LA-UR-80-2433

TITLE: "Relativistic Effects In The Atomic and Nuclear Few-Body Problems"

MASTER

AUTHOR(S): J. L. Friar

SUBMITTED TO: Review talk given at "International Conference on the Few-Body Problem," Eugene, Oregon, August 1980.

University of California

LOS ALAMOS SCIENTIFIC LABORATORY
Post Office Box 1663 Los Alamos, New Mexico 87545
An Affirmative Action/Equal Opportunity Employer

Form No. 836 R3
St. No. 2629
12/78

UNITED STATES
DEPARTMENT OF ENERGY
CONTRACT W-7405-ENG-36

RELATIVISTIC EFFECTS IN THE ATOMIC AND NUCLEAR FEW-BODY PROBLEMS

J. L. FRIAR

Theoretical Division, Los Alamos Scientific Laboratory
Los Alamos, New Mexico 87545

Abstract: Relativistic effects in the atomic and nuclear few-body systems are classified and discussed with the emphasis on electromagnetic transitions. The size of relativistic corrections, calculational techniques and ambiguities, and comparison of theory and experiment are considered.

1. Introduction

The purview of my talk covers the influence of relativity on the atomic and nuclear few-body problems. This presents both an opportunity and a problem. The opportunity is to introduce practitioners in related fields to each other's problems, methods, and solutions, while the problem which immediately arises is the layer of formalism, notation, and folklore which surrounds a field, and which is difficult for a nonexpert to penetrate. The attempt to synthesize the two fields is particularly worthwhile for those of us in nuclear physics, since many of our conceptual problems which have arisen have been dealt with in atomic physics. The elegance and richness of the atomic few-body problem must be seen to be appreciated[1]. In general, the ability to calculate wave functions makes the few-body systems the best studied of the various atoms and nuclei, and the first place to look when examining small corrections.

The approach that I have adopted is to treat nuclei and atoms as weakly bound systems, for which the binding energies are small compared to rest masses, and therefore the velocities, v, of constituent particles, nucleons or electrons, are slow compared to the speed of light, c. This is a reasonable approximation for any case we will deal with, since in hydrogenic (one-electron) atoms v/c is $Z\alpha/n$, where Z is the nuclear charge (in units of e, the fundamental charge), α is the fine structure constant ($\simeq 1/137$), and n is the principal quantum number of the bound state in question. For nuclei, we can write $v/c = p/M$ and use a typical nuclear momentum p~100-200 MeV/c to estimate $v/c \sim .10$-$.20$. This means that $(v/c)^2$, which characterizes the size of relativistic effects, is of the order of a few percent in nuclei. We will demonstrate this later in a numerical example. It is for this reason that the effects of relativity will be treated as corrections in basically nonrelativistic systems, rather than as primary effects, distinguishing our weakly bound systems from elementary particles, for example. This does not imply that in every instance relativistic corrections are small, since cancellations can enhance the relative correction; one hopes to find just such cases. This type of treatment is not necessary in the two-body problem, either atomic and nuclear, where a more fundamental approach to the dynamics is possible by using the Bethe-Salpeter equation[2], for example. Nevertheless, the cost of using such a formalism that incorporates relativity manifestly and exactly is a formidable increase in complexity, which makes interpretation difficult. Although we will comment later on this approach, the interested reader is referred to the impressive calculations of Tjon and his collaborators[3,4]. We will follow a more pedestrian path. Outside the few-body problem these nonperturbative methods are not available. Hopefully, the lessons learned in treating a few particles can be extended to many.

In the course of preparing this talk, I became aware that there were subtleties in defining what a relativistic correction is! The reason is that these corrections to the nonrelativistic limit are corrections of order $(1/c^2)$ relative to 1. Classically, this ends the discussion, but in quantum mechanics factors of $\hbar c$ are ubiquitous, so much so that they are ignored by theorists. It is quite common to incorporate this factor into definitions of coupling constants, such as $\alpha = e^2/\hbar c$. Doing this can be misleading. For example, the nonrelativistic

Coulomb Hamiltonian is $(p^2/2M-Ze^2/r)$, which can be rewritten as $(p^2/2M-Z\alpha\hbar c/r)$, and can be made in this way to involve c! Many years ago Hideki Yukawa showed that the exchange of a meson of mass μ between two nucleons produced a potential V which depended on their separation r and varied as g^2e^{-x}/r, where $x = \mu cr/\hbar$. Thus the Compton wave length of the exchanged meson, $\hbar/\mu c$, which sets the length scale of the potential, involves c, and in this context has nothing to do with relativistic effects in a nucleus. Consequently, our rules for determining the order of a relativistic correction will be to ignore the c's that occur in the dimensionless combination, x, above, and to avoid incorporating c's in defining coupling constants (i.e. treat g^2, not $g^2/\hbar c$, above). Thus, the Yukawa potential defined above is nonrelativistic. It is not clear that this prescription suffices for all problems that arise, but it will be sufficient for our use.

This procedure is still too complicated for easy usage, because it requires paper and pen; since v/c is p/Mc, I therefore count powers of 1/M, where M would be either the nucleon or electron mass, according to the problem. Consequently $(v/c)^2$ will be dimensionally reckoned as $1/M^2$. To make this prescription consistent, in addition to <u>explicit</u> powers of (1/M), it is necessary to use the weak binding assumption and count powers of the potential as powers of (1/M), also. The kinetic energy of a nonrelativistic particle, $p^2/2M$ (and explicitly (1/M)), is nearly equal in magnitude and opposite in sign to the potential energy; both contain no powers of (1/c) and will be treated as (1/M). (Skeptics should take a brief look at the virial theorem and convince themselves this argument isn't totally crazy). Relativity, of course, treats all forms of energy, potential or kinetic, on an equal footing, an idea that has had enormous consequences for our era. We must therefore expect some relativistic corrections to be of the kinetic type and involve momenta and masses, while others will be potential-dependent. Our counting procedure treats them equally. Corrections of the former type are called nonstatic because they vanish as the mass of the constituents becomes very large (static limit). In addition, because of the weak binding assumption, these two types of terms will tend to be of the same magnitude and opposite in sign, if they are the same order in (1/M). As an example, we expand the kinetic energy $T = \sqrt{p^2c^2+M^2c^4} = Mc^2+p^2/2M-p^4/8M^3c^2$. The leading-order relativistic correction is therefore of order $(1/c^2)$ or $(1/M^3)$ and is manifestly a $(1/M^2)$ correction to $p^2/2M$. Note also that it is attractive. If a nonrelativistic reduction is made of the Dirac equation [5,6]) with a Coulomb potential V_c for a charged particle with magnetic moment μ (in magnetons), the spin-orbit and Darwin-Foldy potentials are the two leading-order relativistic corrections to the potential V_c which result:

$$V_{SO} = (2\mu-1)\frac{\hbar\vec{\sigma}\cdot\vec{\nabla}V_c\times\vec{p}}{4M^2c^2} \qquad (1a)$$

$$V_{DF} = (2\mu-1)\frac{\hbar^2\nabla^2V_c}{8M^2c^2} \qquad (1b)$$

Both are manifestly $1/M^2$ and $1/c^2$ corrections to V_c, and together with the correction to the kinetic energy, generate the complete $1/c^2$ correction to the Bohr energy of a hydrogen atom which is contained in the Dirac eigenvalue. The Darwin-Foldy interaction is a purely quantum mechanical effect, while the spin-orbit interaction is the sum of two classical phenomena: the Thomas precession, and the electric dipole interaction of a moving magnetic dipole.

In addition to the Hamiltonian, which generates the eigenvalues and wave functions of bound states, our interest is directed at transitions between two such states mediated by transition operators. The two most prevalent such operators are the charge (ρ) and current density ($\vec{J}$) operators, which can couple to

external electric and magnetic fields, and are responsible for such phenomena as photon decay or absorption by a system and electron scattering from the system, both elastic and inelastic. Just as we made an expansion of the Hamiltonian in powers of 1/M (1/c), we do the same with ρ and $\vec{J}$ as indicated in fig. 1, neglecting henceforth factors of M and c. The nonrelativistic charge operator shown there has the form

$$\rho_0(\vec{x}) = \sum_i e_i \delta^3(\vec{x}-\vec{x}_i) \tag{2a}$$

while the nonrelativistic current is given by

$$\vec{J}_0(\vec{x}) = \sum_i \frac{e_i}{2M_i} \{\vec{p}_i, \delta^3(\vec{x}-\vec{x}_i)\} - \sum_i \frac{\mu_i}{2M_i} \vec{\sigma}(i)\times\vec{\nabla}\delta^3(\vec{x}-\vec{x}_i) \tag{2b}$$

for a composite system whose ith particle has coordinate, momentum, charge, mass, magnetic moment, and (Pauli) spin: $\vec{x}_i$, $\vec{p}_i$, e_i, M_i, μ_i, and $\vec{\sigma}(i)$. Most electromagnetic transitions in both atoms and nuclei are calculated using these operators. Both the convection (first) and magnetization (second) terms in $\vec{J}_0$ are manifestly (1/M), while the charge operator is $(1/M)^0$. One ingredient enters the nonrelativistic current in nuclear physics which is not present in the atomic problem: interaction currents. This is an important distinction and we must momentarily digress.

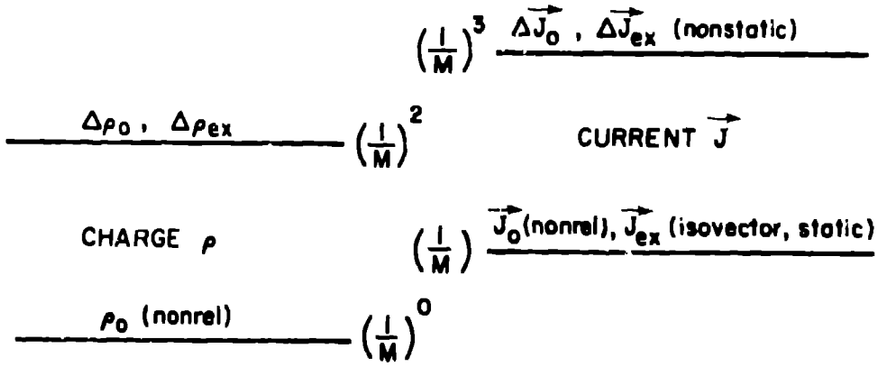

Fig. 1. Categorization of charge and current operators according to relativistic content.

The main qualitative difference of the nuclear and atomic force problems that we will deal with is the fact that the nuclear force is mediated by the exchange of mesons which may be charged, as opposed to the neutral (and massless) photon. This is illustrated in fig. 2, where the upper row of processes depicts electrons (solid line) interacting via photon exchange (wiggly line), possibly in the presence of an external interaction (cross plus wiggly line). The lower row depicts nuclear processes mediated by charged pion exchange, which converts a proton (neutron) into a neutron (proton). The change of nucleon charge (isospin) states is reflected in an isospin-dependent nuclear force in (d). Clearly the flow of charge associated with the meson constitutes a current, and therefore the current operator must have an interaction or exchange component. In fact, without it, the current is not conserved. That is, the current continu-

ity equation

$$\vec{\nabla}\cdot\vec{J}(\vec{x}) = -\,i[H,\rho(\vec{x})] \qquad (3)$$

is not satisfied because the potential in H = T + V fails to commute with ρ and thereby necessitates an additional (potential-dependent) component. The nonrelativistic exchange current $\vec{J}_{ex}$ is isovector (that is, it vanishes for neutral meson exchange) and static (no explicit powers of 1/M). Nevertheless, because it is potential dependent, we call it of order (1/M), just as $\vec{J}_0$ is. Although we will not discuss it in any detail, $\vec{J}_{ex}$ generates 10-15 percent contributions compared to $\vec{J}_0$. This sounds anomalously small in view of our argument that the kinetic and interaction parts should be roughly comparable, but it does illustrate an important point: dimensionless factors can be large and change a 50 percent effect into a 10 percent effect or vice versa. The typical process that we will deal with is an isovector M1, or magnetic dipole, transition. The spin magnetization current dominates this type of transition because it is proportional to $(\mu_p-\mu_n)$, the isovector nucleon magnetic moment (in nuclear magneton units) which is numerically large: 4.7 n.m. Thus we are comparing 10 percent to the 4.7; were $\mu_p-\mu_n$ of "normal" size (1.0), meson exchange effects in these M1 processes would be typically 30-50 percent.

