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LOS ALAMOS SCIENTIFIC LABORATORY OF THE UNIVERSITY OF CALIFORNIA ° LOS ALAMOS NEW MEXICO

LABORATORY HANDBOOK OF NUCLEAR MICROSCOPY



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LA-1510 INSTRUMENTATION

LOS ALAMOS SCIENTIFIC LABORATORY OF THE UNIVERSITY OF CALIFORNIA LOS ALAMOS NEW MEXICO

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LABORATORY HANDBOOK OF NUCLEAR MICROSCOPY

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PREFACE

Originally this handbook was intended to serve as a guide to the beginning microscopist and as a reference for the supervisor in the nuclear microscopy laboratory at Los Alamos. Considerable interest has been shown in its contents by researchers who are engaged in work with nuclear emulsions, especially those who are forming analysis groups such as that at Los Alamos.

In consideration of this interest the handbook has been revised to include additional material, some of it more recent than the original report. Various corrections have been made along with some deletions, particularly when information was not firmly established.

It seems desirable to emphasize the necessity for flexibility in the procedure of analysis. Though the techniques described here are in general applicable to a variety of experimental situations, they can best serve as a guide in the formulation of procedures for other experimentation.

Because of the large number of sources of information on the analysis techniques for nuclear emulsions, no effort has been made to document completely the contributions of various workers in the field. It seems fitting, however, to acknowledge the work of the nuclear microscopists at Los Alamos who have proved the success of the methods described. Also we must acknowledge our indebtedness to Louis Rosen, who is responsible for many of the ideas presented here, though not for any errors in the presentation. We wish also to thank Leona Stewart for her considerable help in the preparation of this report.

Finally, we wish to call the attention of the reader to two review articles, one by Arthur Beiser, Revs. Mod. Phys. <u>24</u>, 273 (1952), and the other by Louis Rosen, "The Measurement of Neutron Energy Spectra Using Nuclear Emulsion Techniques" (in preparation).

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Chapter 1

THE MICROSCOPE

Several views of the Leitz Ortholux microscope are shown in Figs. 1a and 1b and the various parts are labeled. The optical system proper consists of the eyepieces, binocular tube, the objective, the condenser, the field diaphragm and condenser, and the light source. In Fig. 1b, A indicates the protractor eyepiece attachment for measurement of angles between tracks; B is a mirror system which gives a direct view of the scale of the fine adjustment knob; and C and D are the micrometer stage movements in the x and y directions, respectively. The alignment of this system is very important, and detailed instructions for alignment are given at the end of this chapter.

The mechanical features of the microscope provide a convenient means of handling the nuclear plate, which is placed on the stage. Two focusing adjustments are provided. The coarse adjustment is used to bring the plate up to the objective, and the fine adjustment is used in fine focusing and the measurement of the dip of tracks, when this is required.

In spite of its rugged appearance and weight, the microscope is a very delicate instrument. The results which are obtained from a microscope depend to a very great degree on the care with which it is treated.

Absolute cleanliness is probably the single most important factor in the care of the microscope. Our laboratory is air-conditioned with filtered air for the purpose of keeping dust--and especially grit--out of the mechanisms of the microscope. Many of our measurements have to be made with extremely high accuracy and the wearing of a micrometer screw or a fine adjustment can make an instrument useless in such cases. It is important that you keep the dust cover on your microscope at all times when it is not in use.

Should you notice that any movement on the microscope becomes hard or rough, stop using the microscope immediately and tell your supervisor about the trouble. From time to time it becomes necessary to clean and regrease the fine movements. Unless you know exactly how to do this, ask your supervisor for the proper procedure. Dirt and dust in the optical system must also be removed periodically. Usually such work is done by a skilled optician who comes to the laboratory at intervals for the purpose. Again, unless you are <u>positive</u> that you know the proper procedure, do not attempt this kind of job yourself. The alignment of a binocular system, for example, is a difficult job even for an expert.

For most of your work you will use what is called an oil-immersion objective. To use this objective, one places a drop of cedar oil on the slide and brings the slide and objective



Fig. 1a. Leitz Ortholux microscope

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Fig. 1b. Leitz Ortholux microscope

nearly into contact, so that the oil forms a part of the optical system. In this way, higher magnification and greater clarity are obtained. When cedar oil dries it forms a gummy residue which is rather difficult to remove. It may get into the condenser diaphragm, below the slide, and "freeze" it completely. Be especially careful, therefore, not to drop oil down on the condenser. However, if it should happen, be sure to clean the oil off immediately. A good way to keep from dropping oil on the condenser is to bring the oil in from the side of the plate.

Oil-immersion objectives are constructed to prevent leakage of oil into the lens system. They are not always completely tight, however. For this reason, you should never leave the objective in contact with the oil for any extended time when you are not using your microscope. Lower the stage and wipe the oil off the objective with lens paper. At the same time, the cedar oil should be wiped off the slide.

Xylol is a good solvent for cedar oil and may be used to remove dried oil from a plate or an objective. Unfortunately, xylol is also a solvent for the cement with which the lenses are fixed into the objective mount. Therefore, when you use xylol on an objective, use it sparingly. Merely dampen a lens tissue with xylol, wipe it across the front lens, and wipe off the xylol with a clean, dry lens tissue. Immersion oil should be removed from the objective in this manner after every use.

Xylol dissolves some of the protective plastic coatings put on the photographic plates to retard peeling. Hence it is best to remove the immersion oil from a plate only by wiping it clean with Kleenex. Although plates must be wiped to prevent swelling, a small residual film of oil seems in most cases to be beneficial to the plate.

The statement above concerning leakage of oil into an objective is of even greater import in the case of objectives which are not designed to be used with oil. If you should by accident get oil on a "high dry" objective, clean off the objective as soon as possible. Dry objectives are not, in general, very well sealed and oil leakage into them is much more likely than is leakage into oil-immersion objectives.

On both objectives and eyepieces (or oculars) you will find numbers which indicate the magnifications, as 6x, 12x, 45x, 90x. The magnification of the microscope in diameters is the product of the magnifications of the eyepiece and the objective. For example, with 6x (read "six by") eyepieces and a 90x objective, the magnification is 6×90 , or 540 diameters. Thus a track 10μ long will appear to be 5400μ (about 0.2 in.) long in the field of the microscope.

Oil-immersion objectives can be distinguished from dry objectives in that they usually are marked "oil" or "oel." Also, the numerical aperture (N.A.) of an oil-immersion objective is <u>usually</u> greater than 1, for example, 1.3 or 1.4. A dry objective <u>always</u> has an N.A. less than 1.

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Most objectives used in this laboratory are of a type called apochromats. The name refers to the degree of correction of the objective for color and distortion. With an apochromatic objective, always use compensating eyepieces. These are labeled, for example, 6x Comp or $6 \times B$ Periplan.

Most microscopists find that any tendency to eyestrain is minimized by the use of a blue-green filter in the illuminating system. The filter can be placed just below the condenser in the path of the light beam from the light source (see Fig. 1a). The color of the filter appears to be largely a matter of personal preference. Filters in use in our laboratory range in color from a pale yellow-green to dark blue-green.

In all types of analysis you will use an eyepiece reticle. This is a round, flat glass which has engraved on it a large square subdivided into smaller squares (usually 100). In addition, you may use other similar devices, such as eyepiece scales and angle reticles. These are to be placed into one eyepiece by removal of the top lens mount, which unscrews from the eyepiece. The engraved side of the reticle should be down, otherwise the reticle lines will not be in focus. If a reticle or eyepiece scale is used in only one eyepiece, a clear, round piece of glass should be placed in the other eyepiece. This system provides compensation for the change in effective length of the light path caused by insertion of extra glass in the optical system.

Misalignment of the optical system can be detected by a shifting of the grains and tracks as the fine focus is changed. This effect should be corrected by the alignment procedure described below. The following instructions apply primarily to the Leitz microscope. Alignment of the Cooke microscope follows the same general procedure except for the location of some of the adjustments.

Alignment of Leitz Microscope

1. Place plate, emulsion side up (or numbered side down), on stage. Check to see that the small rotatable lens (top element of condenser) is in the vertical position and that it is almost flush with the glass side of the plate. If the lens is too high, the plate will scrape on it, and perhaps be pushed up as the plate is moved horizontally or vertically. This adjustment is made with the screw under and to the rear of the microscope stage on the lefthand side of the microscope--not to be confused with coarse and fine adjustment screws for raising and lowering the stage.

Focus on plate with low-power objective (10x or 12x).

2. Close the field iris (or diaphragm, or aperture) with the milled ring under the stage. Note the image of the aperture (usually a decagon). It may be necessary to change the condenser focus slightly (by raising or lowering it as in 1) to make the image sharp.

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3. Center the image by means of the two aperture-centering screws just under the stage.

4. Open iris to illuminate field evenly. It is not necessary to open it all the way, but just until the image becomes larger than the field of view.

5. Move lamp condenser over full range using lever in front of lamp housing; meanwhile, look into the eyepieces. If the maximum illumination is off-center, bring it into the center with the lamp-housing centering screws. This centering is not critical, but the field should be well illuminated.

6. Move lamp condenser to position of nice illumination, minimum glare (lever will be near top).

7. Now go to oil-immersion lens and focus on tracks. Close the condenser field diaphragm (milled ring). Note aperture image, if possible. In some cases, even with the milled ring turned to the right as far as possible, the image may be larger than the field of view and, if centered, its edges will not be visible. However, if the image is not centered, the field will probably appear nonuniformly illuminated as the aperture is slowly opened and closed.

The centered aperture image should finally be made somewhat larger than the field of view so that the field is well illuminated.

8. Remove one eyepiece and look down into opening. Do not get your eye too close. You should see a uniformly illuminated round spot of light. Close the aperture iris with lever; round spot should become smaller and have polygon appearance. Now bring lever slowly to left until the spot is almost completely a circle again, i.e., until the aperture iris leaves are just sufficiently open to give a bright inner circle of light surrounded by a narrow darker ring of light.

9. The interpupillary adjustment has probably been made during operation 1, so that the centers of the lenses in the two eyepieces are exactly the same distance apart as the centers of the observer's eyes. The adjustment knob between the eyepieces is calibrated in millimeters. Record this reading at times of calibration, and be sure the setting is maintained since later calibrations depend upon it.

10. A final adjustment which may be made compensates for differences in strength between the right and left eyes. To make this adjustment, focus the microscope so that it is sharp for the right eye, then cover the right eyepiece and turn the adjustment on the left eyepiece until the image is equally sharp for the left eye. Both eyes should then see the image equally well.

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Chapter 2

CALIBRATIONS

One of the fundamental measurements necessary in the analysis of any plate is an accurate measurement of the length of a track. What one measures, in reality, is the <u>projected</u> length of a track in the emulsion. This is the distance between the ends of a track projected perpendicularly onto the surface of a plate. Thus, if the track is tilted in the emulsion, a length less than the true length of the track is measured. However, it is possible to arrive at the true length by measuring also the dip, or the distance in microns in a direction perpendicular to the surface of the plate from the beginning to the end of a track.

It is then necessary to calibrate the reticle in the microscope eyepiece in terms of a known length, as well as to check the calibration of the fine-adjustment screw, which is marked directly in microns. These calibrations are described in detail below.

2.1 Calibration of Eyepiece Reticle

Two types of stage micrometers are used. The more accurate one has a scale 1000 μ long, with lines at 5- μ intervals along the entire scale. No correction factor needs to be applied to this stage micrometer. The second type has a scale 2000 μ long subdivided into 100- μ intervals with one 200- μ section divided into 10- μ intervals. With this type of stage micrometer, always use the finely-divided region. A correction usually has to be made with the latter type, and is obtained from the number stamped on the plate itself or on the box in which it is kept. For instance, if the stage micrometer is marked 200.3 (the length in microns of the 20 spaces in the finely-divided region), each space of this portion is equal to 200.3/20 or 10.015 μ , instead of 10.000 μ .

