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Computer Simulation of the Sequential Probability Ratio Test for Nuclear Safeguards

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by

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ABSTRACT

A Fortran IV computer program called SPRTEST is used to simulate the Sequential Probability Ratio Test (SPRT). The program provides considerably more information than one can obtain from the approximate SPRT theory of Wald. For nuclear safeguards applications SPRTEST permits the equipment designer to optimize the input test parameters and, indeed, to determine whether the SPRT is the statistical test of choice. Using Monte Carlo techniques, SPRTEST simulates the use of the SPRT in a radiation monitor. The accumulation of monitoring data from a normal distribution is simulated by repeated sampling of a random number generator. In this way, SPRTEST determines the expected false-positive (α) and false-negative (β) detection probabilities and the average step number (ASN) for a particular SPRT. The report describes SPRTEST, provides a Fortran listing, and demonstrates SPRTEST applications. The report also compares results with those expected from the single-interval test (SIT) on which the SPRT is based; generally, the SPRT provides better detection probabilities for a wide range of source strengths and, at background levels, it takes less time, on average, to make decisions. To obtain optimal results with the SPRT, it must have the capability to detain the counting subject for longer than the SIT time. The SPRTEST program should be useful in choosing the best statistical test for a wide variety of applications, including safeguards, health physics monitoring, and general nuclear detection.

I. INTRODUCTION

The Sequential Probability Ratio Test (SPRT) of Wald[1] is a statistical analysis method in use at Los Alamos for nuclear safeguards applications.[2-9] The test, as used for portal safeguards monitors,[4-6] consists of examining nuclear counting data sequentially in time and making one of three decisions after each step or increment of data is obtained.

1. Accept the hypothesis H_o (background only).

2. Accept the hypothesis H_1 (count is above background).

3. Accept neither hypothesis; continue counting by obtaining another increment of data.

When either of the first two decisions is made, the counting sequence usually terminates and the result is indicated visually or audibly. Wald shows that eventually acceptance of either H_o or H_1 will occur if the sequence continues long enough.

The average time required to make a decision for a properly designed SPRT may be considerably less than the time required for a single-interval test (SIT) of similar statistical strength for differentiating between background-only and above-background radiation levels.[1] That is the primary reason for using the sequential test. The primary disadvantages of the SPRT are that it is more complex to set up, that the time required for a particular trial or test may be longer than that required for the equivalent single-interval test, and that the analytic equations provided by Wald generally provide only approximate values for the statistical parameters of interest. These parameters are α, β, and the average step number (ASN).

1. α: error of the first kind, or the false-positive detection probability.

2. β: error of the second kind evaluated for a particular or nominal source strength; this is also referred to as the false-negative detection probability.

3. ASN: the average number of increments or steps required to reach a decision to accept H_o or H_1.

The α and β actually obtained using Wald's equations are generally somewhat different from the nominal (input) values (designated with a zero subscript), but the input values provide reasonably good approximations for many problems. However, those approximations may become considerably poorer if the testing sequence is forced to terminate after a set maximum number of steps. In practice, it is often desirable to force a termination to ensure that a counting period does not exceed some predetermined time. Doing so, however, also decreases the ASN, and Wald does not provide a method of estimating the magnitude of that effect.

Furthermore, the input value for β (β_o) only approximates the true value for a particular or nominal source strength. (As described in Sec. II-B, that nominal source strength is determined by the input parameters for α and β, referred to as α_o and β_o, and, of course, the background count rates and counting times.) In safeguards applications, as well as many others, sources (i.e., above-background signals) of different strengths may be present, and it is desirable to know the false-negative detection probabilities for them even though the SPRT is set up to optimally detect the nominal source strength.

To determine the parameters estimated by Wald more accurately, a computer program, called SPRTEST, was devised to simulate the SPRT using

Monte Carlo techniques. While developed independently, presumably SPRTEST is similar in concept to other programs that have been written previously.[9] Alternative methods[10,11] for improving on Wald's theory were not pursued in this study.

Data similar to those obtained with SPRTEST could, in theory, be obtained experimentally, but results can be generated much more quickly by computer, without the potential uncertainties associated with experimental data. Of course, the fluctuations associated with sampling from statistical populations (i.e., sources of nuclear radiation) are preserved using the Monte Carlo technique. Thus, the results obtained with the computer simulation will, if properly performed, represent the best statistical test performance that can be expected experimentally.

Two versions of the Fortran IV code, SPRTEST and SPRTREP, used for simulating the SPRT on the Los Alamos computer system appear, respectively, in Appendixes A and B. These two programs run on a CDC Cyber-176 computer. Los Alamos users can obtain the programs from the MASS storage system under the directory root KLCQ2.

II. COMPUTER SIMULATION OF THE SPRT

This section describes the method used in the SPRTEST program, setting up a problem, and interpreting the program output.

A. Description of the Method

The basic computer program, SPRTEST, is designed to simulate actual experiments by using Monte Carlo sampling techniques described as follows.

The decision levels for accepting hypothesis H_0 and H_1 are set by the user's selection of nominal (input) parameters α_0 and β_0, following Wald's approximations

$$B = \ln[\beta_0/(1 - \alpha_0)] \quad \text{and}$$

$$A = \ln[(1 - \beta_0)/\alpha_0]$$

At the start of any step in the sequential analysis, SPRTEST calls a random number generator RANF(1)* twice to obtain two numbers uniformly distributed between 0 and 1. It uses these numbers to calculate Y, which corresponds to a point on the abscissa of a normal distribution with a mean of zero and a standard deviation of 1. This value is always positive; the probability of

*RANF(1) is a standard random number generator widely used at Los Alamos, written by M. Steuerwalt. The generator uses the algorithm $S' = S * F \bmod 2^{48}$, and delivers $2^{-48} * S'$ as a normalized fraction. It uses $F = 553645_8$ and starts with $S = 1274321477413155_8$. The value 1 in parentheses following RANF is a dummy argument of no significance.

obtaining a value from any region of the positive abscissa is proportional to the corresponding ordinate of the normal distribution. A third call to the random number generator is then made to determine whether to assign a positive or negative value to the abscissa, depending on whether the third random number is larger or smaller than 0.5.

This value, in nuclear counting applications, then corresponds to the detection of a number of photons or nuclear particles. Thus, it is assumed that in each step of the actual test being simulated, enough events are detected to approximate the population sampled by a normal distribution; fifty or more events detected per step would be adequate for most experimental applications. The SPRTEST never actually refers to a specific number of counts, but as will be described in Sec. II-B, the results can be related to a particular mean number of counts per step.

SPRTEST is set up such that the normal distribution just described, which has a mean of zero, corresponds to the background-only distribution. To simulate counts obtained from populations with means greater than zero (i.e., background plus a radiation source), a value, UADD, is added to the Y obtained previously to obtain the sum U. (The units of UADD are standard deviations of the normal distribution.) Thus, it is assumed that the standard deviation of all the populations sampled--background only and above background--are the same, which is a good approximation for many safeguards applications. For example, if one wishes to detect a source giving an average count per step of 100 plus a background mean of 1000, the approximate standard deviations are $(1000)^{1/2} = 31.6$ for the background and $(1100)^{1/2} = 33.2$ for the background plus source. Differences of this magnitude will generally not appreciably affect comparisons of experimental results derived from these calculations.*

Next, the program computes $Z = \ln [f(U,\Theta_1)/f(U,\Theta_0)]$, which is the logarithm of the quotient of the two normal distributions' ordinates evaluated at the abscissa value, U, obtained previously. In the case of the normal distribution, Z takes the simple form $Z = \Theta_1 \times U - 0.5\ \Theta_1^2$, where Θ_1 is the abscissa of the distribution mean of a nominal (user-selected) source and U is the abscissa value obtained using the random number generator, as described previously.

Then Z is added to the Z value obtained in the previous step of the sequence and the sum is compared to A and B. If the sum is less than or equal to B, the hypothesis H_0 (background only) is accepted; if the sum of Z is greater than or equal to A, the hypothesis H_1 (above background) is accepted. In either case, the result is recorded by incrementing by +1 the value of the decision matrix IH0(i) or IH1(i), respectively, where i corresponds to the step number where the decision is made. Then another independent trial is begun.

*SPRTEST program could be changed, rather easily, so that the effective width of the normal distribution would become a function of the mean count. This could be done by recasting the program to make counts the unit for the abscissa, instead of fractions of the standard deviation, as it now is. For very low count rates, it would be more appropriate to sample from a Poisson distribution,[12] instead of the normal distribution.

If neither decision to accept H_o or H_1 occurs, then another step is made by sampling again from the normal distribution. Another Z is computed and added to the previous value. Then that sum is compared to A and B to determine whether to accept hypothesis H_o or H_1, or to continue the trial. This process can be repeated for up to 98 steps (as now programmed), if necessary, to reach a decision to accept H_o or H_1.

SPRTEST also provides for forcing a decision after NSTEP steps; the forced result is stored in IH0(100) or IH1(100), respectively, depending on whether H_o or H_1 was accepted. The criterion used for this forced decision is to determine whether the sum of Z is equal to or less than 0.0 (accept H_o) or greater than zero (accept H_1), as suggested by Wald.* Other criteria can readily be substituted by editing SPRTEST, and might be more appropriate in particular cases; see Ref. 8 for examples of such criteria. Whereas a decision can be forced at any step number and the result recorded as indicated, the trial also continues until a decision is made using the original, nonforcing decision points (A and B) or until step 98 is completed. In the sample tests described in Sec. III, step 98 seldom is reached. However, if it is, a decision is forced (using the same criterion as at NSTEP) with the result recorded in IH0(99) or IH1(99), respectively, depending on whether H_o or H_1 is accepted.

After completion of a trial, another independent trial begins and the process repeats until a total of 100,000 trials have been made. This typically takes less than 30 s of computer time, including compilation.

The value of 100,000 can, of course, be readily changed by editing SPRTEST. Increasing the number of trials may be necessary to obtain sufficient statistical precision in some cases, such as, for example, when α_o is less than 10^{-3}.

B. Setting Up Problems

The usual method for setting up an SPRT is to base it on a single-interval test with false-detection probabilities of α_o and β_o, as the SIT is relatively easy to visualize and set up. The intent, then, is that the SPRT will have a better α or β or will require less time to run, on average, even though the nominal α_o and β_o are the same as for the SIT.

The following example will illustrate the general approach to setting up the SPRT based on a single-interval test. Assume that a safeguards radiation monitor has a mean background of 500 counts/s; you want to set up a 30-s single-interval test with an $\alpha = 0.01$ and a $\beta = 0.05$. Thus, in 30 s the mean background will be $30 \times 500 = 15{,}000$ and the standard deviation will be $\sigma = (15000)^{1/2} = 122.5$. From a table of areas under the normal curve[13] you

*Comparison of the sum of Z with 0.0 corresponds, in nuclear counting applications, to making a decision at a count level halfway between the background mean and the nominal source mean.

find that the abscissa for $\alpha = 0.01$ is 2.326 and for $\beta = 0.05$ is 1.645 standard deviations. Therefore, the mean of the source that can be detected in 30 s with these errors must be $(1.645 + 2.326)\sigma = 486$ counts/30 s above background. These relationships are illustrated in Fig. 1. A source whose count rate is greater than 486 counts/30 s will give a smaller β, and vice versa. The decision level, of course, always remains at a count rate of 15000/30 s + 2.326σ/30 s = 15285/30 s. Every count will be 30 s in length, regardless of a source's presence or size.