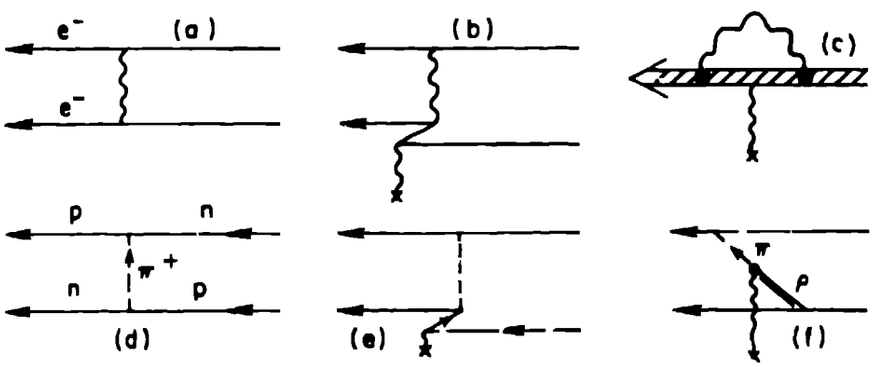

Fig. 2. Graphs of physical processes which contribute to binding and transitions in atoms and nuclei.

In addition to the nonrelativistic components, both ρ and $\vec{J}$ have relativistic correction pieces of kinetic type ($\Delta\rho_0$ and $\Delta\vec{J}_0$) and potential-dependent type ($\Delta\rho_{ex}$ and $\Delta\vec{J}_{ex}$). The former are nonstatic and contain explicit powers of (1/M), while the latter type may or may not be static. An example of a static exchange contribution to $\Delta\rho_{ex}$ is the two-boson-exchange charge density of Ilyuga and Ohtsubo[7], which can be shown to be of order $1/c^2$, according to our rules, and $1/M^2$ by inspection, since it has the schematic form V^2 and we treat V~1/M. This type of term is isovector and vanishes for neutral meson exchanges[8].

The most important contributions to $\Delta\rho_0$ are the same ones we saw earlier in the potentials in eq. (1): the spin-orbit and Darwin-Foldy interactions. In the present context, one has to replace V_c in eq. (1) by the external electromagnetic potential which generates the electric field, but the physics is the same. If one probes the density of charge in an atom or nucleus by placing it in an electric field, the fact that the constituents are moving internally generates a contribution to ρ(x) via ρ_{SO}, while ρ_{DF} results because of zitterbewegung. In nuclei, the large neutron magnetic moment, -1.91 n.m., generates a small but non-negligible component of ρ from neutrons which leads to a difference between the

proton density and the charge density, and historically led to a clarification of the anomalous charge radius differences of the ^{3}Ca isotopes[9]).

A wide spectrum of contributions of order $1/M^3$ to ΔJ_0 can occur, and we mention only one: the Lorentz contraction of the magnetic moment of a moving magnetic dipole[10]) by a factor $\sqrt{1-(v/c)^2} \sim 1-p^2/2M^2$. This nonstatic term has a classical origin and will play an important role later, when we discuss M1 transitions in few-electron atoms.

A variety of mechanisms lead to the transition operators we have discussed. The most important for the interaction currents are the processes (b) and (e) in Figure 2, which illustrate virtual electron-positron pairs, or nucleon-antinucleon pairs, which are created and destroyed by the potential and the external interaction. If the formalism one uses does not explicitly include negative energy (Dirac) components, contributions of this type to operators will occur. This is the mechanism by which the $\vec{A}^2/2M$ term in the Schroedinger equation is generated from the Dirac equation, which is linear in the electromagnetic vector potential, $\vec{A}$. More exotic interaction currents are generated in (c) and (f). In (c) an electron in the Coulomb field of a nucleus (shaded line) can interact with its own electromagnetic field (Lamb shift) while emitting or absorbing a photon[11]). In (f) the external field can interact with mesons being transmitted between nucleons; even the nature of the meson can be changed in mid air[12])! Many other mechanisms are also possible.

Our discussion above has centered on a few phenomena of relativistic origin which generate transition operators of (relative) relativistic order $(v/c)^2$. Earlier we discussed contributions to the Hamiltonian of order $(1/M^2)$. Clearly these terms generate wave function components of order $(v/c)^2$, and they will contribute to transition matrix elements in the same order. Two types of terms are generated in the wave function: corrections to the center-of-mass or internal wave function, and motional corrections caused by the overall motion of the system as a whole. Thus we find 3 sources of $(v/c)^2$ terms in matrix elements: (1) the transition operators; (2) the internal wave function; (3) the motional corrections. The latter correction is relatively unimportant in atoms, because of the large nucleus mass.