Clamp a stage micrometer in position on the micrometer stage and locate the scale. It may be possible to find it directly with the high-power oil-immersion lens. If trouble is experienced, however, shift temporarily to a low-power objective (10x or 12x) until you locate the scale; then return to the oil-immersion objective. For ease in locating the lines the next time calibration is necessary, note "V" (vertical) and "H" (horizontal) stage vernier positions of the marked end of the stage micrometer.

Place the reticle in the eyepiece used by your dominant eye (see Appendix K). Insert a clear round of glass in the other eyepiece. With the horizontal movement, bring one of the lines of the stage micrometer into coincidence with the left-hand line of the reticle. Then look at the right-hand side of the reticle. If the right-hand line is also lined up exactly with a stage-micrometer line, the reticle length is an integer (usually 10) times the stage-micrometer spacing, S. Thus to get the length of the reticle in microns, count the number, n, of

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stage-micrometer divisions within the reticle, and multiply this number by the calibration factor, S, for the stage micrometer. R. L. (reticle length) = $n \times S$.

NOTE: Because the lines of the stage micrometer are wider than those of the reticle, you should always bring a reticle line into coincidence with the middle of the stagemicrometer line, the left edge, or the right edge--it does not matter which, as long as you are consistent. Also, always make the horizontal motion in the same direction when you bring reticle and stage-micrometer lines into coincidence. If you overshoot, back up and approach again. Now rotate the reticle so that the top and bottom are in the positions where the sides were, and repeat the above observations. This dimension gives the reticle width, which is also the swath width. If the reticle length and width are each exactly an integral number of stage-micrometer divisions, the calibration is completed. Return the reticle to its original position.

If both reticle dimensions are not integral multiples of stage-micrometer divisions, replace the clear round of glass in one eyepiece with one having a scale etched on it. After calibration has been completed, the scale may be removed and the clear round of glass returned to that eyepiece. (Either a scale or a clear glass is needed to compensate for the reticle in the other eyepiece which alters the path of the light coming up that eyepiece.) With the horizontal movement, bring one of the fine lines of the stage micrometer into coincidence with the zero line of the scale. Then, starting at the 100 end of the scale, follow back toward the zero end with your eye until you find a line of the scale which coincides with a micrometer line. Count the number of stage-micrometer divisions between zero on the scale and this maximum scale reading at which coincidence of scale and micrometer lines occurs. Divide the number of stage-micrometer divisions found in this interval by the scale reading at the end of the interval. Call this quantity C, the scale calibration factor.

EXAMPLE: There are seven stage-micrometer divisions between 0 and 86, where 86 is the maximum scale position at which a scale line and a micrometer line coincide. Then C = 7/86.

Now bring a stage-micrometer line into coincidence with the left-hand line of the reticle. Count the number, N, of <u>whole</u> stage-micrometer divisions within the reticle. Then, with the length of the scale parallel to the horizontal lines of the reticle, determine the number of scale divisions between the right-hand edge of the reticle and the last stage-micrometer line falling inside the reticle. Call this number of scale divisions M. Then $(N + C \times M)$, multiplied by the stage-micrometer calibration factor, S, gives the reticle length in microns.

Reticle Length = $S \times (N + C \times M)$.

Repeat this procedure with the reticle turned through 90⁰ to find the reticle length.

If the reticle divisions are not numbered and if the two dimensions are not the same, put an identifying mark, such as an inkspot, near one side of the reticle so that it will always be in the same position in the eyepiece.

2.2 Calibration of Fine Adjustment

The fine-adjustment knob of the microscope is marked in divisions which would, ideally, correspond to one micron movement of the stage. In general, however, the stage does not move exactly 1μ per division on the fine adjustment, but somewhat more or less, depending on the particular microscope. Also, the fine adjustment of a microscope is to some extent nonlinear over its range. That is, one division may correspond to 1μ at one part of the scale, but not at another part. These deviations from exact depth measurements ordinarily do not exceed 10 per cent. For this reason, we sometimes accept this error and use the fine-adjustment scale as though it were truly calibrated in microns. Since the fine-adjustment screw is somewhat nonlinear, it is best always to make measurements of depth in a plate by using the central portion of the adjusting mechanism. You will find that there are about 20 turns of the fine-adjustment knob from one extreme end of its range to the other, corresponding to a movement of the stage of about 2 mm. For the calibrations which follow, and for all depth measurements, you should use the central range, about 10 turns from either end. So that you can continue to use the central range, always bring the emulsion on the plate in focus first with the coarse adjustment.

Fortunately, it is not necessary to have an absolute calibration of the fine adjustment for much of our work. A relative calibration will serve practically the same purpose. The important feature, in general, is a knowledge of the ratio of thickness of the undeveloped emulsion to the <u>apparent</u> thickness of the developed emulsion. This ratio we will call the apparent shrinkage factor, and it should be used as K in the formulae in which the shrinkage factor is involved.

K is determined as follows:

Before a plate is exposed, two opposite corners of the emulsion on the plate are cut off, and the true thickness of the emulsion is measured at each end of the plate by means of a dial gauge. This is done in the darkroom during the preparation of the plate for exposure, and the measured thicknesses are recorded.

After exposure and development, the thickness of the emulsion is measured on the microscope by focusing on the top and bottom of the emulsion. (Remember to work against gravity and focus in the center of the field for this measurement. With a Leitz microscope, turn the

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fine-adjustment knob away from you; with a Cooke, turn the knob toward you.) Then the apparent shrinkage factor will be given by

The only assumption made here is that the stage movement is linear, that is, that one division on the fine-adjustment knob moves the stage the same distance as does any other division.

<u>NOTE</u>: This method takes into account any systematic error introduced by a difference in refractive indices of processed emulsion and immersion oil.

A knowledge of the apparent shrinkage factor, K, permits the measurement of absolute dip angles in the unprocessed emulsion. For the most accurate work, it is necessary to make a new determination of K for each plate analyzed, since the shrinkage is not always the same for two plates even though they come from the same batch and are processed together.

2.3 Use of Depth of Focus for Dip Measurement

The optical system of the microscope focuses in a range such that all grains within a vertical distance appear to be equally in focus. This distance is called the depth of focus of the objective. It is about 0.9μ for an oil-immersion 90x objective, but will vary somewhat depending on the microscope-observer combination. The larger the numerical aperture of an objective, the smaller will be the depth of focus.

It is possible to measure the dip of a track--or at least to determine whether or not a track falls within a given dip acceptance angle--by using the depth of focus criterion. This criterion is that a certain minimum length of the track must be in focus if the track is to be accepted.

To calibrate for the dip measurement, consider, for example, a track within the emulsion which has a dip of 1 in 10, as measured with the fine adjustment and reticle width, both calibrated previously. A second requirement for such a track is that it be straight, that is, it must not scatter detectably in the reticle width over which the dip is measured. This can be ascertained by focusing along the track length and noting that the same length of track is in focus for any portion of the track. In this measurement, as in the dip measurement above, it is important because of curvature of field that the center of the section of track on which measurements are made be at the center of the field.

The next step is to focus on the portion of the track at the center of the field, and observe the length of the track which is in focus. The depth of focus is given by

d = (length in focus) tan θ_d ,

where θ_d is the dip angle. Tan θ_d is the vertical dip distance from one side of the reticle to the other, divided by the reticle width. For example, if the reticle is 100 μ wide, and the dip is 10 μ

$$\tan \theta_{\rm d} = \frac{10 \ \mu}{100 \ \mu} = 0.10.$$

Then, if 9μ are in focus at once, the depth of focus in

$$d = 9 \mu \times 0.1 = 0.9 \mu$$
.

For a given angular criterion, the minimum length of track which should be in focus at one time will be

$$\mathcal{L}_{\min} = \frac{d}{\tan \theta_{\max}} \, .$$

If, for example, the dip criterion is 2 in 10, then

$$l_{\min} = \frac{d}{2/10} = 5d$$
.

In the case above, where $d = 0.9 \mu$, the minimum length which can be in focus is

$$\ell_{\rm min} = 5 \ge 0.9 \ \mu = 4.5 \ \mu.$$

Chapter 3

NUCLEAR EMULSION PLATES

The photographic emulsions used in nuclear-particle detection are, to some extent, quite similar to those used in an ordinary roll of film. Both consist of a light-sensitive chemical (silver bromide) distributed fairly uniformly throughout a gelatin. When light falls on the emulsion, changes in the silver bromide grains occur. These changes can be emphasized by other chemicals (development) so that the bromine is removed from the activated silver bromide grains. In its place are left small particles of silver. These appear black in their finely-divided form.

An action similar to that caused by light occurs when a charged nuclear particle passes through an emulsion. Development then makes visible the path of the particle in the emulsion. In the microscopy laboratory, the analyst investigates the various characteristics of such tracks, counting their number, and measuring their lengths, angular distributions, or other features which may be needed for the interpretation of a particular experiment.

A fundamental difference between nuclear emulsions and ordinary photographic emulsions is that the nuclear emulsion contains a great deal more silver. About four-fifths of the weight of the emulsion before development consists of silver. This feature makes a track more dense and continuous in nuclear emulsion, hence it can be followed more easily. Nuclear emulsions are also much thicker than ordinary photographic emulsions, and many particles will lose all their energy in the emulsion. When this happens, it is possible to determine the initial energy of the particle by measuring the length of the track (range) in the emulsion, provided the identity of the particle is known.

Energy, in the sense that we use it, is a measure of the mass of a particle and its velocity. That is, the greater the mass and velocity of an object, the greater its energy. When a nuclear particle enters the emulsion it begins to slow down, which is the same thing as saying that it starts to lose energy. But the greater the energy, the greater will be the distance required to stop the particle. A fixed relation exists between the energy and range in emulsion for all charged particles. For protons and alpha particles, this relation is well known. It is important to note that this relation varies with the nature of the particle, being different for a proton than for a deuteron, for example. Range-energy tables are given for the more common particles in Appendix A.

The unit of range which we use is the micron, one-thousandth of a millimeter. Energy is expressed in a less familiar unit, Mev (million electron volts).

Several types of nuclear emulsions are available. These differ mainly in their sensitivity to the various kinds of particles and one or another type of emulsion is used, depending on the particle to be studied.

Chapter 4

CHARGED PARTICLES AND NEUTRONS

From the nuclear-plate point of view it is convenient to divide nuclear particles into two groups--those which are electrically charged, and those which are not. The latter group, for our purposes, consists of only one kind of particle, the neutron. The former group includes many particles, of which the simplest is the proton. The neutron and the proton have essentially the same mass, and differ only in that the proton has a positive charge. In describing the charge of other particles, we use the charge of the proton as the basic unit. If we say a nucleus has a charge of 2, we mean that it has twice the charge of a proton.

Only charged particles produce tracks in the emulsion. Therefore, it is impossible for a neutron to record its path in the emulsion directly. When it is necessary to observe neutrons in an experiment, the neutrons can be "converted" to charged particles by various means. These methods are discussed in Chapter 6.

The most common light particles, together with their charge and mass in units of the proton charge and mass, are indicated in the following table:

Symbol	Name	Charge	Mass
N	Neutron	0	1
Р	Proton	1	1
D	Deuteron	1	2
Т	Triton	1	3
He	Helium three	2	3
He ⁴	Helium four, or alpha	2	4

Occasionally you may see in a plate a star formation of tracks about 25 μ long, originating at a point inside the emulsion. This is usually a thorium star. It is produced by a small amount of thorium, a radioactive element which emits alpha particles (charged helium nuclei). Thorium sometimes occurs as an impurity in the glass or in the emulsion.

Again, you may find a star with many branches of various lengths and degrees of darkness. This is a cosmic-ray star, produced by a highly energetic particle which had its origin directly or indirectly in outer space. However, thorium and cosmic-ray stars are not of any particular interest in our present work.

Chapter 5

MATHEMATICS

As a microscopist you are not required to know or use any but the simplest mathematics--most of it arithmetic. There are a few easy ideas from more advanced mathematics, however, which may prove useful to you in your work. In the experiments which produce the plates you analyze, these ideas are used frequently. Some understanding of trigonometry, for instance, may help you better to understand the reasons for certain measurements you will make. Consequently, you will be able to do your analysis more efficiently. Here, then, are some of the concepts most often used in this laboratory.

