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TITLE SHORT TIMES AND SHORT DISTANCES IN NUCLEAR AND PARTICLE PHYSICS - A PEDAGOGICAL REVIEW

LA-UR--88-4285

DE89 005468

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SUBMITTED TO CONFERENCE PROCEDINGS OF NUCLEAR AND PARTICLE PHYSICS ON THE LIGHTCONE HELD IN LOS ALAMOS AT LAMPF. (TO BE PUBLISHED BY WORLD SCIENTIFIC)

Los Alamos

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FORM NO. 836 R4
ST. NO. 2629 5/81

SHORT TIMES AND SHORT DISTANCES IN NUCLEAR AND PARTICLE PHYSICS - A PEDAGOGICAL REVIEW

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ABSTRACT

The formalism relevant to deep inelastic processes in both non-relativistic and relativistic systems is reviewed with an emphasis on scaling and its violations. In the former case we show how a systematic expansion in $1/q^2$ (q being the momentum transfer) can be derived which delineates the incoherent scattering from bound state and potential corrections. We demonstrate how this exact many-body non-relativistic formalism corresponds to the light-cone operator product expansion in quantum field theory. As examples, scaling in liquids, nuclei and nucleons is discussed with emphasis on the EMC effect, shadowing and the relationship to the photo absorption limit.

I INTRODUCTION

An elementary argument based on the uncertainty principle clearly demonstrates that high momentum transfer processes are sensitive to physics on the light cone: just simply taking $q^2 \rightarrow \infty$ probes the region $x^2 \rightarrow 0$. Since QCD is asymptotically free (i.e. its effective coupling constant becomes vanishingly small when $q^2 \rightarrow \infty$), things simplify considerably in this regime: perturbation theory, at least naively, is a justifiable approximation. Indeed, it was precisely this property that led to the establishment of QCD as the theory of the strong interactions. In particular, as will be reviewed below, its prediction of logarithmic violations of exact scaling in deep inelastic lepton scattering was a striking success. The arguments, which technically relied on the behaviour of products of currents near the light cone, justified not only the use of the parton model but the identification of partons with the quark and gluon fundamental degrees of freedom. It was natural to try to extend such arguments to other high momentum processes, such as form factors, lepton pair production, wide-angle scattering, and heavy quark decays. However, although the light cone certainly plays an important (and possibly even a dominant) rôle in all of these processes the application of perturbation theory alone to describe them is generally impossible to justify. The point is that in deep inelastic scattering it is possible to make a clean separation of the infrared (i.e. the non-perturbative) from the ultraviolet(the perturbative). In almost all other processes such a separation is generally not possible even in the extreme ultra-violet limit. Typically non-perturbative physics creeps in. Indeed one of the major challenges in QCD physics is to understand how to graft non-perturbative infrared or bound state effects onto perturbative ones controlled by light-cone physics.

Particle and nuclear physics are beginning to come together in this endeavour although their emphases have in the past been quite different. The emphasis in particle physics was originally to try to substantiate QCD as the theory of the strong interactions[1]. Having done so (at least to the satisfaction of most physicists) the emphasis shifted to using it as a probe of new physics (i.e. new interactions beyond the standard model or new particles such as the top quark). This meant understanding phenomena such as jet structure, multiparticle production, decay processes and so on[2]. This has been accomplished almost entirely within the context of perturbation theory (and, by implication, physics on the light cone). Phenomenologically, this has proven to be successful in spite of the fact that non-perturbative effects ought to play some rôle.

Ironically, even though all particle physicists may believe that QCD is <u>the</u> theory, nevertheless, it is worth remembering that the self-interaction of the gluons (and, subsequently the presumed existence of a glueball state) has yet to be experimentally substantiated! Non-perturbative physics, i.e. physics away from the light cone, has by and large become the province of lattice QCD though important analytic efforts have been made. The major effort thus far has been in trying to understand the hadronic spectra.

Until relatively recently nuclear physics worked almost exclusively within the context of meson and nucleon degrees of freedom. However as energies have increased and the realization that QCD is here to stay has crystallized, the emphasis has begun to shift to the question of the rôle of quarks and gluons inside the nucleus. Here the fundamental questions revolve around how the description of low energy phenomena described in terms of mesons and nucleons evolves into quarks and gluons as the energy scale increases. A central question for example is the existence and experimental signal of a quark-gluon plasma. In coming to grips with some of the serious problems raised by going to higher energies considerable work has focused on phenomenological descriptions in terms of relativistic nucleons and effective relativistic field theories of mesons and nucleons (QHD). Whether this is a more useful, economical or physical way of dealing with some of the problems rather than trying to come to grips directly with the rôle of QCD in nuclei remains an open question at this time[3]. In the last few years a central battleground for the advocates of these rather different approaches has been the EMC effect[4]. This is the experimental observation that the deep inelastic structure functions do not simply scale with A as one changes the target. All sides have adequate explanations of the effect, which is not too surprising since both descriptions are to some degree valid and the experiments, after all, only measure gross features.

The rest of this talk will in fact, concentrate on the theoretical description of the scaling phenomena observed in classic deep inelastic scattering. As intimated by the title, the emphasis will be pedagogical, and, as such, will for the most part, be a review of well-known theoretical techniques and results. I shall, however, give the discussion in terms of two rather different contexts: (a) many-body non-relativistic potential theory and (b) fully relativistic quantum field theory. The latter encompasses QCD whereas the former applies to nucleons bound in a nucleus by inter-nucleon potentials. At the end I shall briefly discuss applications to the EMC effect and some questions of shadowing.