To set up the SPRT, use the same α and β (referred to here as α_0 and β_0, the input values) and divide the 30-s interval, somewhat arbitrarily, into a number of steps. If the number of steps is too small, say 3 or less, the average length or time to make the test may be unnecessarily long. On the other hand, if there are too many steps, say more than 30 or 40, you may need to modify SPRTEST to keep the number of forced decisions after step 98 to a small fraction of the total. There is usually little, if anything, to gain by increasing the number of intervals beyond 30 or so. For purposes of illustration, let us choose to divide the 30-s interval into 10 steps and choose the step number, NSTEP = 15, to force a decision if neither hypothesis H_0 or H_1 is accepted based on the A or B decision criterion at the completion of the step. The forced result, as stated previously, is stored in IH0(100) or IH1(100), and the trial continues.

Another input parameter required is the location on the abscissa, in units of σ, of the mean of the source distribution of interest. If you wish to determine the actual α and ASN for background only, the abscissa location is 0.0.

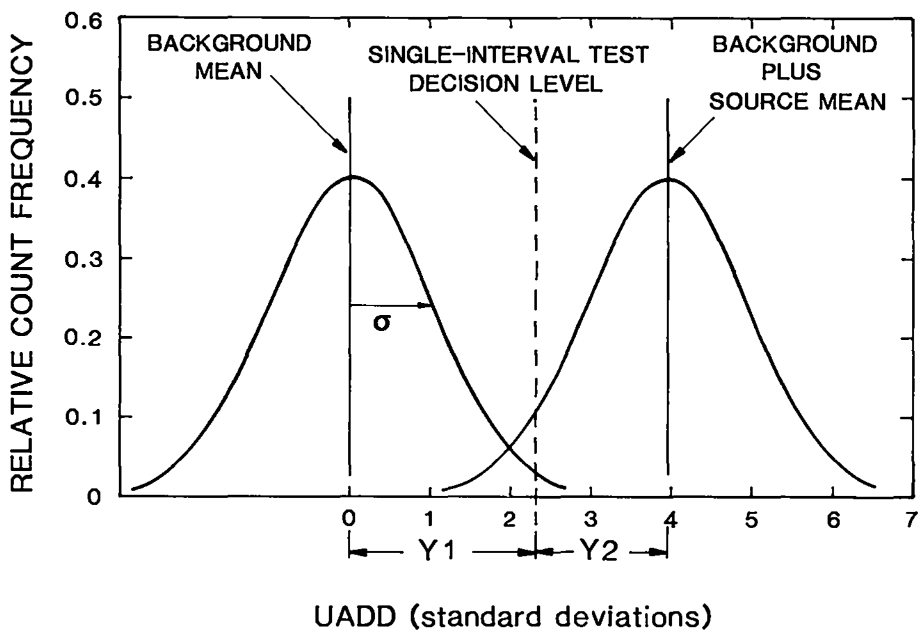

Fig. 1.
Sketch of normal distributions with means of background-only and above-background, as appropriate for a single-interval test with $\alpha_0 = 0.01$ and $\beta_0 = 0.05$.

To test for the ASN and β for the nominal source strength giving 486 counts/30 s above background, use an abscissa value of 1.645 + 2.326 = 3.971. Of course, you can select other values in between or even greater than 3.971 to determine the ASN and β for other source-strength values; you should do this for a complete comparison with other statistical tests. SPRTREP does this automatically for background and 10 other incremented values of the source strength (see Appendix B for a listing).

The last parameter to select is the starting argument for the random number generator. Normally, this is input as 0 (zero), which causes the generator to start at its default value. At the end of each run, a number related to the current argument of the random number generator is printed out. If this number is reinserted at the start of a subsequent run, the random number sequence will start at that point. This would be useful, for example, if you wish to compare two different runs using the same parameters, but using a different subset of random numbers. If you use 0 in both runs, the results will be identical, because the random numbers used are the same.

The preceding paragraphs give the complete set of parameters required to run a simulated SPRT. They are shown in Table I.

TABLE I

INPUT VALUES FOR SAMPLE PROBLEM 1

Fortran Name	Value for Example	Meaning
ALPHA	0.01	Nominal α_o (false-positive detection probability)
BETA	0.05	Nominal β_o (false-negative detection probability for UADD = 3.971)
Y1	2.326	Abscissa value corresponding to α_o, in standard deviations
Y2	1.645	Abscissa value corresponding to β_o, in standard deviations
UADD	0.0 or 3.971	Abscissa value of the mean of the source to be sampled
NO	10	Number of steps corresponding to the nominal single-interval test length
NSTEP	15	Step after which a decision is forced
NSEED	0	Number that provides the starting argument for the random number generator

To run SPRTEST at the Los Alamos Central Computing Facility on the Livermore Time Sharing System (LTSS), store SPRTEST as a local file and issue the command

FTN (I=SPRTEST,GO) / t p

The letters t and p stand for the maximum time in minutes allowed for the run and the priority assigned; normally, values of 1 (the default value) for both parameters will suffice.

After compilation, SPRTEST prompts the user for the parameter values, in the order listed in the table, with the Fortran name of the parameter. During and after completion of the run, the results are printed at the user's terminal, as explained in Sec. II-C.

C. Interpreting the Computer Output

The first 10 lines of output data constitute the IHO matrix, which is a record of decisions for accepting the H_0 hypothesis; i.e., decisions that the population sampled was background only. A sample printout appears in Fig. 2. The first element of the first row is the number of times, out of the 100,000 trials, that H_0 was accepted after step 1. The second element is the number of times H_0 was accepted after step 2, etc. Row 2 contains the number of decisions for H_0 after steps 11 through 20; row 3, steps 21 through 30; etc., for rows 4 through 9. In row 10, the ninth element corresponds to forced decisions for H_0 after completion of 98 steps in which no decision for either H_0 or H_1 was reached using the normal (A and B) decision criteria. Hence, IH0(99) is the number of decisions made to accept the hypothesis H_0 (background only) based on the sum of $Z \leq 0.0$ after step 98. Finally, IH0(100) represents the number of decisions for H_0 after step NSTEP, where a decision was forced (using the sum of $Z \leq 0.0$).

The next 10 rows of data represent the decisions for H_1 (above background), arranged in the same manner as for H_0. Elements 99 and 100 represent forced decisions after steps 98 and NSTEP, based on the sum of $Z > 0.0$. Examination of the elements of these matrices can be very instructive regarding when decisions (correct or incorrect) are made in the sequential analysis.

The next row contains values labeled ASN and ASN(FORCED). The first is the average step number, when the only forced decisions, if any, occur after step 98. ASN(FORCED) is the average step number resulting from termination of the sequence after step NSTEP, made by forcing a decision after that step if a decision to accept H_0 or H_1 is not made sooner. Both are obtained by appropriate calculations using the IH0 and IH1 matrix elements. These values, divided by N0, give the fraction of the single-interval test length that the average SPRT takes to make a decision, shown in the next row. It is, of course, best that these fractions be less than 1 over the range of UADD values of most interest to the user.

MATRIX IHO(BACKGROUND-ONLY):

3914	17776	19423	15662	11579	8302	6013	4414	3231	2349
1730	1225	1038	739	557	394	272	211	157	131
108	68	34	43	25	21	20	15	14	10
9	2	2	3	2	1	1	0	1	0
0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1120

MATRIX IH1(ABOVE-BACKGROUND):

1	28	60	69	71	65	45	36	33	19
24	13	9	7	4	7	4	2	2	1
1	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	444

ASN= 4.969 ASN(FORCED)= 4.905

ASN/NO= .4969 ASN(FORCED)/NO= .4905

NHO= 99497 NH1= 503

ALPHA= .005030

FNHO= 99072 FNH1= 928

ALPHA(FORCED)= .009280

LAST RANDOM NO. STARTING SEED= 274451029000570645

Fig. 2.
Computer printout of calculated results for UADD = 0.0 for problem 1. See the text for details.

The next row contains NH0 and NH1, which are simply the total number of decisions in the matrices IH0 and IH1, respectively, excluding elements 100 in both cases. Then NH1/(NH0 + NH1) is the fraction of decisions accepting the hypothesis H_1. This represents α (the false-positive probability) when the population being tested in the SPRT simulation is the background; i.e., for runs with UADD = 0.0. For runs with UADD > 0.0, NH0/(NH0 + NH1) is equal to β, the false-negative probability. The computed ALPHA or BETA is shown in the next row. The β obtained for UADD = Y1 + Y2, and the α can be compared with the input, nominal β_o and α_o, respectively, to determine how the statistical performance of the SPRT compares with the single-interval test. These calculated α and β values, of course, are based on no forced decisions (except possibly after step 98).

The next row contains FNH0 and FNH1, which are the sums of the IH0 and IH1 matrix elements, respectively, from elements 0 through NSTEP, plus elements 100. Thus, they represent decisions made for an SPRT with forced decisions made after step NSTEP. An α or β can be obtained with these values in analogous fashion to the preceding calculations; they are shown in the next row as ALPHA(FORCED) or BETA(FORCED). These values can be compared to the α and β calculated previously to determine the effect of truncating the sequential test at step NSTEP. Of course, these values for α and β can also be compared directly with α_o and β_o of the nominal single-interval test.

For the program SPRTREP, the next value shown is UADD, which is the mean (in standard deviations) of the distribution being sampled.

Finally, the LAST RANDOM NO. STARTING SEED appears. Insertion of this value into the input of a subsequent run will start the random number generator at this point.

III. RESULTS FOR SAMPLE PROBLEMS

This section contains results for three sample problems, and a brief discussion of the results. The problems explore how different combinations of initial input parameters affect the SPRT results.

Sample Problem 1: $\alpha_o = 0.01$, $\beta_o = 0.05$,
Sample Problem 2: $\alpha_o = 0.01$, $\beta_o = 0.01$,
Sample Problem 3: $\alpha_o = 3.16 \times 10^{-5}$, $\beta_o = 0.5$.

A. Problem 1

Problem 1 ($\alpha_0 = 0.01$, $\beta_0 = 0.05$) uses the values from Table I as input parameters to SPRTEST. (The problem is discussed in Sec. II.) Two runs were made: the first with UADD = 0.0, corresponding to background only, and the second with UADD = 3.971, which corresponds to a source giving a mean count of 486/30 s above the background mean. The computed results for UADD = 0.0 and 3.971 are shown in Figs. 2 and 3, respectively. Figure 4 shows selected portions of the printout obtained at the data input stage when the program was compiled and run for UADD = 0.0, showing the input of the parameters from Table I.

For the first run (UADD = 0.0), it can be seen (Fig. 2) that the ASN is just less than 5, regardless of whether a decision is forced after NSTEP = 15. Because the SPRT is based on a single-interval test of 10-step length, this means that for background only the SPRT requires, on average, just one-half the length of the single-interval test, as shown by ASN/N0.

The false-positive probability, α, is ALPHA = 0.00503 for the unforced case and ALPHA(FORCED) = 0.00928 for the test when the sequence is terminated no later than step 15. These values can be compared with the nominal α_o of 0.01 for the single-interval test. Thus, both versions of the SPRT give a lower (better) value for α, with the nonforced value considerably better than that obtained when the decision is forced after step 15.

MATRIX IHO(BACKGROUND-ONLY):

122	458	500	343	283	175	153	100	73	49
33	31	24	13	10	12	6	8	3	0
3	2	1	0	0	2	0	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	270

MATRIX IH1(ABOVE-BACKGROUND):

139	4626	11783	14588	13764	11841	9644	7465	5826	4440
3336	2576	1892	1473	1156	792	575	450	322	254
177	128	90	59	52	36	25	29	17	11
7	4	4	1	3	2	0	2	2	1
0	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2814

ASN= 6.660 ASN(FORCED)= 6.542

ASN/NO= .6660 ASN(FORCED)/NO= .6542

NHO= 2405 NH1= 97595

BETA= .024050

FNHO= 2637 FNH1= 97363

BETA(FORCED)= .026370

LAST RANDOM NO. STARTING SEED= 274530846076037529

Fig. 3.
Computer printout of calculated results for UADD = 3.971 for problem 1.

