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MASTER

A Lecture on Detonation-Shock Dynamics

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Abstract

We summarize some recent developments of J. B. Bdzil and D. S. Stewart's investigation into the theory of multi-dimensional, time-dependent detonation. These advances have led to the development of a theory for describing the propagation of high-order detonation in condensed-phase explosives. The central approximation in the theory is that the detonation shock is weakly curved. Specifically, we assume that the radius of curvature of the detonation shock is large compared to a relevant reaction-zone thickness.

Our main findings are: (1) the flow is quasi-steady and nearly one dimensional along the normal to the detonation shock, and (2) the small deviation of the normal detonation velocity from the Chapman-Jouguet (CJ) value is generally a function of curvature. The exact functional form of the correction depends on the equation of state (EOS) and the form of the energy-release law.

1. Introduction

In this lecture we will describe a theory for unsteady, unsupported, multi-dimensional detonation propagation for the standard explosive model; the reactive Euler equations for a prescribed EOS and rate law. For this model, the detonation structure is ZND, i.e., a shock followed by a reaction zone which contains an embedded, trailing sonic locus. See Figure 1. In laboratory frame coordinates, the governing equations for this model are

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot u) = 0 \tag{1}$$

$$\rho \frac{Du}{Dt} = -\nabla P \quad , \tag{2}$$

$$\frac{DE}{Dt} + P\frac{D(1/\rho)}{Dt} = 0 \quad , \tag{3}$$

$$\frac{D\lambda}{Dt} = r \quad , \tag{4}$$

where in the above ρ, u, P, E, λ and r are respectively the density, particle velocity, pressure, specific internal energy, single reaction progress variable and the rate of forward reaction. To complete the specification of the problem we need to choose constitutive relations for the internal energy function $E(P, \rho, \lambda)$ and the rate law $r(P, \rho, \lambda)$. For illustrative purposes we select the polytropic form for E,

$$E = \frac{P}{\rho}(\gamma - 1)^{-1} - q\lambda \tag{5}$$

where γ is the polytropic exponent, and q is the specific heat of reaction. The solution of these equations must satisfy the standard normal shock relations at the leading detonation shock.

The theoretical developments are carried out in the limit that the radius of curvature of the shock front (R) is much greater than a characteristic reaction-zone length (r_ℓ), i.e.

$$\delta^2 \equiv |r_\ell \ /R| << 1 \quad . \tag{6}$$

With appropriate assumptions, the main result is that the velocity of the leading detonation shock along its normal deviates from the Chapman-Jouguet value by a small amount that is proportional to curvature (in the simplest cases) and more generally is a function of curvature, i.e.

$$D_n = D_{CJ} - \alpha\kappa \text{ where } \alpha = \text{ constant or } \alpha = \alpha(\kappa) \quad . \tag{7}$$

We were led to the discovery of (7), by our desire to formulate a rigorous theory of the evolution of the detonation shock in complex, two-dimensional (2D) and three-dimensional

(3D) geometries, which retained full reaction-zone effects, time dependence, and which was a physically correct and simple-to-use method for correcting detonation velocity. This study was aimed at gaining a fundamental understanding of multi-dimensional detonation.

Our theory is closely related to Whitham's theory of Geometrical Shock Dynamics [1]. Similarly, our theory stresses the dynamics of the shock. However, unlike Whitham, we have a systematic theory of the following flow that supports the shock that is strictly valid when the radius of curvature is large compared to the reaction-zone length.

In Section 2, we give a brief history of earlier developments in 2D detonation theory We sketch the fundamental approximations and our recent theoretical developments, in Section 3. In Section 4, we give some examples of fundamental detonation interactions, while in Section 5, we extend or modeling by examining an energy-release rate that is strongly dependent on state. Finally in Section 6, we comment on the practical implications of the theory for explosive engineering.

2. History of the development

The line of the development of the research presented here can be traced back through the work of Wood and Kirkwood [2] in 1954, Bdzil [3] in 1981, and through the recent collaboration of Bdzil and Stewart from 1984 to present. See references [4] and [5].

