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Optimum Hypersonic Airfoil with Power Law Shock Waves

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Introduction

In the present paper the flow field over a class of two-dimensional lifting surfaces is examined from the viewpoint of inviscid, hypersonic small-disturbance theory (HSDT). It is well known that a flow field in which the shock shape $S(x)$ is similar to the body shape $F(x)$ is only possible for $F(x) = x^k$ and the freestream Mach number $M_\infty = \infty$. This self-similar flow has been studied for several decades as it represents one of the few existing exact solutions of the equations of HSDT. Detailed discussions are found for example in papers by *Cole*[3], *Mirels*[5], *Chernyi*[7] and *Gersten* and *Nicolai*[8] but they are limited to convex body shapes, that is, $k \leq 1$. The only study of concave body shapes was attempted by *Sullivan*[6] where only special cases were considered. The method used here shows that similarity also exists for concave shapes and a complete solution of the flow field for any $k > \frac{2}{3}$ is given. The effect of varying k on $\frac{C_L^{3/2}}{C_D}$ is then determined and an optimum shape is found. Furthermore, a wider class of lifting surfaces is constructed using the streamlines of the basic flow field and analysed with respect to the effect on $\frac{C_L^{3/2}}{C_D}$.

We neglect viscous effects and assume boundary layers to be thin and attached

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to the surface. The surfaces are considered to correspond to the lower compression surface of a two-dimensinal wing. Since the pressure difference across the shock induced by this surface is of higher order than that of the shock induced by the upper expansion surface we neglect the contribution of the upper surface to the lift or drag.

Similarity Solution

This section is a formulation of our problem in the framework of hypersonic small-disturbance theory. If we substitute the scaled variables $y = \frac{\overline{y}}{\delta}$ and $x = \overline{x}$, with δ = thickness ratio, together with the asymptotic representations for velocity, pressure and density into the equations of motion and neglect $O(\delta^2)$ terms we obtain a reduced problem with the longitudinal momentum equation uncoupled from the rest of the problem. This longitudinal momentum equation can later be determined using the Bernoulli equation.

For a slender airfoil we write for the body surface $\overline{y} = \delta F(x)$ with associated shock shape $\overline{y} = \delta S(x)$. See figure 1.

Next, we change the (x, y)-coordinate system to the (x, ψ)-coordinate system, where ψ **is the** stream function. See figure 2. We further change from (x, ψ) to (x, ξ) **coordinates**, where ξ is the shock location $x = \xi$, i.e. the x-location where an incoming streamline crosses the shock. See figure 2 and 3. Note, that ψ and ξ are related by $\theta(\xi)\frac{\partial}{\partial\psi} = \frac{\partial}{\partial\xi}$, where $\theta(\xi) = \frac{dS(\xi)}{d\xi}$ can be determined by a separation of variables argument together with the continuity and momentum equation as $\theta(\xi) = ka\xi^{k-1}$. Also note that now the density can be written in terms of the pressure using the entropy equation and the shock conditions. Finally, we obtain for our basic problem:

$$\text{Continuity} \qquad \frac{c^2\theta(\xi)^{\frac{2}{\gamma}+1}}{p^{*\frac{1}{\gamma}}}\frac{\partial p^*}{\partial x} + \frac{\partial v^*}{\partial \xi} = 0 \tag{1}$$

$$\text{Momentum} \qquad \frac{\partial p^*}{\partial \xi} + \theta(\xi)\frac{\partial v^*}{\partial x} = 0 \tag{2}$$

$$\text{Shock conditions} \quad p^*(\xi,\xi) = p_S^* = \theta^2(\xi) \tag{3}$$

$$v^*(\xi,\xi) = v_S^* = \theta(\xi) \tag{4}$$

$$\text{Boundary Condition} \quad v^*(x,0) = \frac{\gamma+1}{2}\frac{dF(x)}{dx} \tag{5}$$

where $c^2 = \frac{\gamma-1}{2\gamma}, v = \frac{2}{\gamma+1}v^*, p = \frac{2}{\gamma+1}p^*$.