Angles are commonly measured in degrees, a unit derived by dividing the central angle of a circle into 360 equal parts. Some angles and their values are shown below:



A 90° angle is called a right angle; a triangle having one angle of 90° is called a right triangle. Trigonometry is the mathematics of triangles and, in particular, of right triangles.

The sum of the three angles of a right triangle is 180° . Thus, if we are given one acute (<90°) angle of a right triangle, all three angles are known.

Now let us look at two right triangles having equal angles, but with sides of differing lengths.



These are called "similar" triangles. The sides of these triangles are in exact proportion to each other, no matter how large or how small the triangle, and names are given to the ratios of the sides. We can refer to the sides of the triangle in relation to angle A as opposite A, and adjacent to A. The third side, opposite the 90° angle, is always called the hypotenuse.

It is now possible to name the ratios of the sides:

sine
$$A = \sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$
;

cosine A = cos A =
$$\frac{\text{adjacent}}{\text{hypotenuse}}$$
;
tangent A = tan A = $\frac{\text{opposite}}{\text{adjacent}}$.

The values of these ratios for all angles are available in tables.

As an example of the use of these functions, suppose it is desired to measure the length of a track which dips into an emulsion, starting at the surface, as shown:



Usually it is possible only to measure \mathcal{L} , the "projected" length of the track in the plane of the surface of the emulsion. L is the true length of the track. By the above definition for the cosine of an angle,

$$\frac{l}{L} = \cos B$$

or

$$L = \frac{l}{\cos B}$$

Now, if angle B is known (and usually it is in an experiment of this kind), all that is necessary is to look up cos B in the tables. Then the true length is found by dividing \mathcal{L} by cos B.

Suppose, for example, that \mathcal{L} is 1000 μ and B is 10⁰. From the tables, cos 10⁰ = .9848, and

$$L = \frac{1000 \ \mu}{.9848} = 1015 \ \mu.$$

There will be many such applications of trigonometry in your work. Usually most of the data necessary for such calculations will be given.

There is one other relation which you will be called upon to use rather frequently. This is, again, a relation between the sides of a right triangle and the hypotenuse. It states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides, or $l = \int_{V} v$



This is a useful relation in finding the true length of a track which has scattered in the emulsion. One measures the horizontal and vertical components of a track by micrometer screw or by reticle, and can then arrive at a value for the true length. The table of squares and square roots of numbers in most mathematical-table collections is useful here.

For tables of the various trigonometric functions, see Appendix J.

Chapter 6

EXPERIMENTAL TECHNIQUES AND ANALYSES OF PLATES

Just as we have divided the particles of interest into two divisions--charged particles and neutrons (Chap. 4)--so we can classify the experimental techniques of photographic emulsions. There may be charged particles to begin with, or it may be necessary to convert neutrons into charged particles which can produce tracks in the emulsions. The analysis techniques are considerably different for the two types of experiment, hence they will be treated separately.

6.1 Charged Particles

At Los Alamos there are at present two types of particle accelerators which produce charged particles of high energy. These are the Van de Graaff accelerator and the cyclotron. Usually, the particles which are accelerated in these machines are protons, deuterons, and alpha particles. In the Van de Graaff accelerator it is possible to accelerate other particles, including tritons and helium threes.

In the usual experimental arrangement, a charged particle beam passes through the nuclear multiple-plate camera, a photograph of which is shown in Fig. 2. To be more specific, let us suppose that we are using the beam of deuterons from the cyclotron.

The multiplate camera is loaded with plates, closed tight, evacuated, and then filled with a very pure gas, say, helium four. If the cyclotron is turned on and deuterons are passed through the camera, most of them will go straight through. But for every thousand or so deuterons which pass through the camera, one will collide with a He^4 atom and be scattered. When this happens, the deuteron will give a part of its energy to the He^4 and both particles will go off at angles to the original deuteron direction.

Some of those particles which are scattered will pass through the slits, fall on the surfaces of the various plates, and produce tracks which can be seen after development. The situation is shown in Fig. 3, except that the deuterons and He^4 's are shown very much larger than they actually are, and the slit systems are also somewhat enlarged.

Now you may see the reasons for the standards which are set up for the analysis of a plate of this kind. First, the tracks can go in only one direction--away from the numbered end of the plate, which is always pointed toward the center of the camera. Second, the tracks must start on the surface of the emulsion, since the charged particles which produce the tracks enter at the surface. Third, the slits limit the angle which the path of a particle can make with the axis of the plate. You are thereby given certain angular limits for the spread



Fig. 2. Nuclear multiple-plate camera

1 22 1



Fig. 3. Multiplate camera

and dip of a track. Finally, the slits are small enough so that only the central portion of the plate can be struck by the scattered particles. Tracks should appear only in the central portion of the plate. The reason for recording the position of tracks on the plate is that later you will be able to make a plot of track density vs position along the plate. Only those tracks which fall in the proper area on the plate are acceptable.

In the analysis of a plate of this type, tracks are counted in swaths, which are strips taken lengthwise along the plate. The width of a swath is defined by the reticle height as it appears in the microscope field. If a track begins within the reticle it is counted; otherwise it is not. Even if the track begins within the reticle and then scatters out, it is still counted. Swaths should be spaced far enough apart so as not to overlap. This means that if the reticle height is less than 100 μ , 0.1 mm is sufficient spacing between swaths. If the reticle height is more than 100 μ , 0.15 or 0.2 mm spacing should be used. In order that the swath be properly defined, it is essential that the top and bottom edges of the reticle be parallel to the direction of the stage motion in the H direction. Make sure this is the case by placing a silver grain on the top of the reticle and moving the H adjustment. If necessary, rotate the eyepiece containing the reticle until the grain stays on the reticle ruling across the entire width of the reticle.

It is possible to calculate the energy of any particle scattered at any angle in the camera, provided the initial energy of the beam particle is known. From this energy and the range-energy curve for the particular particle, it is possible to predict the range of the particle in the emulsion. On some plates there might be several range groups, corresponding to various kinds of particles scattered at the same angle. This is the reason for making a range analysis--that is, measuring all tracks, tabulating the number of tracks in equal range intervals, and plotting a graph. Such a graph indicates whether the particles have their proper range, and whether there are extraneous particles which may have an influence on the experiment.

As we have seen, neutrons cannot themselves produce tracks in an emulsion since they have no electric charge. In order to observe the effects of neutrons, it is necessary to convert the neutrons into charged particles. The charged particles can then be observed, and it is possible to arrive at conclusions concerning the neutrons which produce the charged particles.

6.2 Radiator Technique

One convenient way in which neutrons can be converted is to place a thin foil of material that contains hydrogen (paraffin, for instance) in the path of the neutrons. Some of the neutrons will strike hydrogen nuclei in the foil, and the hydrogen nuclei, or protons, will take up some of the energy of the striking neutron. If the proton goes in a direction parallel to that

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of the neutron direction, it will receive all the energy of the neutron. If there is an angle, θ , between the neutron and proton directions, the proton receives a part of the neutron energy, according to

$$E_p = E_n \cos^2 \theta$$
.

This equation says, for example, that if there is an angle of 10° between the directions of neutron and proton, the proton will have 97 per cent of the initial neutron energy.

Now it is necessary to let the protons fall on a photographic plate and produce tracks. The apparatus for such an experiment is shown in Fig. 4. From the figure it is possible to see how the analysis criteria are set up. These are:

1. The track must start on the surface of the emulsion.

2. The track must proceed away from the numbered edge of the plate.

3. The track must enter the emulsion within certain angular limits which are determined by the placement of the plate and the radiator, the size of the radiator, and the area to be counted. (See footnote page 36.)

The camera is evacuated so that the protons produced in the radiator will not lose any of their energy before they strike the plate.

In an experiment of this kind one usually wants to find the number of neutrons at each energy. To do this, note first that in measuring the length of a track in the microscope, we really measure the "projected length"--only the length of the track parallel to the surface of the emulsion. In order to get the true length we must divide by $\cos \theta$.

Now it is possible to find the energy of the proton which produced the track by using the range-energy curve for protons. Knowing the proton energy, we use the equation above in reverse, that is,

$$E_n = \frac{E_p}{\cos^2 \theta}$$

Thus we arrive at the neutron energy by dividing proton energy by $\cos^2 \theta$.

There is some slowing-down of protons in the radiator and in aluminum absorbers, which are sometimes placed between the radiator and the plate. Small corrections are necessary for these factors; they occur as a correction to the true range of the proton.

The probability that a neutron will collide with a proton varies with the energy of the neutron. The higher the neutron energy, the less likely it is that a neutron-proton collision will occur. This probability is expressed by a number called σ_{n-p} , the neutron-proton scattering cross section. σ_{n-p} represents the effective proton area available to a neutron for making a collision. A table of σ_{n-p} plotted against neutron energy is given in Appendix B.



Fig. 4. Neutron camera

Since it is less likely that neutrons of high energy will produce recoil protons, it is necessary to divide the number of protons produced by σ_{n-p} in order to arrive at a true picture of the neutron distribution. σ_{n-p} always enters into our neutron measurements.

The instructions for reading and calculating the neutron spectra observed in a camera with a radiator are summarized in Sect. 6.3.

Measure only tracks starting on the surface in the central inch of the plate, and proceeding away from the numbered edge. Acceptable tracks must satisfy given horizontal and dip-angle criteria.

6.3 <u>Calculation of Neutron Spectra for Plates in Camera</u> -- i.e., with radiator (and, possibly, absorber)

1. Column I. E_n in 0.5-Mev intervals.

2. Column II. $E_p (= E_n \cos^2 \theta)$. Use four significant figures. θ will depend upon position of plate in camera.

3. Column III. R_p , for each value of E_p in Column II. Obtain from range-energy curves for protons in <u>dry</u> emulsion (J. Rotblat, Nature <u>167</u>, 550 (1951)). Read to four significant figures.

4. Column IV. $R_{pcorr.} = R_p - (A + Rad/cos \theta)$ where A = correction for absorber, and Rad/cos θ = correction for radiator.

- 5. Column V. $R_{obs} (= R_{pcorr.} x \cos \theta)$.
- 6. Column VI. ΔE_n . Tabulate Column I in 0.5-Mev steps, e.g., 0-0.5, 0.5-1.0, etc.
- 7. Column VII. $\triangle R_{obs}$. Tabulate Column V in corresponding steps.
- 8. Column VIII. N_p (= number of proton tracks in ΔR_{obs} intervals). Tabulate from microscope data.

9. Column IX. σ_{n-p} . (n-p cross section for mid-point of ΔE_n intervals, e.g., for 9.25 Mev, 9.75 Mev, etc.).

Obtain from large graph of σ_{n-p} vs E, or from tables.

10. Column X. $N_n (= N_p / \sigma_{n-p})$. 11. Plot N_n vs E_n (Mev). For E_n use mid-point of ΔE_n intervals, as in Column IX (see Table 6.1).

6.4 Bare Plate Technique

Another way in which neutrons can be made to produce charged particles is the so-called bare plate technique. Here the principle is much the same as in the radiator technique in that neutrons collide with protons, the protons then producing tracks in the emulsion. The difference

Table 6.1

I	II	III	ΙV	v	VI	VII	VIII	IX	X
En	Ep	R _p	R pcorr.	Robs	ΔE _n	ΔR_{obs}	Np	σ _{n-p}	N n
0.5 1.0 1.5	$= E_n \cos^2 \theta$	from range- energy curve for protons in dry emulsion	$= \mathbf{R}_{\mathbf{p}} - \left(\mathbf{A} + \frac{\mathbf{Rad}}{\cos \theta}\right)$	=R _{pcorr} . x cos θ	0-0.5 0.5-1.0 1.0-1.5	corres- ponding steps	no. of observed proton tracks in Δ R _{obs} intervals from microscope data	from graph of σ_{n-p} vs E, at midpoint of ΔE_n interval	$=\frac{Np}{\sigma_{n-p}}$

SUGGESTED TABULATION OF DATA FOR NEUTRON SPECTRUM ANALYSIS WITH RADIATOR TECHNIQUE

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is that the hydrogen in the gelatin of the emulsion is the radiator.