The fact the detonation propagation speed is dramatically affected by diverging geometry is illustrated by a standard experiment in a rate stick. In that experiment, a cylindrical stick confined by an inert tube is ignited at the bottom by means of a planewave explosive lens and a pad of high pressure booster explosive A nominally plane, overdriven detonation is thus introduced at the bottom of the stick. As time passes, the detonation shock in the stick becomes curved, because the high-pressure flow expands the tube walls into the relative vacuum surrounding the experiment (i.e., room pressure air). As a result, the plane character of the wave is destroyed. When a steady detonation develops in the stick it has an elliptical-like shape. The final steady 2D-detonation velocity can be measured by simple means and is found to be a function of the radius of the stick and the degree of confinement, i.e., tube wall material and thickness. The steady detonation velocity is reduced from the 1D Chapman-Jouguet value, D_{CJ}, by an amount that becomes greater as the radius of the stick, R_s, is reduced (see Figure 2 for a schematic diagram). At some critical radius, experiments using witness plates show that a steady detonation is not propagated in the stick. Presumably some form of extinction occurs.

The first theoretical calculations that explained these experimentally observed effects were carried out by Wood and Kirkwood [2]. They used the basic model described in the introduction specialized to a steady, radially symmetric flow. By restricting their analysis to the central streamline, and by further assuming that the 2D radial flow divergence, $\nabla \cdot u$, was known, they reduced the problem to a system of nonlinear ordinary-differential equations for the steady detonation structure. In particular, they assumed that the quantity, $\nabla \cdot u$ was related to a single *ad hoc* parameter (e.g., R) that measures the divergence of the flow. In these equations the detonation velocity, D, is an unknown constant parameter and R is a specified parameter. Fickett and Davis [6] further showed that this system of equations could be reduced to a single equation for $U^2 \equiv |u - D|^2$, the kinetic energy in the main flow direction, as a function of the reaction progress variable λ.

A qualitative analysis of this governing equation can be carried out quite conveniently in the (U^2, λ)-phase plane. A given value of D defines the starting value for U^2 at the shock. The task is to determine an integral curve in this plane, that follows U^2 as λ changes from $\lambda = 0$ at the shock to $\lambda = 1$ at complete reaction. In the limit that the flow divergence is zero, the integral curve terminates at a singular point at $\lambda = 1$. When the flow divergence is non zero, an additional singular point is found in the phase plane that corresponds to the intersection of the thermicity line and the sonic line. The reaction is incomplete at this new saddle-type singular point. The integral curve will pass through this point, for only a single value of D for a given R, i.e., $D(R)$. In general, this relationship must be found by numerical shooting techniques. An excellent account of the details of this work is found in Fickett and Davis's book (1979) [6], Section 5g3.

The next contribution to the development of the current theory is due to Bdzil [3]. He analyzed the problem of a steady-state 2D detonation in rate-stick geometry. This analysis was rigorous and not *ad hoc* as was that of Wood and Kirkwood. It was not restricted to the central streamline, but considered the entire 2D problem. This theory is an asymptotic theory which is consistent with the assumption that the stick radius, R_s, is large compared to a 1D reaction-zone length. Once again a parameter equivalent to

$$\delta^2 \equiv |r_\ell \;/ R_s| << 1 \quad ,$$

can be defined. (In Bdzil's account δ is related directly to the angle of the streamline deflection at the confinement boundary.) This assumption is equivalent to a small shock slope, with an O(1) change in the shock position Z_s (measured on the scale of reaction-zone lengths) taking place over the lateral distance scale $r\delta \sim O(1)$ (many reaction-zone lengths).

Bdzil found that all the leading features of the flow could be determined, and that they were parameterized by the shock locus function, Z_s. In turn, the shock locus was a function of the scaled transverse coordinate $\varsigma = r\delta$ and, for a particular example involving the choice of EOS and rate law, satisfied the second-order ordinary-differential equation

$$\frac{D_{CJ}}{2}\left[\frac{dZ_s}{d\varsigma}\right]^2 = \alpha\frac{d^2 Z_s}{d\varsigma^2} - D^{(4)} \quad , \tag{8}$$

where $D^{(2)}$ is identified by the expansion

$$D = D_{CJ} + \delta^2 D^{(2)} \quad ,$$

and measures the deviation of the steady detonation velocity from D_{CJ}.

The position of the shock, Z_s, is measured from a plane, $Z =$ constant, which moves with the steady detonation velocity, D. The function $Z_s(\varsigma)$ determines the local detonation velocity normal to the shock along its extent. Indeed, even though this is not made explicit in Bdzil's paper, equation (8) is equivalent to the coordinate-independent statement

$$D_n = D_{CJ} - \alpha\kappa + o(\kappa) \quad , \tag{9}$$

where D_n is the velocity along the shock normal. In the above, α is a constant (the assumptions about the EOS and rate law in [3] give α a specific value).