II Non-Relativistic Systems[5]

We begin by considering spinless non-relativistic scattering from a target composed of Z scattering centers such as is the case of a nucleus or of a macroscopic liquid. The formalism that I shall review applies in fact almost precisely to the case of thermal neutron scattering from liquids. In general, the process to be discussed is illustrated in Fig. 1: the scattered probe particle (an electron, say) is detected without regard to the fate of the target final states. In terms of the energy loss (ν) and momentum transfer (q) it is convenient to introduce the structure function (appropriate to Coulomb scattering).

$$W(\nu, q^2) = \frac{(d^2\sigma/d\Omega\, dE)}{(d\sigma/d\Omega)_{Ruth}} \tag{1}$$

$(d\sigma/d\Omega)_{Ruth}$ is the classical Rutherford scattering cross-section for structureless

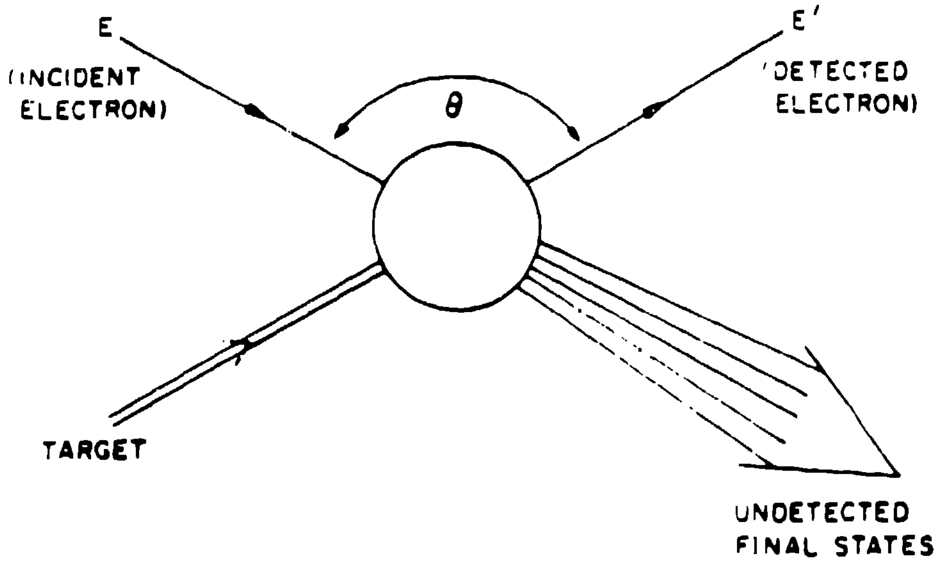

Figure 1: General graph illustrating inclusive scattering from an arbitrary target.

particles. From the Fermi golden rule W is given by

$$W(\nu, q^2) = \sum_f |< \Psi_f| \sum_{i=1} Q_i e^{i q \cdot r_i} |\Psi_0 > |^2 \delta(E_f - E_o + \nu) \tag{2}$$

where Q_i is the charge of the i'th constituent whose position is r_i. $\Psi_{0(f)}$ is the initial (final) state of the target. Using the Heisenberg equations of motion together with the completeness of the set of final states f (i.e. the conservation of probability) one can express (2) as a ground state expectation value

$$W(\nu, q^2) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i\nu t} < \Psi_0 | \sum_{ij} Q_i Q_j e^{i q \cdot r_i(t)} e^{-i q \cdot r_j(0)} | \Psi_0 > \tag{3}$$

The price paid for eliminating the sum over final states is the need for knowledge of the time development of $r_i(t)$. This, of course, is governed by the Hamiltonian of the system whose general structure is taken to be

$$H = -\sum_i \frac{\nabla_i^2}{2\mu} + V(\underline{r}_1, \cdots\cdots, \underline{r}_Z) \tag{4}$$

where μ is the mass of the constituents. Although we will not need to do this in what follows, it is usually assumed that the potential V can be expressed as a sum of 2-body potentials.

$$V(\underline{r}_1 \cdots \underline{r}_Z) = \sum_{i<j} v(\underline{r}_i - \underline{r}_j) \tag{5}$$

Indeed this usually leads to a 2nd quantized many-body description in terms of creation-destruction operators $a_{\underline{k}}$:

$$W(\nu, q^2) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i\nu t} < \Psi_0 | [\rho_q(t), \rho_q(o)] | \Psi_0 > \tag{6}$$

The density operator is given by

$$\rho_q \equiv \sum_{\underline{k}} a^{+}_{\underline{k}+q} a_{\underline{k}} \tag{7}$$

Its time development is controlled by the Hamiltonian, eq. (4) which, in this formalism, can be expressed as

$$H = \sum_{\underline{k}} \frac{\underline{k}^2}{2\mu} a^{+}_{\underline{k}} a_{\underline{k}} + \frac{1}{2} \sum_{\underline{k}} v(\underline{k}) \rho^{+}_{\underline{k}} \rho_{\underline{k}} \tag{8}$$

$v(\underline{k})$ is just the Fourier transform of the 2-body potential $v(r)$ defined through eq. (5). This field theoretic description of eqs. (2) and (3) allows one to think of W as the imaginary part of the corresponding (virtual) photon, forward Compton scattering amplitude as illustrated in Fig. 2. The question we wish to address is what is the behaviour of W when q becomes very large?