The third category has received a lot of attention[13,14]), however, in nuclear physics, because this is the mechanism by which such phenomena as the Lorentz contraction, Thomas precession, etc., for the entire composite system are manifested. An impressive series of papers by Foldy[15]) and his collaborator Krajcik[16]) has detailed the constraints that the wave function of a many-body system must satisfy in order to satisfy the constraints of special relativity. These constraints must be so formulated that a system of mass M in the system's rest frame generates an energy $\sqrt{\vec{P}^2+M^2}$ when the system is given a momentum $\vec{P}$. This is more than a nicety for practical calculations, since almost every external interaction changes a system's momentum. To order $(v/c)^2$ Foldy showed that the wave function in a general frame had to have the form

$$\psi_{\vec{P}} \sim (1 - i\chi(\vec{P}))\psi_0 e^{i\vec{P}\cdot\vec{R}} , \qquad (4)$$

where ψ_0 is the rest frame wave function and $\chi(\vec{P})$ accounts for all the motional effects. The operator χ can be divided into kinetic (χ_0) and potential-dependent terms (χ_V). The former accounts for Lorentz contraction[17]), etc., while the latter can be different in every formalism[18]); we will deal with this problem later. Foldy also showed that the potential energy of a system of particles changes when the system begins to move with momentum $\vec{P}$:

$$V(\vec{P}) = V(0) - \vec{P}^2 V_0/2M^2 - i[\chi_0, V_0] - i[\chi_V, H_0] , \qquad (5)$$

where V_0 and H_0 are the nonrelativistic potential and Hamiltonian. This relation is important, because it demonstrates how the potential between 2 particles,

for example, is modified by relativity when both particles are moving together relative to the center-of-mass of a many-body system. It does not deal with the corrections in $V(Q)$, however. This relation, or formalisms equivalent to it, has been much used[19,20]).

The theoretical mechanism which takes a system at rest and makes it move is the "boost" operator, a fundamental operator in any theory, together with the Hamiltonian, momentum and angular momentum operators. The boost is intimately related to χ and knowing one you essentially know the other. Having calculated ψ_0 by some means, knowing the boost allows you to deduce ψ in a general frame. This is illustrated in fig. 3, which depicts a spherical system at rest with a mass M and a radial density $\rho(r)$. When the "boost" causes the object to move relative to a fixed frame, the energy becomes $\sqrt{\vec{P}^2+M^2}$ and Lorentz contraction occurs. This is also one of the mechanisms which contributes to Foldy's relation (5). Another manifestation is the fact that the form factor F, the Fourier transform of $\rho(r)$, develops an argument $q^2 = \vec{q}^2 - q_0^2$, which is a Lorentz invariant[17]); nonrelativistically we would have only $\vec{q}^2$.

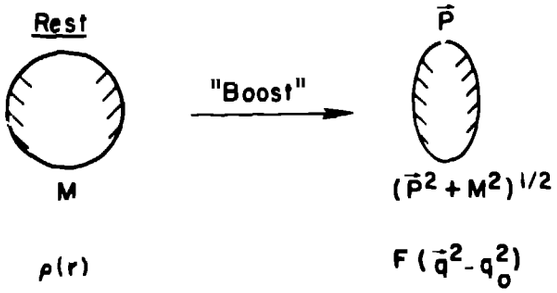

Fig. 3. Effects of the "boost" operator on a composite system.

2. Calculational methods

The methods used to calculate wave functions, transitions rates and eigenvalues are as varied and numerous as the people who use them. I like to divide them into two basic categories. One category subsumes atomic physics calculations and ab initio nuclear calculations and uses a model for exchanges of quanta which produce binding. The other type uses Foldy's relation (5), or its equivalent to calculate the corrections which arise for a fixed two-body potential in the two-body center-of-mass. The latter type therefore cannot calculate all relativistic effects, particularly electromagnetic ones[16]) and those arising from three-body forces. We will concentrate on the former type.

Most begin in principle with the Bethe-Salpeter equation, its Feynman graphical equivalent, or the underlying field theory. Although the former approach can stand by itself, most efforts are devoted to mapping this four-dimensional equation into an equivalent three-dimensional equation, called a quasi-potential equation, or a Blankenbecler-Sugar reduction[21]) in some cases. This is not a unique procedure. It is not even clear that any one method is better than all the others. The problem is easily illustrated by examining the form of the exchange of a single meson of mass μ and four momentum $q = (q_0,\vec{q})$ between two nucleons in any Feynman diagram: $V/(\vec{q}^2 - q_0^2 + \mu^2)$. The vertex factor in the numerator is unimportant, and ignoring the q_0-term leads directly to the Yukawa potential we saw earlier. The relative energy variable q_0 has no non-relativistic analogue[22]), and is required to accomodate the retardation of the nuclear force due to the finite propagation speed of signals. It cannot be ignored in many cases since it leads directly to relativistic corrections, but there is no unique method for eliminating q_0 in favor of other quantities. A

similar situation exists in the Coulomb problem if we set μ=0. There the ambiguities are simply the choices of gauge used to describe the electromagnetic field of the exchanged photon. Fifty years ago this ambiguity first showed up in the atomic potentials of Breit[23]) and Gaunt[24]). For most applications Breit's form is preferred[25]).

If there are ambiguities in potentials, is there an ambiguity in the physics? The answer is no, if calculations are performed consistently, but interpretation may be ambiguous. To understand this we have to understand the sources of the problem. In mapping the four-dimensional formalism into a three-dimensional one, many methods generate an effective potential which depends on the energy (eigenvalue of the system). This energy dependence has so many unpleasant consequences when it comes to calculating transition matrix elements, that I (and many others) regard it as a serious technical defect. The reason is that the energy is actually a time derivative in the Schroedinger equation, and it is this derivative term that leads to the definition of the probability density. Adding potential-dependent terms of this type modifies the very nature of the probability density in an unpleasant way. It also modifies the wave function orthonormality condition[26]). Consequently, considerable effort has been devoted to formalisms that eliminate this problem. Among these techniques are: (1) the renormalization method, similar to the renormalization methods of field theory[5]) and used by the present author in the nuclear problem[17,27]) and Drake[28]) in the atomic problem; (2) the FST method[29]), which uses projection operators; (3) the folded diagram method popularized by Mikkel Johnson[30]); (4) certain of the quasipotential methods, such as Franz Gross's[31]) in the one-boson-exchange approximation.