The apparatus is quite simple. A plate is wrapped in black paper to prevent blackening by light, and the plate is aimed at the source of neutrons, with the numbered end of the plate pointed toward the source. Neutrons pass through the plate, producing recoil protons whose energy depends on the energy of the neutron and the angle of collision ($E_p = E_n \cos^2 \theta$). Figure 5 shows a typical experimental arrangement, and a greatly enlarged view of what happens in the plate during and after an exposure.

The criteria for analysis of a plate exposed in bare exposure are somewhat more involved. In this case the tracks must start and end within the emulsion. To make this certain, the requirement is made that the beginning and end of a track be 2μ or more below the surface and 2μ or more above the bottom of the emulsion.

Since we ordinarily want to know the energies of the neutrons which give rise to the recoil protons, it is necessary to set angular limits for acceptance of tracks both in dip and in horizontal angles. Usually, the angles which we use are about 10° , to either side and up and down. The reason for this is that for a 10° n-p collision, the proton will have 97 per cent of the neutron energy ($\cos^2 10^{\circ} = .9698$). If the angle for the collision is less than 10° , the proton will have a greater fraction of the neutron energy.

We know that the emulsion shrinks vertically during development by a factor of about 2.3. This means that, in general, the dip angle for acceptance of tracks must be reduced by about this factor. Sometimes plates may be soaked in glycerin in order to swell them back to their original, pre-development thickness. In such a case the dip angle is usually the same as the horizontal angle.

As noted previously, it is important to remember to measure the dip, especially of short tracks, by placing the section of track where the dip is to be measured in the center of the field of the microscope. Also, always remember to turn the fine adjustment knob in the direction which lifts the stage (see Chap. 1).

There is another very important difference to bear in mind in the measurement of track length in bare plate exposures. The track length must be measured along the long dimension of the plate. In Fig. 6a, for example, the two tracks illustrated would be recorded as having the same length, provided both fell within the proper angular criteria.

A correction factor is always applied to the range measured in this manner. This factor is $1/\cos \theta$, where the bar above $\cos \theta$ means that $\cos \theta$ is averaged over all angles in the pyramid for acceptance.

Similarly, the factor for conversion of proton energy to neutron energy is $1/\cos^2 \theta$, where $\cos^2 \theta$ is averaged over the pyramid for acceptance. In calculating $\cos^2 \theta$, it is necessary



Fig. 5



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also to take into account the size of the neutron source, so that $\cos^2 \theta$ may be averaged over somewhat larger angles than $\cos \theta$. For this reason, $(\cos \theta)^2$ may not have the same value as $\cos^2 \theta$. Details of the calculation are given in Appendix C.

Particularly in neutron work, but also in any plate in which protons are the particles which produce the tracks you are observing, you may find an occasional track which looks like that shown in Fig. 6b. If the angle between the branches of the tracks is exactly 90° , you may be sure that the proton which made the beginning part of the track has struck a second proton in the emulsion. The first proton has lost a part of its energy to the second, and both produce tracks.

The proper way to find the true range of the initial proton is as follows:

1. Measure the range of the first proton to the point of branching.

2. Measure the true length of each of the branches. Using the range-energy curve for protons, find the energy corresponding to each branch of the track.

3. Add the two energies from (2) together and find the range of a proton of the combined energies, again using the range-energy curve.

4. Now add the range which you got from (3) to the range of the first proton up to the branching point. This new value is the range that the first proton would have had, had there not been a collision.

As in the case of the radiator technique, it is necessary to divide the number of protons per energy interval by the appropriate n-p scattering cross section (σ_{n-p}) in order to take into account the lower probability of scattering for higher energy neutrons.

Another criterion for acceptance of tracks is that they must begin and end in the emulsion. This gives rise to the correction factor, P, related to the probability that a track of a particular length will go out of the emulsion when certain angular criteria are applied. Clearly, the longer a track and the greater the angular limits allowed, the more likely it is that the track will go out of the emulsion. The methods used in evaluating P are described in Appendix G.

A concise summary of the criteria for measuring tracks in plates exposed in "bare" geometry is given below.

6.5 Plates in Bare Exposure

6.5.1 Instructions for Reading Plates

Measure only tracks originating in the emulsion, 2μ or more below the surface and 2μ above bottom, and proceeding away from the numbered edge. Since a track may be traveling in any direction and at an angle either up or down in the emulsion, the direction in which it is proceeding must be determined by its character--tenuous at the start, and becoming heavier toward the end. In general, and unless otherwise instructed, measure only those tracks which also end in the emulsion.

Allow a spread of 11.3° (1 in 5) on either side of the horizontal axis. The dip angular criterion is 7° (1 in 8). Tracks may dip either up or down and be accepted.

In determining whether or not a track falls within these angular criteria, the important point to notice is the initial direction of the track. A track may scatter outside the angular limits, for example, only 5 μ from its beginning, or it may remain within these angular limits throughout its entire length. What counts is that the <u>first</u> part of the track must be within the angular limits.

If a track remains within the specified angles through its entire length, the length to be measured is a projection of the track length parallel to the long axis of the plate. This means that for such a track, you need only take a reading with your horizontal micrometer screw.

If, however, a track begins within the angular limits and scatters out of either or both of them, it is necessary to measure the true length of the scattered portion of the track. If the scattered length is short, measurement can be made most accurately by turning the eyepiece reticle parallel to the track. When the scattered portion is long, use the micrometer screw. If you do not have a vertical micrometer screw, use the reticle to measure the vertical component of the track length. Should the track scatter up or down, it is necessary to measure the difference in dip (in microns) with the fine-adjustment scale. Having measured the horizontal, vertical, and dip components of the track, you can arrive at the true length of the scattered portion of the track by using the formula

$$\mathcal{L} = \sqrt{h^2 + v^2 + (Kd)^2}$$

The factor K is the shrinkage factor of the emulsion. If you have measured the track length with your reticle, substitute the square of that measurement for $h^2 + v^2$ in the above equation.

Having found the true length of the scattered portion of the track, multiply it by the factor .985 (for the criteria given above), and add this corrected length to the length of the track which falls within the criteria. The sum is the proper length to use in your tabulation.

If the scattered portion of the track falls outside the horizontal but not the dip angular limit, you need only measure the horizontal and vertical components, and omit d from the formula above. Similarly, if the track falls outside the dip angle only, you can ignore the vertical component, measuring only the horizontal and dip components.

In all dip measurements, be sure to turn the fine-adjustment knob in the direction which lifts the stage (Leitz) or nosepiece (Cooke). For the Leitz, turn the knob away from you; for the Cooke, toward you. You should always have the segment whose dip is to be measured in the center of the field to avoid error caused by curvature of the field.

While measuring tracks, make about 10 determinations of the thickness of the emulsion at different regions of the total area covered. This figure will determine which graph of P is to be used in the calculations.

The above instructions and those for calculations of the spectrum below apply to most bare plates. There may be occasional exceptions to these instructions. For example, if you have a plate which has been soaked in glycerin subsequent to development, the dip angle for acceptance of tracks will be altered. In item 2 below, the value which you should use for θ_{av} will depend on the experimental setup. Consult your supervisor about which value to use.

- 6.5.2 Calculation of Neutron Spectra from Bare Plate Data
 - 1. Column I. E_n in 0.5-Mev intervals.
 - 2. Column II. $E_p (= E_n \overline{\cos^2 \theta}).$

3. Column III. R_p , for each value of E_p in Column II. Obtain from range-energy curves for protons in <u>wet</u> emulsion (Lattes, Fowler, and Cuer, Ilford Technical Data, 1947). Read to four significant figures.

- 4. Column IV. $R_{obs} (= R_p \times \overline{\cos \theta})$.
- 5. Column V. ΔE_n . Tabulate Column I in 0.5-Mev steps, i.e., 0-0.5, 0.5-1.0, etc.
- 6. Column VI. $\triangle R_{obs}$. Tabulate Column IV in corresponding steps.
- 7. Column VII. N_p (= number of recoil proton tracks in ΔR_{obs} intervals). Tabulate from microscope data.

8. Column VIII. σ_{n-p} . (n-p cross section for mid-point of ΔE_n intervals, e.g., for 9.25 Mev, 9.75 Mev, etc.).

Obtain from large graph of σ_{n-p} vs E, or from tables made from this graph.

- 9. Column IX. $N_p \ge 1/\sigma_{n-p}$.
- 10. Column X. F

Obtain from probability correction curve for the appropriate angle and for the thickness of unprocessed emulsion of your plate. Record values for mid-points of ΔR_{obs} intervals.

- 11. Column XI. $N_n = P N_p / \sigma_{n-p}$.
- 12. Plot a graph of N_n vs E (Mev). (See Table 6.2.)

Table 6.2

Ι	II	III	IV	v	VI	VII	VIII	IX	x	XI
En	Ep	Rp	Robs	ΔE _n	Δ R _{obs}	N p	σ _{n-p}	$\frac{N_p}{\sigma_{n-p}}$	Р	N n
0.5 1.0 1.5	$=\mathbf{E}_{\mathbf{n}}\overline{\cos^2\theta}$	from range- energy curve for protons in <u>wet</u> emulsion	$= R_p \overline{\cos \theta}$	0-0.5 0.5-1.0 1.0-1.5	corres- ponding steps	no. of recoil proton tracks in ΔR_{obs} intervals from micro- scope data	from graph of σ_{n-p} vs E, at midpoint of ΔE_n intervals		from probability correction curve for given θ and measured thickness, for midpoints of ΔR_{obs} intervals	$= \Pr{\frac{N_p}{\sigma_{n-p}}}$

SUGGESTED TABULATION OF DATA FOR NEUTRON SPECTRUM ANALYSIS WITH PLATES IN BARE EXPOSURE

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6.6 Neutron Spectra Measurements Using Nuclear Reactions

In both the radiator and bare techniques described above, it is difficult to measure accurately the energy of a neutron when it lies in the range 0.5-1.0 Mev, and it is practically impossible if the neutron energy is less than 0.5 Mev. In the energy region 0-2 Mev, a supplementary technique is being attempted. Called the Li^6 (lithium-six) technique, this method makes use of the fact that an atom of Li^6 can react with a neutron to produce an alpha particle (He⁴), a triton (H³), and a considerable amount of energy, even though the neutron may have almost zero energy. This reaction is written

$$Li^{6} + n \longrightarrow He^{4} + H^{3} + 4.78$$
 Mev.

When a neutron of very low energy (called a thermal neutron) produces this reaction, the energy available from the reaction is divided between the alpha particle and the triton in such a manner that the total range of the disintegration tracks is about 42 μ . If the neutron energy is increased, the length of the disintegration tracks increases in a calculable way.

A complete discussion of the ${\rm Li}^6$ method can be found in LA-1303, a training manual by J. H. Roberts and J. Haugsnes.

We have observed with some emulsions that reticulation during development of a very thin surface layer may lead to shifting of the first few grains of a track. It is important that this fact be taken into account in determining the acceptability of a track since its neglect may lead to the rejection of tracks which would otherwise be acceptable.