For UADD = 3.971, the ASN from Fig. 3 is about 6.6 for both the forced and unforced cases, whereas β is about 0.025. So again, the average trial time is less than the SIT time and the β is about half the nominal β_o.

Examination of the matrices shows that because element 99 is always zero, the nonforced decisions were all made before the completion of step 98. Element 100 contains the number of decisions forced at the completion of step 15 (NSTEP). For example, of the forced decisions in Fig. 3, 2814 were made to accept H_1 and 270 were made to accept H_o.

In summary, these results show that the SPRT for this case gives a better α and β, and requires less time, on average, for both the nonforced and forced

```
FTN (I=SPRTEST.GO) / 1 1



 TYPE IN ALPHA (F10.8)
? .01

 TYPE IN BETA (F10.8)
? .05

 TYPE IN Y1 (F7.5)
? 2.326

 TYPE IN Y2 (F7.5)
? 1.645

 TYPE IN UADD (F7.5)
? 0.0

 TYPE IN NO (I2)
? 10

 TYPE IN NSTEP (I2)
? 15

 TYPE IN NSEED (I18)
? 0
     RANDOM NO. STARTING SEED= 0
```

Fig. 4.
Computer printout at the data input stage for problem 1. The question marks are computer prompts, requiring the user to type in the particular input parameter values.

(NSTEP = 15) decision cases than the nominal single-interval test on which it was based, for the two distributions tested. For other values of source strength, the SPRT may or may not be a better test than the single-interval test; problem 2 illustrates this point.

B. Problem 2

Problem 2 (α_o = 0.01, β_o = 0.01) uses the input parameters shown in Table II. Thus, this SPRT is based on a single-interval test with $\alpha_o = \beta_o = 0.01$, having a nominal length of 12 steps. Decisions will be forced after step 12; i.e., for the forced-decision situation, no trial will be longer than the single-interval test. To solve the problem took a total of 11 runs, starting with UADD = 0.0 and incrementing by Y1 + Y2 = 4.652/5 = 0.9304 for succeeding runs. These incremental runs will provide a range of source strengths ranging from zero to 9.3 times the standard deviation of the single-interval background. The run for UADD = 4.652 corresponds to the source strength on which the single-interval test was based; i.e., for that source strength the single-interval test is expected to result in β = 0.01. By varying the source strengths in the above manner, we can determine the variation in actual ASN and the actual α and β; they can then be compared with the single-interval test values.

This result could be accomplished by running SPRTEST eleven times with the appropriate value of UADD input for each run. However, this type of problem can more readily be handled by the program SPRTREP, which is simply SPRTEST with a DO-LOOP added to automatically increment UADD and repeat the test for a total of 11 runs. Each run starts with the next random number, so that a different set of random numbers are sampled for each run. The input UADD is 0.9304, the increment value we want.

TABLE II
INPUT VALUES FOR SAMPLE PROBLEM 2

Fortran Name[a]	Value
ALPHA	0.01
BETA	0.01
Y1	2.326
Y2	2.326
UADD	[(Y1 + Y2) * J]/5., J = 0, 10[b]
NO	12
NSTEP	12
NSEED	0

[a]See Table I for the definition of the parameters.
[b]The actual input value is 0.9304, as discussed in the text.

Selected results are shown in Table III. The single-interval data were calculated by hand using standarized tables[13] of the cumulative area under a normal curve.

The value of α can be derived from the first row (UADD = 0.0000) of Table III as described previously. For the unforced case, α = 0.0045; for the forced, it's 0.0118; and for the single-interval test, α = 0.0100. Thus α for the unforced problem is considerably better than that for the single-interval test and slightly worse for the forced SPRT case.

By examining the second, fourth, and last columns of the other rows in Table III, whose values are all equal to $\beta \times 10^5$, one can compare the false-negative detection probabilities for the three different tests. For UADD less than about 2, the forced and single-interval tests give similar values for β, whereas the unforced test gives poorer values. In the range of UADD from about 2 to 6, the unforced SPRT gives better results for β, whereas for larger UADD, the single-interval test appears to give a smaller β. (Because the statistics in the table are poor for small β, runs using SPRTEST were made with 10^6 trials at UADD = 6.5128 and 7.4432 to confirm the latter conclusion.)

Figures 5-7 show the computer output for runs with UADD = 0.0, 2.7912, and 9.3040, respectively. Comparison of the matrices in Figs. 5 and 6 shows that decisions are generally made more quickly in the case of background only (UADD = 0.0), as can also be seen from the ASN values. From Fig. 5, in fact, it is evident that all decisions are made before step 50, whereas in Fig. 6, that is not the case. Based on this observation, it is apparent that the unforced

TABLE III

RESULTS FOR SAMPLE PROBLEM 2

	SPRT Unforced[a]		SPRT Forced[b]		Single-Interval Test[c]
UADD	NHO	ASN	FNHO	ASN Forced	$\beta \times 10^5$
0.0000	99554	6.22	98815	5.99	99000
0.9304	96045	9.49	91243	7.87	91900
1.8608	74346	14.87	67561	9.35	67900
2.7912	25610	14.87	32520	9.34	32100
3.7216	3877	9.43	8543	7.85	8100
4.6520	426	6.17	1166	5.95	1000
5.5824	45	4.54	96	4.52	56
6.5128	2	3.62	2	3.62	1
7.4432	1	3.02	1	3.02	0
8.3736	0	2.62	0	2.62	0
9.3040	0	2.33	0	2.33	0

[a]Decisions were actually forced after step 98 if the trial continued that long; this occurred only 33 times out of 100,000 trials, in the worst case.
[b]Decisions were forced after step 12, if the trial continued that long.
[c]Based on a single-interval test corresponding in length to 12 steps.

SPRT could be improved somewhat, by forcing a decision at, say, step 50 to accept H_1; i.e., if the sequence does not terminate before reaching step 50, force termination with the decision that the trial is sampling background plus a source (above background). Not only would that result in a somewhat decreased β for UADD between 2 and 3, but the ASN in that region would also decrease slightly. Moreover, the maximum possible length of a trial would be reduced by a factor of 2. So, there would appear to be several advantages to making such a forced termination of the sequence, and no apparent disadvantages.

Figure 6 shows that a few trials did not result in a decision after completion of 98 steps. Thus, a decision was forced and the result recorded in element 99. In this case, the SPRT made 11 decisions to accept H_0 (background only) and 16 to accept H_1 (above background). Generally, the SPRT has the most difficulty making a decision--and thus, the largest ASN--for UADD values about midway between 0 and (Y1 + Y2). When the corresponding mean count rates are lower or much higher, the SPRT can make decisions more quickly, which, at higher count rates, are more frequently correct. It can be seen, for example, in Fig. 7, where UADD = 9.304, that all decisions are made before step 9, with the majority made at the end of step 2, and all decisions were made correctly to accept H_1.

MATRIX IHO(BACKGROUND-ONLY):

297	6717	14331	16414	14307	11510	8977	6827	5298	3874
2791	2097	1549	1207	925	651	463	336	240	213
152	95	65	61	34	30	25	13	13	15
8	5	2	1	3	0	2	1	3	0
0	1	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	5375

MATRIX IH1(ABOVE-BACKGROUND):

2	42	70	70	57	39	44	29	29	14
10	9	6	8	4	4	1	1	3	1
2	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	770

ASN= 6.222 ASN(FORCED)= 5.987

ASN/NO= .5185 ASN(FORCED)/NO= .4989

NHO= 99554 NH1= 446

ALPHA= .004460

FNHO= 98815 FNH1= 1185

ALPHA(FORCED)= .011850

UADD= 0.00000

LAST RANDOM NO. STARTING SEED= 274554561552738961

Fig. 5.
Computer output for problem 2, with UADD = 0.0.

MATRIX IHO(BACKGROUND-ONLY):

16	463	1083	1463	1612	1624	1501	1469	1348	1177
1118	1068	1000	897	807	724	659	664	546	538
439	466	431	371	379	302	284	242	253	227
207	171	171	153	141	126	121	109	104	89
76	84	69	61	59	52	48	51	41	42
46	45	27	26	24	26	20	19	21	26
18	13	7	8	13	9	6	4	5	4
6	3	10	5	7	8	6	4	5	5
4	4	2	1	2	2	1	0	3	2
3	1	2	0	0	0	1	0	11	18582

MATRIX IH1(ABOVE-BACKGROUND):

56	1203	3176	4303	4663	4702	4475	4147	3813	3618
3292	2962	2783	2571	2370	2164	2002	1834	1668	1493
1426	1309	1136	1157	1020	903	851	724	715	665
576	576	503	441	435	406	324	313	291	275
234	246	209	194	173	152	154	141	110	123
126	109	81	99	82	58	70	59	39	44
36	47	31	41	31	32	21	25	32	19
23	13	21	18	18	3	6	13	13	7
10	6	5	9	4	8	8	2	1	2
3	4	6	4	0	3	1	3	16	27066

ASN= 14.869 ASN(FORCED)= 9.344

ASN/NO= 1.2391 ASN(FORCED)/NO= .7787

NHO= 25611 NH1= 74389

BETA= .256110

FNHO= 32524 FNH1= 67476

BETA(FORCED)= .325240

UADD= 2.79120

LAST RANDOM NO. STARTING SEED= 274706265348229153

Fig. 6.
Computer output for problem 2, with UADD = 2.7912.

```
MATRIX IHO(BACKGROUND-ONLY):

   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0

MATRIX IH1(ABOVE-BACKGROUND):

7997 58621 27086  5327   834   126     8     1     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0
   0     0     0     0     0     0     0     0     0     0


ASN=        2.328           ASN(FORCED)=       2.328

ASN/NO=    .1940            ASN(FORCED)/NO=   .1940


NHO=          0       NH1=   100000

BETA= 0.000000


FNHO=          0       FNH1=   100000

BETA(FORCED)= 0.000000

UADD=     9.30400

LAST RANDOM NO. STARTING SEED=  27446371285041I425
```

Fig. 7.
Computer output for problem 2, with UADD = 9.3040.

C. Problem 3

Problem 3 (α_0 = 3.16 x 10^{-5}, β = 0.5) involves computer simulations of a vehicle portal monitor used in a nuclear safeguards application, as the monitor was initially set up. The monitor's decision logic requires some changes in SPRTEST. Only part of the results are described in this report; a listing of the modified program is not included because of the program's specialized nature.

The actual monitor consists of four detector modules, each performing the SPRT using identical parameters. The simulated SPRT for a single module is described first, then the simulation for the four modules combined.

For the single module, N0 = 12 and NSTEP = 15. But, SPRTEST was modified so that A is equal to 8.0, and after step 15 the forced decision always accepts hypothesis H_0 (background only). The results for β and the ASN as a function of UADD are plotted in Fig. 8.

The ASN for background only (UADD = 0.0) is 2.4, meaning an average time savings of a factor of 5 over the nominal (12-step) single-interval test for a monitoring situation where no source is present. The ASN increases to almost 9 for UADD = 2.0, then declines for higher values of UADD. Because the actual monitoring that is being simulated is almost always of vehicles without sources, the value of the ASN for UADD = 0.0 is, by far, the most important one.