The second source of ambiguity is more subtle, and certainly more confusing to the uninitiated. In the first complete treatment of the $(v/c)^2$ contributions[17,27]) to $\Delta\rho_{ex}$ from one-pion-exchange, it was pointed out that different approaches produced charge, current, and Hamiltonian operators (and wave functions) which were different, but unitarily equivalent, ensuring that the difference in the form of operators does not affect observables such as energies, magnetic moments, form factors, etc. Recently it was shown that the four different techniques[15]) listed above lead to (in general) different looking, but unitarily equivalent, treatments of the deuteron charge form factor. Several other observations follow from the above discussion: (1) the many physical phenomena that we discussed in the introduction are spread throughout various Feynman diagrams differently in different representations; (2) the amount of meson-exchange contribution of order $1/c^2$ to a given process is not an observable and can be varied by changing representation; (3) the percentage of D-state in the deuteron is not an observable, (i.e. unmeasurable[32])) and varies from representation to representation; (4) consistency demands that both wave functions and operators be calculated to the same order[33]). All of these problems are basically of order $(1/c^2)$ and don't affect the nonrelativistic limits. Almost every nuclear calculation of $\Delta\rho_{ex}$ or $\Delta\vec{J}_{ex}$ has violated (4), and consequently is useless. For the atomic problem, the usual choice of Coulomb gauge to perform calculations fixes the representation to order $(1/c^2)$; for one-photon-exchange $\Delta\rho_{ex}$ is zero[34]) (although $\Delta\vec{J}_{ex}$ is not), and this simplifies many calculations. There is in fact a close connection between the choice of gauge in the atomic problem and the choice of representation.

We summarize this section by stating that there is no shortage of methods (each with its own champion), but this abundance has not always led to an increase in insight.

3. Experiment

Every physics student is exposed to the hydrogen atom problem repeatedly and in detail. Nevertheless, this most studied of the simple quantum systems continues to be a research topic, because of the rich variety of states and the transitions between them, as well as the great experimental precision of many of the measurements. It is reasonable to look to this system for examples of relativis-

tic effects in transitions. The fine structure splittings are examples of relativistic corrections to eigenvalues, as we have discussed. Figure 4 shows the four lowest-lying states of a hydrogenic ion (1 electron, arbitrary Z), including the 2p3/2-2p1/2 fine structure and the 2s1/2-2p1/2 Lamb shift. Electric dipole transitions dominate the s-p transitions, including the exotic 1000 MHz (in H) one(see Fig. 2c). More interesting for our purposes is the 2s1/2 → 1s1/2 transition, which prefers two-photon (2E1) decay, but can also proceed via the retarded M1 decay by a single photon, although this has not been observed in light elements. The leading-order (nonrelativistic) M1 decay amplitude vanishes

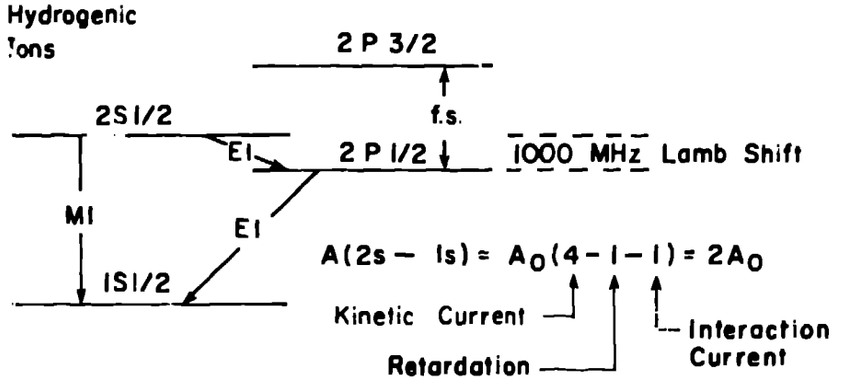

Fig. 4. Low-lying states and transitions in hydrogenic ions. The $2s_{\frac{1}{2}}$-$1s_{\frac{1}{2}}$ M1 amplitude is decomposed into components.

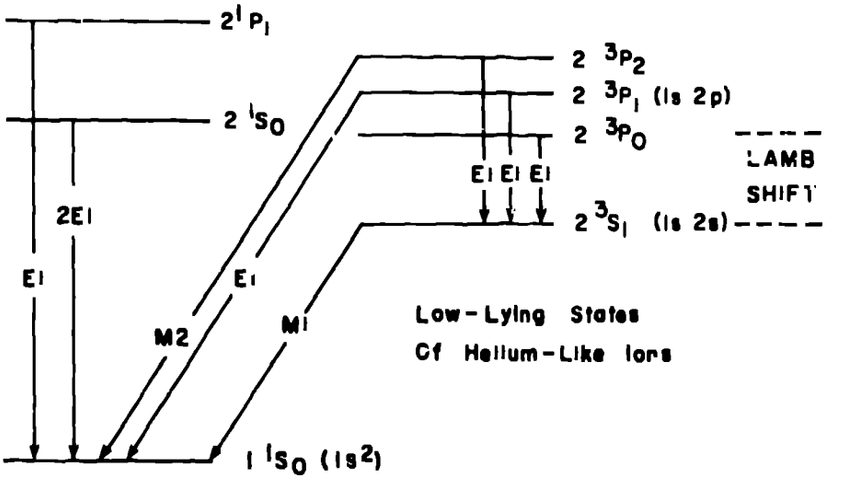

Fig. 5. Low-lying states and transitions in helium-like ions.