Appendix A

E (<u>Mev</u>)	R(μ) Proton (Wet Emulsion)	R(μ) Proton (Dry Emulsion)	R(μ) Deuteron Dry	R(μ) Triton Dry	R(μ) He ³ Dry	R(μ) He ⁴ Dry
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1	0.8	0.8	0.8	0.8	1.6	0.3
0.2	1.8	1.8	1.7	1.6		0.7
0.3	2.9	2.8	2.6	2.5		1.0
0.4	4.2	4.0	3.6	3.4		1.2
0.5	5.6	5.3	4.6	4.4		1.5
0.6	7.1	6.8	5.6	5.4	3.2	1.9
0.7	8.7	8.4	6.8	6.4		2.2
0.8	10.5	10.1	8.0	7.4		2.6
0.9	12.5	12.0	9.3	8.5		2.9
1.0	14.5	14.0	10.7	9.7		3.2
1.1	16.6	16.1	12.1	10.9	5.1	3.6
1.2	18.8	18.4	13.6	12.0		4.0
1.3	21.1	20.7	15.2	13.2		4.3
1.4	23.5	23.0	16.8	14.5		4.7
1.5	26.1	25.5	18.5	15.9		5.0
1.6	28.7	28.1	20. 2	17.3	7.0	5.3
1.7	31.3	30.8	22. 1	18.8		5.7
1.8	34.1	33.7	24. 0	20.3		6.1
1.9	37.0	36.6	25. 9	21.9		6.5
2.0	40.0	39.7	27. 9	23.5		6.8
2.1	43.1	42.9	30. 0	25.2	9.0	7.2
2.2	46.4	46.2	32. 2	27.0		7.7
2.3	49.7	49.4	34. 5	28.8		7.9
2.4	53.1	52.7	36. 8	30.6		8.3
2.5	56.5	56.0	39. 1	32.3		8.7
2.6	60.0	59.5	41.4	34.1	11.5	9.1
2.7	63.5	63.1	43.7	35.9		9.5
2.8	67.3	66.8	46.0	37.9		10.0
2.9	71.2	70.7	48.4	39.9		10.4
3.0	75.2	74.6	51.0	42.0		10.9
3.1	79.3	78.6	53.6	44.0	14. 3	11.3
3.2	83.6	82.7	56.2	46.0		11.7
3.3	88.0	87.0	58.9	48.2		12.1
3.4	92.5	91.3	61.6	50.5		12.6
3.5	97.0	95.7	64.5	52.9		13.0
3.6	101.5	100.3	67.4	55.3	17. 1	13.5
3.7	106.1	104.9	70.3	57.7		14.0
3.8	110.9	109.5	73.2	60.0		14.4
3.9	115.7	114.2	76.3	62.2		14.9
4.0	120.6	119.0	79.4	64.4		15.3

RANGE-ENERGY RELATIONS FOR PROTONS, DEUTERONS, TRITONS, He^3 , AND He^4

Appendix A (continued)

E (Mev)	R(μ) Proton (Wet Emulsion)	R(μ) Proton (Dry Emulsion)	R(μ) Deuteron Dry	R(μ) Triton Dry	R(μ) He ³ Dry	R(μ) He4 Dry
<u> </u>	195 5	<u>(</u>			<u></u>	<u></u>
4.1	125.5	124.0	82.6	66.7		15.8
4.2	130.6	129.0	85.8	69.0		16.2
4.3	135.8	134.1	89.1	71.3		16.8
4.4	141.0	139.3	92.4	73.9		17.3
4.5	146.2	144.5	95.6	76.5	20.2	17.9
4.6	151.5	149.7	98.8	79.0		18.3
4.7	156.8	154.9	102.1	81.6		19.0
4.8	162. 2	160. 3	105.3	84.3		19.7
4.9	167.6	165.7	108.6	87.0		20.3
5.0	173.0	171.3	112.0	89.7	23.5	20.9
5.1	178.5	177.1	115.4	92.4		21.5
5.2	184.2	183.0	118.9	95.2		22.0
5.3	190. 0	189.0	122.5	98.1		22.5
5.4	196.0	195.2	126.2	101.1		23.0
5.5	202.1	201.3	129.9	104.0	26.9	23.8
5.6	208.3	207.4	133.6	106.9		24 5
5.7	214.5	213.5	137.4	109.9		25 2
5.8	220.9	219.7	141.3	113.0		25.9
5.9	227.4	226.1	145.2	116.2		26.4
6.0	234.0	232.5	149.2	119.5	30.6	27.0
6.1	240.8	238 9	153 2	122 6		277
6.2	247.7	245.2	157 2	125.6		28.3
6.3	254.7	251.5	161.3	128.7		29.0
6.4	261.8	258.0	165 4	131 9		29.6
6.5	269.0	264.8	169.7	135.1	34.6	30.2
6.6	276 3	271 6	174 0	138 4		30 0
6.7	283 6	278 5	178.3	141 5		30.5
6.8	291.0	285 5	182 7	141.0 144 Q		32 1
6.9	298.4	292 5	187 0	148 2		32.8
7.0	305.9	299.7	191.3	151.3	38.6	33.5
71	313 5	306 9	105 0	154 6		21 9
7 2	391 9	314 1	200 5	157.0		25 0
73	320 1	201 2	200.0	101.9		25.7
7 4	323.1	322.5	204.9	101.4		00,1 90,1
7.5	345 0	335 8	209.0	168 0	49 g	30.4
	050,0	040.4	411, U		12, 0	
7.6	353.0	343.1	219.0	171.4		37.9
7.7	361.0	350, 5	223.8	174.9		38.7
7.8	369.0	358.1	228,6	178.3		39.4
7.9	377.0	366.0	233.3	181.9		40.1
8.0	385.0	374.0	238.1	185.6	47.6	40.9

Appendix A (continued)

F	R(μ) Proton	R(μ) Proton	R(μ) Deuteron	R(μ) Triton	R(μ) He ³	R(μ) He4
(Mev)	(Wet Emulsion)	(Dry Emulsion)	Dry	Dry	Dry	Dry
8.1	393.0	382.0	243.0	189.3		41.6
8.2	401.1	390.1	248.0	193.2		42.3
8.3	409.3	398.2	253.0	197.0		43.1
8.4	417.6	406.4	258.0	200.6		44.0
8.5	426.0	414.7	263.0	204.4	52.5	44.8
8.6	434.4	423.0	268.2	208.1		45.6
8.7	442.8	431.5	273.4	212.0		46.4
8.8	451.3	440.2	278.6	216.0		47.2
8.9	460. 0	449.0	283.8	220. 0		48.0
9.0	468.7	457.9	289.0	223.9	57.5	48.9
9.1	477.7	466.9	294.1	227.9		49.7
9.2	487.0	475.8	299.2	231.9		50.4
9.3	496.4	484.8	304.4	235.8		51.2
9.4	505.8	493.7	309.7	239.9	_	52.0
9.5	515.3	502.5	315.0	244.0	62.7	52.9
9.6	524.9	511.5	320.4	248.1		53.8
9.7	534.5	520.5	325.9	252.4		54.7
9.8	544.3	529.5	331.3	256.7		55.5
9.9	554.2	538.4	336.7	261.0		56.3
10.0	564.1	547.5	342.1	265.1	67.9	57.1
10.1	574.0	557.0	348.6	269.2		58.0
10.2	584.0	566.5	354.4	273.5		59.0
10.3	594.0	576.2	360.2	277.9		60.0
10.4	604.0	586.0	366.0	282.3		61.0
10.5	614.0	596.0	372.0	286.8	73.7	62.0
10.6	624.0	606.0	378.0	291.5		63.0
10.7	634.0	615.9	384.1	296.2		63.9
10.8	644.5	625.9	390.3	300.9		64.8
10.9	655.1	636.0	396.5	305.4		65.7
11.0	665.6	646.1	402.6	310.0	79.6	66.7
11.1	676.3	656.4	408.7	314.7		67.7
11.2	687.1	666.7	414.8	319.3		68.7
11.3	698.0	677.0	420.9	324.0		69.7
11.4	709.0	687.3	427.0	328.6	_	70.7
11.5	720.0	697.7	433.2	333.3	85.5	71.7
11.6	731.0	708.2	439.5	338.0		72.7
11.7	742.0	718.7	445.9	342.7		73.7
11.8	753.3	729.3	452.3	347.3		74.7
11.9	764.6	740.0	458.7	352.0		75.6
12.0	776.0	750.9	465.1	357.0	91.4	76.5

Appendix A (continued)

E (Mev)	R(μ) Proton (Wet Emulsion)	R(μ) Proton (Dry Emulsion)	R(μ) Deuteron Dry	R(μ) Triton Dry	R(μ) He ³ Dry	R(μ) He ⁴ Dry
12.1	787.5	761.8	471.5	361 9		77 5
12.2	799.0	772.8	477.8	366.9		78 5
12.3	810.5	783.8	484.1	372.0		79 5
12.4	822.0	794.8	490.4	377.0		80.5
12.5	834.0	805.8	496.8	382.0	97.7	81.6
12.6	846.0	816.8	503.2	387.0		82.7
12.7	858.1	828.0	509.7	392.0		83.9
12.8	870.2	839.4	516.2	397.0		85.0
12.9	882.4	850, 8	522.9	402.1		86.0
13.0	894.5	862.1	529.7	407.1	104.1	87.0
13.1	906.6	873.8	536.5	412.3		88.1
13.2	918.8	885.5	543.3	417.6		89.3
13.3	931.0	897.2	550.1	423.0		90.5
13.4	943.1	909.1	557.0	428.2		91.7
13.5	955.0	921.0	564.0	433.6	111.0	92.9
13.6	966.8	932.9	571.0	438.5		94.1
13.7	978.8	945.0	578.0	443.6		95.1
13.8	990.9	957.3	585.0	448.8		96.1
13.9	1003.0	969.7	592.1	454.0		97.2
14.0	1015.0	981.9	599.3	459.3	118.1	98.4
14.1	1027.0	994.2	606.6	464.7		99.6
14.2	1039.0	1006.9	613.8	470.0		100.8
14.3	1051.0	1019.9	621.0	475.3		102.0
14.4	1063.1	1032.9	628.1	480.6		103.1
14.5	1075.2	1045.9	635.3	485.8	125.2	104.3
14.6	1087.3	1059.0	642.6	491.1		105.6
14.7	1099.4	1072.1	649.8	496.7		106.8
14.8	1111.6	1085.7	657.0	502.1		108.0
14.9	1123.8	1099.9	664.5	507.4		109.2
15.0	1136.0	1114.6	672.2	513.0	132.3	110.4

Note:

Proton wet emulsion data taken from experimental results of Lattes, Fowler and Cuer, Nature 159, 301 (1947).

Proton dry emulsion data taken from experimental results of J. Rotblat, Nature 167, 550 (1951).

The ranges of deuteron, triton, He³, and He⁴ particles are calculated.

Appendix B

•

		1
E	σ _{n-n}	σ_{n-n}
	(harns)	$(harns)^{-1}$
	(bains)	(
0 - 0.25		
0.25 - 0.50	6.91	0,145
0.50 - 0.75	5.52	0.199
0.75 - 1.00	4.54	0.220
1.00 - 1.25	3.89	0.257
1.25 - 1.50	3.54	0.282
1.50 - 1.75	3, 29	0.312
1.75 - 2.00	3,06	0.327
2.00 - 2.25	2.86	0.350
2.25 - 2.50	2.69	0.372
2 50 - 2 75	2 53	0 395
2.75 - 3.00	2.39	0.418
3,00 - 3,25	2,25	0.444
3 25 - 3 50	2.14	0.467
3,50 - 3,75	2.03	0.493
	1.04	0 515
3,75 - 4,00	1.94	0.515
4.00 - 4.25	1.84	0.543
4.25 - 4.50	1.76	0. 568
4.50 - 4.75	1.69	0. 592
4.75 - 5.00	1.03	0.013
5.00 - 5.25	1.57	0.637
5.25 - 5.50	1.51	0.662
5.50 - 5.75	1.46	0.685
5.75 - 6.00	1.41	0.709
6.00 - 6.25	1.36	0.735
6.25 - 6.50	1, 32	0.758
6.50 - 6.75	1.28	0.781
6.75 - 7.00	1.24	0.806
7.00 - 7.25	1.20	0,833
7.25 - 7.50	1.16	0.862
7.50 - 7.75	1.13	0,885
7.75 - 8.00	1.10	0,909
8.00 - 8.25	1.06	0.943
8, 25 - 8, 50	1.04	0.962
8.50 - 8.75	1.01	0.990

n-p scattering cross section per 0.25 MeV

Appendix B (continued)