The actual α determined by the simulation is (1.07 ± 0.10) x 10^{-4}, which is considerably larger than the nominal α_0. This larger α is due primarily to the use of the modified value of 8.0 for A (instead of the value 9.67, which would have been calculated by the normal equation used in SPRTEST and SPRTREP).

To compare the power of the SPRT with the (12-step) single-interval test, the latter was calculated using the same α as determined above; i.e., α_0 = 1.07 x 10^{-4}. The results for β are also plotted in Fig. 8, where it can be seen that they are very close to the SPRT values for UADD less than 4.0. At higher values of the abscissa, the single-interval values of β are superior (i.e., lower).

To model the simultaneous use of the four detector modules, further modifications of SPRTEST were made to simulate the logic of the system controller. That logic is basically as follows. A background indication is given only when all four modules accept hypothesis H_0. An alarm results as soon as any of the modules makes a decision to accept H_1. Thus, for the H_0 hypothesis, the length of time required to complete the trial is governed by the module that takes the longest time to make a decision. For the H_1 hypothesis, the module making the decision in the shortest time controls the overall time for the trial.

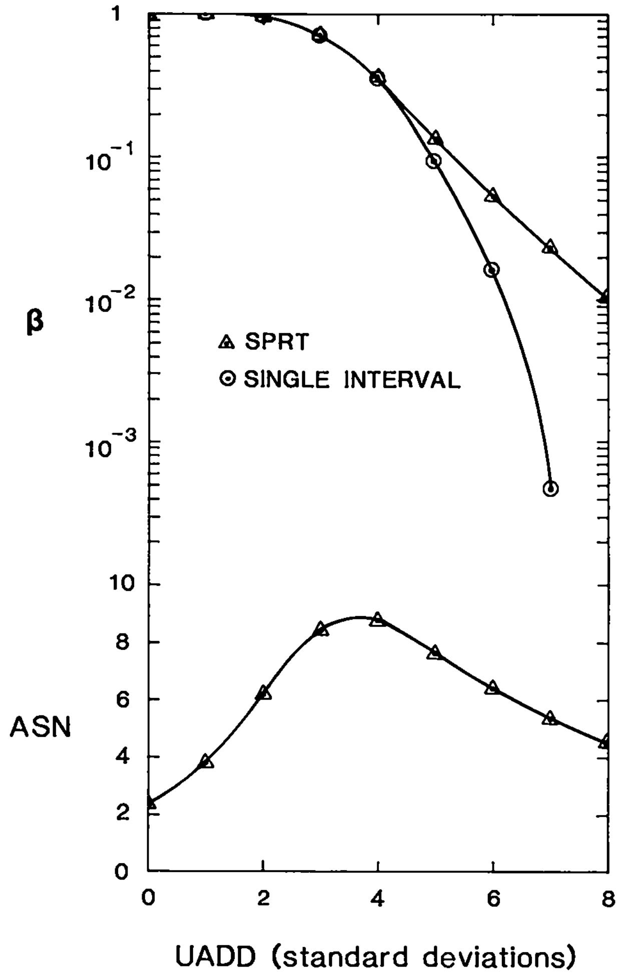

Fig. 8.
Plots of the computer results for problem 3, for a single detector module. The top plot shows the false-negative detection probability, β; the bottom shows the average step number, both as a function of UADD. Input parameter values N0 = 12, NSTEP = 15, and β_o = 0.5.

The results of this simulation are shown in Fig. 9. The problem assumed that all modules had the same background intensity and were exposed to the same source strength; the plot is in terms of the UADD for a single detector module. A comparison of Fig. 9 to Fig. 8 shows that the ASN goes up considerably for small values of UADD, and is smaller for large values, as would be expected based on the controller logic. The ASN for UADD = 0.0 is 4.8, which is twice the single-module value. Still, it is only 40% of the nominal single-interval time. The calculated α for the four-module SPRT is $(4.3 \pm 0.2) \times 10^{-4}$, which, as would be expected, is four times the single-module value.

The single-interval test results for β are also plotted in Fig. 9 for comparison with the SPRT values. Again, for UADD less than about 4 they are quite similar to the SPRT values, but diverge at larger values with the single-interval β being lower. The single-interval values shown here for β were simply calculated from the single-interval values in Fig. 8 by taking those values to the fourth power. The 4-module SPRT values for β were obtained from the computer simulation, but similar values could also have been obtained from the one-module SPRT values by the same method used to calculate the single-interval results.

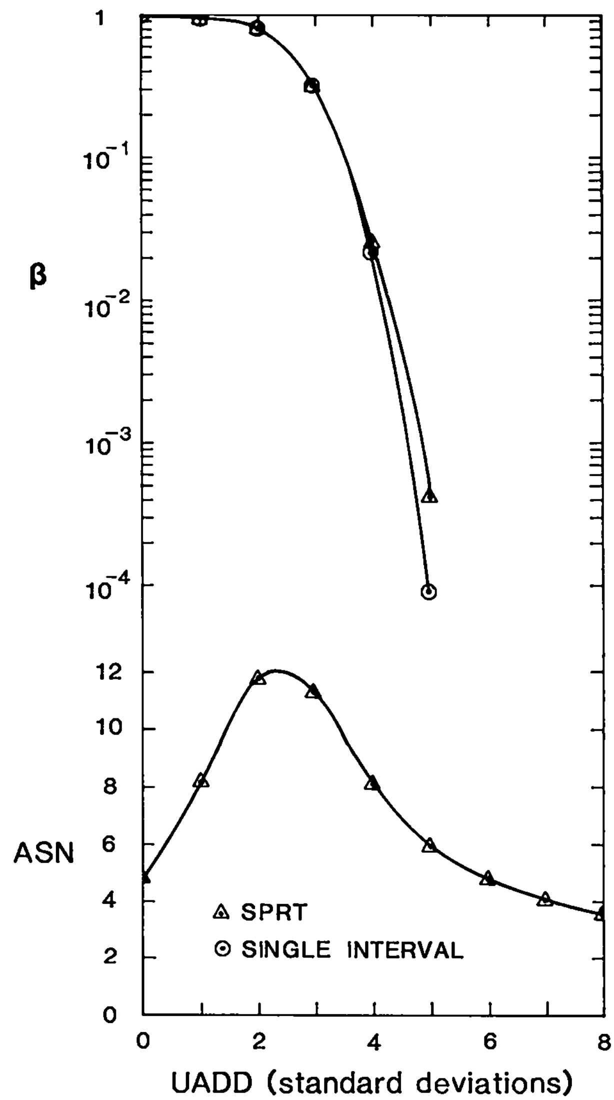

Fig. 9.
Plots of results for problem 3 for four detector modules operating simultaneously. See caption of Figure 8 for details.

IV. PARAMETER COMPARISONS

This section describes selected results of a series of runs made with SPRTEST to provide a systematic comparison of the parameters α, β, and the ASN. Runs were made for α_o = 0.1, 0.05, 0.01, 0.001, and 0.0001, while for each α_O, β_O took on the values of 0.5, 0.1, 0.05, and 0.01. For each of these combinations, a run was made with UADD = 0.0, corresponding to background, and UADD = Y1 + Y2, corresponding to background plus a source that would give $\beta = \beta_o$ for the nominal single-interval test.

One-hundred thousand trials were made for each run, except for those with α_o = 0.001 and 0.0001 with UADD = 0.0, where the number of trials was set at 4×10^5 and 2×10^6, respectively. Changes were made in the Fortran code to obtain reasonable statistical precision for the low-probability tallies in IH1 for those values of α_o and UADD. N0 and NSTEP were set at 10 and 15 respectively, for all the runs.

The values of α_o and β_o chosen cover a range of practical use in most safeguards applications. The NO and NSTEP were selected somewhat arbitrarily, but again they are typical of what might be used in actual applications. Although the results in the following paragraphs strictly apply only for these parameter values, similar results and conclusions would be expected for other parameter choices similar to these.

A. False-Positive Probability

Table IV shows the values obtained for α for various values α_o and β_o from the various computer runs when no forced decisions were made (except in a few rare and insignificant number of trials where a decision was forced after step 98).

In all cases α is less than α_o, ranging in value from about 30 to 98% of α_o. The ratio of α/α_o is largest for large β_o and decreases as β_o decreases. Although not shown in the table, runs were made for the extreme cases of $\beta_o = 0.5$ and $\alpha_o = 0.25$ and 0.40; even in those cases α was not greater than α_o, within the statistical uncertainties of the 100,000-trial runs.

Table V shows the results for α when a decision is forced after step 15. In many cases α is greater than α_o; indeed, in some cases it is greater by more than an order of magnitude. On the other hand, for some sets of α_o and β_o, α is less than the nominal α_o by almost 50%. This wide difference in the α/α_o ratio for forced decisions clearly illustrates the need for caution when you force the sequential test to terminate prematurely.

TABLE IV

CALCULATED VALUES FOR α FOR UNFORCED DECISIONS

	β_o			
α_o	0.5	0.1	0.05	0.01
0.1	0.098	0.064	0.062	0.051
0.05	0.048	0.031	0.028	0.024
0.01	0.0091	0.0056	0.0046	0.0042
0.001	0.00084	0.00052	0.00042	0.00038
0.0001	0.00009	0.00005	0.00004	0.00003

B. False-Negative Probability

Table VI shows the calculated values of β for various values of α_o and β_o for unforced decisions. These are the calculated β values for a source strength corresponding to Y1 + Y2; i.e., a source that would give the nominal β_o in the single-interval test used to set up the particular SPRT.

TABLE V

CALCULATED VALUES FOR α FOR FORCED DECISIONS AT NSTEP = 15

α_o	β_o			
	0.5	0.1	0.05	0.01
0.1	0.152	0.081	0.072	0.054
0.05	0.096	0.045	0.037	0.026
0.01	0.038	0.013	0.0085	0.0057
0.001	0.011	0.0026	0.0016	0.00069
0.0001	0.0078	0.0020	0.00036	0.00012

TABLE VI

CALCULATED VALUES FOR β FOR UNFORCED DECISIONS[a]

α_o	β_o			
	0.5	0.1	0.05	0.01
0.1	0.392	0.064	0.030	0.0056
0.05	0.367	0.059	0.028	0.0053
0.01	0.322	0.053	0.024	0.0046
0.001	0.273	0.046	0.021	0.0038
0.0001	0.239	0.041	0.018	0.0033

[a]Evaluated at a source strength corresponding to Y1 + Y2 for each β_o.

The values for β are all less than the β_o values, ranging from about 33 to 78% of β_o. In Sec. IV-A for the unforced case, α was always less than α_o for the range of α_o and β_o covered, therefore it follows that $\alpha + \beta \leq \alpha_o + \beta_o$, which is the relationship derived by Wald[1] for the general case. The trend observable in the table is for β/β_o to decrease as α_o decreases.

Table VII shows the calculated values of β when a decision is forced after NSTEP = 15. The trend here is the same as in the preceding table, namely, β/β_o decreases as α_o decreases. However, for $\beta_o \leq 0.1$, the values of β here are somewhat greater than those in the preceding table, and in the case of $\alpha_o = 0.1$ and $\beta = 0.01$, β/β_o is greater than 1. For $\beta_o = 0.5$, the values of β are less than those in Table VI. So, the actual β for forced decisions can be smaller or larger than the unforced β values, depending on β_o.

A different decision criterion for forced decision could markedly change the results shown in Tables V and VII for α and β, respectively. For example, if hypothesis H_o is always accepted after NSTEP (= 15 or otherwise), then the forced-decision values for α will be lower than those shown in Table V, while the values for β will be higher than in Table VII; in fact, the forced-decision α values will be equal to or lower than the unforced values.