Although much formal and phenomenological work has been based on this 2nd quantized representation it is more convenient for our purposes to stay with the

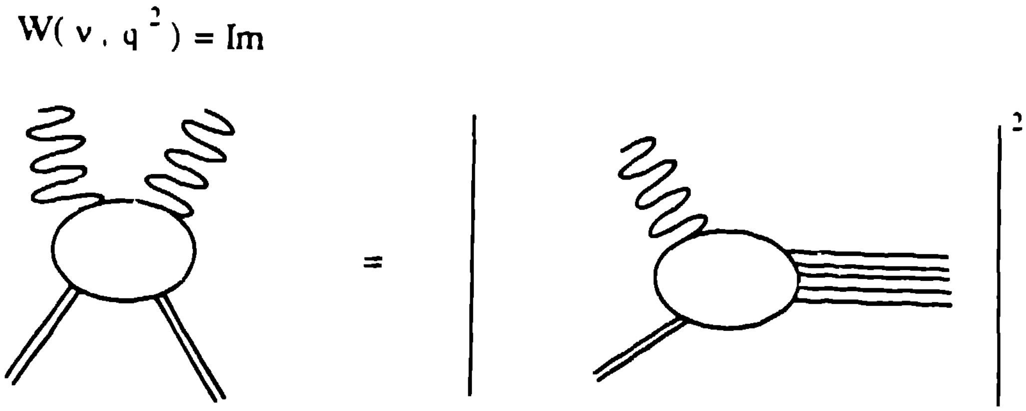

Figure 2: The optical theorem relating W to the imaginary part of the vital forward Compton scattering amplitude.

equivalent 1st quantized form, eq. (3). From a judicious use of operator identities coupled with the equations of motion

$$\frac{d\underline{p}_i}{dt} = i[H, \underline{p}_i] = -\nabla_i V(\underline{r}_1, \cdots \underline{r}_Z) \equiv F_i \tag{9}$$

[p_i is the momentum operator for the i'th constituent] one can derive the following <u>exact</u> representation

$$W(\nu, q^2) = \langle\Psi_0| \sum_{i,j=1}^{Z} Q_i Q_j e^{iq(z_i - z_j)} T \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i\frac{qt}{\mu}[\mu y - p_{iz} - \int_0^t (1 - t'/t) F_{iz}(t') dt']} |\Psi_0\rangle$$

(10)

Here we have defined the z-direction as that of q and introduced the dimensionless variable

$$y \equiv \frac{2\mu\nu - q^2}{2\mu q} \tag{11}$$

This expression has a lot of nice properties, not least of which is that it delineates three separate aspects of the physics:

(i) The degree of coherence in the target: this is represented by the term

$$\sum_{ij=1}^{Z} Q_i Q_j e^{iq\cdot(\mathbf{r}_i-\mathbf{r}_j)} = \sum_{i=1}^{Z} Q_i^2 + \sum_{i\neq j} Q_i Q_j e^{iq\cdot(\mathbf{r}_i-\mathbf{r}_j)} \qquad (12)$$

The point is that the incoherent contribution coming from terms with $i = j$ contains no phase factor and so is not damped when $q \to \infty$. On the other hand the terms with $i \neq j$ which represent the coherent contribution do contain a phase factor and so fall rapidly with increasing q^2 just like a form factor; [see Fig. 3].

(ii) Quasielastic scattering: if the constituents were free and at rest then the probe scatters elastically from them and so $q^2 = 2\mu\nu$ requiring $y = 0$. Thus deviations from $y = 0$ are a measure of the bound state of the target. This can be seen explicitly in (10) by setting $F = 0$ and performing the integration over t. The term $(\mu y - p_{iz})$ in the exponent shows that y is a measure of the internal momentum of the constituents inside the target - as will be shown explicitly below.

(iii) Dynamical corrections: these are completely represented by $F_i(t')$ in the exponent. To evaluate them is of course, very complicated. However in the deep inelastic limit, the expression simplifies considerably as we shall now demonstrate.

Introduce $\beta \equiv qt$ (and $\beta' \equiv qt'$) then the incoherent part of eq. (10) can be re-expressed as

$$qW(\nu, q^2) = \langle\Psi_0|\sum_{i=1}^{Z} Q_i^2 T \int_{-\infty}^{\infty} \frac{d\beta}{2\pi} e^{i\beta[y - \frac{p_{iz}}{\mu} - \frac{1}{q}\int_0^\beta d\beta'(1-\frac{\beta'}{\beta})\frac{F_i(\beta'/q)}{\mu}]}|\Psi_0\rangle \qquad (13)$$

Apart from suppressing the coherent contribution which vanishes rapidly with q^2, this expression is exact. Now, if we take $q \to \infty$ at fixed y, it is clear that the term in the exponent containing F_i also eventually vanishes and we are left with

$$\mathcal{F}(y, q^2) \equiv qW(\nu, q^2) \to < \Psi_0|\sum_{i=1}^{Z} Q_i^2 \delta(y - \frac{p_{iz}}{\mu})|\Psi_0 > \qquad (14)$$

