NCI $=32767$ |
| $20 \in 3$ |  | 1F（NC1－NT1）31，26，31 |
| 2064 | 26 | －ETUNT |
| 2065 | 31 | 1F（mT1－mCl）43，41：43 |
| 2066 | 43 | NC＝0 |
| 2067 |  | $\cdots=0$ |
| 2068 |  | D⿴囗 $1201 \pm 1$ ，mCE |
| 2069 |  | Do $120 \mathrm{y}=1$ MCTL |
| 2070 |  | $x=w+1$ |
| 2071 | 120 | YNE゙（1，J，LN）＝\％N1（K，LN） |
| 2072 |  | 1F（ITEST．ME．1）EOTC E1 |
| E073 |  | M－ITE（NG，EO1）NCEPMCTL，NCC，NCIDPLNPMANE1 |
| 2074 | 201 |  |
| 2075 |  |  |
| こ076 | 51 | $\cdots \times 1=1$ |
| 2077 |  |  |
| 2078 |  | TESTE FLOAT（nCe）$/ 8.0 \uparrow .999$ |
| 2079 2080 |  | LMA天：TE <br> Do 145 L＝1．Lemx |

```
    IF(NN2-NC6)232,232,233
    232 1F (ITEST.ME.1) GOTD 49
        M= ITE (NE, 24E.) (Emp,J, J=NW1, MNC)
    245 Fammat(7m-mas mT-8(6x,A4,:3))
    y0 241 i=1,NCTL
```



```
    20E FOmmA (2,4,1-EE13.5)
    49 NNI= NN2+1
    145 NNZ=an1+7
        IF (ITEST.NE.1) GOTD 9999
        HWITE (N6,75)
    TE FOmmat (1m(0)
9999 CONTINUE
41 60 TO 50
499 EETUMN
2098
2099 c
2100 c
2101 c
2102 c
2103 c
21104
```



```
2107
2107
2108
2109
2110
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2114
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2116
2117
2118
2119
2120
2121
2122 C -* CNLCULATE DELTR SIMMAS DST NWD DSL
8123
DST(G) = xs(G, 8MT,1)
2125 1F(<ITYR.EE.1).MNL.(1DES.EE.0))
```



```
2127 1F((ITYF.ER.1).ANL.(1DES.ER.1))
2128 - DST(G) = 0.01*xs(G,IMT,1)
2129 Axs(G) = Es(G;imn,1)
    FIsxs(G)=xs(6, 100N+1,1)
    DO 40 cm=1.G
    DO40L=1.LL
    ISL(E+1-GD,GOL)=xS(E,CD+IMT,L)
    IF(ITYO.EE.1), ANE. (IDES.EE.U))
```



```
    1F((1TYR.EQ.1).MNL.(1DES.EQ.1))
    * LsL(G+1-@P,GOL)=0.010xs(G,EF+3MT,L)
    40 CONTINUE
C -O NOW TME LSL-AmmNT IS CONUENIENTLY ONDEEED FOM TNE AE-FOMONLATIDN,
C -O* ISL(GPIGIL) CAM LIPECTLY EE INTEMPMETED AS SCATTENING FMOM GAOUP
C - GP INTO GNOUP E, ONEENED SO TMNY OP STMMTS WITM 1 GND INCMEASES
C -\infty TO GPEG, TWE WEST MNE 2EmOS.
c
C CO CALCULATE DSLFD FON FDGFOMmLLATION
    DO 30 E=1, icm
    DO 30 cp=sizem
    N=8MT+GP-E+1
    DO 30 L=1.LL
    ESLFD(G,GP:L) = xS(EP,K,L)
    IF((ITYF.EQ.t).,NWL. (IDES.EE.0))
    - DSLFD(G,CO,L)=xs(GP,K,L) - xsmeme(eO,K,L)
    3F((ITYF.EE.1)..ND.(IDES.EM.1))
    - isLFD(G,Ge,L)=0.010xs(@R,K,L)
    30 CONTINUE
```




```
C ** CALCULATE TOTOL MACNOSCODIC SCATTENING COOSS SECTIDN OEN EmOUN
    1F(ITrP.EE.1) EO TO 203
    20 60 1=1.16m
    sxs(1) = 0.0
```

```
2161
2162
```



```
2164
2165
2166
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2171
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2190
2191
2192
E193
2194
<
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2199
2200
2201
2202