TABLE VII

CALCULATED VALUES FOR β
FOR DECISIONS FORCED AT NSTEP = 15[a]

α_o	β_o = 0.5	0.1	0.05	0.01
0.1	0.380	0.081	0.043	0.0126
0.05	0.356	0.069	0.036	0.0095
0.01	0.316	0.056	0.027	0.0058
0.001	0.272	0.047	0.021	0.0041
0.0001	0.238	0.041	0.019	0.0034

[a]Evaluated at a source strength corresponding to Y1 + Y2 for each β_o.

C. Average Step Number

Table VIII shows the ASN values versus α_o and β_o for unforced decisions with UADD = 0.0 (background). These values range from 24 to 75% of NO, the nominal length of the single-interval test on which the SPRT is based. The obvious trends are that the ASN decreases as α_o decreases and as β_o increases. The lowest ASN is for α_o = 0.0001 and β_o = 0.5.

For UADD = Y1 + Y2, the results are shown in Table IX. These values are higher, on average, than for UADD = 0.0, but they are always less than N0 (= 10). However, for some values of UADD between 0.0 and Y1 + Y2, the ASN might be greater than N0, as is apparent from some of the sample problems discussed in Sec. III.

As expected, for those entries corresponding to $\alpha_o = \beta_o$, the ASN values in Tables VIII and IX are equal, because the analysis of UADD = 0.0 and UADD = Y1 + Y2 is symmetrical in that situation. Similarly, the values for α in Tables IV and V are equal (within statistical variations) to the values of β in Tables VI and VII, respectively, for $\alpha_o = \beta_o$.

TABLE VIII

THE AVERAGE STEP NUMBER FOR UADD = 0.0
(BACKGROUND)

	β_o			
α_o	0.5	0.1	0.05	0.01
0.1	7.1	7.3	7.4	7.5
0.05	6.1	6.3	6.5	6.7
0.01	4.3	4.7	4.9	5.3
0.001	3.0	3.5	3.7	4.1
0.0001	2.4	2.8	3.0	3.4

TABLE IX

THE AVERAGE STEP NUMBER FOR UADD = Y1 + Y2

	β_o			
α_o	0.5	0.1	0.05	0.01
0.1	9.7	7.3	6.3	4.7
0.05	9.7	7.4	6.5	4.9
0.01	9.7	7.5	6.7	5.3
0.001	9.8	7.7	6.9	5.6
0.0001	9.9	7.9	7.2	5.9

In fact, for α_o and β_o in Tables IV, V, and VIII equal to β_o and α_o in Tables VI, VII, and IX, respectively, the entries should be equal, within statistical variation. For example, the entry in Table VIII for $\alpha_o = 0.01$, $\beta_o = 0.1$ is equal to the Table IX entry for $\alpha_o = 0.1$, $\beta_o = 0.01$. As another example, the entry in Table IV for $\alpha_o = 0.01$, $\beta_o = 0.05$ is 0.0046, whereas the equivalent value in Table VI for $\alpha_o = 0.05$, $\beta_o = 0.01$ is 0.0053. Because these values are each based on 10^5 trials, they represent approximately 460 and 530 decisions, respectively. Thus, their standard deviations are approximately $(460)^{1/2} \simeq 21$ and $(530)^{1/2} \simeq 23$. To determine if these entries are within reasonable agreement, the normal distribution test[13] may be applied to yield $t = |530 - 460|/(530 + 460)^{1/2} = 2.22$. This means that a difference at least this large would be expected with a frequency of 2.6%. Considering the number of entries being compared in the tables, these two entries seem to be in reasonable agreement. Most of the other entries appropriate for comparison are in closer agreement.

V. EFFECT OF VARYING THE NOMINAL STEP NUMBER

To gain some insight into the effect of varying N0, the number of steps corresponding to the nominal single-interval test length, a series of runs was made with N0 = 1, 2, 4, 8, 16, and 32. For all runs the value $\alpha_o = \beta_o = 0.01$ was used, while UADD took on values from 0.0 to 6.0 in increments of 1.0. Each run was 100,000 trials in length.

The results for α and β are shown in Table X for the unforced decision case. (Although a decision was actually forced after step 98 for some trials, this did not have a significant effect on the results shown except for N0 = 32, where the values for UADD = 2.0 and 3.0 would have been, respectively, somewhat larger and smaller.) It can be seen that smaller N0 values resulted in smaller values for α. However, for small values of UADD, β is poorer (larger) for smaller N0 values; this is, of course, always the case for very small values of UADD, because in the limit as UADD goes to zero, $\beta = 1 - \alpha$.

Because $\alpha_o = \beta_o = 0.01$, it follows that for UADD = Y1 + Y2 = 2.326 + 2.326 = 4.652, $\beta = \alpha$; and for UADD = 2.326, $\beta = 0.5$ for all values of N0. Also, for any N0, the β for any UADD′ = 4.652 − UADD is equal to $1 - \beta$ for UADD. For example, the β for UADD′ = 4.652 − 2.0 is equal to 1 − 0.685 = 0.315 for N0 = 8. Thus additional values for β may be derived from the table for UADD′ = 0.652, 1.652, 2.652, 3.652, and 4.652.

Based on these characteristics, it follows that for values of UADD between 2.326 and 4.652, the smaller N0 is, the smaller (relatively) is β. This is clear from the table for UADD = 3.0 and 4.0, and, indeed, the table indicates that this might be the trend for considerably larger values of UADD.

The statistical cost of the lower α as a function of lower N0 is demonstrated in Table XI, where the ratio of the ASN to N0 is shown for the unforced decision case. (Again, a decision was actually forced after step 98, if no decision had been reached by then. This only had a noticeable effect on the runs with NO = 32 and with UADD = 2.0 and 3.0, where otherwise the values for ASN/NO would have been somewhat larger.)

The average time for a test (relative to the nominal single-interval test) increases with decreasing NO. For example, if these tests were based on a single-interval test that took 10 s, the average length of the SPRT test for UADD = 0.0 would be 10.9 s for NO = 1, but only 4.7 s for NO = 32. Actually, every trial for the SPRT test for NO = 1 takes as long or longer than the single-interval test because no decision can be made until the end of step 1, which is exactly the length of the single-interval test.

TABLE X

CALCULATED RESULTS FOR α AND β FOR UNFORCED DECISIONS

NO	UADD						
	0.0[a]	1.0	2.0	3.0	4.0	5.0	6.0
1	0.0004	0.986	0.736	0.106	0.0048	0.0002	$<10^{-5}$
2	0.0016	0.975	0.713	0.134	0.0098	0.0006	$<10^{-4}$
4	0.0027	0.967	0.699	0.153	0.0149	0.0012	0.0001
8	0.0038	0.959	0.685	0.165	0.0185	0.0016	0.0002
16	0.0048	0.952	0.675	0.179	0.0213	0.0021	0.0003
32	0.0061	0.946	0.664	0.191	0.0255	0.0025	0.0003

[a]Entries under the column with UADD = 0.0 are the calculated values for α; all other columns contain the calculated β values.

TABLE XI

ASN/NO VALUES FOR UNFORCED DECISIONS

NO	UADD						
	0.0	1.0	2.0	3.0	4.0	5.0	6.0
1	1.09	1.48	2.56	2.12	1.28	1.05	1.00
2	0.75	1.14	1.96	1.66	0.96	0.68	0.56
4	0.62	0.97	1.60	1.39	0.81	0.55	0.42
8	0.55	0.87	1.38	1.21	0.72	0.48	0.36
16	0.50	0.79	1.24	1.10	0.66	0.44	0.33
32	0.47	0.74	1.11	1.00	0.63	0.41	0.31

So, although α is better for small NO than large, the length of time required to make a decision is larger. It is, thus, not apparent from these two tables that there is a universally best NO for the SPRT with $\alpha_o = \beta_o = 0.01$. This general problem of a best NO requires further study.

For the same runs discussed previously, but for forced decisions at NO = NSTEP, the results are shown in Tables XII and XIII. Setting NSTEP = NO ensures that the SPRT never takes longer than the single-interval test on which it is based. In fact, because of the forced-decision criteria used in the program, for $\alpha_o = \beta_o$, the run with NO = NSTEP = 1 is exactly equivalent to the single-interval test. In Table XII, the theoretical results of the single-interval test, as determined from cumulative probability tables for the normal distribution, are shown in the first row, while the values obtained from the computer program are shown in the second row (NO = 1). The agreement between the two rows is excellent. The trends noticeable in Table XII are that α increases slightly with increasing NO, and the β values for particular source strenths are very similar for a large range of UADD values, increasing somewhat with NO as UADD increases above 2.326.

Table XIII shows that for NO = 1, ASN/NO = 1; in fact, one and only one step is always required. For the other values of NO, the ASN is always less than 1. Of particular interest is the ASN/NO ratio for UADD = 0.0. This is, for example, equal to 0.48 for NO = 16; i.e., the SPRT with a decision forced after step 16 takes only half as long on average, as the single-interval test. It never takes longer than the single-interval test for any value of UADD, and

TABLE XII

CALCULATED RESULTS FOR α AND β FOR FORCED DECISIONS AT NSTEP + NO

	UADD						
NO	0.0[a]	1.0	2.0	3.0	4.0	5.0	6.0
(1)[b]	(0.0100)	(0.908)	(0.628)	(0.250)	(0.0470)	(0.00375)	(0.00012)
1	0.0104	0.908	0.627	0.250	0.0454	0.0038	0.0001
2	0.0108	0.907	0.629	0.255	0.0492	0.0042	0.0002
4	0.0112	0.905	0.629	0.255	0.0496	0.0045	0.0002
8	0.0115	0.903	0.627	0.252	0.0504	0.0049	0.0004
16	0.0122	0.900	0.622	0.256	0.0502	0.0049	0.0004
32	0.0133	0.899	0.624	0.254	0.0533	0.0052	0.0004

[a]Entries under the column with UADD = 0.0 are the calculated values for α; all other columns contain the calculated β values.
[b]Values in parentheses are for the nominal single interval test; β values were obtained from standard statistical tables.

TABLE XIII

ASN/NO VALUES FOR FORCED DECISIONS AT NSTEP = NO

	UADD						
NO	0.0	1.0	2.0	3.0	4.0	5.0	6.0
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.70	0.83	0.92	0.89	0.79	0.66	0.62
4	0.59	0.75	0.85	0.83	0.69	0.54	0.42
8	0.52	0.69	0.81	0.78	0.63	0.47	0.36
16	0.48	0.65	0.78	0.75	0.59	0.43	0.33
32	0.46	0.62	0.75	0.72	0.56	0.41	0.31

has similar β values (Table XII) for a range of UADD of interest to many safeguards problems. The α is, however, somewhat larger, and β for large values of UADD is also larger than that for the single-interval test. Tests such as this may well be useful in particular applications, because they allow considerably faster tests on average, are never longer, and have only a slight decrease of statistical power, compared to the single-interval test.

VI. SELECTION OF THE INPUT FALSE-NEGATIVE PROBABILITY VALUE

The input parameter α_o is selected to provide the (approximate) desired false-positive detection probability; to maximize detection sensitivity, it is generally chosen to be as large as tolerable for field conditions. However, selecting the input false-negative probability value β_o may be less straightforward, especially if you expect to encounter a range of source strengths. This difficulty arises because the choice for β_o affects the value of β for all source strengths (in contrast to the single-interval test, where the choice of α_o fixes β for all source strengths).