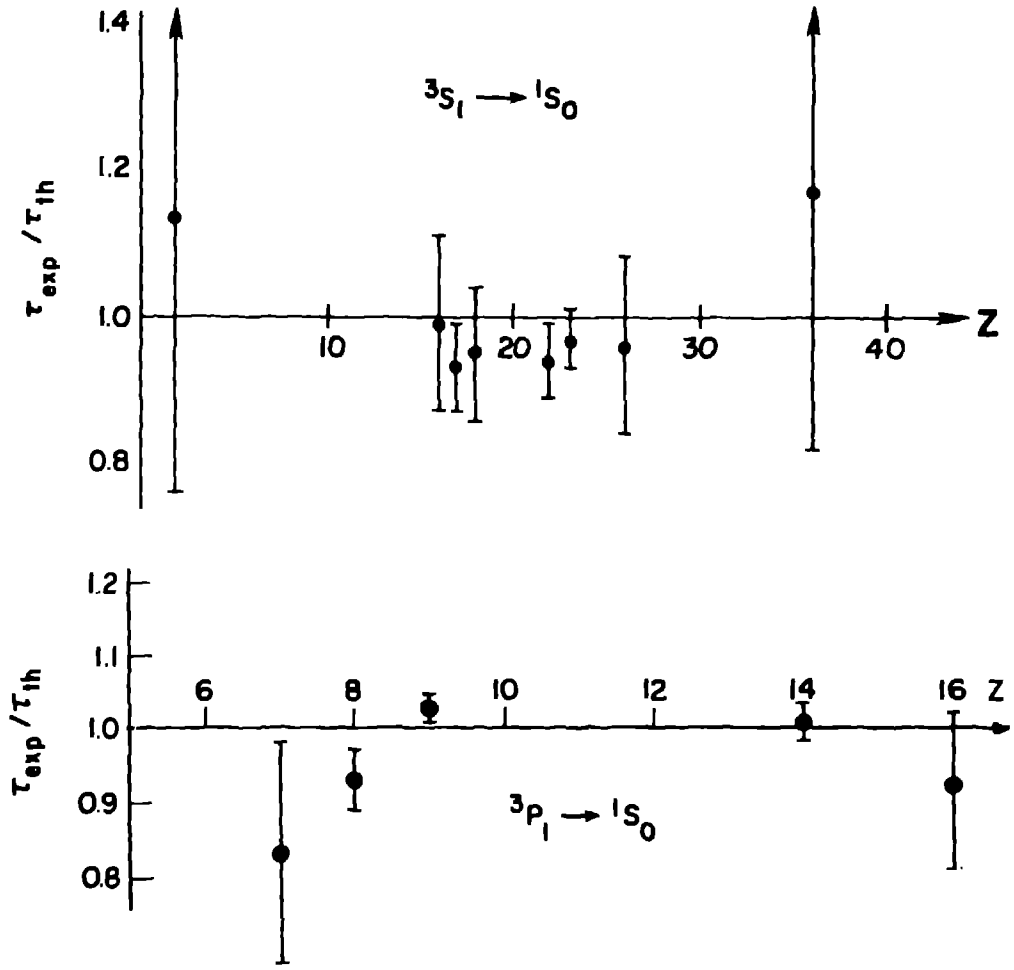

Fig. 6. Ratio of experiment to theory for the $^3S_1 \rightarrow {}^1S_0$ and $^3P_1 \rightarrow {}^1S_0$ transitions in helium-like ions.

because we are dealing with orthogonal s-states. The amplitude A for this decay to order $(1/c^2)$ involves three contributions[35]: (a) the retardation correction to the nonrelativistic matrix element; (b) the Lorentz contraction correction to the magnetic moment operator; (c) a number of contributions from the spin-orbit interaction, which includes an interaction current component as well as a kinetic part (the sum of which vanishes). These have been arranged as illustrated in fig. 4; the factor A_0 contains all the parameters involved. There is actually an ambiguity of the type we discussed earlier. It is possible to use the Schroedinger equation to eliminate half the interaction current component in terms of a kinetic contribution. The simple structure of one-electron atoms allows us to calculate all the matrix elements analytically for this problem.

Much more interesting are the helium-like (2 electrons, arbitrary Z) atoms illustrated in fig. 5, showing the types of transitions. Particularly relevant

is the $^3S_1 \rightarrow {}^1S_0$ transition which has virtually the same structure as the 2s1/2-1s1/2 transition we analyzed earlier. The ratio of experiment to theory is shown above and is in excellent agreement[36,37]) with 1. The other interesting transition is the $^3P_1 \rightarrow {}^1S_0$ transition, which is E1 in nature. However, because the nonrelativistic electric dipole operator has no spin flip component, the transition induced by this operator is greatly inhibited, proceeding only through relativistic components of the wave function ($^3P_1$-$^1P_1$ mixing) induced by the spin-orbit potential. In addition, retardation and relativistic corrections to the dipole operator play a comparable role; the latter two contributions cancel[34]). The agreement of theory and experiment is again excellent[36]). This transition offers excellent lessons for nuclear physics because the relativistic corrections to the wave function cannot be neglected (as they usually are in our problems) and because the relativistic corrections to the dipole operator violate Siegert's theorem (actually an approximation), which has played such an important role in photonuclear physics.

After listing the successes of atomic physics in the relativistic regime, it is with some trepidation that I discuss the nuclear problem. In the past, two approaches have been used when discussing relativistic corrections. Strictly speaking one should compare the results of a complete calculation with a purely nonrelativistic one, as is done in atomic physics. Because the strong interaction dynamics are poorly understood, every ab initio calculation contains at least a few adjustable parameters. It is not clear whether some of these should be readjusted before making comparisons. Thus various calculational approaches could produce different answers to the same question: how large are relativistic corrections? Consequently, meaningful experimental comparisons with theory are difficult to formulate. In most cases, three-body calculations of relativistic effects adjust the two-body Hamiltonian to reproduce the known two-body data in the two-nucleon center-of-mass. Some don't do this, however.

One problem in nuclear physics is the lack of clean, "forbidden" transitions. The $^1S_0 \rightarrow {}^3S_1$ radiative transition rate involving a proton and thermal neutron is not small as it is in atoms, because the neutron and proton magnetic moments are different. The thermal n+d $\rightarrow {}^3$H+γ transition, however, is very small for essentially the same reason that the atomic M1 transitions that we discussed are small[38]). This transition proceeds through wave function components that are induced by the spin-dependence of the nucleon-nucleon force (which can be non-relativistic in origin), and through (nonrelativistic) meson-exchange currents[39]). Although this is an extremely interesting process, there is no direct evidence of relativistic effects.