$$q\,W(q^2,\nu) \equiv F(q^2,y)$$

$$\approx \mathrm{Im} \sum_i$$

$$+ \mathrm{Im} \sum_{i \neq j}$$

$$+ \ldots\ldots$$

$$+ \mathrm{Im} \sum_{i \neq j}$$

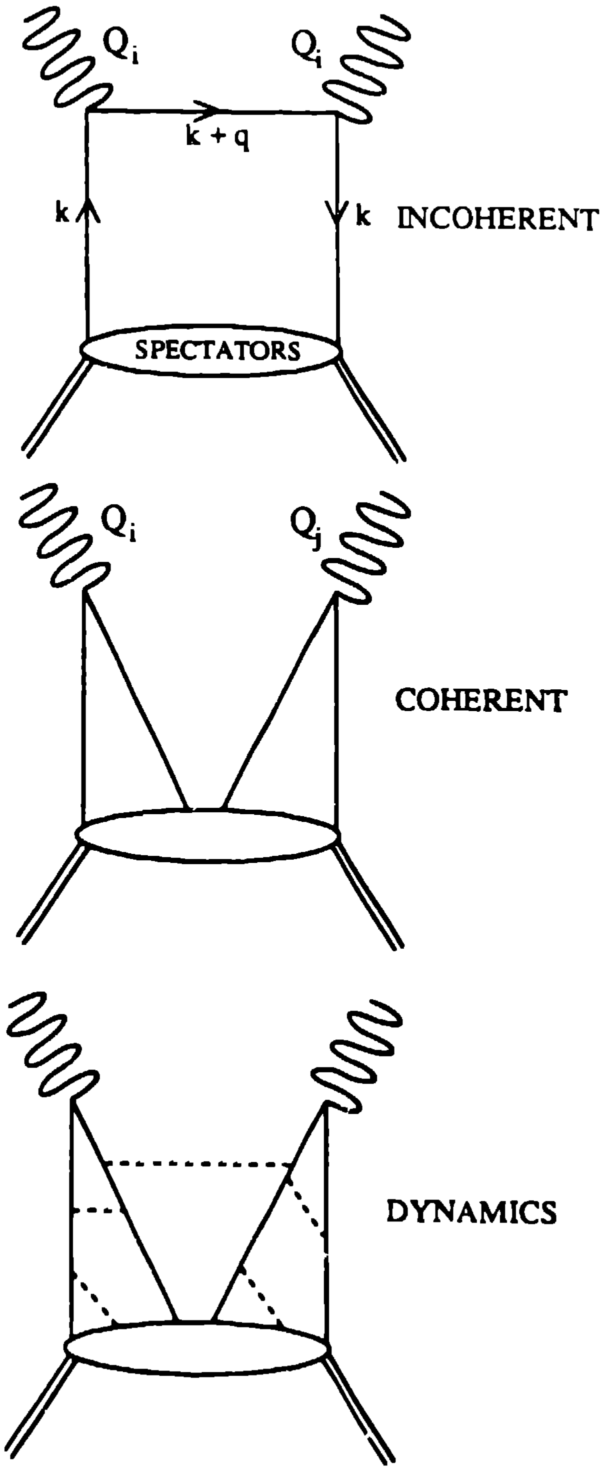

Figure 3: Generic expansion of $\mathcal{F}$.

i.e. the incoherent quasielastic scattering. In an explicit momentum space representation this reads

$$\mathcal{F}(y,q^2) \approx \sum_i Q_i^2 \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_Z}{(2\pi)^3} | < \Psi_0 | \underline{k}_1 \cdots \underline{k}_Z > |^2 \delta(k_{iz} - \mu y) \quad (15)$$

or, if $|f(\underline{k}_i)|^2$ is a single-particle momentum distribution defined by

$$|f(k_i)|^2 \equiv \left[\int \frac{d^3k}{(2\pi)^3} \right]_i | < \Psi_0 | \underline{k}_1 \cdots \underline{k}_i \cdots \underline{k}_Z > |^2 \quad (16)$$

where the integration symbol means integrate over the momenta of <u>all</u> the constituents <u>except</u> the i'th, then (in the symmetric case), (15) reduces to

$$\mathcal{F}(y,q^2) \approx Z \int \frac{d^2k_\perp}{(2\pi)^3} \int_{-\infty}^{\infty} dk_z |f(k_\perp,k_z)|^2 \delta(k_z - \mu y) \quad (17)$$

Thus for large q^2 , $\mathcal{F}(y,q^2) \equiv qW(\nu,q^2)$ scales to a function of y which measures the longitudinal momentum distribution of constituents inside the target.

It is clear from this discussion that the approach to $y-$ scaling is governed by correlations as well as explicit dynamics. The expression given in eq. (10) or (13) allows for a systematic expansion in powers of $1/q$. [Actually, with some reasonable approximations, one can translate this into an expansion in power of $e^{1/q}$]. Thus the scaling phenomenon simply reflects the fact that the target can be well described by Z scattering centers. In this sense, it is the correction and the approach to scaling that contain the really interesting physics. On the other hand, in the high energy case, where it was not known that hadrons were definitely composed of quarks, the scaling phenomena (discussed below) was the clearest evidence for the ultimate establishment of the quark model.

Typical scaling curves for electron scattering from nuclear targets ($\lesssim GeV$ range) and for neutron scattering from liquids (($\lesssim KeV$ range) are shown in Fig. 4. The theoretical discussion above leads to many interesting results which are in agreement with these data some of which are the following:

(i) The dynamical corrections (from F_i) dominate the correlations at large q with the result that scaling should be approached from above.

(ii) The leading correction requires $\partial\mathcal{F}/\partial q^2|_{y=0} \approx 0$; i.e. there is virtually no correction near the maximum at $y = 0$

(iii) For a symmetric system $\mathcal{F}(0, q^2) = \langle \mu/2k \rangle \approx 2 - 3$.

(iv) Scaling results whether F is a confining force or not. Thus even for potentials $V(r) \sim r^n$ for $r \to \infty$, the system behaves as if the constituents were free.

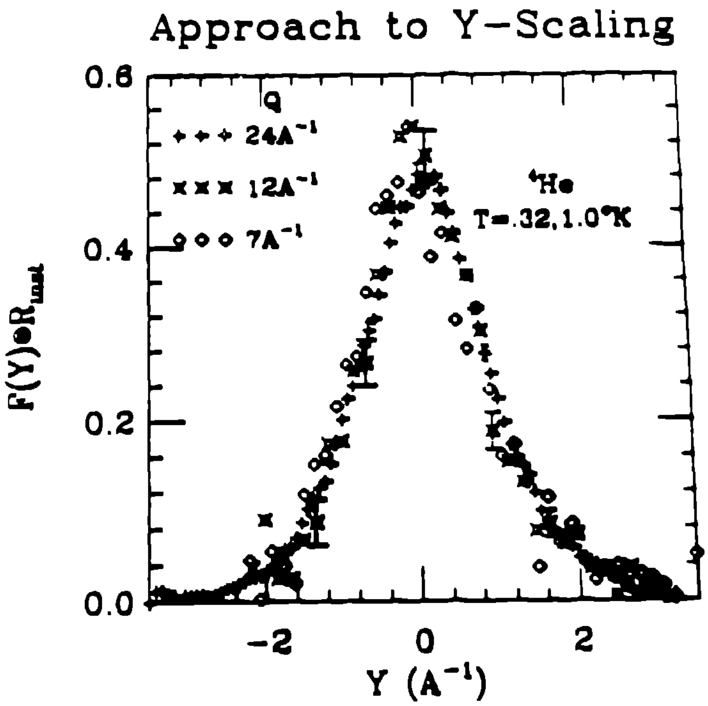

Figure 4: (a) y-scaling curve for thermal neutron scattering from liquid helium for various values of q^2(in $\AA^{-2}$).