2204
2205
2206
2207
2208
2E09
2210
2211
221
OO CONTINUE. IFNCOUQLEE.O) GO TG 2OE
** cO##ECT DINGGMAL sums FOO
    DO 204 c=1,1%m
    204 sxSNG(E)=0.0
    NG1 =NCOURL+1
    DO 200 E=1,NCOUPL
    20 201 EPENG1:&G*
    I=1MT+\epsilon--G+1
    201 ExSNG(E)=smSNE(E)+xS(E-:1,1)
```



```
    200 contimue
    202 1F(1TEST.ME.1) E0 T0 26
    MnitE (6,1050)
```



```
    -SECTION PE界 EmOUR. IN 1/CM*/)
    -SECTIDN PEG Gm
        w#1TE (6.1051) 6.5x5(6)
```



```
    70 cONTIMNE
        w|yTE (6,1052)
```



```
        4N Gmoupe)
            ma 71 c*i.NCOUPL
            MWITE (6,1053) &, sxsmG(&)
```



```
    71 CONTINUE
    203 1F(ITEST.NE.1) Ea Ta 26
+** TEST P贯INTGUT OF DELTM SI&mAS
    H目价(6,7004)
```



```
        WGITE (6,1)(42) (DST(©),G=1, % ©M)
    1042 F0mat(1m.9 (2x,10E12.5))
        IF(IGM.ET.9) ©0 T0 805
        DO 41 L=1.LL
        DC 41 L=1,LL
```



```
        D0 41 6 =1.1%*
        HmyTE(6,1016) 5
    1016 FOMMAT (1M &MNEN EF@, & 3)
```



```
    805 CONTINUE
        IF(NCOUPL.EE.O) s0 T0 26
        DO 800 L=1%LL
        NelTE(6,801)L
```



```
        ** &3)
        malTE(6,807)
```




```
        20 610 &=1,NCOUML
```




```
    BUU cONTIONE
```



```
    803 FgmmAT (1M *TEST F#INTOUT FOD TOTAL N-GAmMA MACmOSCDPIC CmOSS
        -SECTION PEO NEUTBON GNOUO, IN 1/CM* )
        wa g02 g=1.mCOUPL
    80E WMITE(6,804) E,SxSmE(5)
```



```
    26 OETUNM
        END
c
2227c
22&8 c
2229 c
2230 c
2230
2231
SUPmOUTIME TEXT
```



```
C EDITEDIN SUD8
        HOITE(6,801)
    801 (0mmar(1m 111x, 87(*-*):ノ)
        ##TE (6, 802)
```



```
        1 TUNE * 
        MGITE (6, B05)
    805 FOmmat(1m,11x,87(*-)
```




```
2401
    265 conTINUE
2402
```



```
2404 205 1F(al.EO.1) DELi=0.0
2405 5.0 E L*d,LMAMP
2405 2406 IFO (Wxs.EB.E) TWOLM1(L)=1.0
        IF (KxS.EE.E) TWOLM1(L)=1.0
        5 CONTIMUE
2407
c
C +** USE A SIMPLE SUMMATIDN DNEW L
2411 C +* USE DO 9G E=j1.IEml
2411 DO 99 E= 
2413 va 98 L=1.Lmmx*
2414 z0 9S GP=31.6
```




```
2417 IEんD = 0% - DELI
2418
C
```



```
2421 c
2421 C IF(J1.EO.1) DELIFD = 0.0
2423 DO 89G591.16+11
2424 FFD(G)=0.0
2425 Da 68 LEIOLMAXP
2426 2, E8 cp=G|i&ml
2427
2428
2429
2430
2431
2432
2433
C434
24%6
C
C OQ STANT EDITINE DESICN SENSITIVITY INFOMNATIDN
    iF (\Omega1.NF.1) EO TO 1110
    1F(1DES.EO.1) WOITE (6,1109)
1109 FOMmAT(1m,06(1m*), mESULTS MEE FOD ASSUNED 1 PEO CENT FLAT MS-IN
```



```
1110 2F(W.GT.O) 60 T0 70
    1F(J1.NE.1) GO TO ?1
    1F(J1.NE.1) E0
```



```
    1 * NLL ENEMEIESS * 人
```



```
    W#ITE (6,1101) DELI, DELIFE
```




```
        &0 T0 73