To gain some understanding of this effect, a series of runs was made using SPRTREP for α_o = 0.0228, and with β_o = 0.5, 0.1587, 0.0228, 0.00135, and 3.167×10^{-5}, corresponding to Y2 = 0.0, 1.0, 2.0, 3.0, and 4.0, respectively. For each of the five runs, NO equaled 10 while UADD varied from 0.0 to 6.0 in increments of 0.5.

The results for α and β are shown in Table XIV for all five runs and are plotted in Fig. 10 for three runs. Examination of these data shows that, in general, each column has one region with a β lower than in any other column; this is near the region of UADD corresponding to the mean of the distribution appropriate for β_o. Thus, for example, in Fig. 10 the curve for β_o = 0.0228 is best in the vicinity of UADD = Y1 + Y2 = 2.0 + 2.0 = 4.0. The other obvious generality is that the larger β_o is, the better (lower) β is at lower source strengths and the poorer it is at high source strengths. The converse is also

TABLE XIV

VALUES FOR α AND β VERSUS β_o

UADD	β_o				
	0.5	0.1587	0.0228	0.00135	3.167×10^{-5}
0.0[a]	0.0213	0.01447	0.01125	0.00914	0.00760
0.5	0.9165	0.9486	0.9660	0.9761	0.9825
1.0	0.7686	0.8462	0.9043	0.9402	0.9612
1.5	0.5447	0.6351	0.7530	0.8517	0.9113
2.0	0.3457	0.3839	0.5001	0.6697	0.8070
2.5	0.2056	0.1960	0.2456	0.3842	0.5962
3.0	0.1243	0.0917	0.1116	0.1501	0.3011
3.5	0.07265	0.0426	0.0340	0.0430	0.0902
4.0	0.04379	0.0191	0.0112	0.0105	0.0177
4.5	0.02681	0.0088	0.0038	0.0023	0.00320
5.0	0.01581	0.0043	0.0011	0.00061	0.00043
5.5	0.00942	0.0020	0.0040	0.00012	0.00008
6.0	0.00582	0.00094	0.00015	0.00003	0.00001

[a]Values in columns 2-6 of this row correspond to α; all other rows are β values.

true; i.e., small β_o results in relatively high values of β for small UADD and low β values for large UADD. The choice of β_o also affects α, as described in Sec. IV. The values for α are shown in the first row of Table XIV, for UADD = 0.0.

Table XV shows the ASN/NO values obtained for all five runs and Fig. 11 shows plots for three of them. It appears that for each run there is a region of UADD where the ASN/NO value is less than for any other run. This is near, but not identical to the region corresponding to β_o for that run.

From this limited amount of data, it is obvious that the choice of β_o can significantly influence the statistical parameters α, β, and ASN. To determine the exact effect to expect for a particular α_o, you might think it necessary to perform a series of Monte Carlo runs as I did. However, to the extent that these data can be generalized, it appears that a particular choice for β_o gives the best test for source strengths corresponding to that value, as expected from the theory. If your concern is primarily with detecting sources of that

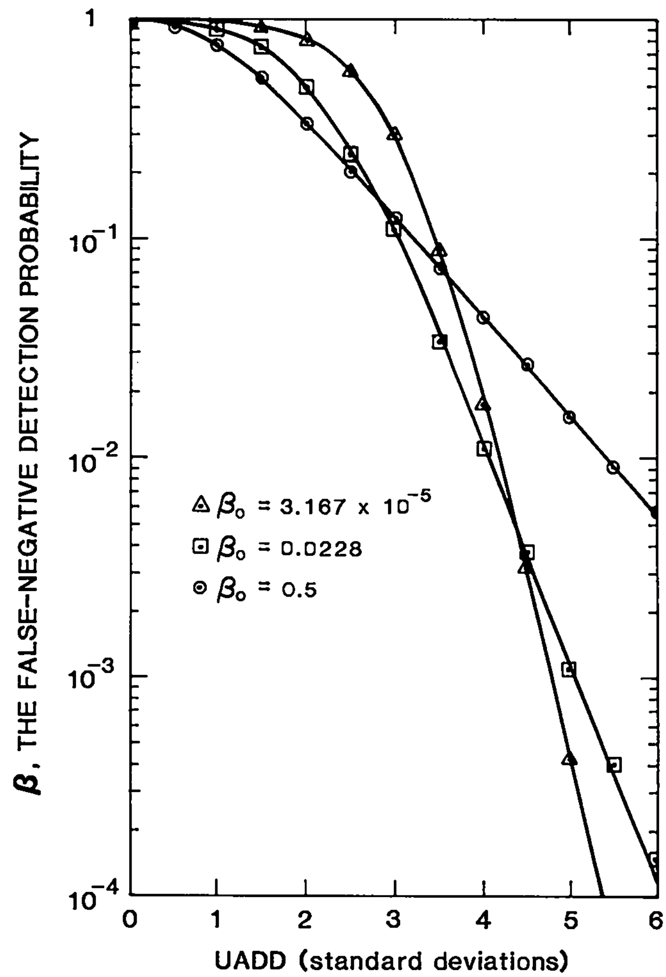

Fig. 10.
Plot of β versus UADD for selected SPRT runs with $\alpha_o = 0.0228$ and N0 = 10.

intensity, the choice of β_o then is obvious. Because the actual problem is not always (or even usually) that simple, a more detailed examination of the expected results, using the technique demonstrated here may be appropriate.

For example, examination of the curves in Fig. 10 shows that the one for $\beta_o = 3.167 \times 10^{-5}$ has the poorest detectability at low values of source strength. In most safeguards applications, this would be undesirable and, therefore, a larger β_o would be chosen. However, this feature may be useful in some radiation monitoring applications, when, as here, it is coupled with very good capabilities at larger source strengths. Such features might be useful, for example, in a contamination monitor where only significant levels of contamination are of interest, and you don't want an alarm for levels just above background.

TABLE XV

CALCULATED VALUES FOR ASN/NO VERSUS β_o

UADD	β_o 0.5	0.1587	0.0228	0.00135	3.167×10^{-5}
0.0	0.506	0.536	0.581	0.623	0.660
0.5	0.706	0.712	0.730	0.752	0.775
1.0	0.930	0.934	0.936	0.931	0.929
1.5	1.022	1.121	1.176	1.172	1.140
2.0	0.974	1.123	1.287	1.411	1.408
2.5	0.866	0.985	1.168	1.443	1.646
3.0	0.751	0.814	0.932	1.177	1.577
3.5	0.655	0.676	0.725	0.863	1.168
4.0	0.572	0.567	0.577	0.639	0.794
4.5	0.506	0.486	0.477	0.496	0.564
5.0	0.453	0.424	0.403	0.405	0.433
5.5	0.410	0.377	0.351	0.343	0.353
6.0	0.375	0.338	0.312	0.298	0.300

VII. SUMMARY AND CONCLUSIONS

SPRTEST simulates the SPRT for populations described by the normal distribution. SPRTEST and its variation SPRTREP are listed in the appendixes; Los Alamos users can obtain them directly from the MASS storage system using the command GET/KLCQ2/name.

The SPRTEST program should prove useful in deciding whether to use the SPRT or another statistical test in various applications, in selecting parameters for the test, and in determining what experimental results would be expected ideally using a particular SPRT. Its current use is primarily for nuclear safeguards testing, but it should also be useful in other fields involving random sampling from populations approximated by the normal distribution. The various tables and figures in this report provide some insights into the usefulness and limitations of the SPRT for such applications.

For the domain of α and β of most interest in safeguards applications, it was shown that for NO = 10, α is always equal to or less than the nominal α_o for unforced decisions, and $\beta < \beta_o$ for UADD = Y1 + Y2. For other values of UADD, β may be greater or lesser than the single-interval test β, but a number of trends were noted.

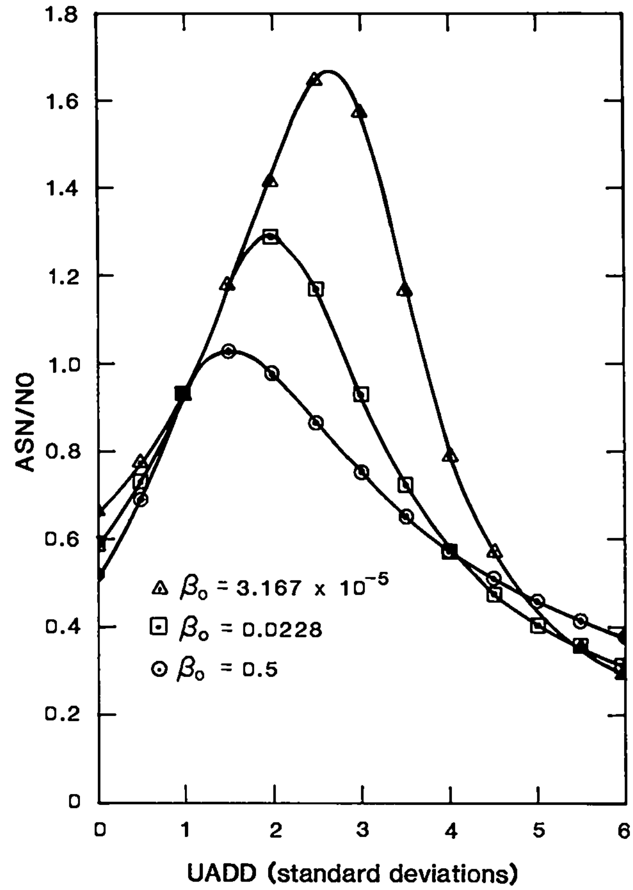

Fig. 11.
Plot of the fractional average step number, ASN/NO, for selected SPRT runs with α_o = 0.0228 and NO = 10.

The average length of time required to complete an SPRT is usually less than that for the single-interval test on which it is based for background (UADD = 0.0) sampling and for UADD $\geq$ Y1 + Y2. In between, however, it is often longer.

The effect of dividing the nominal single-interval period into different numbers of steps, NO, was investigated and trends were noted. For NSTEP = NO = 1, the SPRT was shown to be equivalent to the nominal single-interval test on which it is based, for the forced decision criteria used in the program.

A maximum time may be imposed on the SPRT by forcing a decision after NSTEP steps of the sequence. This never improves α and β simultaneously and may increase both, while the ASN decreases (or in extreme cases, remains the same). In general, NSTEP should be as large as tolerable to maximize the power of the SPRT. However, even when NSTEP = NO, the SPRT may be preferred to the single-interval test for particular applications; this choice for NSTEP ensures that the SPRT is never longer than the single-interval test on which it is based.

The effect of varying β_0 was investigated over a limited range. In general, if it is most important to detect the source strength corresponding to a particular β_0, then input of that value provides the best SPRT. However, if a broad range of source strengths is of more or less equal importance, then it may be desirable to investigate the effect of varying β_0, using the Monte Carlo technique, before deciding on which β_0 to use in the particular safeguards monitor. That type of investigation was demonstrated in this report.

While not described in this report, SPRTEST can be easily modified to examine more complex safeguards problems. For example, the source strength can be varied during a test sequence to simulate passage of a source through a radiation monitor.[4] The frequency of detection with the SPRT can then be compared with that for the single-interval test, or other commonly used tests such as the sliding-interval procedure.[14] SPRTEST may also be readily modified to use a Poisson distribution[8] instead of the normal distribution used in this report.

ACKNOWLEDGMENTS

I am grateful to Paul E. Fehlau of Los Alamos who introduced me to the subject of the SPRT. The Monte Carlo Theory and Application Course, taught by Tom Booth also of Los Alamos, provided me with the background necessary to conceive this study and the basic technique to carry it out.