		1
En	σ n-p	σ n-p
(Mev)	(barns)	$(barns)^{-1}$
8.75 - 9.00	0.99	1.01
9.00 - 9.25	0.97	1.03
9.25 - 9.50	0.95	1.05
9.50 - 9.75	0.93	1.08
9.75 - 10.00	0.91	1.10
10.00 - 10.25	0.89	1.12
10.25 - 10.50	0.87	1,15
10. 50 - 10. 75	0.86	1,16
10.75 - 11.00	0.84	1.19
11.00 - 11.25	0.82	1.22
11.25 - 11.50	0.81	1.23
11.50 - 11.75	0.80	1.25
11.75 - 12.00	0.78	1.28
12.00 - 12.25	0.77	1.30
12.25 - 12.50	0.76	1.32
12.50 - 12.75	0.75	1.33
12.75 - 13.00	0.73	1.37
13.00 - 13.25	0.72	1.39
13.25 - 13.50	0.71	1.41
13.50 - 13.75	0.70	1.43
13.75 - 14.00	0.69	1.45
14.00 - 14.25	0.67	1.49
14.25 - 14.50	0.66	1.52
14.50 - 14.75	0.65	1.54
14.75 - 15.00	0.64	1.56
15.00 - 15.25	0.64	1.56
15.25 - 15.50	0.63	1.59
15.50 - 15.75	0.62	1.61
15.75 - 16.00	0.61	1.64
16.00 - 16.25	0.60	1.67

Appendix B (continued)

		1
E	σ _{n-n}	σ _{n-n}
(Mev)	(barns)	(harns)-1
(mev)	(bains)	(bai iib)
0 - 0.50	8.0	0.125
0.50 - 1.00	5.04	0.185
1.00 - 1.50	3.70	0.270
1.50 - 2.00	3.17	0.315
2.00 - 2.50	2.77	0.361
2.50 - 3.00	2.46	0.406
3.00 - 3.50	2.20	0.454
3.50 - 4.00	1.98	0.505
4.00 - 4.50	1.80	0.556
4.50 - 5.00	1.66	0.602
5.00 - 5.50	1.54	0.649
5.50 - 6.00	1.43	0.699
6.00 - 6.50	1.34	0.746
6.50 - 7.00	· 1.26	0.794
7.00 - 7.50	1.18	0.847
7.50 - 8.00	1.11	0,901
8.00 - 8.50	1.05	0.952
8.50 - 9.00	1.00	1.00
9.00 - 9.50	0.96	1.04
9.50 - 10.00	0.92	1.09
10.00 - 10.50	0.88	1.14
10.50 - 11.00	0.85	1.18
11.00 - 11.50	0.82	1.22
11.50 - 12.00	0.79	1.27
12.00 - 12.50	0.77	1.30
12.50 - 13.00	0.74	1.35
13.00 - 13.50	0.72	1.39
13.50 - 14.00	0.69	1.45
14.00 - 14.50	0.67	1.49
14.50 - 15.00	0.65	1.54
15.00 - 15.50	0.63	1.59
15.50 - 16.00	0.62	1.61
16.00 - 16.50	0.60	1.67

n-p scattering cross section per 0.50 MeV

Appendix C

BARE EXPOSURE--CALCULATION OF AVERAGED
$$\cos \Theta$$
 ($\cos \Theta$)
AND AVERAGED $\cos^2 \Theta$ ($\cos^2 \Theta$)

1. $\cos^2 \Theta$

Tracks are accepted in a pyramid of horizontal half angle B and dip half angle A. Tracks will occur at Θ with an intensity W, where

$$W \sim \sigma_{n-p} (C.M.) \frac{I(\Theta)}{I(\Omega)} 2\pi \sin \Theta d\Theta.$$

At neutron energies of 14 Mev and below, the n-p scattering cross section is isotropic in the center-of-mass system, and so

 $W \sim (4 \cos \Theta) (2\pi \sin \Theta \ d \Theta),$

$$W \sim \sin \Theta \cos \Theta d \Theta$$
.

Hence, in averaging $\cos^2 \Theta$ over the pyramid considered, we must weight the averaged quantity by the factor $\sin \Theta \cos \Theta d \Theta$.



In the figure, Θ is the angle between the path of the neutron, αO , and the path of the proton, $O\beta$, with O being the point of collision, such that

$$E_p = E_n \cos^2 \Theta$$

and

 $\cos \Theta = \cos \theta \cos \theta' - \sin \theta \sin \theta' \cos (\phi - \phi').$

Angles A and B define the pyramid for track acceptance in the plate, and angle C defines a circular source of neutrons, assumed to be of uniform intensity across its face.

$$\frac{\int_{-\infty}^{C} \int_{-\infty}^{2\pi} \int_{-\infty}^{1} \int_{-\infty}^{1} \int_{-\infty}^{2\pi} \int_{-\infty}^{2\pi} \cos^{2} \Theta \cos \Theta \sin \theta}{\sin \theta' \, d\phi \, d\theta' \, d\phi' \, d\theta}}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_$$

The expansion of $\cos^3 \Theta$ contains two terms which are odd in $\cos (\phi - \phi')$; these will integrate to zero with the limits 0 and 2π .

After the ϕ integration, we have

$$\iiint \int_{\theta=0}^{C} \left[2\pi \cos^{3} \theta \cos^{3} \theta' \sin \theta \sin \theta' + 3\pi \cos \theta \cos \theta' \sin^{3} \theta \sin^{3} \theta' \right] d\theta d\theta' d\phi'$$

which yields

,

Thus

$$\oint_{\phi'=0}^{\phi'=2\pi} \int_{\theta'=0}^{\theta'=f(\phi')} \left[\frac{\pi}{2} \cos^3 \theta' \sin \theta' (1 - \cos^4 C) + \frac{3\pi}{4} \sin^4 C \cos \theta' \sin^3 \theta' \right]$$

$$\oint_{\phi'=0}^{\phi'=0} \int_{\theta'=0}^{\theta'=0} d\theta' d\phi'.$$

It is convenient to divide the limits of ϕ' into parts corresponding to the corners of the base of the pyramid of acceptance, that is, for

$$0 < \phi' < \tan^{-1} \left(\frac{\tan B}{\tan A} \right)$$

$$0 < \theta' < \sin^{-1} (\tan A \sec \phi'),$$

and for

$$\tan^{-1} \left(\frac{\tan B}{\tan A} \right) < \phi' < \frac{\pi}{2}$$
$$0 < \theta' < \sin^{-1} (\tan B \csc \phi')$$

The final integration gives for the numerator

~

$$2\pi (1 - \cos^4 C) \left[\tan A \tan B - \frac{1}{3} (\tan^3 A \tan B + \tan A \tan^3 B) \right]$$
$$+ \pi \sin^4 C (\tan^3 A \tan B + \tan A \tan^3 B).$$

Integration of the denominator by a similar process gives

$$4\pi \sin^2$$
 C tan A tan B.

The final result is

$$\overline{\cos^2 \Theta} = 1 - (\frac{1}{3} - \frac{5}{12} \sin^2 C) (\tan^2 A + \tan^2 B) - \frac{1}{2} \sin^2 C.$$

2. $\cos \Theta$

Here the considerations are somewhat simpler, since the dimensions of the source need not be taken into account. We may write

$$\frac{\phi' = 2\pi}{\cos \Theta} = \frac{\phi' = 0}{\int \int \phi' = 0} \frac{\phi' = 0}{\int \phi' = 0} \frac{\phi' = 0}{\int \int \phi' = 0} \frac{\phi' = 0}{\int \phi' = 0} \frac{\phi' = 0}{$$

٠

Again we apply the limits

$$0 < \phi' < \tan^{-1} \left(\frac{\tan B}{\tan A} \right)$$
$$0 < \theta' < \sin^{-1} (\tan A \sec \phi')$$

and

$$\tan^{-1}\left(\frac{\tan B}{\tan A}\right) < \phi' < \pi/2$$

 $0 < \theta' < \sin^{-1} (\tan B \csc \phi').$

The first integration of the numerator yields

$$N = 4/3 \begin{cases} \tan^{-1} (\tan B/\tan A) \\ \int_{0}^{\pi/2} \left[1 - (1 - \tan^{2}A \sec^{2}\phi')^{3/2} \right] d\phi' \\ + \int_{\tan^{-1} (\tan B/\tan A)}^{\pi/2} \left[1 - (1 - \tan^{2}B \csc^{2}\phi')^{3/2} \right] d\phi' \end{cases}$$

This form cannot be integrated and must be evaluated numerically. Integration of the denominator gives

 $D = 4 \tan A \tan B$

and

$$\overline{\cos \Theta} = \frac{N}{4 \tan A \tan B}$$

Numerical integrations have been performed for two cases. The results are

For A = B =
$$10^{\circ}$$
, $\overline{\cos \Theta} = 0.9893$.
For A = B = 20° , $\overline{\cos \Theta} = 0.9561$.

Appendix D

HEALTH MONITORING

Bare Exposure--Calculation of isotropic neutron flux which produced a given number of

$$F_{n}(E) = \frac{4\pi N_{p}(E)}{n_{o} t A \sigma_{n-p}(E)\Omega},$$

where

F _n (E)	=	integrated neutron flux
N _p (E)	=	number of protons observed
n	=	number of hydrogen atoms per unit volume in plate
t	=	thickness of plate analyzed
Α	=	area of plate analyzed
σ _{n-p} (Ε)	=	n-p scattering total cross section at energy E
Ω	=	solid angle in which measurements were made. $\Omega = 4\pi$ if no direction discrimination is made, as in health monitoring.

Reduced to our common units, this formula gives

$$F_{n}(E) = 3.70 \times 10^{8} \frac{N_{p}(E)}{t A \sigma_{n-p}(E) \Omega}$$

where

 $F_{n}(E) = neutrons/cm^{2}$ t = microns A = square millimeters $\sigma_{n-p}(E) = barns$ $\Omega = steradians.$

If measurements are made of tracks falling in a rectangular pyramid of half-angles α and β , respectively, then

$$\Omega = 2\pi - 4 \left[\sin^{-1} \left(\frac{\cos \alpha \tan \beta}{\sqrt{\tan^2 \alpha + \tan^2 \beta}} \right) + \sin^{-1} \left(\frac{\cos \beta \tan \alpha}{\sqrt{\tan^2 \alpha + \tan^2 \beta}} \right) \right]$$

where the angles derived are expressed in radians, and $\boldsymbol{\Omega}$ is in steradians.

If $\alpha = \beta$, this reduces to

$$\Omega = 2\pi - 8 \sin^{-1} (.7071 \cos \alpha)$$
. (See Appendix F.)

Appendix E

BARE EXPOSURE -- HIGH RESOLUTION METHOD

The usual analysis of tracks which fall within a rectangular pyramid inside the emulsion gives an energy resolution determined by the size of the pyramid chosen and the size of the neutron source. In this method, averaged correction factors are applied, both for the short-ening of a track ($R_{obs} = R_T \cos \theta$) and for the conversion from proton energy to neutron energy ($E_{neut} = \frac{E_{proton}}{\cos^2 \theta}$).

Although a rather large amount of effort is required, it is possible to achieve somewhat better resolution in energy by measuring the true range and true θ for each track, then calculating separately for each track the energy of the neutron which produced it. The method follows:

1. As in ordinary bare exposures, measure only tracks which fall within a defined rectangular pyramid. (The angular limits will be given to you by your supervisor.)

2. For long tracks which satisfy the proper criteria, make three measurements. We will call these the horizontal, vertical, and dip distances, and denote them by h, v, and d.

3. For tracks shorter than 100 μ , turn the eyepiece reticle and measure the projected length with the reticle lines parallel to the track. Call this length p. Measure also the projected length, h, of a short track in the H direction, and the dip.

4. Compute the true lengths of the tracks as follows:

a. Long tracks

 $L = \sqrt{h^2 + v^2 + (Kd)^2}$ where K is the shrinkage factor of the emulsion. b. Short tracks $L = \sqrt{p^2 + (Kd)^2}.$

5. Having calculated the true length of each track, find the energy of the proton which produced the track from the range-energy curve for protons.

6. From the proton energy thus found, calculate the energy of the neutron which produced it by the formula

$$E_{n} = \frac{E_{p}}{\cos^{2} \theta}$$
$$\cos^{2} \theta = (h/L)^{2}$$
$$E_{n} = E_{p}(L^{2}/h^{2})$$

The calculation for $\cos^2 \theta$ given here assumes the source to be of negligible size. If this is not the case, an additional factor is required (see Appendix C).