    71 WPITE (6,1102) DELIO DELIFD
```




```
    HaITE (6,1105) mom
```







```
    M@ITE (6,1105) MAD, xFD
```




```
            60 T0 73
        70 1F(د1.NE.1) sa T0 72
            w|ITE (6,1106)*
    1106 FO#mAT (IM PCOMTMIBUTIDNS TO DELI-AD MND DELI-FD FOOM PEPTUMEED Q,
            1 EONE K =0, 13 / )
            W#ITE (6,110T) DELI, DELIFD
    1107 FOM#NT (Sx. & MOM NEUTMON GmOUNS ONLYI *110x,
```



```
            \sigma0 ro }7
    72 weyTE (6,1108) DELI, DELIFD
```




```
    73 CONTIMUE
    C OCOND EDITING DESIEN SENSITIVITY INFOMNATION
C
            *0 T0 900
    100 CONTINUE
```




```
2641 c
2G42 C 也O zEmO OUT TME NEN NmmNYE
2643 vo 2 %=1,16m1
2644 जSEDGe(6)=0.0
2645 -sED&(ब) = 0.0
2646 ssED(E) =0.0
2646 ssED(E)
2648 sMat(ब) = SOLD(\epsilon)=0.0
2649 DeseD(E)=0.0
2650 &o 1 GR=1,1GN1
26.52 2 cINTINuE
et.53 c
eES4 C +0 compuTE mLL mmmaYs TO EEEDITED
2655 cos com%UKEIMLL
2656 iF (KxS.EE.2) TWOLM1 (L)=1.0
2657
```



```
2660 DO 32 F =51. 1enl
2661 DO 31 L =1.LL
2662 31
2664 32 cONTIMuE
2665 33 CDNTINUE
2666 60 t0 50
2667 C EN% OF COMPUTATIDN OF IASIC PsED(EP,E)
```



```
2669 SO DO 5E E=1,zeml
2670 DO E1 Ef=1.6
```



```
2672 E{ CONTImuE
Z673 C *** INTEGOATE OSEV OUEN ML FINNL GODUPS #, FOD MLL INCIDENT GODUPS EP
2674 DO 6ご EP=1,16m1
2675 DO 62 G =00.16m1
C676 61 -SEDG(E) = FEDE(GP) + PSED(GP,E)*DELU(G)
2677 Eट CONTINUE
```



```
C679 IF INTEGNEDEEE.O) SDTO 93
2680 DO 72 GP=1.14%M1
2681 GmEDAN = EmEM(EO)
2682 iF (GNELAN.EE.0) 60 T0 }7
2683 IF (GNEDAN.LT.EP) <0 T0 256
2684 DO 71 EEGO,GMEDAN
2685 71
    SmOT(EP) = SmOT(EP) * ESED(EP,G)EDELU(G)
    SmOT (EP) = gMOT (EP)也DELU(GP)
2686
2687
C ** INTEGMATE OSED DNLY ONEW COLD FINML EROUPS
    INTEGMATEDSED D
    IF (GMED(EO).EEA.0) 60 T0 82
    IF (GMED(EP).EE.IGMl) ED TO BE
    GMEE|1 = GMED(ङ0) + 1
```



```
    sCOLD(CP) = sCOL&(CP)*DELU(GP)
2694
2695 Eट CONLD(ED
C - COMPUTE INTECAN SED SENSITIUITY CDEFFICIENTS QN& OESPDNSE UNCE#T,
    TSSED =0.0
    TSMOT = 0.0
    TSCOLD= 0.0
    TDPSED= 0.0
    Da 91 cp=1,14eml
    ssED (बि) = SMOT(sp) - scald(e\rho)
    SSED (GP) = SMOT(ED) - SCDLD(GO)
    LOSED(EQ) FSED(EP)OSSED(GO)
    C*O CONDUTE TOTAL INTEGMNS
    COMOUTE TOTAL INTECONLS
    TSSED (TSSED SSED(E)
    TLOSED= TDOSED* DOSED(EP)
    TSm@T = TSMDT &mOT(SD)
    91 TsCOLD= TSCOLD* sCDLD(ee)
    ge CONTINUE
    C © CO COMOUTE TOTAL INTEGMANS OF SINELT DIFFEMENTIAL PNOFILES
    93 Tesee =0.0
        TOSE =0.0
        on 94 I=1.{%**1
        TPSEP : TOSEP * ©SEDGP(I)*DELU(I)
        TPSE = Tmse * -SEDS(I) - DELU(I)
    94 CONT IMNE
718
2719 c
    C *** WOM MI ETAmT EDITImE
```



