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AUTHOR(S): John K. Dienes

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LOS ALAMOS SCIENTIFIC LABORATORY
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IN EXPLOSIVE FRAGMENTATION

John K. Dienes
Theoretical Division, Group T-3
University of California
Los Alamos National Laboratory
Los Alamos, NM 87545

ABSTRACT

There appears to be a growing interest in the use of statistical theories to characterize the behavior of rock. In many cases such theories can be used to represent the reduction in elastic modulus and the decrease in strength that result from the presence of cracks. In the current approach we are attempting to characterize the behavior of rocks at large deformations, including the effects of crack growth when unstable, the effects of anisotropy, the distinction between open and closed cracks, the influence of crack intersections, the role of pore pressure, and a calculation of permeability. The theory is quite general, and is intended for use in a computer program rather than as a vehicle for obtaining analytic results, though some such results have been reported in the previous symposium.

When a spherical explosive charge is embedded in oil shale it produces an aspirin-shaped cavity at late times as a result of the bedded structure of the rock. In this paper a calculation of the cavity produced by a spherical explosive is compared with a radiograph, showing remarkable agreement between the two. The shape of the cavity is explained by the behavior of cracks lying in the bedding planes.

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John K. Dienes
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INTRODUCTION AND SUMMARY

An alternative to the use of the plasticity theories to characterize the dynamic behavior of rock is to represent the effect of flaws by statistical methods. We have taken such an approach to study the fragmentation of oil shale because it appears to have a large number of advantages. Foremost among these is that by considering the effect of cracks on rock behavior it becomes possible to address the underlying physics directly and to understand the phenomena that occur during fragmentation. In addition, in such an approach the parameters that characterize material properties have a straightforward physical interpretation and can often be determined by direct measurements. Mean crack size and the number density of cracks are important examples. In a more physical approach it should also be possible to encompass a wider range of scales with a few parameters than in a phenomenological theory. For instance, we note that plasticity theory does not naturally account for rate effects, though they can be artificially introduced through additional functions, which are determined empirically. In our SCM (Statistical Crack Mechanics) theory, however, rate effects are naturally accounted for by introducing the speed of crack growth to characterize the behavior of unstable cracks. It was shown by Dienes and Margolin (1980) that this is sufficient to represent the observed rate effects in oil shale, which are very large. Similarly, size effects are known to be important in rock mechanics, with small samples showing much higher strength than large ones, and such effects are accounted by statistical methods without introducing any new physics. Another serious concern is that the dilatancy observed when rocks are sheared is not modelled in a natural way by plasticity theories. In a micromechanical approach, however, cracks open during loading and may remain open on unloading, and this appears to be the essence of dilatancy. An additional advantage is that it becomes possible to compute permeability from an analysis of intersecting cracks with such a theory.

An attempt to formulate an isotropic statistical theory by Dienes (1978a) was abandoned because it could not incorporate the effects of shear cracks, which we believe are important under compressive loading, and because it appears to be important to permit cracks with certain orientations to grow while others remain fixed in size. This is particularly the case in oil shale, in which bedding cracks play an important role. The current theory, which allows for anisotropic crack distributions, has now been coupled to the SALE hydrodynamic code, making it possible to compute explosion, impact or other dynamic processes. In addition to summarizing the theory, the object of this paper is to show that a spherical explosive calculation with SCRAM using published mechanical properties of oil shale agrees well with experiment.

SCRAM

In the preceeding symposium a theoretical approach to Statistical Crack Mechanics war described and some results based on the SCM computer program were presented. SCM returns the stress as a function of time when supplied with a tensor strain rate history, accounting for growth and coalescence of cracks. In SCRAM the SCM subroutine is coupled to a general purpose code, SALE, written by Amsden, Ruppel and Hirt (1980) which integrates the equations of continuum motion. Although in the current work SALE is used as a Lagrangian code, it has the capability to calculate deformation using an Eulerian mesh, or with a mesh which passes through the continuum in an arbitrary manner. Hence, SALE is an acronym for Simplified Arbitrary Lagrangian Eulerian.