In 1984 we started work on the simplest, most straightforward extension of this steady theory that would include time dependence. We noticed that in order to include time dependence in a quasi-steady theory, it was necessary to introduce a slow-time scale such that the time dependence entered the theory at the same order as the shock curvature. In particular if on the reaction-zone length scale the shock locus, Z_s, is an $O(1)$ function, then the relevant slow-time scale is

$$\tau = \delta^2 t \quad , \tag{10}$$

where t is measured with the reaction-zone time scale. Calculations with these scaling assumptions show that at leading order, the flow through the reaction zone has the same form as it does in the steady-state problem, i.e., it is quasi-steady. However, the shock locus, which is what parameterizes the solution, is now a function of both the scaled transverse coordinate ς and the scaled time τ

In contrast to (8), the shock locus, Z_s, now satisfies the partial-differential equation

$$\frac{\partial Z_s}{\partial \tau} - \frac{D_{CJ}}{2}\left[\frac{\partial Z_S}{\partial \varsigma}\right]^2 = \alpha \frac{\partial^2 Z_s}{\partial \varsigma^2} - D^{(2)} \quad , \tag{11}$$

where Z_s is measured from a constant velocity plane. The above equation is a nonlinear heat equation. Indeed for α = constant, equation (11) can be reduced to a Burgers' equation for the shock slope, $\partial Z_s/\partial \varsigma$. On these length and time scales ς and τ, the evolution of the shock is not governed by a hyperbolic equation, but by the parabolic equation (11). A natural question to ask is why do we find a parabolic evolution equation for a system of hyperbolic equations?

The answer is found in Bdzil and Stewart's [4] (1986) paper on time-dependent 2D detonation. In that paper, we studied the transients that carry an initially 1D detonation into a steady-state 2D detonation. In the example we considered, an initially steady 1D detonation encounters an unconfined corner in the explosive (see Figure (3a)). After the wave reached the corner, the explosive products expanded into the vacuum and the detonation shock began to curve. Because the problem is hyperbolic, a traveling wave head was defined on the detonation shock to the left of which there was no disturbance of the 1D detonation.

We selected the explosive EOS and rate law with the goal of achieving a 1D detonation that was linearly stable to both transverse and flow-direction disturbances. With this goal in mind, we adopted a polytropic EOS model and a rate law for which most of the chemical heat release is given up immediately behind the shock. This was followed by a smaller resolved heat release that took place over a finite distance behind the shock and which controled the dynamics of the problem. For this "small resolved heat-release model," the dynamics of the 1D detonation occur on the "fast" time scale δt. Our results showed that disturbances on the shock propagate according to a hierarchy of two distinct flow regions which occur on the time scales δt and $\delta^2 t$.

In the first region the displacement of the shock is small and the dynamics, which occur on the δt time scale, is wave-like (hyperbolic). This region contains the hydrodynamic wave head, i.e., the leftmost point of the shock disturbance. The magnitude of the shock displacement, length and time scales for this region are given by

$$Z_s \sim O(\delta) \text{ with } \delta^{1/2} r, \delta t.$$

The second region is a diffusion-like region (parabolic). In this region the shock displacement from plane is the largest and the disturbance extends over both the greatest length and time scales. The magnitude of the shock displacement, length and time scales for this region are given by

$$Z_s \sim O(1) \text{ with } \delta r, \delta^2 t.$$

Figures 3a and 3b shows a schematic diagram of both the initial configuration and the evolutionary phase of the detonation shock for these two regions.

What we learned from [4] is that the parabolic flow is naturally embedded in the hyperbolic system. The hyperbolic region while defining the wave head of the disturbance is associated with a small amplitude shock deflection. In contrast the parabolic region is associated with a large scale shock deflection and is the most important region to characterize and measure. The advantage of this description is the relative simplicity of the parabolic region, which involves at most the solution of a simple second-order partial-differential equation (the nonlinear heat equation) Additionally, practical experience with the technologically important case of condensed phase propellants and explosives shows that they have broad well defined detonation shocks. To check the validity of the steady theory for condensed phase explosives, Engelke photographed the shock loci and compared them with the predictions of the steady theory. See Bdzil [3] and Engelke and Bdzil [7]. The theory and experiment were shown to be in qualitative and even quantitative agreement. Therefore, consistency of the unsteady and steady theories then also argues for the parabolic scales.