| $\begin{aligned} & 2801 \\ & 2802 \end{aligned}$ | $4$ |  <br> －COADECT INPUT DATA！＊） |
| :---: | :---: | :---: |
| ç．03 | 999 | CONTINUE |
| こと0．04 |  | OETUNON |
| こと05 |  | END |
| 2806 | c |  |
| 2807 | c |  |
| ce 48 | c |  |
| こ609 | c |  |
| こE10 | C |  |
| 2811 |  |  |
| 2812 | 1 |  |
| 2813 | c |  |
| 2E14 | c $\rightarrow$ T | THIS MDUTINE COMPUTES MWD EDITS SENSITIUITY PROFILES FDO UECTOE |
| ć815 | C $\rightarrow+$ c | Cmoss－sECTIONS im \＃nios of cil |
| 2916 | C $\rightarrow+1$ |  |
| 2917 | C $\rightarrow+1$ | ITS COLAMIMNCE MATMIX． |
| 2818 | c |  |
| 2819 |  | Level zican |
| こ620 |  |  |
| 2821 | 1 | 1 De（1）PE（1） |
| こ6ご2 |  | COMmON／ITE／ITESTMITYO |
| こと23 |  | COMmON／PLOT／TITLE（8） |
| 2824 |  | INTESER GPE |
| 2825 |  | －EAL ${ }^{\text {an }}$ |
| 2826 |  | WOITE（ 6,1000 ）（ 78 TLE（8），1＝1，8） |
| 28ご | 1000 | Famme（1m． $8 \mathrm{Cm} 10,1$ ） |
| こ628 |  | W－ite（6，1100） 12 |
| E829 | 1100 F |  |
| 2830 | 1 |  |
| 2831 |  | W\％ITE（6，1200）mm |
| 2¢32 | $1200=$ |  |
| 2833 | 1 |  |
| C834 |  | W－ITE（6，1300）DEWI．DEME |
| 2Eこ5 | 1300 |  |
| C836 | 1 |  |
| CES7 | 2 |  |
| ¢838 | 3 |  |
| 2839 |  | weztE（6，14（0） |
| 2840 | 1400 |  |
| 2841 | c $+\infty$ c | COMPUTE SENSITIUITT PMOFILES MNL INTEGOAL SENSITIUITIES |
| çac |  | $s+1=0.0$ |
| ¢843 |  | $s-\bar{z}=0.0$ |
| 2844 |  | Do 1 c＝1， 1 enl |
| CE45 |  |  |
| 2846 |  |  |
| 2847 |  | SP1＝Spl＋－1（E）¢ DELU（E） |
| 2848 |  |  |
| 2E49 | 1 C | CONTINUE |
| こと玉0 | c ${ }^{+\infty}$ | －In Int Pmoriles |
| 2851 |  | D0 $2=1.16 \mathrm{ml}$ |
| ことざ |  |  |
| ことらる | 1500 |  |
| C\＆゙」 | 2 c | CONTINUE |
| E¢5゙5 |  | W－8TE（6，1600） |
| ごも¢ | 1600 |  |
| CE57 |  | Wette（6，1700）spl，spl |
| 2¢ち8 | 1700 |  |
| せE゙5 | c + ＋ |  |
| 2860 | C + ＋ |  |
| 26E1 |  |  |
| 2862 |  | －1（E）＝－1（E）DELU（E） |
| 2663 | 3 | －Ė（ब）$=-2(G)$－DELU（E） |
| 2664 | c $\rightarrow$ c | CALCULATE EOUBLE SUN（USE AmAnY UKSE（G）AS INTEMAEDIATE SINELESEUM |
| 2865 |  | 2moumse 00.0 |
| 2867 |  | $v \times s \bar{c}$（s）$=0.0$ |
| 2E68 |  | D 5 cesifieml |
| 2669 |  | vxs己（E）＝vxs己（6）＋－2（ce）econ（6，en） |
| C870 | 4 D |  |
| 2E71 |  | IF（zmounse．EE．0．0）©0 T0 6 |
| EヒフE |  |  |
| EE73 | 1600 F |  |
| 2874 | 1 |  |
| 2875 | 1 |  |
| 2876 | E | －UAOIANCE IS SET TO 2EDO FDW LATED TOTAL UAOIANCE CAL |
| 2877 | 3 | 3 －CULATIDN 0 ） |
| 2678 |  | Den（nys）$=0.0$ |
| 2879 |  | 60 T0 99 |
| 2880 |  |  |






```
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3E06
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3こ20
3221 c
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32256
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3237 c
3238 c
3こ39
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3己44
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3E69
3270
3271
3272c
SE73 C
3274c
3275 c
3E76 c
3277
3278 c
3279 C
3こ80 C
120 comTINuE
    @EAD (Nzċ,90) (xsi(1), IF10NEP)
    IF (MT1.NE,MTZ) ED TC 130
    DO 125 z=1%***
    xsट(1)=xsi(%)
    1ごS CONTIMUE
    G0 T0 150
    130 mEAD (ND2,20) (A(z), &=1,7),mAT,MF,MT,MSE|
    IF (mT.LT.MT2) E0 TO 130
    IF (MT.EQ.mT2) &o to 140
    MNITE (NOUT,60) NDT,ME,NT
    140 CONTINUE
    mEAD (NDE.90) (xS2(1): %=1,NGE)