Lastly, I would like to discuss the rôle of sum rules since these play a crucial rôle when we turn to the relativistic analysis. Returning to the representation (3) it is clear that

$$I(q^2) \equiv \int_{-\infty}^{\infty} d\nu W(\nu, q^2) \quad = \quad \sum_{ij=1}^{Z} \langle \Psi_o | Q_i Q_j e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} | \Psi_o \rangle$$

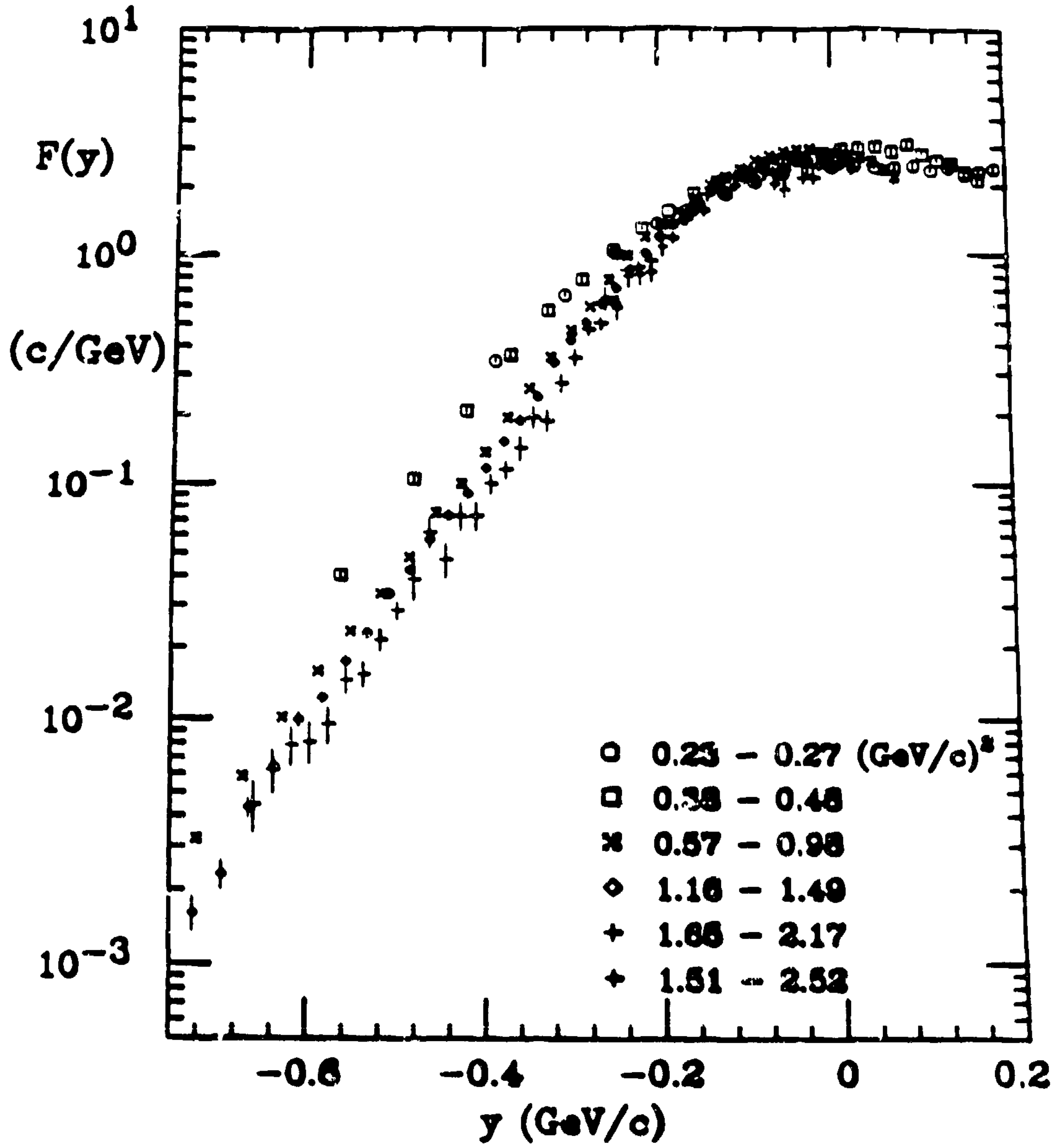

Figure 4: (b) Similar curve for electron scattering from iron nuclei with q^2 in $(\mathrm{GeV/c})^2$.

$$= \sum_{i=1}^{Z} Q_i^2 + \sum_{i \neq j=1}^{Z} Q_i Q_j \langle \Psi_o | e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} | \Psi_0 \rangle \quad (18)$$

where in the last line I have separated the terms into the coherent and incoherent contributions (as in Fig. 3). Notice that for identical particles with $Q_i = 1$.

$$I(q^2) \rightarrow \begin{cases} Z \text{ when } q^2 \rightarrow \infty \\ Z^2 \text{ when } q^2 = 0 \end{cases} \quad (19)$$

showing how the two extreme regimes pick out the incoherent from the coherent. In general, the **sum rule has integrated out the explicit dependence on dynamics so that the approach to scaling for $I(q^2)$ is completely governed by correlations alone**. In terms of the scaling variable we can write

$$\int_{-\infty}^{\infty} dy \mathcal{F}(y, q^2) \approx \sum Q_i^2 \quad (20)$$

[Notice that this is in agreement with eqs. (14) - (17) since the state $|\Psi_o\rangle$ is normalized to unity].