Appendix F

CALCULATION OF SOLID ANGLE SUBTENDED BY A RECTANGULAR PYRAMID



The half angles of the pyramid are α and β . ϕ and θ are measured as indicated. The transformation from rectangular to spherical coordinates is then

and the element of the solid angle is

$$d^{2}\Omega = \frac{\rho \sin \phi \, d \, \theta \, \rho \, d \, \phi}{\rho^{2}} = \sin \phi \, d \phi \, d \, \theta.$$

 θ must go through the range $0-2\pi$; ϕ , from 0 to an upper limit which depends on θ . It is convenient to divide the θ range into intervals corresponding to the diagonals of the face of the pyramid, integrating θ over the limits

$$-\tan^{-1}\left(\frac{\tan\beta}{\tan\alpha}\right) \quad \text{to} \quad \tan^{-1}\left(\frac{\tan\beta}{\tan\alpha}\right)$$
$$\tan^{-1}\left(\frac{\tan\beta}{\tan\alpha}\right) \quad \text{to} \quad \pi - \tan^{-1}\left(\frac{\tan\beta}{\tan\beta}\right)$$

and

$$\tan \alpha / \tan \alpha / \tan \alpha / \tan \alpha / \tan \alpha$$

In the first θ range, ϕ along the edge of the pyramid is given by

z = const = $\rho \cos \phi$ x = const = $\rho \sin \phi \cos \theta$

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$$x/z = \tan \alpha = \tan \phi \cos \theta$$

 $\phi = \tan^{-1} (\tan \alpha \sec \theta),$

while in the second range, the upper limit for ϕ is, similarly,

$$\phi = \tan^{-1} (\tan \beta \csc \theta).$$

Thus we have

$$\Omega = 2 \int_{-\tan^{-1}(\tan \beta/\tan \alpha)}^{\tan^{-1}(\tan \beta/\tan \alpha)} \int_{0}^{\tan^{-1}(\tan \beta \propto \theta)} \int_{0}^{\tan^{-1}(\tan \beta \propto \theta)} \int_{0}^{\pi^{-1}(\tan \beta/\tan \alpha)} \int_{0}^{\pi^{-1}(\tan \beta/\tan \alpha)} \int_{0}^{\tan^{-1}(\tan \beta \propto \theta)} \int_{0}^{\tan^{-1}(\tan \beta \propto \theta)} \int_{0}^{1} \int_{0}$$

Performing the first integration with respect to ϕ , substituting limits, and simplifying, we have

$$\Omega = 2\pi - 2 \left[\int_{-\tan^{-1}(\tan \beta/\tan \alpha)}^{\tan^{-1}(\tan \beta/\tan \alpha)} \int_{-\tan^{-1}(\tan \beta/\tan \alpha)}^{\cos \alpha \cos \theta d \theta} \int_{-\tan^{-1}(\tan \beta/\tan \alpha)}^{\pi - \tan^{-1}(\tan \beta/\tan \alpha)} \int_{-\frac{\cos \beta \sin \theta d \theta}{\sqrt{1 - \cos^2 \beta \cos^2 \theta}}}^{\pi - \tan^{-1}(\tan \beta/\tan \alpha)} \right]$$

The final integration yields

$$\Omega = 2\pi - 4 \left[\sin^{-1} \left(\frac{\cos \alpha \tan \beta}{\tan^{2} \alpha + \tan^{2} \beta} \right) + \sin^{-1} \left(\frac{\cos \beta \tan \alpha}{\sqrt{\tan^{2} \alpha + \tan^{2} \beta}} \right) \right]$$

If $\alpha = \beta$, the expression becomes

$$\Omega = 2\pi - 8 \sin^{-1} (0.7071 \cos \alpha).$$

Appendix G

BARE EXPOSURE--CALCULATION OF PROBABILITY THAT A TRACK OF GIVEN LENGTH PRODUCED IN EMULSION REMAINS IN EMULSION FOR ITS ENTIRE LENGTH*

If $\alpha = dip angle$

t = thickness of emulsion

 \mathcal{L} = length of track

w = probability that the track ends in emulsion

 $P = \frac{1}{w}$ = reciprocal probability that the track ends in emulsion,

and

I.
$$\ell \sin \alpha \ge t$$

 $w = \frac{t}{2\ell \sin \alpha}$

 \mathbf{or}

Ι

I.
$$\ell \sin \alpha \leq t$$

w = 1 - $\frac{\ell \sin \alpha}{2t}$

In the above corrections, no account is taken of the fact that actual tracks in emulsion are not straight, but are scattered and, further, that the effect of multiple scattering is not compensated for completely because of the large cross section for multiple scattering through small angles. Rosen** has made an empirical determination of the probability that tracks of various ranges will leave the emulsion. His work was based on observed track trajectories of recoil protons from 14-Mev neutrons. The results of this study are shown in Fig. 7, in which the solid lines represent the probability of leaving the emulsion for acceptance angles of 10° and 15° , respectively. The dashed lines are the corresponding probabilities calculated by the method of Richards.

^{*}H. T. Richards, Phys. Rev. <u>59</u>, 796 (1941).

^{**}L. Rosen, "The Measurement of Neutron Energy Spectra Using Nuclear Emulsion Techniques," to be published. See also L. Stewart, M.A. Thesis, University of Texas, 1953.



Fig. 7. Corrections for escape of tracks from emulsion for 200 and 400μ emulsions and for acceptance dip angles of 10° and 15° . Dashed lines show rectilinear theory. Solid lines are empirical results of Rosen.

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Appendix H

MOISTURE AND HYDROGEN CONTENT OF EMULSION AS A FUNCTION OF RELATIVE HUMIDITY

We are indebted to Ilford, Ltd., who have supplied us privately with the following data:

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Emulsion Condition (% R.H.)	H ¹ Content (gm/cc)
0	0.052
32	0.054
50	0.056
58	0.056
84	0.062

Appendix I

DEVELOPMENT TECHNIQUES

The methods which follow have been found to give the best results of several which have been investigated. The Berkeley method has the advantages that only stock solutions are used and that it requires less total time for processing than is required for most developing procedures. Its disadvantage is that it produces nonuniform grain development, which is a handicap when the direction of tracks is being observed critically, or when one is trying to detect high-energy particles within the emulsion. The A and B method produces uniform development throughout the emulsion. For emulsion thicknesses up to 200 μ , it has been found to be superior to the "cold technique" of Lattes, et al.

Berkeley Method (A. Oliver, U.C.R.L., private communication)

Presoak plates for 10 minutes. Develop in one part D-19 to five parts water for 30 minutes without agitation. Stop bath, 2 per cent acetic acid, 15 minutes, with agitation. Wipe off surface silver. Fix in acid hypo until clear, plus one-half as long as it takes to clear (with agitation). Wash for at least 2 hours. Place in 2 per cent glycerine bath for 10 minutes. All solutions at 70° F (Berkeley uses 68° , but 70° seems better). Hypo changed at hourly intervals.

A and B Method (M. Blau and J. A. De Felice, Phys. Rev. 74, 1198 (1948))

Presoak plates for 10 minutes. Solution A for 30 minutes; solution B for 30 minutes. (We use no agitation; Blau uses slight agitation in solution A.) Stop bath, 2 per cent acetic acid, 15-20 minutes. Stop bath and all following solutions with agitation. Wipe off surface silver. Neutral hypo until clear, plus one-half again as long. Then acid hypo for 30 minutes. Wash at least 2 hours. Place in 2 per cent glycerine solution for 10 minutes. All solutions at 70° F. Neutral hypo changed at hourly intervals.

For	mulae

Solution A:	Solution B:
2.2 gm Elon 48.0 gm sodium sulfite 8.8 gm hydroquinone 4.0 gm potassium bromide Water to make 4 liters	800 cc Stock D-19 3200 cc water 32 gm sodium carbonate
Neutral Hypo:	

300 gm sodium thiosulfate 1 liter water

All solutions mixed with distilled water.

With additional refinements of the techniques of neutron spectrum measurements by the use of thicker emulsions, improvements have been made in our developing techniques. These are described in the following extract from a paper by L. Rosen:*

Although each manufacturer suggests adequate techniques for processing his emulsions, Dr. A. H. Armstrong and Mrs. M. Gibson, of this laboratory (LASL), have found that for neutron spectrum work in which it is important to determine the sense of the particle producing the trajectory, the following developing procedures may be used to best advantage:

The 100 μ and 200 μ plates are developed by a combination of the two-solution method** and the temperature-development method devised by Dilworth, Occhialini, and Payne.*** The plates are first soaked in distilled water for 1 hr; during this time the temperature is lowered from 21^o to 5^oC. They are then placed consecutively in solution A**** for 50 min and in solution B**** for 50 min, both solutions being maintained at 5^oC.

After cold development in solution B, part of the solution is poured off, leaving enough to barely cover the plates. The solution is then allowed to warm up to $24^{\circ}C$ and remain at this temperature for 20 min. The plates are then placed consecutively into a stop bath of 2% acetic acid for 15 min (at the end of which time the surface silver is removed by rubbing lightly with the finger tips), into neutral hypo for 4 hr (hypo changed every hour), acid hypo for 30 min, and finally into wash water for 2 hr. The acetic acid, hypo, and wash water are all maintained at approximately $22^{\circ}C$ and all solutions are agitated mechanically. The plates are permitted to dry at room temperature and 50% relative humidity while lying horizontally, emulsion side up.

Our method for processing 400 μ plates follows closely the procedure of Dilworth, Occhialini, and Payne as modified by the Naval Research Laboratories***** and is as follows:

Solution A

Solution E

2.2	gm of	metol
48.0	gm of	sodium sulfite
8.8	gm of	hydroquinone
4.0	gm of	potassium bromide
	water	to make 4 liters

800 cc of stock Kodak D-19 3200 cc of distilled water 32 gm of sodium carbonate

*****B. Stiller, NRL, private communication.

^{*}L. Rosen, The Measurement of Neutron Energy Spectra Using Nuclear Emulsion Techniques, to be published.

^{**}M. Blau and J. A. DeFelice, Phys. Rev. 74, 1198 (1948).

^{****}C. C. Dilworth, G. P. S. Occhialini, and R. M. Payne, Nature 162, 102 (1948). ****The formulae used for the two-solution technique are as follows:

		Initial Temperature (^O F)	Final Temperature (^O F)
100	min presoak in distilled water	70	40
50	min in developing solution A	40	40
50	min in developing solution B	40	40
20	min dry developing (see procedure for 200 μ plates)	40	70
5	min dry cooling	70	40
100	min in 2% acetic acid	40	40
	Remove surface silver		
22	hr in neutral hypo with agitation and continuous renewal of hypo*	40	40
24	hr in hypo with constant water dilu- tion at the same rate as above	40	40
3	hr to raise the temperature	40	60
24	hr wash	60	60
30	min in 6% Flexogloss	40	40
1	week drying at 50% R.H.	75	75

If the plates have been in a vacuum, or otherwise desiccated, there is a tendency for the emulsion to separate from the glass backing. This so-called peeling may occur while the plate is being desiccated, during development or after development, even though the plate is stored at 68° F and 50% relative humidity. Peeling may be inhibited by coating the edges of the plate with collodion or clear lacquer. Dr. G. M. Frye, of this laboratory (LASL), has found that branding the edges of the plate with a hot wire at any stage of its history (before desiccation, before development or after development) greatly inhibits peeling from that time on.

^{*}Agitation is performed by bubbling nitrogen through the solution. During the first 18 hr neutral hypo is used, new hypo being added constantly while old hypo is flowing out. During the last 4 hr acid hypo is added rather than neutral hypo. The rate of flow of hypo is such as to transfer each hour a volume of new (and old) hypo equivalent to one-half the volume of the bath.