    150 CONTINUE
    DO 15S N=1, wer
    DO 155 < =1, NGO
    cam (K,w)=0.
    CE1 ( }~,N)=0
    CEC (w,w)=0.
    155 comTIMuE
    nEAE cav. DATA.
160 mend (NDT,20) (A(1), z=1,7),MAT,MF,MT,NSEC
    iF (MF.LT.MF2) 50 T0 160
    IF (MT.NE.MT1) ©0 T0 160
    IF (HT.NE.MT1) EO TG 160
    W#ITE (MCUT,60) NDT,MN,MT
    5T0%
    170 CONT IMUE
    NEAD (NDT,50) C1,C2,L1,L2,LS,L4,MAT,WF,MT,NEEE
    IF (MT.LT.MT1) GO TO 160
    IF (HT.E®.MT1) EO TO 180
    W#ITE (NOUT,60) NDTIM,0MT
    sTOP
    180 CONTIMUE
    MTx=L2
    *OP=L4
```



```
    SEE LETTEO DATED 1 JWONBO ANL MEFEmENCE T-2-L-384S.
    DO 250 m=1.mep
    DO 250 m=1.me*
    2#0 com(N;N)=0.
    D0 190\alpha=1.\alpha<0
    mEAD(NDT:50) C1,C2,L1,L2,L3.L4
    MEAD(NDT
    LGP1=L?
    NGNOEL4
    *LEL4
    AEAD (NDT,90) (COM(MLOL),LELEP1,LEPE)
    IF (NONO.GE.KeP) SO TO 200
    190 CONTIMUE
    2OO 1F (mTX.LT.mT2) SO TO 170
    IF (लTY.EE.MT2) &0 T0 210
    W#ITE (NDUT,60) NDTOMF,mT
    NWITE
    210 cantymun
    DO 230N=1,N40
    *NENGPH*1
    xs^(**)=xsl(m)
    msN(***)=xsl(k)
    msP(m*)=xs2(x)
    mo 2E0 N:1.NGO
    NN=NGP-N+1
```



```
    220 conTzNuE
    zち! cDNTINUK
        IF (NOM.ET,O) DETUMN
    DO 240 N=1,N&O
    80 240 NE1.NGE
```



```
    そ40 CONTINUE
    NETUMN
    EMD
    SUPMDUTIME EETID
    SU&#guTimE gETS CDOmECT mat;mF,MT EIUEN mmx
```

```
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3282
383
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3360
```




```
    MFI=S & MF2=33 S mmxmx=6
    IF (ल##,\leqslantT,6) &0 T0 20
    lF (m青品
```



```
    MTI=1 $ MT2=1
    if (mmx, Ee.5) m+2=2
    IF (m@y.EQ.3) mT2=107
    WETUMN
O CONTINUS
    MTI=2 - mT2=2
    IF (M,m.GE.5) MTV=107
    IF (MOX.ER.6) HTI =107
    mETUNO
20
    IF (MAX.GT.11) 60 T0 40
    MFHT1=306
    MAT1=306
    zF (NWx.6T.08) so TO 30
    MT1=1 & MTE=1
    IF (円#x,EE,8) нT2=2
    #ETUMN
30 conTINuS
    MT1=2 & m+2=2
    IF (M⿴x.EO.10) MT1=4
    if (NO..EO.10) MT2=4
    if (M&#.EO.11) MT1=107
    1F (MOx.EE.11) MT2=107
    #ETU*N
4 0
    1F (MOM.GT.18) &0 T0 70
    1F (HPx.
    MAT1=324
    1F (m0n.बT.13) 60 T0 50
    MT1=1 % Mr2=1
    1F (m#x.FE.13) mT2=2
    #ETURN
50 CONTINUE
    IF (NOX.GT.15) sa To 60
    MT1=2 (MTE゙=?
    1F (M0ッ.EE.15) MTE=4
    #ETUNW
60 cawtimue
    MT1=4 S MTE=4
    iF (M⿴\zh11.बE.17) MT2=102
    102
    IF (~Wx.EEO.18) MT1=102
    @ETUON
70 CONTINU
    IF (NOX.ET.31) CO T0 110
    MAT 1 = 3^6
    iF (MAK.GT. 21) s0 T0 80
    MT1=1 S MTE=1
    IF (Mmx.EQ.20) MTE=2
    IF (m@x.E®.21) MTE=102
    #ETUNN
GETUNWN
    80 CONTINUE
    2F (HN%.ET.24) 50 T0 90
    MT1=ぞS MT2=え
    if (~由ッ.E@.そ3) Mr2=4
    &F (mmx.EEA.24) MT2=102
    #ETUNN
90 CONTINU
    2F (M⿴x.ET.28) 50 T0 100
    MT1=4 % MTE゙ミ4
```



```
    if (ल⿴\zh11.E日.टे7) 円TE=103
    IF (M由x.EE.28) -T2=107
    mETUMN
100 CONTIMUE
    MT1=102 * mT2=102
    iF (M@ッ.EE.30) MT1=103
    2F (M由N.EE.30) MTI=103
    IF (MPM.EQ.31) MrĖ=107
    iF (M@x.EE.31) MT1=107
    AFETUNM
110 CONTINU
    if (mmx.ET.36) E0 ta 120
    MAT1=328
    Mr1=1 S mre=1
    MTI=1 MTC=1
```

```
3361
3362
3363
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3400
3400
3401
3402