A central idea of SCM is to represent the strain rate as the sum of several parts. The first is a strain rate due to distortion of the matrix material, which is characterized as a Maxwell solid. The second is a strain rate due to distortion of an ensemble of open microcracks. The third results from interfacial sliding of shear (closed) cracks. The fourth is a strain rate due to unstable crack extension, and the fifth is the result of material rotation. Expressions for these quantities have been derived in Los Alamos oil shale quarterly reports, and here we shall only summarize the main results. For the matrix strain rate we put

$$d^{*}_{ij} = C_{ijk\ell}\dot{\sigma}_{k\ell} \qquad (1)$$

where the dot is used to denote the rate of change of stress. It is not the Zaremba-Jaumann-Noll stress rate, nor its generalization described by Dienes (1979b), for the rotation terms are accounted for separately below.

In addition to the matrix deformation we account for the influence of microcracks, which are considered as an ensemble of flat circular cracks. The theory is exact in the sense that the opening of an isolated circular (penny-shaped) crack under an arbitrary (static) state of stress is known. Though one might consider more general cracks, the influence of shape appears to be quite small. We consider the crack statistics to be defined by a distribution function $N(c,\Omega,t)$ in which Ω designates, symbolically, crack orientation. Then $N(c,\Omega,t)\Delta\Omega$ represents the number of cracks with orientation near Ω whose radii exceed c with normals in the range of solid angle represented by $\Delta\Omega$. It is shown by Dienes and Margolin (1980) that the strain rate due to opening of these cracks is given by

$$d^{o}_{ij} = \beta^{o}\,\dot{\sigma}_{ij}\int_{\Omega} d\Omega\, n_i n_j n_k n_\ell \int_{0}^{\infty} dc\, \frac{\partial N^{o}}{\partial c}\, c^{3} \qquad (2)$$

where the n_i are the components of the unit crack normal, N^o denotes the distribution of open cracks, and

$$\beta^o = 8(1-\nu)/3\mu \quad , \tag{3}$$

with ν and μ the Poisson ratio and shear modulus. Since the general solution for a crack in an anisotropic material is not known, we have to use average values for μ and ν if the matrix is considered anisotropic. In the current calculation, however, we will be considering the material to be isotropic, and will assume that the observed anisotropy arises from cracks in the bedding planes, so that the theory is self-consistent in this example. The superscript in N^o is used to denote the distribution of open cracks. The criterion for cracks to be open is that the normal component of traction $\sigma_{ij}n_in_j$, be positive (tensile).

In addition to the strain resulting from crack opening there may be a significant contribution from interfacial sliding of closed cracks. By arguments a little more complicated than for open cracks, we have obtained the expression

$$d^S_{ij} = \beta^S \dot{\sigma}_{k\ell} \int_\Omega d\Omega \, b_{ijk\ell} \int_0^\infty dc \, \frac{\partial N^S}{\partial c} c^3 \tag{4}$$

for the rate of strain due to shear cracks in which

$$\beta^S = 8(1-\nu)/3\mu(2-\nu) \tag{5}$$

and

$$b_{ijk\ell} = n_in_k\delta_{j\ell} + n_jn_k\delta_{i\ell} - 2n_in_jn_kn_\ell \quad . \tag{6}$$

Here N^S denotes the distribution of closed (shear) cracks - those for which the normal component of traction is negative (compressive).

If the normal force on a closed crack is significantly greater than the tangential force, friction may prevent interfacial sliding. In this case we say that the crack is locked, and N^S is set to zero. The normal component of traction is given by

$$B = \sigma_{ij}n_in_j \tag{7}$$

and the tangential stress by

$$\tau = \sqrt{A - B^2} \tag{8}$$

where

$$A = \sigma_{ij}n_j\sigma_{ik}n_k \tag{9}$$

is the magnitude squared of the traction acting on a crack. The locking criterion is simply

$$A < (\bar{\mu}^{-2}+1)B^2 \tag{10}$$

where $\bar{\mu}$ is the coefficient of friction.