The results of [4] confirmed the importance of evolution equations of the parabolic type which were discovered earlier. The earlier work was eventually recorded in a paper by Stewart and Bdzil [5], where some examples of relationships between the normal detonation-shock velocity and the curvature were derived for the first time.

The simplicity of the parabolic description makes it possible to do routine calculations of a class of unsteady detonation problems. The detonation-wave spreading problems of greatest interest occur in explosives with complicated shapes. If we are to apply the parabolic description outlined above to such problems, we need to carry out the analysis in a system of intrinsic (or problem determined) coordinates. These calculations are the subject of the next section.

3. Sketch of the analysis

In this section we sketch the analysis and explain the approximations used in deriving the shock-evolution equation and the flow description. The model equations are the reactive Euler equations, subject to the shock Hugoniot conditions for a specific EOS and rate law. The presentation here is an outline of the more detailed discussion found in Bdzil and Stewart [8].

The coordinates we choose are shock-attached coordinates, and the problem is three dimensional. Here ξ_i represents arc length along the shock in the directions of the principle curvatures ($i = 1, 2$) defined by the instantaneous shock surface. The variable n represents the distance normal to the shock. The coordinates ξ_i and n form a locally orthogonal coordinate system. A picture of the intrinsic-coordinate system for 2D is shown in Figure 4. Because we have chosen an intrinsic-coordinate system, the shock curvature appears explicitly in the governing equations of motion. These equations become

Mass: $$\rho_{,t} - \left[\rho\left(D_n - u_n\right)\right]_{,n} + \kappa\rho u_n + \ldots = 0, \tag{12}$$

Energy: $$E_{,t} - \left(D_n - u_n\right)E_{,n} - (P/\rho^2)\left[\rho_{,t} - \left(D_n - u_n\right)\rho_{,n}\right] + \ldots = 0, \tag{13}$$

Momentum

n: $$u_{n,t} + \left(D_n - u_n\right)u_{n,n} + (1/\rho)P_{,n} + \ldots = 0, \tag{14}$$

ξ_i: $$u_{\xi_i,t} - \left(D_n - u_n\right)u_{\xi_i,n} + \ldots = 0, \quad i = 1,2 \tag{15}$$

Rate: $$\lambda_{,t} - \left(D_n - u_n\right)\lambda_{,n} = r + \ldots \quad . \tag{16}$$

Note that D_n is the instantaneous shock velocity along the shock normal. u_n and u_{ξ_i} are laboratory-frame particle velocities in the n and ξ_i-directions respectively. The curvature that appears in the above equations is the sum of the principal curvatures, $\kappa \equiv \kappa_1 + \kappa_2$. Higher order terms in the shock curvature are indicated by ellipses.

To these equations we add the shock relations

$$\rho_- D_n = \rho_+\left(D_n - u_{n_+}\right), \qquad P_+ = \rho_- u_{n_+} D_n, \qquad \lambda_+ = 0,$$
$$\frac{D_n^2}{2} = E_+ + \frac{P_+}{\rho_+} + \frac{1}{2}\left(D_n - u_{n_+}\right)^2, \qquad u_{\xi_{i+}} = 0, i = 1,2. \tag{17}$$

The minus subscript refers to the state ahead of the shock, the plus subscript refers to the state behind the shock. In these relations we have adopted the strong shock approximation and have set terms proportional to P_- to zero (we have anticipated that $E_- \sim P_-/\rho_-$).

We make the explicit assumption that the curvature is

$$\kappa \equiv \delta^2\tilde{\kappa}, \qquad \delta^2 << 1 \quad . \tag{18}$$

where $\hat{\kappa}$ is the scaled shock curvature and δ^2 measures the magnitude of curvature relative to the 1D reaction-zone length. The length and time scales required are

$$\tau = \delta^2 t, \qquad n, \text{ and } \varsigma_i = \delta \xi_i, \text{ for } i = 1,2 \quad . \tag{19}$$

We introduce the formal expansions for the dependent variables

$$\begin{aligned}
&u_n = u_n^{(0)} + \delta^2 u_n^{(2)} + \ldots, \quad u_{\varsigma_i} = \delta^2 u_{\varsigma_i}^{(2)} + \ldots, \\
&P = P^{(0)} + \delta^2 P^{(2)} + \ldots, \quad \rho = \rho^{(0)} + \delta^2 \rho^{(2)} + \ldots, \\
&\lambda = \lambda^{(0)} + \delta^2 \lambda^{(2)} + \ldots, \quad D_n = D_{CJ} + \delta^2 D_n^{(2)}(\varsigma_i, \tau) + \ldots.
\end{aligned} \tag{20}$$