```

    IF (ल⿴囗十,EE.34) MT2=4
    ```
    IF (ल⿴囗十,EE.34) MT2=4
    IF (M, x.EE.34) MTI=4
    IF (M, x.EE.34) MTI=4
    iF (M%x.EE.34) MTI=4 (M%.EE.35) MTI=102
    iF (M%x.EE.34) MTI=4 (M%.EE.35) MTI=102
    1F (M曾=.ER.35) MTZ=102
    1F (M曾=.ER.35) MTZ=102
    1F (m由%.E0.36) MT2=103
    1F (m由%.E0.36) MT2=103
    1F (m@x.EO.36) MT1=103
    1F (m@x.EO.36) MT1=103
    mETumen
    mETumen
120 contimun
120 contimun
    1F (max.GT.47) Ea T0 160
    1F (max.GT.47) Ea T0 160
    MAT1=329
    MAT1=329
    IF (max, ET.38) &0 To 130
    IF (max, ET.38) &0 To 130
    MT1=1 % мT己=1
    MT1=1 % мT己=1
    IF (M由x.EO.38) нT2=2
    IF (M由x.EO.38) нT2=2
    #ETUNW
    #ETUNW
130 contrmul
130 contrmul
    IF (MNX.ET.40) 60 T0 140
    IF (MNX.ET.40) 60 T0 140
    Mrl=2 % mT2=2
    Mrl=2 % mT2=2
    IF (M粐.E者,40) mT2=4
    IF (M粐.E者,40) mT2=4
    mETUNW
    mETUNW
140 CONTINUE
140 CONTINUE
    IF (m贯x.ET.44) s0 T0 150
    IF (m贯x.ET.44) s0 T0 150
    MT1=4 $ mT2=4
    MT1=4 $ mT2=4
    IF (M左.EE.4E) MTE=102
    IF (M左.EE.4E) MTE=102
    1F (m#x.E0.43) MTE=103
    1F (m#x.E0.43) MTE=103
    1F (M|x.E0.44) m+2=107
    1F (M|x.E0.44) m+2=107
    #ETTUNON
    #ETTUNON
150 CONTINU
150 CONTINU
    MT1=10己 S MT2=102
    MT1=10己 S MT2=102
    1F (m贯^.E0.46) WT1=103
    1F (m贯^.E0.46) WT1=103
    iF (M⿴⿻冂土.E-.46) mT2=103
    iF (M⿴⿻冂土.E-.46) mT2=103
    IF (M|n.EE.47) MT1=107
    IF (M|n.EE.47) MT1=107
    1F (円由x.EE.47) MTE=107
    1F (円由x.EE.47) MTE=107
    mETUMN
    mETUMN
160 contymul
160 contymul
    1F (m@x.\epsilont.55) sa To 190
    1F (m@x.\epsilont.55) sa To 190
    MAT 1=382
    MAT 1=382
    1F (m,x.ET.50) s0 T0 170
    1F (m,x.ET.50) s0 T0 170
    MTIF1 % MTC=1
    MTIF1 % MTC=1
    if (M*ッ.E0.49) MT2=2
    if (M*ッ.E0.49) MT2=2
    iF (m@x.EO.50) MTZ=102
    iF (m@x.EO.50) MTZ=102
    mETUNN
    mETUNN
170 comTImun
170 comTImun
    IF (m@r.GT.52) 60 T0 180
    IF (m@r.GT.52) 60 T0 180
    MT1=E S MT2#2
```

    MT1=E S MT2#2
    ```