For the similarity solution we have

$$p^*(x,\xi) = k^2a^2x^{2k-2}\eta^{2k-\frac{2}{\gamma+1}}R(\eta) \tag{6}$$

and

$$v^*(x,\xi) = kax^{k-1}\eta^{k-\frac{2}{\gamma+1}}U(\eta) \tag{7}$$

where $\eta = \frac{\xi}{x}$ is the similarity variable. Therefore we obtain , together with the shock conditions (3) and (4) , $R(1) = 1$ and $U(1) = 1$. The boundary condition (5) can be used to determine the constant a. If we substitute (6) and (7) into equations (1) and (2) we obtain

$$\text{Continuity} \qquad -\frac{2\gamma}{\gamma+1}\frac{c^2}{R^{\frac{1}{\gamma}}} - \frac{c^2}{R^{\frac{1}{\gamma}+1}}\eta\frac{dR}{d\eta} + (k - \frac{2}{\gamma+1})U + \eta\frac{dU}{d\eta} = 0 \tag{8}$$

$$\text{Momentum} \qquad (2k - \frac{2}{\gamma+1})R + \eta\frac{dR}{d\eta} - \frac{\gamma-1}{\gamma+1}U + \eta\frac{dU}{d\eta} = 0 \tag{9}$$

The initial value problem we obtained can be solved numerically using a Runge-Kutta method, where we are interested in the cases where $k > 1$. Note that the special case of the Newtomian limit $\gamma = 1$ can be solved completely analytically.

Evaluation of $C_L^{3/2}/C_D$

At first we will study the case where $\xi = 0$ which is the case of the original power law shape. Observe that as $\eta \to 0$ we find that

$$R(\eta) = c_1\eta^{\frac{2}{\gamma+1} - 2k} \qquad\qquad U(\eta) = c_0\eta^{\frac{2}{\gamma+1} - k}. \tag{10}$$

The coefficients c_0 and c_1 are determined using equations (8) and (9). From the definition of the lift and drag coefficients and equations (6) and (7) we obtain the following formula:

$$\frac{C_L^{3/2}}{C_D} = \sqrt{\gamma+1}\frac{3k-2}{(2k-1)^{3/2}}\frac{\sqrt{c_1}}{c_0}. \tag{11}$$

We find that a maximum value of $\frac{C_L^{3/2}}{C_D} = 1.569$ is attained at $k = 1.13$ for $\gamma = \frac{7}{5}$. This result agrees with a result by *Cole* and *Aroesty*[4] who suggested that body shapes which are slightly more concave than a flat plate have superior performance.

Next, we wish to investigate the behavior of $\frac{C_L^{3/2}}{C_D}$ for $\xi \neq 0$. The underlying idea for constructing a wider class of lifting surfaces is to use the streamlines of our basic flow field as the elements of the surface. Then the lifting surface is formed by those streamlines that penetrate the basic shock surface through the points on the leading edge curve. See figure 1. Let us now define the lift and the drag coefficients as functions of ξ.

$$C_{L(\xi)} = -\frac{4}{\gamma+1}k^2a^2\delta^2\xi^{2k-1}\int_1^{\frac{\xi}{\xi+1}} \eta^{-\frac{2}{\gamma+1}} R(\eta)d\eta \tag{12}$$

The integral in the last equation can be found by using the momentum equation. Hence

$$C_{L(\xi)} = \frac{4}{\gamma+1}k^2a^2\delta^2\frac{1}{2k-1}(\xi+1)^{2k-1}\left(\frac{\xi}{\xi+1}\right)^{2k-\frac{2}{\gamma+1}} \left(R\left(\frac{\xi}{\xi+1}\right) - U\left(\frac{\xi}{\xi+1}\right)\right). \tag{13}$$

Similarily, we have for the drag coefficient

$$C_{D(\xi)} = -\frac{8}{(\gamma+1)^2}k^3a^3\delta^3\xi^{3k-2}\int_1^{\frac{\xi}{\xi+1}} \eta^{\frac{\gamma-3}{\gamma+1}} R(\eta)U(\eta)d\eta \tag{14}$$

Using momentum and continuity we can integrate and obtain

$$C_{D(\xi)} = \frac{8}{(\gamma+1)^2}k^3a^3\delta^3\frac{1}{3k-2}(\xi+1)^{3k-2}\left(\frac{\xi}{\xi+1}\right)^{3k-\frac{4}{\gamma+1}}$$

$$\left(U\left(\frac{\xi}{\xi+1}\right)R\left(\frac{\xi}{\xi+1}\right) - \frac{U\left(\frac{\xi}{\xi+1}\right)^2}{2} - \frac{1}{2}R\left(\frac{\xi}{\xi+1}\right)^{\frac{\gamma-1}{\gamma}}\right). \tag{15}$$