Cracks become unstable when the far-field stress is large enough. The effect of unstable crack growth on a microscopic level is to produce additional strain rate at the macroscopic level. According to data collected by Stroh (1957), under high stresses cracks propagate at about a third of the longitudinal wave speed. We denote this constant speed by $\dot{c}$. The stability criterion we use is given by Dienes and Margolin (1980) as

$$A > \nu B^2/2 + \zeta \tag{11}$$

where

$$\zeta = \pi E\gamma(2-\nu)/4c(1-\nu^2) \quad . \tag{12}$$

and is an extension of the Griffith criterion. For shear cracks the stability criterion is significantly more complicated because of the effect of interfacial friction. Dienes (1978c) finds

$$c < \pi\gamma\mu(2-\nu)/2(1-\nu)(\tau-\bar{\tau})(\tau-3\bar{\tau}) \tag{13}$$

where

$$\bar{\tau} = \bar{\mu}B \tag{14}$$

and τ is the shear stress defined above.

Crack growth is quite different from crack opening, as it involves a change in crack diameter rather than crack width. If we write

$$\epsilon^o_{ij} = \beta^o\sigma_{k\ell} \int_\Omega d\Omega \, n_in_jn_kn_\ell F^o(\Omega) \tag{15}$$

as the strain due to open cracks, it may change because of either changes in stress, $\sigma_{k\ell}$, or changes in the distribution, $N^o(c,\Omega,t)$ which influences ϵ^o_{ij} through

$$F^o(\Omega) = \int_0^\infty dc \, c^3 \frac{\partial N^o(c,\Omega,t)}{\partial c} \quad . \tag{16}$$

The rate of change of strain due to the crack growth can be written as

$$d^{go}_{ij} = \beta^o \, \sigma_{k\ell} \int_\Omega d\Omega \, n_in_jn_kn_\ell \, \dot{F}^o(\Omega) \tag{17}$$

where

$$F^{o} = \int_{o}^{\infty} dc\ c^{3} \frac{\partial N^{o}}{\partial c} \quad . \tag{18}$$

There is a similar contribution due to unstable extension of shear cracks which we write as

$$d_{ij}^{gS} = \beta^{S} \sigma_{k\ell} \int_{\Omega} d\Omega\ b_{ijk\ell} \dot{F}^{S} \tag{19}$$

where

$$F^{S} = \int_{o}^{\infty} dc\ c^{3} \frac{\partial N^{S}}{\partial c} \tag{20}$$

and the integral is taken over all unstable, closed cracks that are not locked. The total strain rate due to crack extension, d_{ij}^{g}, is the sum of the expressions in (17) and (19).

The need to account for material rotation was mentioned above, and arises from the effect of material rotation on the stress tensor. It is shown by Dienes (1979d) that the strain rate due to material rotation is given by

$$d_{ij}^{r} = Y_{ik}W_{kj} - W_{ik}Y_{kj} \tag{21}$$

where

$$Y_{ij} = \beta^{o}Z_{ijk\ell}\sigma_{k\ell} + \beta^{S}(z_{ik}\sigma_{kj} + \sigma_{ik}z_{kj} - 2Z_{ijk\ell}^{S}\sigma_{k\ell})$$

$$+ C_{ijk\ell}\sigma_{k\ell} \tag{22}$$

and for moderately small distortions W_{ij} is the vorticity. If the deformation is large it is shown by Dienes (1979b) that W_{ij} should be replaced by a rate of material rotation, Ω_{ij}, and the current code has this capability. In (22)

$$Z_{ijk\ell}^{o} = \int_{\Omega} d\Omega\ n_i n_j n_k n_\ell F^{o} \quad , \tag{23}$$

$$Z_{ijk\ell}^{S} = \int_{\Omega} d\Omega\ b_{ijk\ell}\ F^{S} \quad , \tag{24}$$

and

$$z_{ij} = \int_{\Omega} d\Omega\ n_i n_j F^{S} \quad . \tag{25}$$

At high pressure the cracks are closed and locked and material behavior is governed by a high-pressure equation of state which we assume to be isotropic and have the Mie-Gruneisen form described by McQueen et al. (1970). However, since material behavior in the linear regime is already accounted for by the preceeding (anisotropic) representation, the linear behavior must be subtracted out of the equation of state. If we take the general form of the equation of state to be