```

    mETUNN
    ```
    mETUNN
180 CONTIMUS
180 CONTIMUS
    #T1=4 $ MT2=4
    #T1=4 $ MT2=4
    if (mEx.6E.54) mT 2=102
    if (mEx.6E.54) mT 2=102
    if (MOx.EE.55) MTI=102
    if (MOx.EE.55) MTI=102
    NETUMON
    NETUMON
190 cONTIMUE
190 cONTIMUE
    IF (MNA.ET.58) &0 Ta 200
    IF (MNA.ET.58) &0 Ta 200
    #NT1=1301
    #NT1=1301
    MT1=1 S MT2=1
    MT1=1 S MT2=1
    IF (M由ッ.EE.E7) जTE゙=2
    IF (M由ッ.EE.E7) जTE゙=2
    1F (m@x.EE|.58) लT1=2
    1F (m@x.EE|.58) लT1=2
    GETUNOM
    GETUNOM
まOO CONTIMUE
```

まOO CONTIMUE

```




```

    5T0D
    ```
    5T0D
    EWD
```

    EWD
    ```

\section*{APPENDIX B}

TRDSEN

This appendix was provided by T. J. Seed and is a summary of the changes made in TRIDENT-CTR in order to obtain angular fluxes compatible with SENSIT-2D. In order to make a distinction between this version of TRIDENT-CTR and the normal version, it was renamed TRDSEN.

First UPDATE
```

*ID SENS1T
C1 SEEKIN. 2
C SENSIT
COTTON /SENST/ FNSEN(20). HOLTH(23)
C SENSIT
*D CD2;4
2LTOH. IPXS.LTC.IPCT.LTIT.LTSS.1PFSM. IPFSMR.LTFS.IPSEN.LTSEN
C SENSIT
CHTR1DED.26
DATA FNSEN/6HSNSTB1.GHSHSTB2.6HSNSTB3.6HSHSTB4.6HSNSTES.6HSNSTB6.
I GHSNSTR7.6HSIISTEQ.6HSNSTO9.6HSNST10.6HSNST11.GHSNST17.GHSNST13.
2 GHSNSTI4.6HSNST15.6HSNSTIG.6HSNST17.6HSNSTIE.GHSHETII.6HSNST13.
C SENSIT
\&INPUT11.04
C SEMSIT
EQUIVALENCE (IA(164).LSEN)
C SENSIT
\#1- 1MPUT11.230
C SENSIT
1HOLTH(1) = HHTR1D
1HDLTH(2) = 4H-SEN
1HDLTH(3)=4HSIT
IHOLTH(4) = 4HLINK
IHOLTH(4)=4HLIH
C SENSIT
\#1 1HPUT11.242
C SENSIT
IF(K.ME.1) CO TO 158
DO 15S 1=1. 10
1HOLTH(1+5) - IDUSE(1)
15S CONTINUE
15S CONTINUE
C SENSIT
ED INPUT1I.682
C SENSIT
LSEN = LFL * 3 \#MY \# 1 TMRX
LTLM = LSEN + 3 \# 1THAX
C SENSIT
OD INPUT11.017
C SENSIT
LTSEN = 3*NTC \# 1TH
2PSEN = 2PFSMA * NGFS8 = LTFS
LASTEC = IPSEN + LTSEH + 512
IF(1TH.ED.0) IPSEN = IPPI
C SEHSIT
\#D INPUT11.912. INPUT11.913
C SENSIT
520 FDRIRTY7OH THIS CASE LRS PROCESSED BY THE TRIDENT-CTR SENSIT P
IRDCESSOR ON .2x.A1E)
C SENSIT
C SENSIT
C SENSIT
7SO FORHAT(/ノIX.37NTRIDENT-CTR SENSIT PROCESSOR. DATE - - . AION
C SENSIT
*l GEOCON. }1
C SEMSIT
EOUIVILEMCE (1A(1).17M)
C SENSIT
-1 GEOCON.S9
C SENSIT
1F(1TH.EO.B) RETURN
DO 120 J = 1. JT
CALL LREED(A(LIP).A(L1PG).P1.J.1.3.1PP1.JT)
IMAX = 1T(J)
DO 110 1 = 1. IMAX
V1 - P1(1,1) + P1(2.1) + P1(3.1)
VO 110KK:1) + P1(2.1)
PI(K.1) = PI(K.1)/V1
110 CONTINUE
CALL LR1TE(A(LIP),A(L1PG),P1,J,1,3.1PSEN.JT)
120 CONTINUE
120 CONTINUE