Finally, we obtain for the formula for a general two-dimensional waverider

$$\frac{C_{L(\xi)}^{3/2}}{C_{D(\xi)}} = \sqrt{\gamma+1}\frac{3k-2}{(2k-1)^{3/2}}\sqrt{\xi+1}$$

$$\frac{\left[\eta^{2k-\frac{2}{\gamma+1}}\left(R(\eta)-U(\eta)\right)\right]^{3/2}}{\left[\eta^{3k-\frac{4}{\gamma+1}}\left(U(\eta)R(\eta) - \frac{U(\eta)^2}{2} - \frac{1}{2}R(\eta)^{\frac{\gamma-1}{\gamma}}\right)\right]} \tag{16}$$

at $\eta = \frac{\xi}{\xi+1}$. An examination of the behavior of $\frac{C_{L(\xi)}^{3/2}}{C_{D(\xi)}}$ shows an increase of the maxima as ξ increases while the k where the maxima are attained also increase. The highest maximum value of $\frac{C_{L(\xi)}^{3/2}}{C_{D(\xi)}}$ is the limiting case $\xi \to \infty$ and $k_{max} \to \infty$ which corresponds to the body shape supporting exponential shock shapes. The limiting value is $\frac{C_{L(\xi)}^{3/2}}{C_{D(\xi)}} = 1.5795$. This special case was worked out earlier by *Cole* and *Areosty*[1].

Concluding Remarks

It is my great pleasure to express at this point my gratitude to Professor Julian D. Cole who suggested this problem to me and provided me with very helpful advice and guidance.

Details of above investigations can be found in *Wagner*[9]. The analysis is part of a study of optimum lifting surfaces using HSDT and will be used, in a subsequent paper, to design three-dimensional waveriders supported by two-dimensional flow fields. This represents a generalization of the idea by *Nonweiler*[2] to design three-dimensional inverted-V wings supported by the two-dimensional flow field generated by a flat plate.

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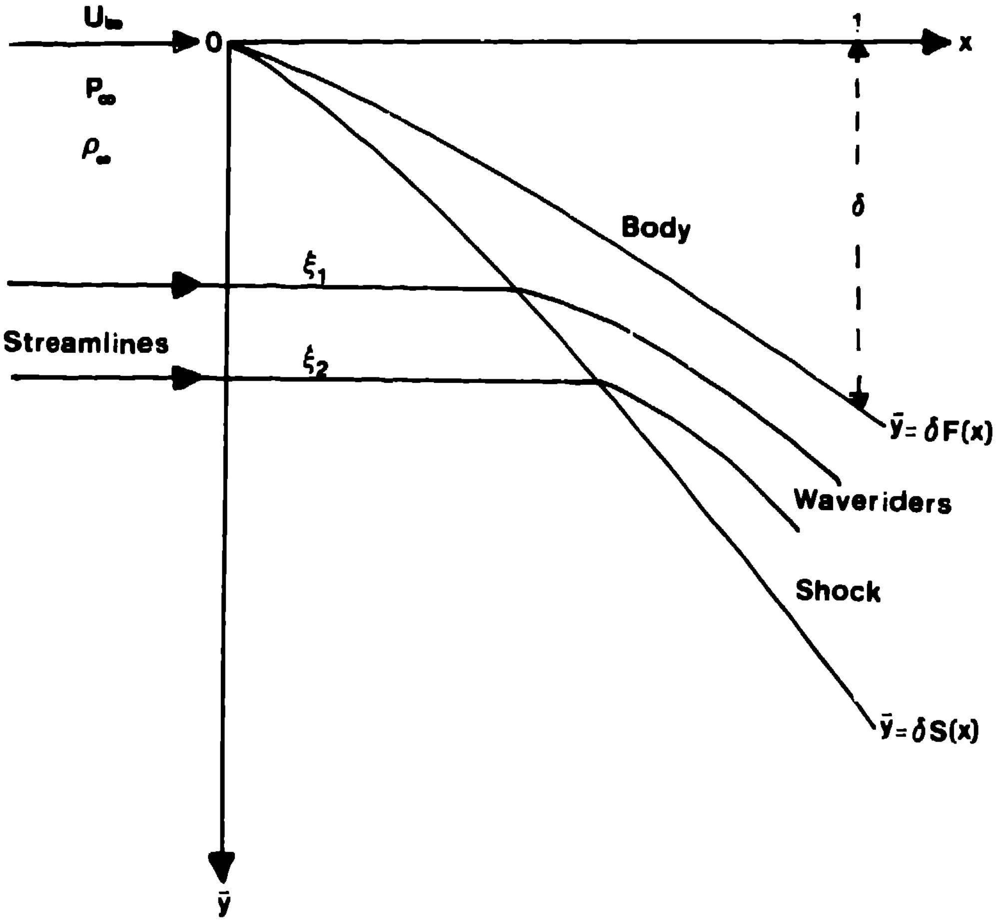