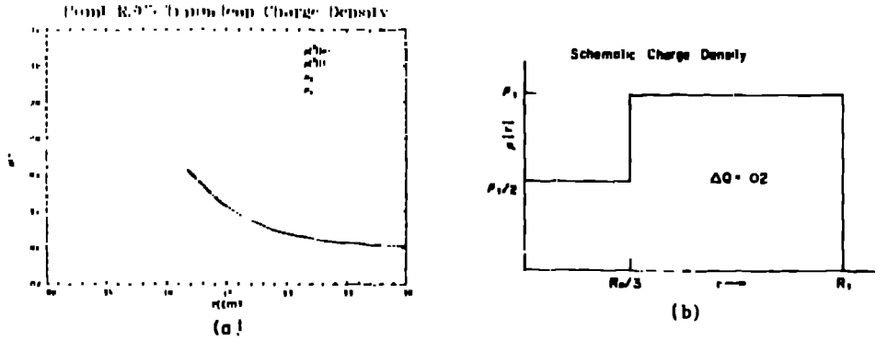

Fig. 7. Theoretical (a) and schematic (b) densities for the trinucleon system.

Much attention has been directed recently at the experimental ^{3}He charge density, which is reputed to have a small "hole" in the middle, if the effect of the

nucleon finite size is removed from the data[40]). This hole is intimately related to the behavior of the form factor F at large momentum transfers as the following sum rule[41]) shows:

$$\rho(0) = \frac{1}{2\pi^2} \int_0^\infty dq q^2 F(q^2) \qquad (6)$$

A large negative contribution to q^2F suppresses $\rho(0)$. Since the behavior of F at large q is a reasonable place to look for relativistic phenomena, have we seen the evidence we are looking for? The answer to this is not known. One thing is clear, however; standard potential models do not produce a hole, as indicated by fig. 7, which shows the results of a Faddeev calculation[42]) for the Reid Soft Core potential model for the scalar, vector, ^{3}He, and ^{3}H densities. Unfortunately little is known experimentally about the ^{3}H density, except the radius. Next to it we have illustrated schematically what a density with a strong central depression might look like. The first observation is that the amount of charge involved is small (1/54), because the volume element d^3r suppresses small-r contributions; similarly, the fractional change of the rms radius is only .008. Although pictorially the depression is a massive change, only a small change is made in global properties such as the radius.

In order to further explore this question I have cooked up a model of two identical nucleons in their center-of-mass frame which is totally unrealistic and unphysical, but has certain features that are illustrative of relativistic effects in general, and that will provide us with a numerical example and possibly some insight. The model Hamiltonian is

$$H = 2(M^2 + H_0 M)^{1/2} \cong 2M + H_0 - \frac{H_0^2}{4M} + \ldots \qquad (7a)$$

$$= 2M + p^2/M + V - p^4/4M - [p^2,V]/4M - V^2/4M \qquad (7b)$$

where $H_0 = p^2/M + V$ is the nonrelativistic Hamiltonian. This model has been constructed so that the exact and the nonrelativistic solutions are identical; moreover the eigenvalues differ by a trivial amount (1 KeV) if H_0 yields the deuteron binding energy E_D (we have used the Malfliet-Tjon 1V potential[43]), for which E_D = 2.24 MeV). In addition the form has been chosen so that if V = 0, one is left with the correct kinetic energy. Terms of the form $[p^2,V]/M$ are extremely common relativistic corrections, as V^2/M-terms are, although signs and factors of two may be different[20,31]). In our model the first potential correction is repulsive, while the second is attractive. Cancellations of the type we have forced in this model may be possible between relativistic corrections of the kinetic and potential types, as noted by Coester[14]) and others[19,20]). Many people neglect terms of the form V^2, preferring to use only one-boson-exchange potentials. We can investigate the effect of such an approximation in our crude model by adding $V^2/4M$ to V and solving H_0 for ψ. This neglects a number of higher order (in 1/M) terms but should give roughly the correct wave function for the relativistic model without the V^2-term. The difference in ψ for the two cases is shown in the next figure. Although $V^2/4M$ is numerically small for large separations, it is very repulsive for small r. Thus, adding or subtracting small relativistic terms may have nonnegligible effects on small parts of the wave function[4]), even producing nodes from nonlocalities in the potential[45]). Leaving out $V^2/4M$ in our model could be compensated by adjusting the parameters in V to reproduce E_D; this would alter $\psi(r)$ in yet a different way. In the three-body problem, one must also worry about three-body forces[46]), which have the schematic form V^2 of our model and can have a nonrelativistic origin. For all these reasons the ^{3}He density is inconclusive evidence of what we seek.

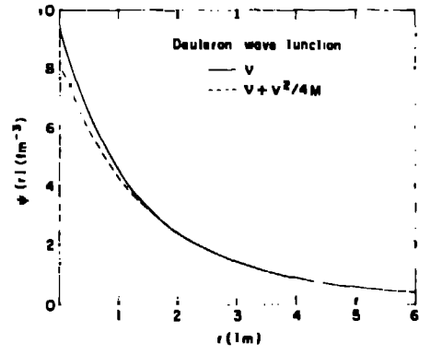

Fig. 8. Deuteron wave function with (solid) and without (dashed) the $V^2/4M$ relativistic correction.

If there is scant experimental evidence of relativistic effects in light nuclei, we will have to resort to theoretical studies. Of all the numerical calculations to date of relativistic corrections in few-nucleon systems, in my opinion only those of Tjon and his collaborators are sufficiently comprehensive to be considered "realistic". They include spin, for example, which is nontrivial and probably necessary, and go beyond the ladder approximation in the Bethe-Salpeter treatments. Although they do not make a pure relativistic-nonrelativistic comparison, they find small relativistic corrections for the two-[3,4]) and three-[47,48]) body problems. In the former case, the consensus[45,49]) is that the effects are repulsive. The calculations by Gross and his collaborators of the deuteron form factor show relativistic corrections which are very similar to those of Tjon. The various calculations for the three-body system[19,20,47,48,50] indicate a small residual attraction if the two-body Hamiltonian is fixed to conform to experimental two-body properties. Unfortunately, the various calculations all compare their results to different limits. Because of cancellations which occur, it is not clear that a small residual repulsion is ruled out[2].