Using these expansions in equations (12) – (16) we find that through and including $O(\delta^2)$, the equations that govern the flow reduce exactly to the equations for quasi-steady flow in cylindrical geometry

$$-\left[\rho\left(D_n - u_n\right)\right]_{,n} + \kappa \rho u_n + \ldots = 0, \tag{21}$$

$$\left(D_n - u_n\right) E_{,n} - (P/\rho^2)\left[\left(D_n - u_n\right)\rho_{,n}\right] + \ldots = 0, \tag{22}$$

$$\left(D_n - u_n\right) u_{n,n} + (1/\rho) P_{,n} + \ldots = 0, \tag{23}$$

$$\left(D_n - u_n\right) u_{\xi_i,n} + \ldots = 0, \quad i = 1,2 \tag{24}$$

$$-\left(D_n - u_n\right) \lambda_{,n} = r + \ldots, \tag{25}$$

since from equation (24) and the shock conditions it follows that $u_{\xi_i} = 0$.

In Section 2 we mentioned that Wood and Kirkwood [2] treated the central streamline problem. Equations (21) – (25) taken together with the normal shock relations are equivalent to the problem they treated. Now, the terms due to the flow divergence are rigorously identified as being proportional to the local shock curvature, κ. The above problem then admits an eigenvalue detonation as its solution. As Wood and Kirkwood showed, it defines a relation between the two parameters D_n and κ as a condition necessary for the integral curve in the (u_n^2, λ)-plane to pass through the saddle singular point, where the flow is sonic. Generally speaking, we have the requirement that there exists a relation of the form

$$D_n = D_n(\kappa) \quad . \tag{26}$$

To illustrate this point we give the equation. Let $U_n \equiv u_n - D_n$, and consider the polytropic EOS

$$E = \frac{P}{\rho}(\gamma - 1)^{-1} - q\lambda \quad . \tag{27}$$

Straightforward manipulation of equations (21) – (25) yields the single ordinary-differential equation for U_n^2 in terms of λ, namely

$$\frac{d(U_n^2)}{d\lambda} = \frac{2U_n^2\left\{q(\gamma - 1)r - c^2\left(D_n + U_n\right)\kappa\right\}}{r\left(c^2 - U_n^2\right)} \quad , \tag{28}$$

where the sound speed is given by $c^2 = \gamma P/\rho = (\gamma - 1)\left[\left(D_n^2 - U_n^2\right)/2 + q\lambda\right]$. The shock boundary condition requires that

$$U_{n+} = -\frac{D_n(\gamma - 1)}{(\gamma + 1)} \quad . \tag{29}$$

Following the nomenclature of Fickett and Davis, the { }-term in the numerator of (28) defines the thermicity locus in the (U_n^2, λ)-plane, and $(c^2 - U_n^2)$ defines the sonic locus. These curves, along with $r = 0$, define the seperatrices and their intersections define the singular points in the phase plane. The object in the phase plane is to find the integral curve that starts from the shock value given by (29) and terminates at complete reaction. Typically such curves must pass through a singular point defined by the intersection of the sonic and thermicity loci. Since κ is small, the intersection point is very close to complete reaction. As mentioned before, this point is a saddle. To ensure passage through the saddle, condition (26) must hold.

In order to give a specific form to relationship (26) we must give the rate law. In Stewart and Bdzil [5] it is shown that for the choice

$$r = k(1-\lambda)^{\nu}, \text{ for } 0 < \nu < 1, \tag{30}$$

equation (26) takes the form

$$D_n = D_{CJ} - \alpha\kappa + o(\kappa), \alpha \equiv \frac{\gamma^2 D_{CJ}^2}{k(\gamma+1)^2}\int_0^1 \frac{\left(1+\sqrt{1-\lambda}\right)^2 d\lambda}{(1-\lambda)^{\nu}} \quad . \tag{31}$$