```

```

    CD GRIND28
    #1 OUTER. 19
    C SENSIT
    CCALL INSTAL
    C

```
```

GCALL SEEKTWO
C SENSIT
\#1 OUTER.23
C SENSIT
DITENSION JPARM(1E).ESEN(5)
C SENSIT
\#1 OUTER.35
C SENSIT
EQUIVALENCE (1A(63).NTT),(1A(165),NSNST),(1A(166),JSEN)
C SENSIT
1] OUTER.SI
C SENSIT
DATA ESEN/GHTJOO MA,6HHY SEM,6HS1T DN,GHTP FIL,GHES
C SENSIT
D OUTER.60. OUTER.70
C SENSIT
1DOLD = 0
PLDS = HTC \# MSPO
MGSD = MAXDMP / MIDS
IF (NGSD.LT.1) NGSD = 1
NLNDS = NGSD F MDDS \& 33 + 512
NSDK - (1GM - 1) / NGSD - 1
1F(NSDK.GT.20) CALL ERROR(1,ESEH,5)
NGLD = 1GM - (NDSK-1) \# NGSD
C
JPARM(1) = 1TM
JPARM(1) = 1TM
JPARM(2) = IGM
JPARM(3) = JT
JPARM(4) = NTL
JPARM(S) = THPO
JPARM(7) = NGSD
IPARM(7) = NGSD
C SENSIT
C SENSTT,
C SENSIT
1DSDK = (C - 1) < HGSD + 1
1F(IDSDK.EO.1DOLD) GO T0 13
IF(IDSDK.ED.1) GO TD 137
CALL FILLU(i.FNSEN(IDSDK-1).FNSEN(1DSDK-1). MNDS1)
CALL SRITE(NSNST. ITEHP.0.B.E.4.JSEN)
CALL SEEK (FNSEN(IDSDK-i). IVERS.NSNST, 4)
137 CONTINUE
NLJDS1 = NLJDS
IF(1DSDK.EO_NSDK) NLDS1 = NHDLD
IVERS - IDSDK
JPARH(9) = HLDSS1
JPARM(10) = 1DSDK
CALL F1LLU(1.FNSEN(1DSDK).FNSEN(1DSDK).FMDS1)
CALL F1LLU(2.FNSEN(1DSDK).FNSEN(1DSDK), B)
CALL SEEK (FNSEN(IDSDK). 1VERS.NSNST. 1)
CALL SEEK(FNSEN(IDSDK). IVERS.NSNST.1)
CALL SRITE(NSNST. IHOLTH.0.0.23.1.JSEN)
CALL SRITE(NSNST.JHOL TH,0,0.23,1,JSEN)
130 CONT INUE
C SEMSIT
CD OUTER.303.OUTER.321
CD OUTER.
C SEHSIT
CD DUTER.324.OUTER.334
C SENSIT
\#D OUTER.337.OUTER.379
C SENSIT
*) IHNER.70
C SENSIT
CD 2HNER.O1
C SENSIT
CD INNER.g4
C SEHSIT
CD IHNER.97. IMNER.115
C SENS\T
JFS = 1
C SENSIT
\#D INNER.201.1MNER.207
C SEHSIT
THANK NELAB.ABSORO
\#1 SLEEP. }3
C1 SLEEP. }3
EQUIVALENCE (1A(164).LSEH)
C SEHSIT
0 SLEEP.98
C SEHSIT
CALL LRSNST(AF (1.2).AS.A(LSEN).1T)
C SENEIT
OD SLEEP. 140
C SENSIT
CALL LRSMST(AF (1.2).RS.A(LSEN).1T)

```
```

169 \#D SLEEP.249.SLEEP.254
*1 SLEEP.2S9
SUBRDUTINE LRSNST(AF,CF,SEN,1T)
C
C
CCRLL CD2
DITEENSIDN AF(3,1),SEN(3.1).CF(1)
C EOUIVALENCE (1A(16S).NSNST).(1A(166).JSEN)
DO 10 1 = 1. 1T
CF(1): 0.0.
DO 10 K = 1.
CF(1) = CF(i) + AF (K,1) = SEN(K.1)
CONTINUE
CALL SRITE(NSNST,CF,1T.*.0.2,JSEM)
RETURN
RE TURN
*C TRDCTR.SETBCl

```

Second UPDATE
```

1 - ID SENI
2-g mEMDOF.80
3-I MEMDEF.84
4 NLCM=0
5-I IMNEG.55
6 SEMSIT
7 EDUIUMLENCE (IN(164),LEEN)
8C SEMSIT
9-1 1MNEN. 131
10C SENSIT
11 CALL
1E SENSIT
13 DD IMNTM. 144
14 C SENSIT
15 - I NMEW. 167
16 C SENSIT

```

```

8 C SENSIT
19 [ IMNE田.180
20 C SENSIT
CD SNEEP. 269

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\section*{Los Alanos}```


[^0]:    a central processor unit time
    b input/output time
    C Livermore time sharing system time (total computing time)

[^1]:    * quadratic sums

[^2]:    1 \# spatial intervals for ONEDANT.
    ${ }^{2}$ equal-weight quadrature sets.
    ${ }^{3}$ Gaussian quadrature sets.