$$p = G_o\rho_o I + f(\rho) \tag{26}$$

where I denotes the internal energy and ρ, the density; and also assume the linear form

$$u_S = c + Su_p \tag{27}$$

between shock velocity, u_S, and particle velocity, u_p, then it is straightforward to show that

$$f(\rho) = k\theta(1-G_o\theta/2)/(1-S\theta)^2 \tag{28}$$

where θ is used to denote the compression

$$\theta = 1 - \rho_o/\rho \quad . \tag{29}$$

It follows that the high-pressure portion of the stress tensor is given by

$$\dot{\sigma}_{ij}^{h} = [(k\rho_o/\rho^2 - f_\rho)\dot{\rho} - G_o\rho_o\dot{I}]\delta_{ij} \quad . \tag{30}$$

where k is the bulk modulus.

The preceeding results can be summarized by

$$d_{ij} - d_{ij}^{r} - d_{ij}^{g} = H_{ijk\ell}\dot{\sigma}_{k\ell} \tag{31}$$

where

$$H_{ijk\ell} = \beta^{o}Z_{ijk\ell} + \beta^{S}(z_{j\ell}\delta_{ik} + z_{i\ell}\delta_{jk} - 2Z_{ijk\ell}^{S})$$

$$+ C_{ijk\ell} \quad . \tag{32}$$

In the current version of SCM it is assumed that initially all the cracks are exponentially distributed and active, that is, free to grow if the stress is great enough to make them unstable. As a result of intersections, however, we envisage that many cracks will become inactive. Without intersections failure of rock samples would be catastrophic with the largest cracks free to propagate through the sample. This is not what is usually observed, and we believe that this is, at least in part, because the material behavior is modified by crack intersections. We denote by $L(c,\Omega,t)$ the number of active cracks with orientation Ω whose radii exceed c, with a similar definition of $M(c,\Omega,t)$ for the inactive cracks. The total number density of cracks, N, is the sum of L and M. When cracks with the mean size, $\bar{c}$, are unstable we consider all the cracks with that orientation to be unstable. This is a great simplification, and does not cause a major error since the smallest cracks do not contribute significantly to the overall behavior. It is shown by Dienes (1978a) that L satisfies the Liouville equation

$$\dot{L} + \dot{c}L' = -\dot{M} \tag{33}$$

where L' denotes the derivative with respect to c and

$$\dot{M} = kL \tag{34}$$

is the rate at which active cracks become inactive. It can be shown that for $c > \dot{c}t$

$$L = N_o \exp[-(c-\dot{c}t)/\bar{c}-kt] \tag{35}$$

and

$$M = (N_o k/\bar{c}\beta)[\exp(\beta t-c/\bar{c}) - \exp(-c/\bar{c})] \tag{36}$$

whereas for $c < \dot{c}t$, $L = 0$ and

$$M = (N_o k/\bar{c}\beta)[\exp(-kc/\dot{c}) - \exp(-c/\bar{c})] \quad . \tag{37}$$

Here,

$$\beta = \dot{c}/\bar{c}-k \tag{38}$$

and k describes the rate at which cracks become inactive. Dienes (1978a) gives an estimate for k in the case of an isotropic distribution. Since it has become clear that isotropy is too strong an assumption, we have formulated a more general theory with the parameters depending on orientation. We assume that the crack orientations are lumped into a finite number of bins (currently 9) with average orientation, Ω_j.

For oil shale it is natural to divide the distribution of cracks into a bedded set and an isotropic set. Then, it can be shown that

$$k_j = (4\pi^2\dot{c}/\alpha)(\pi^2 L\bar{c}^2 + N_b\bar{c}_b^2 \sin\theta_j) \tag{39}$$

where L represents the number density of isotropic cracks; N_b, the number density of bedded cracks; θ_i, the angle of the ith bin with the bedding planes; $\bar{c}$, the mean size of isotropic cracks; $\bar{c}_b$, the mean size of bedded cracks, and α, a crack interaction parameter, typically 4.