It is straightforward to derive other sum rules; for example

$$\int_{-\infty}^{\infty} dy y^2 \mathcal{F}(y, q^2) \approx (\sum Q_i^2) \frac{2}{3} \langle \frac{T}{\mu} \rangle \quad (21)$$

where $T \equiv \vec{p}^2/2\mu$ is the kinetic energy operator. Thus this second moment of $\mathcal{F}$ measures the mean kinetic energy of the constituents. More generally one can derive an infinite sequence of sum rules which relate moments of $\mathcal{F}$ to matrix elements of operators:

$$\int_{-\infty}^{\infty} dy y^{2n} \mathcal{F}(y, q^2) \approx (\sum_i Q_i^2) \langle \Psi_0 | \sum_{m=0}^{\infty} a_m p^{2(n-m)} \left(\frac{p}{\mu q} \right)^m | \Psi_0 \rangle \quad (22)$$

Although this is not particularly useful in non-relativistic systems where one can work directly with the original expression such as eq. (10), its analogue in relativistic field theory turns out to be the key to progress. This is because the expression in

(22) factorizes into a probe-dependent target-independent piece (i.e. $\sum Q_i^2$) and a target-dependent matrix element which is probe-independent. This effectively separates the ultraviolet part of the problem from the infra-red bound state aspects. In the relativistic case, to which we immediately turn, this will bring the behavior of currents on the light cone.

III RELATIVISTIC FIELD THEORY (QCD)[6]

Let us first discuss some preliminaries. The relativistically covariant generalization of the structure function W of eq. (2) is given by:

$$W_{\mu\nu}(p,q) = \bar{\sum}_N | < N|j_\mu|p > |^2 \delta^{(4)}(p+q-p_N) \tag{23}$$

where the bar implies an average over target spin and j_μ is the electromagnetic current; (for neutrino scattering this becomes the appropriate weak current). Unlike (2) this incorporates transitions due to both longitudinal and transverse virtual photons. $W_{\mu\nu}$ can be decomposed into scalar amplitudes $W_i(q^2,\nu)$.

$$W_{\mu\nu}(p,q) = -W_1(q^2,\nu)g_{\mu\nu} + W_2(q^2,\nu)p_\mu p_\nu + \cdots\cdots \tag{24}$$

where $\nu \equiv p.q/M$, M being the mass of the target. In the Lab frame where $\underline{p} = \underline{0}$, $\nu = q^0$, the energy lost by the projectile. As before, a use of unitarity (i.e. completeness of the final set of states $|N>$) allows one to express $W_{\mu\nu}$ as a ground state expectation value, analogous to eq. (3):

$$W_{\mu\nu}(p,q) = \int d^4x e^{iq\cdot x} < p|[j_\mu(x), j_\nu(0)]|p > \tag{25}$$

With quarks as the fundamental degrees of freedom which carry charge, the electromagnetic current is $j_\mu = \sum_i \bar{q}_i Q_i \gamma_\mu q_i$ where the sum runs over all quark-types. Note also that $W_{\mu\nu} = Im\ T_{\mu\nu}$, where $T_{\mu\nu}$ is the corresponding Compton amplitude obtained from (25) by replacing the commutator by a time ordered product.

Let us now examine more explicitly why the light-cone plays a crucial rôle when $q^2 \to \infty$. To do so introduce light-cone variables

$$\begin{aligned} q_\pm &= q_0 \pm q_z \\ \text{and } x_\pm &= x_0 \pm z \end{aligned} \tag{26}$$

with the z-direction defined along q, [i.e. $\underline{q}_\perp = \underline{0}$]. Thus $q^2 = q_+q_-$, $x^2 = x_+x_- - x_\perp^2$ and $q.x = 1/2(q_+x_- + q_-x_+)$. Now, in the large q^2 limit

$$\begin{aligned} q_+ \approx 2\nu[\,1 - q^2/4\nu^2 + \cdots] &= 2\nu[\,1 - x^2/q^2 + \cdots] \\ \text{and } q- \approx q^2/2\nu[\,1 - 3/4\ q^2/\nu^2 + \cdots] &= -x[\,1 -^{3x^2}/q^2 + \cdots] \\ \text{where } x \equiv -q^2/2\nu. & \end{aligned}$$

The limit $q^2 \rightarrow \infty$, with x fixed defines the Bjorken limit[1,5]. In this limit $q^2 \approx 2\nu \rightarrow \infty$. By virtue of the properties of Fourier transforms this drives $x_- \sim 0(2/q_+) \sim 0(1/\nu)$ in the representation (26). Similarly the major contribution to the x_+ integration comes from the region $x_+ \sim 0(2/q_-) \sim 0(2/x)$. Clearly, then, the region that dominates the integrand in eq. (25) in the Bjorken limit is given by $x^2 \approx -x_\perp^2 \leq 0$, i.e. whenever x_μ is space-like or null. On the other hand, causality requires that the commutator in (25) vanishes outside of the (forward) light-cone, i.e. the integrand can only be non-zero when x_μ is time-like or null ($x^2 \geq 0$). Thus, in the Bjorken limit, all of the contribution to the integral can only come from x_μ null, i.e. from the light-cone itself $x^2 \approx 0$. We therefore need to know the behaviour of products of currents near $x^2 \approx 0$. To get an idea of what this involves it is useful to consider a toy model:

The toy model consists of treating the fundamental fields $\phi(x)$ (the quarks) as scalars and defining a fictitious scalar current $j(x) = \phi^2(x)$ which is a bilinear in $\phi(x)$ – just as the real current $j_\mu(x)$ is bilinear in the quark fields $q(x)$. We then manipulate the fields as if they were free. In that case the standard Wick expansion leads to

$$\begin{aligned} T[\,j(x)j(0)\,] &= T[\,\phi^2(x)\phi^2(0)\,] \\ &= -2\Delta_F^2(x,m^2) + 4i\Delta_F(x,m^2)\phi(x)\phi(0) + \phi^2(x)\phi^2(0) \end{aligned} \tag{27}$$

where

$$\Delta_F(x,m^2) \doteq \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot x}}{k^2 - m^2 + i\epsilon} \tag{28}$$

is the Feynman propagator, m being the mass associated with $\phi(x)$. Diagramatically, the Compton amplitude, of which W is the imaginary part, is shown in fig. 5. The first term contains no operator and gives rise to a disconnected graph which does not contribute to the physical deep inelastic scattering. The other two terms give contributions which are precisely analogous to the result of the non-relativistic

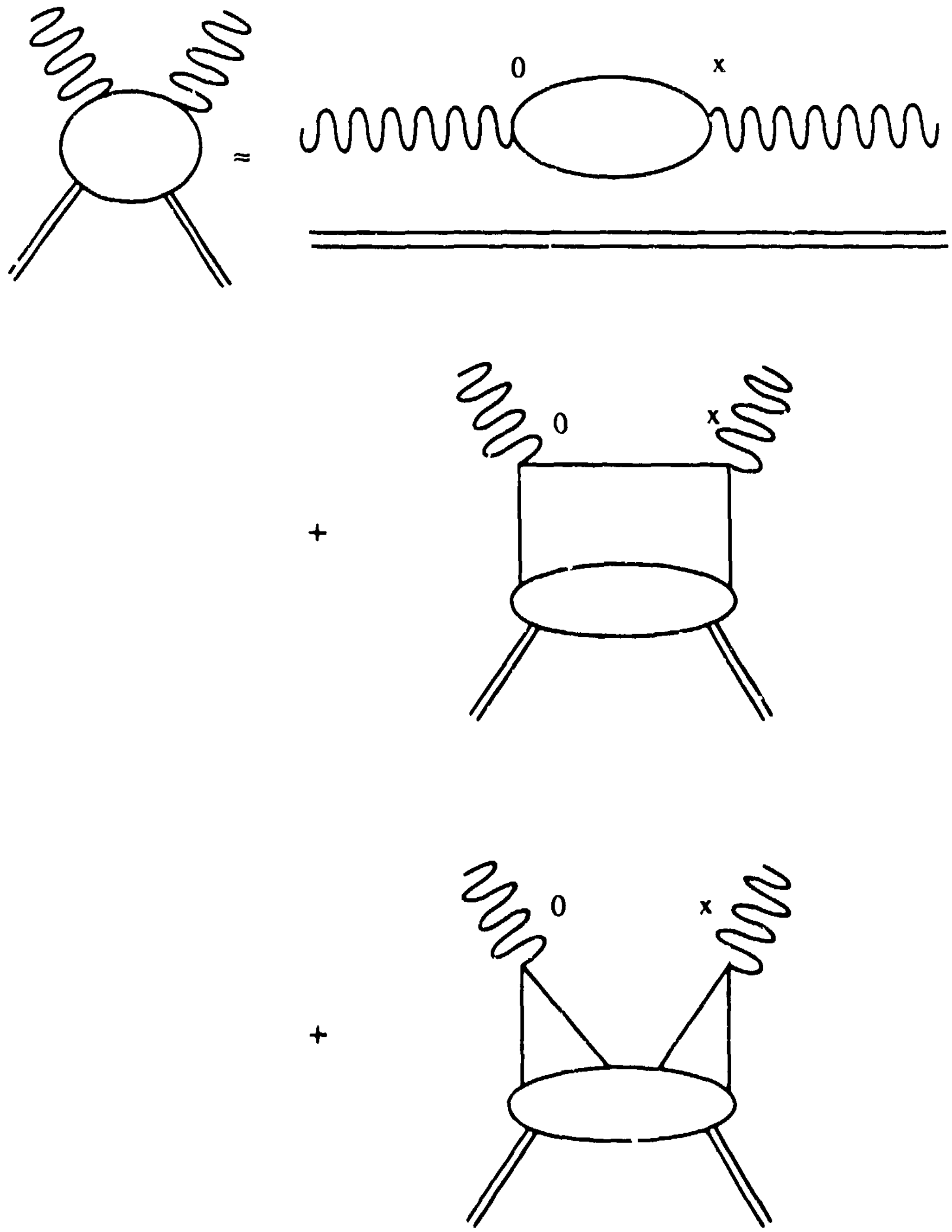

Figure 5: Analog expansion to fig. 3 of F

analysis, and which break up into coherent and incoherent pieces as in fig. 3. In fact, the analogy can be taken even further when we recall that when $x^2 \approx 0$

$$\Delta_F(x, m^2) \approx \frac{i}{4\pi^2} \frac{1}{x^2 - i\epsilon} + O(m^2 x^2) \tag{29}$$

so that the second term in (27) dominates the third when $x^2 \approx 0$. Thus the leading behaviour for W is given by

$$W(q^2, \nu) \approx Im \int d^4 x e^{iq \cdot x} \Delta_F(x, m^2) \langle p|\phi(x)\phi(0)|p\rangle. \tag{30}$$

Suppose now that we introduce a momentum distribution function

$$|f(k)|^2 \equiv \int d^4 x e^{-ik \cdot x} \langle p|\phi(x)\phi(0)|p\rangle \tag{31}$$

then eq. (30) can be re-expressed as

$$W(q^2, \nu) \approx \int \frac{d^4 k}{(2\pi)^4} |f(k)|^2 \delta[(k + q)^2 - m^2] \theta(k_0 + q_0). \tag{32}$$

Now, in the Bjorken limit $(k + q)^2 - m^2 \approx 2\nu(k_- - x)$ which immediately leads to the scaling result

$$\begin{aligned} \nu W(q^2, \nu) &\equiv F(x, q^2) \\ &\approx \int \frac{d^4 k}{(2\pi)^4} |f(k)|^2 \delta(k_- - x) \end{aligned} \tag{33}$$

This is clearly the analogue of the non-relativistic many-body formula derived in eq. (17) and justifies identifying $|f(k)|^2$ of eq. (31) as a momentum distribution function. It shows that νW scales to a function of x which in the Lab frame measures the k_- ("the longitudinal light-cone momentum") distribution of constituents in the target. The situation in this toy model is therefore just like the non relativistic case.