For pedagogical purposes we note that $\langle -p^2/4M^2 \rangle = -.35$ MeV while $\langle p^2/m \rangle = 11.9$ MeV for our potential model, yielding a 3% relativistic correction to the kinetic energy. The two potential terms of order $(1/M^2)$ give +.35 MeV. These numbers confirm our earlier estimates. Clearly much more work needs to be done in this field.

This work was performed under the auspices of the U. S. Dept. of Energy.

References

1) R. Marrus and P. J. Mohr, Adv. in Atom. Mol. Phys. 14 (1978) 181 is a very good review of this problem.
2) N. Nakanishi, Prog. Theor. Phys. (Suppl.) 43 (1969) 1 reviews this subject.
3) J. Fleischer and J. A. Tjon, Phys. Rev. B21 (1980) 84; ibid. B15 (1977) 2537; Nucl. Phys. B84 (1975) 375.
4) J. A. Tjon and M. J. Zuilhof, Phys. Lett. 84B (1979) 31; preprint (4/80).
5) J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics, (McGraw-Hill, N.Y., 1964).
6) W. A. Barker and F. N. Glover, Phys. Rev. 99 (1955) 317.
7) H. Hyuga and H. Ohtsubo, Nucl. Phys. A294 (1978) 348.
8) T. Sato, H. Hyuga, and H. Ohtsubo, Prog. Theor. Phys. 62 (1980) 516; H. Hyuga, H. Ohtsubo, and T. Sato, Nucl. Phys. A300 (1978) 242.
9) W. Bertozzi, J. Friar, J. Heisenberg, and J. W. Negele, Phys. Lett. 41B (1972) 408.

10) J. L. Friar, in Mesons in Nuclei, eds. M. Rho and D. H. Wilkinson (North-Holland, Amsterdam, 1979) 597.
11) E. J. Kelsey, Phys. Rev. A15 (1977) 647.
12) R. J. Adler and S. D. Drell, Phys. Rev. Lett. 13 (1964) 349.
13) J. L. Friar, Phys. Rev. C12 (1975) 695.
14) W. Glöckle and L. Müller, Bochum preprint (1980) is an recent example.
15) L. L. Foldy, Phys. Rev. 122 (1961) 275.
16) R. A. Krajcik and L. L. Foldy, Phys. Rev., D10 (1974) 1777.
17) J. L. Friar, Ann. Phys 81 (1973) 332; ibid. 96 (1976) 158.
18) J. L. Friar, Phys. Rev. C22 (1980) 796.
19) E. Jacob, V. S. Bhasin, and A. N. Mitra, Phys. Rev. D1 (1970) 3496.
20) L. A. Kondratyuk, J. Vogelzang, and M. S. Panchenko, preprint (1980); B. L. G. Bakker, L. A. Kondratyuk, and M. V. Terent'ev, Nucl. Phys. B158 (1979) 497.
21) R. M. Woloshyn and A. D. Jackson, Nucl. Phys. B64 (1973) 269 reviews this topic.
22) I. T. Todorov, in Lectures in Theoretical Physics, eds. W. Brittin and K. Mahanthappa (Gordon and Breach, N.Y., 1971) p. 183.
23) G. Breit, Phys. Rev. 34 (1929) 553.
24) J. A. Gaunt, Proc. Roy. Soc. (London) A122 (1929) 513.
25) H. Grotch and D. R. Yennie, Rev. Mod. Phys. 41 (1969) 350.
26) This is a frequent occurance in Sturm-Liouville equations, which corresponds to a "potential" linear in the energy. See, for example, F. B. Hildebrand, Methods of Applied Mathematics, (Prentice-Hall, Englewood Cliffs, N.J., 1952) p. 96.
27) J. L. Friar, Phys. Lett. 59B (1975) 145; ibid. 69b (1977) 51.
28) G. W. F. Drake, Phys. Rev. A5 (1972) 1979.
29) N. Fukuda, K. Sawada, and N. Taketani, Prog. Theor. Phys. 12 (1954) 156.
30) M. Johnson, Ann. Phys. 97 (1976) 400.
31) F. Gross, Phys. Rev. D10 (1974) 223.
32) J. L. Friar, Phys. Rev. C20 (1979) 325.
33) R. M. Woloshyn, Phys. Rev. C12 (1975) 901.
34) J. L. Friar, Z. Phys. A291 (1979) 1.
35) G. W. F. Drake, Phys. Rev. A3 (1971) 908.
36) W. R. Johnson and C. D. Lin, Phys. Rev. A14 (1976) 565.
37) D. L. Lin, Phys. Rev. A15 (1977) 2324 is very similar in approach to the nuclear calculations of Ref. 17.
38) L. I. Schiff, Phys. Rev. 52 (1937) 242.
39) E. Hadjimichael, Phys. Rev. Lett. 31 (1973) 183.
40) I. Sick, Lecture Notes in Physics 86 (1978) 300.
41) R. E. Krebs and J. J. de Swart, Phys. Rev. 162 (1967) 1729. The sum rule is older than this reference.
42) G. Payne, B. F. Gibson, J. L. Friar and E. L. Tomusiak (to be published).
43) R. A. Malfliet and J. A. Tjon, Nucl. Phys. A127 (1969) 161.
44) F. Coester, S. C. Pieper and F. J. D. Serduke, Phys. Rev. C11 (1975) 1.
45) R. G. Arnold, C. E. Carlson, and F. Gross, Phys. Rev. C21 (1980) 1426.
46) S. A. Coon, J. G. Zabolitzky, and H. W. K. Blatt, Z. Physik A281 (1977) 117.
47) A. D. Jackson and J. A. Tjon, Phys. Lett. 32B (1970) 9.
48) J. A. Tjon, invited talk at this conference, p. 1.
49) P. Danielewicz and J. M. Namyslowski, Phys. Lett. 81B (1979) 110.
50) E. Hammel, H. Haier, and A. S. Rinat, Phys. Lett. 85B (1979) 191.