The fragmentation theory has been incorporated into a family of subroutines called SCM. The simplest use of the subroutine is with a driver that prescribes the strain rate for SCM, which then prints out stress and strain at prescribed intervals. Verification of SCM was described in the preceeding symposium. One method was to run hysteresis loops simulating triaxial test conditions and verify that the behavior was credible and that the residual energy had the correct sign. Another test was to run loading histories to a fixed strain at different strain rates. The final stresses were strongly strain rate dependent, and are in qualitative agreement with experimental data obtained by Grady and Kipp (1980). Quantitative comparisons are not feasible because the crack statistics for the samples tested are not available. The most definitive test of SCM was to determine the moduli of the cracked material for several kinds of loading from computer output and compare with analytic solutions, which can be obtained when the crack distribution is isotropic.

COMPARISON OF SCRAM WITH EXPERIMENT

The original purpose of the spherical shots carried out by Fugelso (1978) was to determine an effective yield strength for oil shale by embedding in it spheres of high explosive and comparing radiographs of the cavity produced with numerical calculations. Such a comparison was made by Dienes (1978d) using plasticity theory and an average yield strength of 100 MPa (14500 psi) for 1.85 g/cc oil shale and showed fair agreement. The discrepancy was due primarily to asymmetry of the cavity which is aspirin-shaped, having vertical sides and rounded top and bottom. In order to explain this curious shape, calculations were made with a number of variations on the anisotropic plasticity theory developed by Dienes (1979a), but in no case were there any significant deviations of the cavity from a spherical shape. It was, therefore, most gratifying to find that SCRAM could calculate the shape accurately. In the remainder of this section we discuss the experiment, details of the calculation, and present an explanation of the cavity shape.

The spherical explosive experiment was set up by machining a one-inch hole with a hemispherical bottom in an irregular block roughly a foot across. A one-inch sphere of PBX-9501 was placed in the hole, which was then filled with clay. The detonation mechanism for the spheres has been carefully designed to result in spherical detonation waves. Tests on shales of different densities were made, but in this paper we will be concerned only with the cavity in 1.85 g/cm^3 material. A radiograph was made at 30 µs and is reproduced in Fig. 1. The horizontal lines are evidence of the layered structure, and the aspirin-shape is evident.

In order to obtain a credible explanation of the cavity shape using numerical calculations it is important to establish a priori the properties of the oil shale. Since the current theory is based on crack statistics, one could go for direct measurements of crack size and number density, and this is the approach taken at SRI, as discussed by Seaman, Curran, and Shockey (1976). There is an alternative, however, which may be more conservative and much easier to implement, and that is to infer crack statistics from simple mechanical properties such as strength and elastic moduli. This allows us to avoid

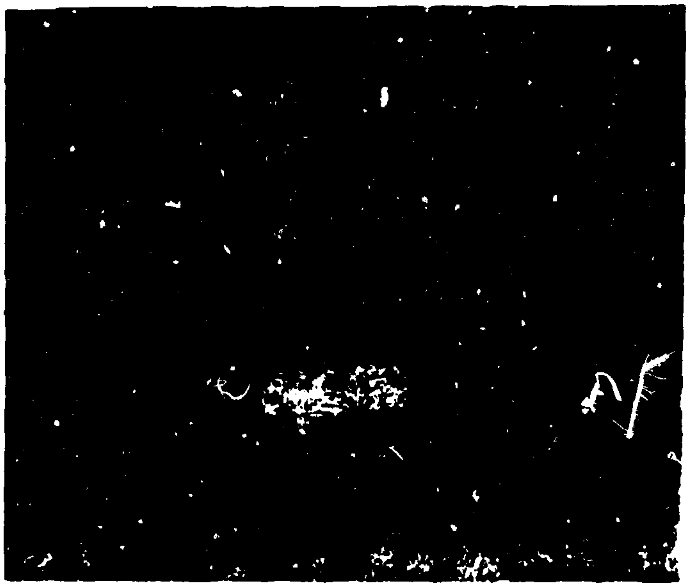

Fig. 1. Radiograph of the cavity produced in oil shale at 30 μs by a one-inch sphere of explosive.