Figure 1: Domain of BVP in dimensional coordinates

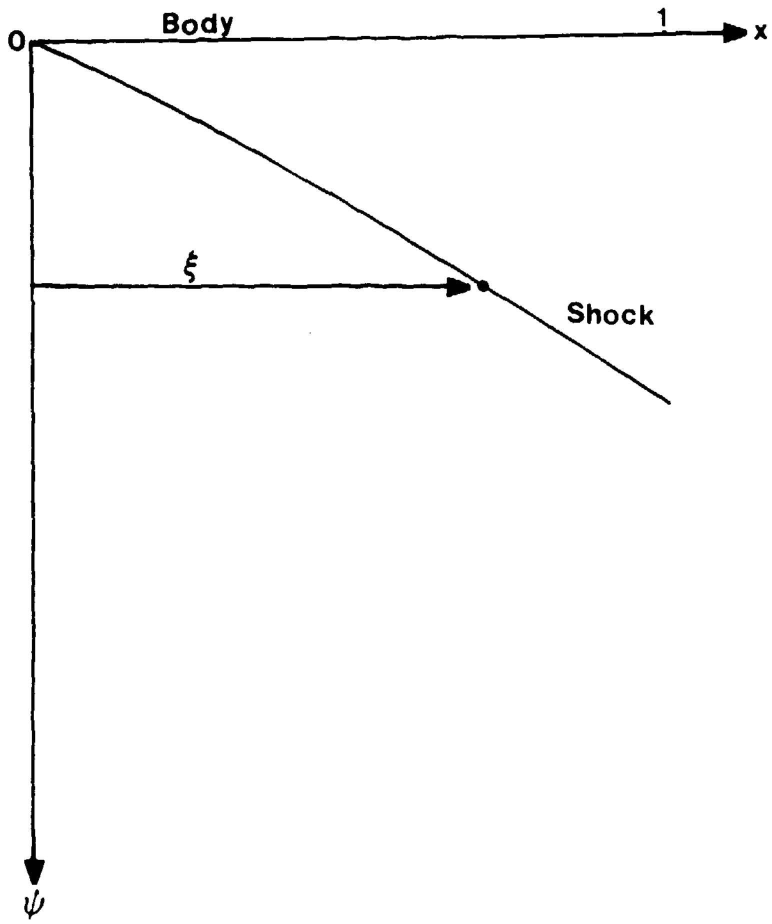

Figure 2: Domain of BVP in (ψ, x)-coordinates

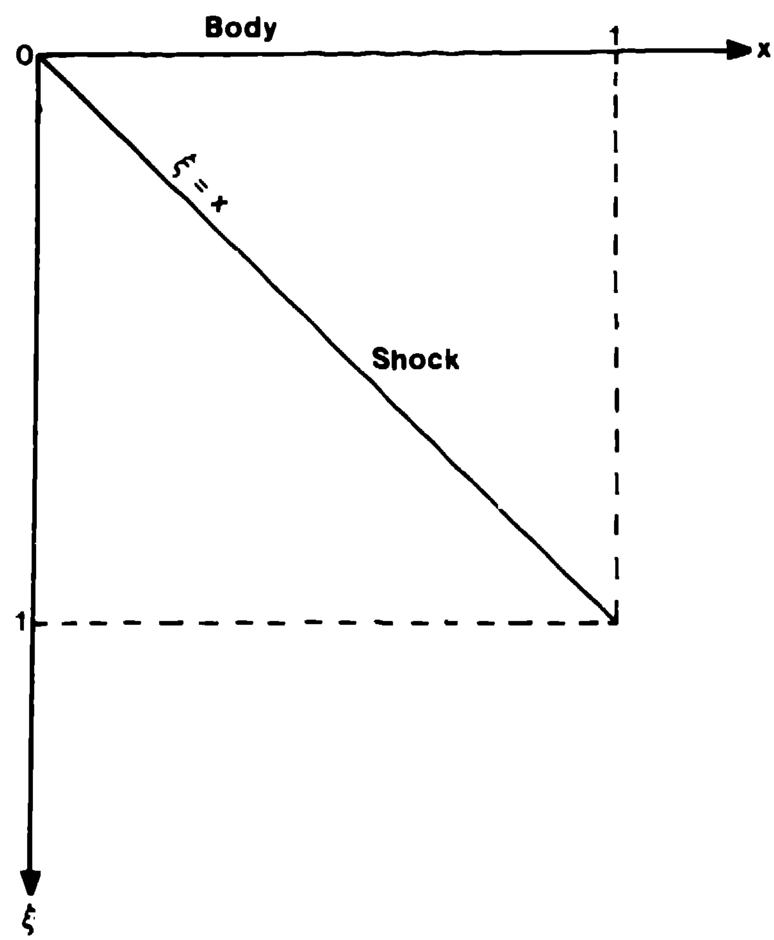

Figure 3: Domain of BVP in (ξ, x)-coordinates