Appendix J

TRIGONOMETRIC FUNCTIONS AND SQUARE AND SQUARE ROOT TABLES

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The following identities may be useful.

secant
$$\theta$$
 = sec θ = $\frac{1}{\cos \theta}$
cosecant θ = csc θ = $\frac{1}{\sin \theta}$
cotangent θ = cot θ = $\frac{1}{\tan \theta}$

Deg.	Sin	Cos	Tan
0.0	. 0000	1.0000	. 0000
0.0	.0087	1.0000	. 0007
1.0	.0174	.9998	. 0175
1.0	. 0262	.9997	. 0262
2.0	.0349	.9994	. 0349
2.5	.0436	. 9990	. 0437
3.0	. 0523	. 9986	. 0524
3.5	. 0611	. 9981	. 0612
4.0	. 0698	. 9976	. 0699
4.5	. 0785	. 9969	. 0787
5.0	. 0872	. 9962	. 0875
5.5	. 0958	. 9954	. 0963
6.0	1045	. 9945	. 1051
6 5	1132	9936	1139
7 0	1219	9925	1228
7.5	. 1305	. 9914	. 1316
	1000	0000	1405
8.0	. 1392	. 9903	. 1405
8.5	.1478	.9890	. 1494
9.0	.1564	.9877	.1584
9.5	.1651	.9863	. 1673
10.0	. 1736	. 9848	.1763
10.5	. 1822	. 9833	. 1853
11.0	. 1908	.9816	. 1944
11.5	. 1994	. 9799	. 2035
12.0	. 2079	. 9781	. 2126
12.5	. 2164	. 9763	. 2217
13.0	. 2250	. 9744	. 2309
13.5	2334	.9724	. 2401
14 0	2419	9703	2493
14 5	2504	9681	2586
15.0	. 2588	. 9659	. 2679
15 5	2672	9636	2773
16 0	2756	9613	2867
16.5	2840	0588	2062
17 0	2040	9563	3057
17 5	2007	. 3503	3153
11.5	. 3001	. 9001	. 01 00
18.0	. 3090	. 9511	. 3249
18.5	. 3173	.9483	. 3346
19.0	. 3256	. 9455	. 3443
19.5	. 3338	. 9426	. 3541
20.0	. 3420	. 9397	. 3640
20.5	. 3502	. 9367	. 3739
21.0	. 3584	. 9336	. 3839
21.5	. 3665	. 9304	. 3939
22.0	. 3746	. 9272	. 4040
22.5	. 3827	. 9239	. 4142
l			

Deg.	Sin	Cos	Tan
23.0	. 3907	. 9205	. 4245
23.5	. 3987	. 9171	. 4348
24.0	. 4067	.9135	. 4452
24.5	. 4147	.9100	. 4557
25.0	. 4226	. 9063	. 4663
25.5	. 4305	. 9026	. 4770
26.0	. 4384	. 8988	.4877
26.5	. 4462	. 8949	. 4986
27.0	. 4540	.8910	. 5095
27.5	. 4617	. 8870	. 5206
28.0	. 4695	. 8829	. 5317
28.5	. 4772	. 8788	. 5430
29.0	. 4848	. 8746	. 5543
29.5	. 4924	. 8704	. 5658
30.0	. 5000	. 8660	. 5774
30.5	. 5075	. 8616	. 5890
31.0	. 5150	. 8572	. 6009
31.5	. 5225	. 8526	. 6128
32.0	. 5299	. 8480	. 6249
32.5	. 5373	. 8434	.6371
33.0	. 5446	. 8387	. 6494
33.5	. 5519	. 8339	. 6619
34.0	. 5592	. 8290	.6745
34.5	. 5664	. 8241	. 6873
35.0	. 5736	. 8192	. 7002
35.5	. 5807	. 8141	. 7133
36.0	. 5878	. 8090	. 7265
36.5	. 5948	. 8039	.7400
37.0	. 6018	. 7986	. 7536
37.5	. 6088	. 7934	. 7673
38.0	. 6157	. 7880	. 7813
38.5	. 6225	. 7826	. 7954
39.0	. 6293	. 7771	. 8098
39.5	. 6361	. 7716	. 8243
40.0	. 6428	. 7660	. 8391
40.5	. 6494	. 7604	. 8541
41.0	. 6561	. 7547	. 8693
41.5	. 6626	. 7490	. 8847
42.0	. 6691	. 7431	. 9004
42.5	. 6756	. 7373	. 9163
43.0	. 6820	. 7314	. 9325
43.5	. 6884	. 7254	. 9490
44.0	. 6947	. 7193	. 9657
44.5	. 7009	. 7133	. 9827
45.0	. 7071	. 7071	1.0000
L	l	<u> </u>	

Deg.	Sin	Cos	Tan		Deg.	Sin	Cos	7
45.5	. 71 33	. 7009	1.018		68.0	. 9272	. 3746	2
46.0	.7193	. 6947	1.036		68.5	. 9304	. 3665	2
46.5	7254	. 6884	1.054		69.0	. 9336	. 3584	2
47 0	7314	6820	1.072		69.5	9367	3502	2
47.5	. 7373	. 6756	1.091		70.0	. 9397	. 3420	2
40 0	7/91	6601	1 111		70 5	0496	2220	9
40.0	. 7431	.0091			70.0	.9420	. 3330	4
40.0	.7490	.0020	1.130		71.0	.9400	. 3430	4
49.0	.7547	. 6561	1.150	1	71.0	. 9483	. 3173	2
49.0	.7604	. 6494	1.171		72.0	. 9511	. 3090	3
9 0, 0	. 7660	. 6428	1.192		72.5	. 9537	. 3007	3
50.5	. 7716	. 6361	1.213		73.0	. 9563	. 2924	3
51.0	.7771	. 6293	1.235		73.5	.9588	. 2840	3
51.5	. 7826	6225	1.257		74.0	.9613	. 2756	3
52.0	.7880	.6157	1.280		74.5	. 9636	. 2672	3
52.5	. 7934	. 6088	1.303		75.0	.9659	. 2588	3
53.0	. 7986	. 6018	1.327		75.5	. 9681	. 2504	3
53.5	. 8039	. 5948	1.351		76.0	. 9703	. 2419	4
54.0	. 8090	. 5878	1.376		76.5	. 9724	. 2334	4
54.5	. 8141	. 5807	1.402		77.0	.9744	. 2250	4
55.0	. 8192	. 5736	1.428		77.5	. 9763	. 2164	4
55.5	. 8241	. 5664	1.455		78.0	. 9781	. 2079	4
56.0	. 8290	. 5592	1.483	1	78.5	. 9799	. 1994	4
56.5	. 8339	. 5519	1.511		79.0	. 9816	. 1908	5
57.0	. 8387	. 5446	1.540		79.5	. 9833	. 1822	5
57.5	. 8434	. 5373	1.570		80.0	. 9848	. 1736	5
58.0	. 8480	. 5299	1.600		80.5	. 9863	. 1651	5
58.5	. 8526	. 5225	1.632		81.0	. 9877	. 1564	6
59.0	. 8572	. 5150	1.664		81.5	. 9890	. 1478	6
59.5	. 8616	. 5075	1.698		82.0	. 9903	. 1392	7
60.0	. 8660	. 5000	1.732		82.5	. 9914	. 1305	7
60.5	. 8704	. 4924	1.767		83.0	. 9925	. 1219	8
61.0	. 8746	4848	1.804		83.5	. 9936	. 1132	8
61.5	. 8788	.4772	1.842		84.0	. 9945	. 1045	9
62.0	. 8829	. 4695	1.881		84.5	.9954	. 0958	10
62.5	. 8870	.4617	1.921		85.0	. 9962	. 0872	11
63 0	. 8910	4540	1,963		85.5	9969	. 0785	12
63 5	8949	4462	2,006		86.0	9976	. 0698	14
64.0	8988	4384	2.050		86.5	9981	0611	16
64 5	9026	4305	2 007		87 0	3800	0523	19
65.0	. 9063	. 4226	2.145		87.5	. 9990	. 0436	22
65 5	01.00	A117	2 10/		88 O	0004	0340	20
66 0	0125	4067	2.194	1	88 5	. 3334	0249	20
66 5	0171	3007	2.440		80 0	0000	0174	50
00,0 67 0	0905	3007	2.300		09.0 80 5	1 0000	0.007	11/
67 5	0220	3297	2.000		09.0	1 0000	0001	114
01.0	. 3439	. 3041	4.414		a0.0	1.0000		1

.

.

					Square I	Root of N	1			
N	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0,9
0	0.00	0.32	0.45	0.55	0.63	0.71	0.77	0.84	0.89	0.95
1	1.00	1.05	1.10	1.14	1.18	1.22	1.26	1.30	1.34	1.38
2	1.41	1.45	1.48	1.52	1.55	1.58	1.61	1.64	1.67	1.70
3	1.73	1.76	1.79	1.82	1.84	1.87	1.90	1.92	1.95	1.97
4	2.00	2.02	2.05	2.07	2.10	2.12	2.14	2.17	2.19	2.21
5	2.24	2,26	2.28	2,30	2,32	2.35	2.37	2, 39	2.41	2.43
6	2.45	2, 47	2.49	2, 51	2, 53	2, 55	2,57	2, 59	2.61	2.63
7	2.65	2.66	2.68	2.70	2,72	2.74	2.76	2.77	2.79	2.81
8	2.83	2,85	2.86	2.88	2.90	2.92	2.93	2.95	2.97	2.98
9	3.00	3. 02	3,03	3.05	3.07	3.08	3.10	3. 11	3.13	3.15
10	3.16	3, 18	3.19	3.21	3, 22	3.24	3.26	3. 27	3.29	3.30
11	3.32	3, 33	3.35	3,36	3, 38	3.39	3.40	3.42	3.43	3.45
12	3.46	3.47	3.49	3.50	3.52	3.53	3.55	3.56	3.58	3.59
13	3.61	3.62	3.64	3.65	3.66	3.67	3.69	3.70	3.71	3.73
14	3.74	3.76	3.77	3.78	3.79	3.81	3.82	3.84	3.85	3.86
15	3.87	3.89	3.90	3.91	3.93	3.94	3.95	3.96	3.97	3.99
16	4.00	4.01	4.03	4.04	4.05	4.06	4.07	4.09	4.10	4.11
17	4.12	4.14	4.15	4.16	4.17	4.18	4.19	4.21	4.22	4.23
18	4.24	4.25	4.27	4.28	4.29	4.30	4.31	4.33	4.34	4.35
19	4.36	4.37	4, 38	4.39	4.40	4.42	4.43	4.44	4.45	4.46
20	4.47	4.48	4.50	4.51	4.52	4.53	4.54	4.55	4.56	4.57
21	4.58	4.59	4.60	4.62	4.63	4.64	4.65	4.66	4.67	4.68
22	4.69	4.70	4.71	4.72	4.73	4.74	4.75	4.76	4.77	4.79
23	4.80	4.81	4.82	4.83	4.84	4.85	4.86	4.87	4.88	4.89
24	4.90	4.91	4.92	4,93	4.94	4.95	4.96	4.97	4.98	4.99
25	5.00	5.01	5.02	5.03	5.04	5.05	5.06	5.07	5.08	5.09

SQUARE ROOT AND SQUARE TABLE

Square Root of N

(Numbers < 0.1)

N	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
√N	0.10	0.14	0.17	0.20	0.22	0.25	0.27	0. 28	0.30	0.32

Appendix K

THE DOMINANT EYE

Oculists have found that people tend to be right- or left-eyed in a similar sense that they are right- or left-handed. In reading, for example, one eye does more work than the other. For this reason, it is felt that it is probably advisable in microscopy to use the dominant eye in making measurements with the eyepiece reticle. Although there is very little eyestrain associated with microscopy, use of the dominant eye will tend to minimize any eyestrain that might occur.

There is apparently no correlation between "handedness" and "eyedness," that is, a right-handed person may be left-eyed. Here is a simple test for determining which is your dominant eye.

1. Look at an object 10 feet or more away. This object should be vertical--it may be, for example, the wall line at the corner of a room, a pipe, or a telephone pole.

2. Holding your finger about a foot from your face, with both eyes open, and <u>still</u> looking at the distant object, align your finger with the distant object.

3. Now close first one eye and then the other. You should observe that for one eye your finger will be aligned with the distant object. With the other eye open, your finger will be misaligned. The eye which, when open, aligns the distant object with your finger is your dominant eye. In step 2, you have subconsciously used the dominant eye to make the alignment.

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