difficult questions such as how to characterize cracks that are not penny-shaped, what to do about characterizing flaws that are not cracks, such as inclusions and crystal boundaries, and how to determine volumetric properties from observations on a surface. Though Dienes (1979c) has addressed the problem of inferring volume statistics from observations on a plane, the circumstances are somewhat idealized and have not been verified by direct observation. Perhaps more to the point, mechanical properties are available for oil shale and crack statistics are not.

According to ultrasonic measurements by Olinger (1976) the elastic properties of oil shale with density 1.85 g/cm^3 considered as a transversely isotropic material are given by c_{11} = 21.7 GPa, c_{33} = 13.2 GPa, c_{44} = 4.2 GPa, c_{12} = 8.0 GPa and c_{13} = 6.0 GPa. In terms of Young's modulus and the generalized Poisson ratios Dienes (1976) finds E = 17.5 GPa, E' = 10.8 GPa, ν = 0.276, ν' = 0.204, and ν" = 0.332. In order to determine crack size, we make use of the Griffith criterion and tensile strength measurements by Youash (1969), who has tested a number of anisotropic materials at varying orientations. He finds that rich oil shale varies in strength from 300 psi when loaded across the bedding planes to 1600 psi when loaded in the bedding planes. The lean varied from 900 to 1400 psi with orientation. Dynamic measurements have been made by Schuler, Lysne and Stevens (1976) who find that rich oil shale has a dynamic tensile strength averaging 2645 psi and the lean, 3340 psi, with no apparent dependence on orientation. For the SCRAM calculation we assume a tensile strength of 1250 psi when loaded across the bedding planes and 2500 psi when loaded in the bedding planes. Much more work needs to be done to characterize the tensile strength.

To determine the crack statistics we begin with the fracture toughness relation

$$\sigma^2 c = K^2/\pi = 2E\gamma \qquad (40)$$

and an estimate by Grady (1980) for K of about 1 MPa m$^{\frac{1}{2}}$. To estimate the surface energy, γ, we need a value of E, which we take as the Young's modulus when loaded in the bedding planes and is given above. Then, γ = 7.57 J/m^2. In the current analysis we make the assumption that the anisotropy is entirely due to the effects of penny-shaped cracks in the bedding planes. Since these cracks do not affect the stiffness measured in their plane, the Young's modulus of the matrix material and the in-plane modulus are the same. Using the theory of the preceeding section and the assumption that the crack radii are exponentially distributed with mean $\bar{c}$ it is straightforward to show that the compliance matrix has the form

$$\bar{H} = \begin{pmatrix} 1/\bar{E} & -\bar{\nu}/\bar{E} & -\bar{\nu}/\bar{E} & 0 \\ -\bar{\nu}/\bar{E} & 1/\bar{E} & -\bar{\nu}/\bar{E} & 0 \\ -\bar{\nu}/\bar{E} & -\bar{\nu}/\bar{E} & 1/\bar{E}+\beta^{U}N_b h & 0 \\ 0 & 0 & 0 & \beta^{S}N_b h+1/2\bar{\mu} \end{pmatrix} \qquad (41)$$

where N_b is the number of bedding cracks per cm^3 and

$$h = 6\bar{c}^3 \qquad (42)$$

The fourth-order material tensor in (32) can be replaced by a 9x9 matrix and the stress and strain tensors redefined as 9-vectors. Because of the symmetry of the stress and strain tensors, there are only 6 independent components, and it is sufficient to consider a 6x6 matrix to characterize the material. For axisymmetric deformations there are only 4 independent stresses and strains, and it is possible to redefine H as a 4x4 matrix. In (41) $\bar{\nu}$, $\bar{E}$, and $\bar{\mu}$ denote properties of the isotropic matrix material. $\bar{H}$ is somewhat different from the general compliance matrix for a transversely isotropic material

$$H = \begin{pmatrix} 1/E & -\nu/E & -\nu'/E & 0 \\ -\nu/E & 1/E & -\nu''/E & 0 \\ -\nu'/E & -\nu''/E & 1/E' & 0 \\ 0 & 0 & 0 & 1/2\mu \end{pmatrix} \qquad (43)$$