For the special case of simple depletion ($\nu = 1$) it can be shown that for diverging geometry ($\kappa > 0$)

$$D_n = D_{CJ} + \beta\kappa \ell n(\kappa) + 2\beta\kappa\left[\ell n\left(\beta/D_{CJ}\right) - 3\right] + \ldots, \ \beta \equiv \frac{\gamma^2 D_{CJ}^2}{k(\gamma+1)^2} \quad . \tag{32}$$

4. Detonation interactions

The formulas given in the last part of Section 3 show that the detonation-shock velocity is a function of the curvature of the shock. In order to describe the evolution of the shock we must have a second relation between D_n and κ. Using the surface compatibility conditions of differential geometry, we have derived such a second relation. We call this relation the kinematic-surface condition

$$\frac{1}{\kappa}\left(\frac{1}{\kappa}D_{n,\xi}\right)_{,\xi} + D_n = -\frac{1}{\kappa}\left(\frac{1}{\kappa}\int_{\xi^*}^{\xi}\kappa_{,t}d\xi\right)_{,\xi} \quad , \tag{33}$$

where ξ^* is a fixed reference position on the shock (see Figure 4). In 2D, the natural representation of the shock locus is in terms of the angle ϕ that the shock normal makes with respect to a fixed reference direction. Then ϕ is related to the shock curvature by

$$\phi \equiv \int_{\xi^*}^{\xi}\kappa d\xi \tag{34}$$

If we consider the simple case given by equation (31) and use the scalings given by equation (19), we find that equations (31) and (33) imply the following equation for ϕ,

$$\phi_{,\tau} + \frac{D_{CJ}}{2}\phi\phi_{,\varsigma} = \alpha\phi_{,\varsigma\varsigma} \quad . \tag{35}$$

Equation (35) is Burgers' equation for ϕ. The constant α plays the role of viscosity. Burgers' equation has analytical exact solution via the Hopf-Cole transformation and its dynamics have been studied extensively. Thus for this example, fundamental shock interaction problems can be studied with these exact solutions. According to our theory, there now exists a catalogue of solutions for detonation-shock interactions, that is similar to the catalogue of solutions to Burgers' equation.

Two simple examples from this catalogue are the step-shock solution and the N-wave solution to Burgers' equation. The step-shock solution corresponds to the solution for two colliding detonations, providing that the detonating material is large enough that the detonation-shock angles are constant in the far field. If two plane detonations are initiated obliquely so as to run into one another, the slope of their common intersected shock locus starts from the left with one value and moves to another value as we pass to the right. Solutions to Burgers' equation show that ultimately a steady-state step-shock solution is attained with a definite shock-shock [1] thickness that depends on α. This interaction mimics a reactive Mach stem. Importantly, it is diffuse (see Figure 5a).

The N-wave solution corresponds to a positive shock imperfection. In the right and left far field, the detonation is flat and hence ϕ is zero In the center the shock is raised, giving rise to an N-shape for ϕ, from left to right. The N-wave solution then shows that this imperfection ultimately "diffuses" away; the time required for "diffusion" of the imperfection depends on the value of α (see Figure 5b)

5. Stronger state dependence of the rate

The results given by equations (31) and (32) show that the exact functional form of the detonation-shock velocity vs curvature relationship depends on the details of the rate law. Bdzil's [3] results, for steady 2D detonation, showed that as the sensitivity of the rate to the local state is increased, a steady solution does not exist when the curvature becomes sufficiently large. This theoretical observation is consistent with experimental observation.

In this section we present a simple model that shows the consequence of increased state sensitivity. Consider the following shock-state dependent rate (shock-state dependence is typical of solid high explosives)

$$r = kf(\lambda) = \hat{k}\ \exp\Big[-\theta\big(D_{CJ} - D_n\big)\Big] f(\lambda) \quad . \tag{36}$$

Since D_n is proportional to the shock pressure, the rate multiplier k is now a function of how hard the particles were hit by the passage of the shock. Individual particles react at a rate that is determined by how hard they were shocked. The fact that the state dependence is sensitive (i.e., large changes in r occur for small changes in D_n), is modeled by requiring that the dimensionless parameter

$$\theta D_{CJ} >> 1 \quad . \tag{37}$$

For the purpose of this illustration, we further consider the following distinguished limit relating the large parameter θD_{CJ} and δ^2