REFERENCES

1. Abraham Wald, Sequential Analysis (Dover Publications, Inc., New York, 1963).

2. P. E. Fehlau, J. C. Pratt, J. T. Markin, and T. Scurry, Jr., "Smarter Radiation Monitors for Safeguards and Security", 24th Annual Meeting of the Institute of Nuclear Materials Management, Vail, Colorado, July 10-13, 1983.

3. J. T. Markin, J. E. Stewart, and A. S. Goldman, "Data Analysis for Neutron Monitoring in an Enrichment Facility," Proceedings of the 4th Annual ESARDA Symposium Specialist Meeting on Harmonization and Standardization for Nuclear Safeguards," Petten, Netherlands, April 27-19, 1982.

4. P. E. Fehlau, K. L. Coop, and J. T. Markin, "Application of Wald's Sequential Probability Ratio Test to Nuclear Materials Control," ESARDA/INMM Joint Specialists Meeting on NDA Statistical Problems, Ispra, Italy, September 10-12, 1984.

5. P. E. Fehlau, K. Coop, C. Garcia, Jr., and J. Martinez, "The Pajarito SNM Monitor: A High-Sensitivity Monitoring System for Highly-Enriched Uranium," Proceedings of the 25th Annual Meeting of the Institute of Nuclear Materials Management, Columbus, Ohio, July 16-18, 1984.

6. P. E. Fehlau, K. L. Coop, and K. V. Nixon, "Sequential Probability Ratio Controllers for Safeguards Radiation Monitors," Proceedings of the 6th Annual ESARDA Symposium on Safeguards and Nuclear Material Management, Venice, Italy, May 14-18, 1984.

7. P. E. Fehlau "Materials Control and Accounting (MC&A) Technology Development: Sequential Decision Logic for Safeguards Radiation Monitors," in "Safeguards and Security Progress Report August 1982 - January 1983," D. R. Smith, Comp., Los Alamos National Laboratory report LA-9821-PR (November 1983).

8. K. L. Coop, "Monte Carlo Simulation of The Sequential Probability Ratio Test for Radiation Monitoring," Proceedings of the IEEE Nuclear Science Symposium, Orlando, Florida, October 31 - November 2, 1984.

9. J. P. Shipley, "Sequential Likelihood-Ratio Tests Applied to Series of Material Balances," in Mathematical and Statistical Methods in Nuclear Safeguards, F. Argentesi, R. Avenhaus, M. Franklin, and J. P. Shipley, Eds. (Harwood Academic Publishers for the Commission of the European Communities, New York, 1982).

10. E. H. Cooke-Yarborough and R.C.M. Barnes, "Rapid Methods for Ascertaining Whether the Activity of a Weak Radioactive Sample Exceeds a Predetermined Level," Proceedings of the Institution of Electrical Engineers 108 B, 153 (1961).

11. L. A. Aroian, "Applications of the Direct Method in Sequential Analysis," Technometries (3) 18 (August 1976).

12. C. J. Everett and E. D. Cashwell, "A Third Monte Carlo Sampler", Los Alamos National Laboratory report LA-9721-MS (March 1983).

13. J. B. Kennedy and A. M. Neville, Basic Statistical Methods for Engineers and Scientists, 2nd ed. (Thomas Y. Crowell Co., New York, 1976).

14. W. H. Chambers et al., "Portal Monitor for Diversion Safeguards", Los Alamos Scientific Laboratory report LA-5681 (December 1974).

APPENDIX A

SPRTEST FORTRAN LISTING

Los Alamos Identification No. LP-1732

```
$ FTN (I=SPRTEST.GO.SET.SYM=^)
      PROGRAM SPRTEST(TTY.INPUT=TTY.OUTPUT=TTY)
C  KEN COOP'S PROGRAM TO TEST WALD'S SEQUENTIAL PROB. RATIO TEST
C    GROUP Q-2. LOS ALAMOS NATIONAL LABORATORY. MAIL STOP J-562
C  WRITTEN IN FORTRAN IV FOR THE LOS ALAMOS LTSS COMPUTER SYSTEM
C    JANUARY 3. 1985 VERSION
C
      INTEGER FNHO,FNH1
      DIMENSION IHO(100).IH1(100)
C
C  INITIALIZE SOME PARAMETERS
      DO 10 J=1.100
      IHO(J)=0
   10 IH1(J)=0
      NH1=0
      NHO=0
      ASN=0.0
      LOOP=-1
C
C  READ IN PARAMETERS FROM KEYBOARD
C
C READ IN THE NOMINAL ALPHA
      PRINT 12
      READ 14.ALPHA
C  READ THE NOMINAL BETA
      PRINT 16
      READ 18.BETA
C READ IN Y1.THE ABSCISSA VALUE CORRESPONDING TO ALPHA(NOMINAL)
      PRINT 20
      READ 22.Y1
C READ IN Y2. THE ABSCISSA VALUE CORRESPONDING TO BETA(NOMINAL)
      PRINT 24
      READ 22.Y2
C READ FROM KEYBOARD VALUE TO ADD TO U TO GET MEAN OF DISTRIBUTION
C     THAT IS BEING TESTED OR SIMULATED
C    PROPERLY LOCATED FOR HYPOTHESIS HO.THE VALUE IS 0.0
      PRINT 30
      READ 60.UADD
C  READ IN NO. NO. OF STEPS CORRESPONDING TO NOMINAL SINGLE-INTERVAL TEST
      PRINT 26
      READ 28.NO
C READ IN STEP NO. AFTER WHICH A DECISION IS FORCED
      PRINT 40
      READ 70,NSTEP
C READ IN SEED FOR RANDOM NO. GENERATOR:
C  USUALLY THIS WILL BE 0 (ZERO)
      PRINT 50
      READ 80.NSEED
      PRINT 90.NSEED
   12 FORMAT(/.30H TYPE IN ALPHA (F10.8)         )
   14 FORMAT(F10.8)
   16 FORMAT(/.30H TYPE IN BETA (F10.8)          )
   18 FORMAT(F10.8)
   20 FORMAT(/.30H TYPE IN Y1 (F7.5)           )
   22 FORMAT(F7.5)
   24 FORMAT(/.30H TYPE IN Y2 (F7.5)           )
   26 FORMAT(/.30H TYPE IN NO (I2)             )
   28 FORMAT(I2)
   30 FORMAT(/.30H TYPE IN UADD (F7.5)         )
   40 FORMAT(/.30H TYPE IN NSTEP (I2)          )
   50 FORMAT(/.30H TYPE IN NSEED (I18)         )
   60 FORMAT(F7.5)
   70 FORMAT(I2)
   80 FORMAT(I18)
   90 FORMAT(5X.25HRANDOM NO. STARTING SEED=.I20)
C ALPHA IS THE FALSE POSITIVE PROBABILITY (ERROR OF FIRST KIND)
C  BETA IS FALSE NEGATIVE PROB. (ERROR OF SECOND KIND)
C  Y1 IS THE ABSCISSA OF THE NORMAL DIST. CORRESPONDING TO ALPHA
C  Y2 IS THE ABSCISSA (ABSOLUTE VALUE) FOR BETA
C   NO IS THE NOMINAL NUMBER OF STEPS CORRESPONDING TO THE SO-CALLED
C   (BY WALD) "CURRENT BEST SINGLE TEST PROCEDURE"
C   I REFER TO IT AS THE "SINGLE-INTERVAL" TEST OR "SIT"
```

```
C
C   CALCULATE SOME VALUES USED FOR ALL TRIALS BELOW
C
       A=ALOG((1.0-BETA)/ALPHA)
       B=ALOG(BETA/(1.0-ALPHA))
       UADD=UADD/NO**.50
       THETA=(Y1+Y2)/NO**0.50
C   INITIALIZE RANDOM NUMBER GENERATOR. USING RANSET( ).IF CALLED
       IF(NSEED.EQ.0) GO TO 100
       CALL RANSET(NSEED)
C
C MAIN LOOP STARTS
C
   100 LOOP=LOOP+1
       X=0.0
       IF(LOOP.GE.100000) GO TO 300
       DO 200 K=1,98
C   FIND EFFECT OF STOPPING AFTER NSTEP STEPS
       IF(K.NE.NSTEP+1) GO TO 120
       IF(Z.LE.0.0) IH0(100)=IH0(100)+1
       IF(Z.GT.0.0) IH1(100)=IH1(100)+1
   120 CONTINUE
C  OBTAIN ABSCISSA VALUES FROM NORMAL DISTRIBUTION SAMPLING
       R=(-ALOG(RANF(1)))**0.5
       TNU=1.5707963*RANF(1)
       Y=1.4142136*R*COS(TNU)
       IF(RANF(1).GT..5000) GO TO 150
       Y=-Y
   150 CONTINUE
C
C   CALCULATE Z. THE LOGARITHM OF THE PROBABILITY RATIO
       M=K
       U=Y+UADD
       X=X+THETA*U
       Z=X - M*THETA*THETA*.50
C  COMPARE Z WITH LIMITS.REPEAT TEST OR STORE RESULT
C
       IF(Z.LE.B) GO TO 280
       IF(Z.GE.A) GO TO 290
   200 CONTINUE
       IF(Z.LE.0.0) IH0(99)=IH0(99)+1
       IF(Z.GT.0.0) IH1(99)=IH1(99)+1
       GO TO 100
   280 IH0(M)=IH0(M)+1
       GO TO 100
   290 IH1(M)=IH1(M)+1
       GO TO 100
C  PRINT OUT MATRICES
C
   300 PRINT 380
       PRINT 400. (IH0(K).K=1.100)
       PRINT 390
       PRINT 400. (IH1(K).K=1.100)
   380 FORMAT(//.10X."MATRIX IH0(BACKGROUND-ONLY): "./)
   390 FORMAT(//.10X."MATRIX IH1(ABOVE-BACKGROUND):"./)
   400 FORMAT(5X.10I6)
```

```
C
C   CALCULATE AVERAGE NUMBER OF STEPS
C     ASN IS THE NUMBER WITH 98 STEPS PERMITTED
C   FASN IS THE NUMBER WITH A MAX. OF NSTEP STEPS PERMITTED
C
C   NHO IS TOTAL NUMBER OF RUNS ENDING WITH HO FOR 98 STEP MAX.
C   NH1 IS TOTAL ENDING IN DECISION H1 FOR 98 STEP MAX.
        DO 500 J=1.99
        IF(J.NE.NSTEP+1) GO TO 450
        FASN=ASN
        FNHO=NHO
        FNH1=NH1
  450   CONTINUE
        NHO=NHO+IHO(J)
        NH1=NH1+IH1(J)
  500   ASN=ASN+(IHO(J)+IH1(J))*J
        ASN=ASN/LOOP
        FASN=FASN+(IHO(100)+IH1(100))*NSTEP
        FASN=FASN/LOOP
C   FNHO IS THE NUMBER OF TESTS ACCEPTING HO FOR A MAX. OF NSTEP STEPS
C   FNH1 IS THE NO. OF TESTS REJECTING HO FOR A MAX. OF NSTEP STEPS
        FNHO=FNHO+IHO(100)
        FNH1=FNH1+IH1(100)
C
C     PRINT OUT CALCULATED RESULTS AND NEXT RANDOM GEN. SEED USING RANGET( )
C
        PRINT 550.ASN.FASN
  550   FORMAT(///.10X.6H ASN= .F10.3.10X."ASN(FORCED)= ".F10.3)
        PRINT 560.ASN/NO.FASN/NO
  560   FORMAT(/.11X."ASN/NO=".F7.4.11X."ASN(FORCED)/NO=".F7.4)
        PRINT 600.NHO.NH1
  600   FORMAT(///.10X.6H NHO= .I7.5X.6H NH1= .I7)
        ANHO=NHO*1.0
        ANH1=NH1*1.0
        AFNH1=FNH1*1.0
        AFNHO=FNHO*1.0
        IF(UADD.GT.0.0) GO TO 635
  620   PRINT 630.ANH1/(ANH1+ANHO)
  630   FORMAT(/.11X."ALPHA=".F9.6)
        GO TO 645
  635   PRINT 640. ANHO/(ANHO+ANH1)
  640   FORMAT(/.10X."BETA=".F9.6)
  645   PRINT 650.FNHO.FNH1
  650   FORMAT(///.10X.6HFNHO= .I7.5X.6HFNH1= .I7)
        IF(UADD.GT.0.0) GO TO 685
        PRINT 680.AFNH1/(AFNH1+AFNHO)
  680   FORMAT(/.10X."ALPHA(FORCED)=".F9.6)
        GO TO 700
  685   PRINT 690.AFNHO/(AFNHO+AFNH1)
  690   FORMAT(/.10X."BETA(FORCED)=".F9.6)
  700   RAN=RANF(1)
        CALL RANGET(NUM)
        PRINT 800.NUM
  800   FORMAT(///.10X.30HLAST RANDOM NO. STARTING SEED=.I20.//////)
        END
```