To bring $\bar{H}$ and H into approximate agreement we take $\bar{E}$ = E, and $\bar{\nu}$ to be the average of ν, ν', and ν", which is 0.27. Then $\bar{H}_{12} = \bar{H}_{13} = \bar{H}_{23}$ = −1.55, whereas H_{12} = −1.58, H_{13} = − 1.16 and H_{23} = −1.90, all in inverse megabars. Thus the error is on the order of 20%. However, $H_{11} = \overline{H}_{11} = H_{22} = \overline{H}_{22}$. To complete the representation of oil shale with bedded cracks, we require that

$$1/E + \beta^{o} N_b h = 1/E' \quad . \tag{44}$$

The number density of bedding cracks can be obtained from this result if $\bar{c}$ (hence h) is known. To estimate $\bar{c}$ we consider the strength of the samples tested by Youash which is determined by the largest cracks they contain. The crack size $\tilde{c}$ such that on average there is just one crack greater than $\bar{c}$ in radius in the volume V is given by

$$V N_b e^{-\tilde{c}/\bar{c}} = 1 \tag{45}$$

If we consider an ensemble of samples of size V, the mean size of the bedding cracks exceeding $\bar{c}$ in radius is given by

$$\bar{\bar{c}} = \int_{c}^{V^{1/3}} Vc \frac{\partial N}{\partial c} dc \cong c \ \ln(VN_b/e) \quad , \tag{46}$$

provided that $V >> \bar{c}^3$.

To complete the estimate of $\bar{c}$ we use the preceeding result and write the fracture toughness relation for penny-shaped cracks (given, for example, by Tetelman (1967)) as

$$\sigma_p^2 \ \bar{\bar{c}} = (\pi/4) K_p^2 = (\pi/2) \gamma E/(1-\nu^2) \quad . \tag{47}$$

Combining these results we have

$$\bar{c} = \frac{\pi \gamma \overline{E}}{2(1-\nu^2)} \frac{1}{\sigma_p^2 \ \ln(VN_o/e)} \tag{48}$$

Though the result depends on assumptions about sample size and crack shape, it is somewhat insensitive to them, and the reader should not infer that different assumptions would lead to very different results. Strictly speaking, we do not have N_b at this point, and the solution should proceed iteratively. We anticipate, however, and note that on the basis of exploratory calculations with SCRAM the cavity shape seems about right for $N_b = 100$. We estimate V the sample volume for the Youash tests, to be 5 cm^3. With the isotropic matrix parameters $\overline{E}$ and $\overline{\nu}$ given above and σ_p = 8.62 MPa (1250 psi) we find $\bar{c}$ = 0.058 cm. With this result (44) can be solid for the number density, and we find N_b = 107 cm^{-3}. In addition to the bedded cracks there are isotropically distributed cracks. Their mean size is taken as 0.0145 cm on the basis that the strength across the bedding planes is half the strength in the bedding planes, and critical crack size goes as the inverse square of strength.

The high pressure behavior is specified by the Gruneisen parameter G = 1.5 and the slope of the $u_S - u_p$ line, S = 1.5. In addition, it is necessary to specifiy the coefficient of friction, which we take to be $\bar{\mu} = 0.2$. The high explosive is represented as an ideal gas expanding adiabatically with a ratio of specific heats $\gamma = 3$ and and initial energy of 1000 cal/g. Though this is not the best representation of the explosive, it gives good results so long as the pressure remains above a few kilobars, as it does in the current problem. The volume of the explosive products, which is necessary and sufficient to compute the pressure in the adiabatic approximation, is obtained by numerical integration.