$$\Big[\theta D_{CJ}\Big]^{-1} = \delta^2 \tag{38}$$

Using the expansion for D_n, the rate law becomes

$$r = \hat{k}\ \exp\Big[D_n^{(2)}/D_{CJ}\Big] f(\lambda) \tag{39}$$

Now it is easy to see that for the case $f(\lambda) = (1-\lambda)^{\nu}$, where $0 < \nu < 1$, equation (31) still holds, with the exception that k is replaced by $\hat{k}\exp\left[D_n^{(2)}/D_{CJ}\right]$. Using the previous definition for scaled curvature, $\kappa = \delta^2\hat{k}$, we find the reduced shock velocity curvature relation becomes

$$-\left(D_n^{(2)}/D_{CJ}\right)\exp\left[D_n^{(2)}/D_{CJ}\right] = \hat{\alpha}\hat{\kappa} \quad , \tag{40}$$

where $\hat{\alpha}$ is given by equation (31) for α, with $\hat{k}$ replacing k. We rewrite equation (40), in order to compare directly with (31) and (32);

$$D_n = D_{CJ} - \alpha\kappa \ \exp\ \left[-\theta\left(D_n - D_{CJ}\right)\right] \quad . \tag{41}$$

From equation (41) it is simple to show that for the reduced curvature $\hat{\kappa}$ in the range $0 < \hat{\kappa} < \hat{\kappa}_{cr}$, that there are two values for $D_n^{(2)}$. Hence the detonation velocity is multivalued for positive (divergent) curvature below a critical value of curvature (see Figure 6). For values of curvature above the critical value, it is not possible to have detonation-shock evolution described by the parabolic scales. A possible consequence of this is extinction of the detonation wave on portions of the curve where the critical curvature is exceeded.

6. Practical implications for explosive engineering

The theory discussed in this lecture pertains to explosive materials in which a broad, well-defined detonation shock is observed in the limit that the radius of curvature is large compared to the distance from the leading shock to the sonic locus. Indeed this is the case of practical interest for a wide class of explosives.

Engineers who design explosive charges typically use the Huygen's rule of detonation propagation whereby the detonation shock is advanced along its normal at the constant Chapman-Jouguet velocity. Our results indicate that this "recipe" should be modified, and that the correction factor is generally a function of the curvature. In addition our results show that the detonation structure from shock to sonic locus is easily calculated and is locally a 1D, cylindrical, quasi-steady flow.

The theory then suggests that the $D_n(\kappa)$ relation may describe the shock evolution for certain explosives for a wide range of initial and confinement conditions. If this theoretical statement is true, then $D_n(\kappa)$ can be determined directly from experiment. For example, $D_n(\kappa)$ could be determined from photographs of steady detonation-shock loci in rate sticks. Suppose the steady detonation velocity, D, along the axis of the stick has been measured. If ϕ is the angle that the shock normal (taken from the photograph) makes with the axis of propagation, then the normal velocity is given by

$$D_n = D \cos \phi \quad .$$

The shock curvature κ could be inferred from the photograph as well. Thus for the extent of the shock locus shown in the photograph, a portion of the $D_n(\kappa)$ curve can be constructed

Other experiments, steady or unsteady, in totally different geometries, properly analyzed, should reproduce the same $D_n(\kappa)$ in regions of overlap. Consider the case of a 1D, unsteady cylindrically or spherically expanding detonation. In this experiment D_n is simply $\dot{R}$, the rate of change of the radius from the central point, while $\kappa = 1/R$.

Thus the experimentally determined $D_n(\kappa)$ curve, would determine the detonation characteristic for many different geometries and configurations without our having detailed knowledge of either the equation of state or the energy-release law

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Figure captions

Figure 1. A schematic representation of the detonation shock with normal and trailing sonic locus displayed.

Figure 2. Rate sticks and the diameter effect. Figures 2a and 2b show schematic diagrams of a standard rate stick experiment. Figure 2a shows the stick prior to initiation. Figure 2b shows steady propagation. Figure 2c shows the steady value of the detonation velocity D minus D_{CJ} plotted versus the inverse of the stick radius, R_s^{-1}. Two different cases showing results for strong and weak confinement are shown. The open circles show extinction points which indicate no steady propagation for small radius tubes.

Figure 3. Figure 3a shows the configuration prior to the 1D detonation reaching the vacuum. Figure 3b shows subsequent detonation evolution at two times.

Figure 4. A sketch of the 2D intrinsic shock-attached coordinate system.

Figure 5. Two examples of detonation shock interactions.

Figure 6. Scaled detonation velocity $D_n^{(2)}/D_{CJ}$ versus scaled detonation shock curvature $\hat{\kappa}$.