APPENDIX B

SPRTREP FORTRAN LISTING

```
$ FTN (I=SPRTREP.GO.SET.SYM=^)
      PROGRAM SPRTREP(TTY.INPUT=TTY.OUTPUT=TTY)
C  KEN COOP'S PROGRAM TO TEST WALD'S SEQUENTIAL PROB. RATIO TEST
C  GROUP Q-2. LOS ALAMOS NATIONAL LABORATORY. MAIL STOP J-562
C  WRITTEN IN FORTRAN IV FOR THE LOS ALAMOS LTSS COMPUTER SYSTEM
C   JANUARY 3. 1985 VERSION
C
C  THIS VERSION REPEATS SPRTEST 11 TIMES WITH INCREMENTED UADD VALUES
C
      INTEGER FNHO.FNH1
      DIMENSION IHO(100).IH1(100)
C
C  READ IN PARAMETERS FROM KEYBOARD
C
C READ IN THE NOMINAL ALPHA
      PRINT 12
      READ 14.ALPHA
C  READ THE NOMINAL BETA
      PRINT 16
      READ 18.BETA
C READ IN Y1.THE ABSCISSA VALUE CORRESPONDING TO ALPHA(NOMINAL)
      PRINT 20
      READ 22.Y1
C READ IN Y2. THE ABSCISSA VALUE CORRESPONDING TO BETA(NOMINAL)
      PRINT 24
      READ 22.Y2
C  READ IN UADD. WHICH IN THIS PROGRAM IS THE INCREMENT FOR THE ABSCISSA
C  USUALLY THIS IS IN THE RANGE FROM ABOUT .5 TO 1.0
      PRINT 30
      READ 60.UADD
C  READ IN NO. NO. OF STEPS CORRESPONDING TO NOMINAL SINGLE-INTERVAL TEST
      PRINT 26
      READ 28.NO
C READ IN STEP NO. AFTER WHICH A DECISION IS FORCED
      PRINT 40
      READ 70.NSTEP
C READ IN SEED FOR RANDOM NO. GENERATOR.
C  USUALLY THIS WILL BE 0 (ZERO)
      PRINT 50
      READ 80.NSEED
      PRINT 90.NSEED
   12 FORMAT(/.30H TYPE IN ALPHA (F10.8)          )
   14 FORMAT(F10.8)
   16 FORMAT(/.30H TYPE IN BETA (F10.8)           )
   18 FORMAT(F10.8)
   20 FORMAT(/.30H TYPE IN Y1 (F7.5)              )
   22 FORMAT(F7.5)
   24 FORMAT(/.30H TYPE IN Y2 (F7.5)              )
   26 FORMAT(/.30H TYPE IN NO (I2)                )
   28 FORMAT(I2)
   30 FORMAT(/.30H TYPE IN UADD (F7.5)            )
   40 FORMAT(/.30H TYPE IN NSTEP (I2)             )
   50 FORMAT(/.30H TYPE IN NSEED (I18)            )
   60 FORMAT(F7.5)
   70 FORMAT(I2)
   80 FORMAT(I18)
   90 FORMAT(5X.25HRANDOM NO. STARTING SEED=.I20)
C ALPHA IS THE FALSE POSITIVE PROBABILITY (ERROR OF FIRST KIND)
C  BETA IS FALSE NEGATIVE PROB. (ERROR OF SECOND KIND)
C  Y1 IS THE ABSCISSA OF THE NORMAL DIST. CORRESPONDING TO ALPHA
C  Y2 IS THE ABSCISSA (ABSOLUTE VALUE) FOR BETA
C   NO IS THE NOMINAL NUMBER OF STEPS CORRESPONDING TO THE SO CALLED
C   (BY WALD) "CURRENT BEST SINGLE TEST PROCEDURE"
C   I REFER TO IT AS THE "SINGLE-INTERVAL" TEST OR "SIT"
C
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C   CALCULATE SOME VALUES USED FOR ALL TRIALS BELOW
C
      A=ALOG((1.0-BETA)/ALPHA)
      B=ALOG(BETA/(1.0-ALPHA))
      UAODIJ=UAOD/NO**.50
      THETA=(Y1+Y2)/NO**0.50
C   INITIALIZE RANDOM NUMBER GENERATOR USING RANSET( ). IF CALLED
      IF(NSEED.EQ.0) GO TO 97
      CALL RANSET(NSEED)
   97 CONTINUE
C   THIS VERSION REPEATS SPRTEST 11 TIMES WITH INCREMENTED UAOD VALUES
      DO 1000 IJ=1.11
      UAOD=(IJ - 1)*UAODIJ
C
C  INITIALIZE SOME PARAMETERS
      DO 98 J=1.100
      IHO(J)=0
   98 IH1(J)=0
      NH1=0
      NHO=0
      ASN=0.0
      LOOP=-1
C
C MAIN LOOP STARTS
C
  100 LOOP=LOOP+1
      X=0.0
      IF(LOOP.GE.100000) GO TO 300
      DO 200 K=1.98
C   FIND EFFECT OF STOPPING AFTER NSTEP STEPS
      IF(K.NE.NSTEP+1) GO TO 120
      IF(Z.LE.0.0) IHO(100)=IHO(100)+1
      IF(Z.GT.0.0) IH1(100)=IH1(100)+1
  120 CONTINUE
C  OBTAIN ABSCISSA VALUES FROM NORMAL DISTRIBUTION SAMPLING
      R=(-ALOG(RANF(1)))**0.5
      TNU=1.5707963*RANF(1)
      Y=1.4142136*R*COS(TNU)
      IF(RANF(1).GT..50) GO TO 150
      Y=-Y
  150 CONTINUE
C
C   CALCULATE Z. THE LOGARITHM OF THE PROBABILITY RATIO
      M=K
      U=Y+UAOD
      X=X+THETA*U
      Z=X - M*THETA*THETA*.50
C  COMPARE Z WITH LIMITS.REPEAT TEST OR STORE RESULT
C
      IF(Z.LE.B) GO TO 280
      IF(Z.GE.A) GO TO 290
  200 CONTINUE
      IF(Z.LE.0.0) IHO(99)=IHO(99)+1
      IF(Z.GT 0.0) IH1(99)=IH1(99)+1
      GO TO 100
  280 IHO(M)=IHO(M)+1
      GO TO 100
  290 IH1(M)=IH1(M)+1
      GO TO 100
C  PRINT OUT MATRICES
C
  300 PRINT 380
      PRINT 400. (IHO(K).K=1.100)
      PRINT 390
      PRINT 400. (IH1(K).K=1.100)
  380 FORMAT(//.10X."MATRIX IHO(BACKGROUND-ONLY):  "./)
  390 FORMAT(//,10X."MATRIX IH1(ABOVE-BACKGROUND):"./)
  400 FORMAT(5X.10I6)
C
```

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C  CALCULATE AVERAGE NUMBER OF STEPS
C   ASN IS THE NUMBER WITH 98 STEPS PERMITTED
C  FASN IS THE NUMBER WITH A MAX. OF NSTEP STEPS PERMITTED
C  NHO IS TOTAL NUMBER OF RUNS ENDING WITH HO FOR 98 STEP MAX.
C  NH1 IS TOTAL ENDING IN DECISION H1 FOR 98 STEP MAX.
      DO 500 J=1.99
      IF(J.NE.NSTEP+1) GO TO 450
      FASN=ASN
      FNHO=NHO
      FNH1=NH1
  450 CONTINUE
      NHO=NHO+IHO(J)
      NH1=NH1+IH1(J)
  500 ASN=ASN+(IHO(J)+IH1(J))*J
      ASN=ASN/LOOP
      FASN=FASN+(IHO(100)+IH1(100))*NSTEP
      FASN=FASN/LOOP
C  FNHO IS THE NUMBER OF TESTS ACCEPTING HO FOR A MAX. OF NSTEP STEPS
C  FNH1 IS THE NO. OF TESTS REJECTING HO FOR A MAX. OF NSTEP STEPS
      FNHO=FNHO+IHO(100)
      FNH1=FNH1+IH1(100)
C
C   PRINT OUT CALCULATED RESULTS.UADD. AND NEXT RANDOM GEN. SEED
C
      PRINT 550.ASN.FASN
  550 FORMAT(///.10X.6H ASN= .F10.3.10X."ASN(FORCED)= ".F10.3)
      PRINT 560.ASN/NO.FASN/NO
  560 FORMAT(/.11X."ASN/NO=".F7.4.11X."ASN(FORCED)/NO=".F7.4)
      PRINT 600.NHO.NH1
  600 FORMAT(///.10X.6H NHO= .I7.5X.6H NH1= .I7)
      ANHO=NHO*1.0
      ANH1=NH1*1.0
      AFNH1=FNH1*1.0
      AFNHO=FNHO*1.0
      IF(UADD.GT.0.0) GO TO 635
  620 PRINT 630,ANH1/(ANH1+ANHO)
  630 FORMAT(/.11X."ALPHA=".F9.6)
      GO TO 645
  635 PRINT 640. ANHO/(ANHO+ANH1)
  640 FORMAT(/.11X."BETA=".F9.6)
  645 PRINT 650.FNHO.FNH1
  650 FORMAT(///.11X.6HFNHO= .I7.5X.6HFNH1= .I7)
      IF(UADD.GT.0.0) GO TO 685
      PRINT 680.AFNH1/(AFNH1+AFNHO)
  680 FORMAT(/.11X."ALPHA(FORCED)=".F9.6)
      GO TO 700
  685 PRINT 690.AFNHO/(AFNHO+AFNH1)
  690 FORMAT(/.11X."BETA(FORCED)=".F9.6)
  700 RAN=RANF(1)
      CALL RANGET(NUM)
      PRINT 750.UADD*NO**0.5
  750 FORMAT(//.11X.7HUADD=   .F9.5.//)
C  THE VALUE PRINTED OUT FOR UADD HAS THE INTERPRETATION OF BEING
C   THE ABSCISSA VALUE OF THE MEAN OF THE DIST. BEING TESTED
      PRINT 800.NUM
  800 FORMAT(11X.30HLAST RANDOM NO. STARTING SEED=.I20.//////)
 1000 CONTINUE
      END
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