The SCRAM calculation was run with a time step of 2×10^{-7} sec using a polar coordinate system, with the (spherical) cavity edge a coordinate surface. Cells near the cavity have a radial dimension of 1 mm and are very nearly square. Away from the cavity the cell dimensions increase geometrically with a growth rate of 10%. The initial pressure in the cavity is 167 kilobars, and it results in an essentially radial motion of the oil shale at early times since the material behavior at high pressure is dominated by its (isotropic) equation of state. At later times cracks begin to open and to grow as a result of tensile hoop stresses. The effect of anisotropy at later times may be seen in Fig. 2 which illustrates the

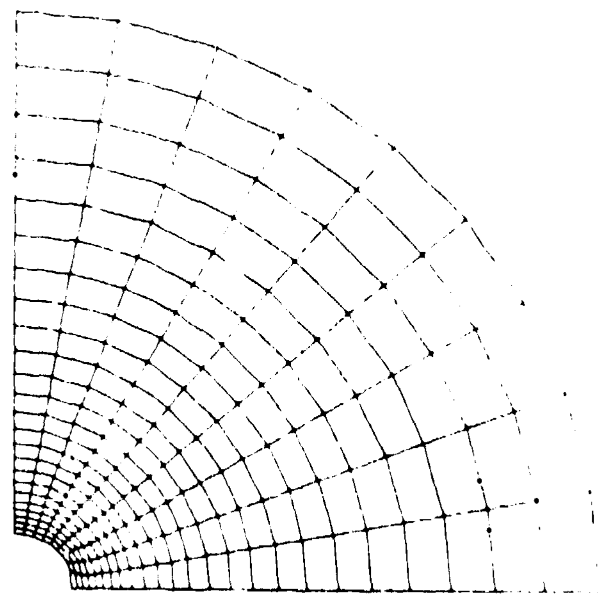

Fig. 2. The grid distortion in oil shale at 30 μs due to detonation of a one inch explosive sphere.

grid distortion. The strain in cell 19, which is adjacent to the intersection of the equator and the cavity, is strongly influenced by the dilatant effect of the bedding cracks. The average density of the cell gets very low, on the order of 1 g/cm^3, at late times. The calculated cavity shape is compared with the radiographic shape at 30 μs in Fig. 3. The difference in the two shapes appears to be within the resolution of the radiograph. The reason for the pill shape of the cavity is the extreme dilatancy of the material in cell 19, which causes the displacement at the edge of the cavity to be dominated by strain normal to the bedding.

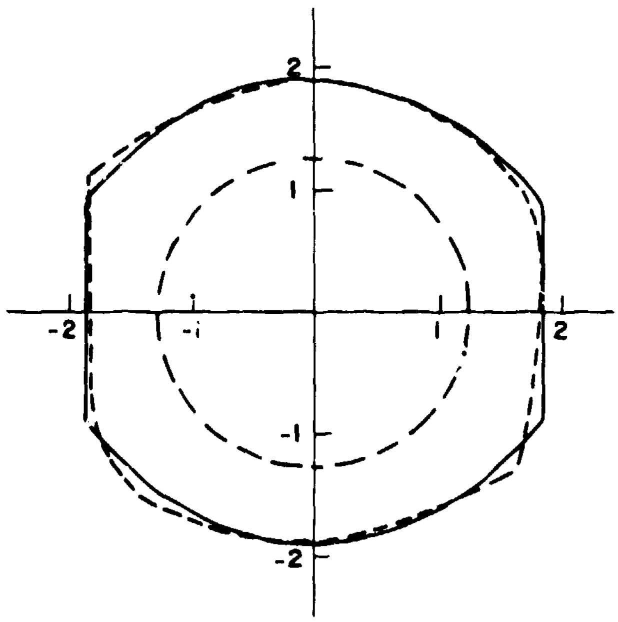

Fig. 3. Comparison of the radiographic cavity shape at 30 μs with a SCRAM calculation.

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