The situation in the real world is more complicated; fields cannot be treated as if they were free. However, the generalization from the free to interacting case is actually quite straightforward. The crucial characteristic of the expansion (27) which was based on treating ϕ as a free field is that it is in the form of c_- number singular functions of x^2 (such as Δ_F) multiplied by (composite) operators [e.g. $\phi(x)\phi(0)$]. Wilson suggested (and it was later proven valid) that this structure is maintained even in the fully-interacting theory; so, for the scalar case, one would write:

$$T[j(x)j(0)] \approx \sum_m C_m(x^2)O_m(x) \tag{34}$$

where the $C_m(x^2)$ are functions like $\Delta_F(x^2)$ which are singular near the light cone and the $O_m(x)$ are the complete set of <u>all</u> possible composite operators occuring in the theory. Notice that the $O_m(x)$ are, like $\phi(x)\phi(0)$ of the toy model, <u>not</u> local operators (i.e. they depend on at least two different space-time points, x_μ and 0 in this case). Near the light-cone, however, the operators $O_m(x)$ can be expanded in a Taylor series whose coefficients <u>are</u> local operators:

$$O_m(x) = \sum_n x_{\mu_1}\cdots x_{\mu_n} O_{mn}^{\mu_1\cdots\mu_n}(0) \tag{35}$$

Inserting this in (34) we obtain the <u>operator product expansion</u>:

$$T[j(x)j(0)] \approx \sum_{m,n} C_m(x^2)x_{\mu_1}\cdots x_{\mu_n} O_{mn}^{\mu_1\cdots\mu_n}(0) \tag{36}$$

From the intuition gained in the toy model, where the operators $O_m(x)$ were interpreted as analogous to the wave-function of the non-relativistic theory the expansion (35) seems a little strange. For it is as if one were expanding a spatial wave-function around the origin ($x \sim 0$) in a Taylor series expansion. However, for the Bjorken limit this is a natural thing to do since knowledge of the most singular behaviour of the $C_m(x^2)$ is in principle sufficient to determine the large q^2 behaviour of W.

From ordinary dimensional analysis one can deduce from (36) that the most singular $C_m(x^2)$ occur for operators $O_{mn}^{\mu_1\cdots\mu_n}$ which are <u>bilinears</u> in the fundamental fields (i.e. quarks and gluons). These are the operators of lowest <u>twist</u> (= its dimension - its spin). Higher twist operators are multilinear in the quark and gluon fields and give rise to less singular $C_m(x^2)$ and therefore to corrections to the leading large q^2-behaviour.

Substituting this light-cone operator product expansion (36) into the definition of the virtual Compton amplitude - of which the physical structure functions are the imaginary parts - leads to an infinite sequence of sum rules:

$$M(q^2,n) \equiv \int_0^1 dx\, x^{n-2} F_2(x,q^2) \approx c(q^2,n)\langle p|O_n|p\rangle \ (n \geq 2) \tag{37}$$

Here, the $c(q^2,n)$ are related to Fourier transforms of the $C_m(x^2)$; they are independent of the target but probe (and therefore q^2) dependent. The operators O_n are basically the invariant scalar components of the $O_{mn}^{\mu_1 \cdots \mu_n}$; their matrix elements are, of course, target dependent, though independent of the probe (and therefore q^2). It is clear that the operator product expansion has allowed one to separate the infrared features of the problem (represented by the matrix elements) from the ultra-violet (represented by the $c(q^2,n)$).

By this ruse the determination of q^2-dependence is disentangled from the knotty problems of dealing with the structure of the target - which, of course, is a non-perturbative infrared problem. The leading q^2 behaviour of the moments is thereby tied to the behaviour of the $c(q^2,n)$ and therefore the the twist-2 quark and gluon bilinear operators. Now, QCD is asymptotically free, which means that as q^2 increases, the effective coupling decreases $[g^2 \sim 1/ln\,(q^2/\mu^2)]$ allowing an accurate perturbative estimate for the $c(q^2,n)$. Technically, this is accomplished by using the renormalization group which effectively sums graphs and leads to

$$c(q^2,n) \sim [\,ln\, q^2/\mu^2\,]^{-\gamma_n} \tag{38}$$

where the γ_n are related to the anomalous dimension of the O_n and are all calculable This behaviour has been brilliantly confirmed by experiment as shown in fig. 6 and (because $\gamma_{n+1} > \gamma_n > 0$) leads to a pattern of scale breaking illustrated in fig 7.

The target dependent piece, $\langle p|O_n|p\rangle$, remains in general undetermined since it requires a solution of the bound state problem. Thus the light-cone only determines the q^2-evolution of the structure functions - their shape and normalization are infrared properties. Remarkably, however, the normalization can in fact be, in some sense, determined. The reason for this is that the lowest moment ($n = 2$) corresponds in eq. (40) to the 2-tensor $O^{\mu_1\mu_2}$ which must contain the energy-momentum tensor. This is not only a conserved quantity (so that its anomalous dimension $\gamma_2 = 0$) but, furthermore, its matrix elements at rest are known, being given by the mass of the target. Thus the complete right-hand-side is known. One finds

$$M(q^2,2) \qquad \int_0^1 F_2(x,q^2)\,dx$$

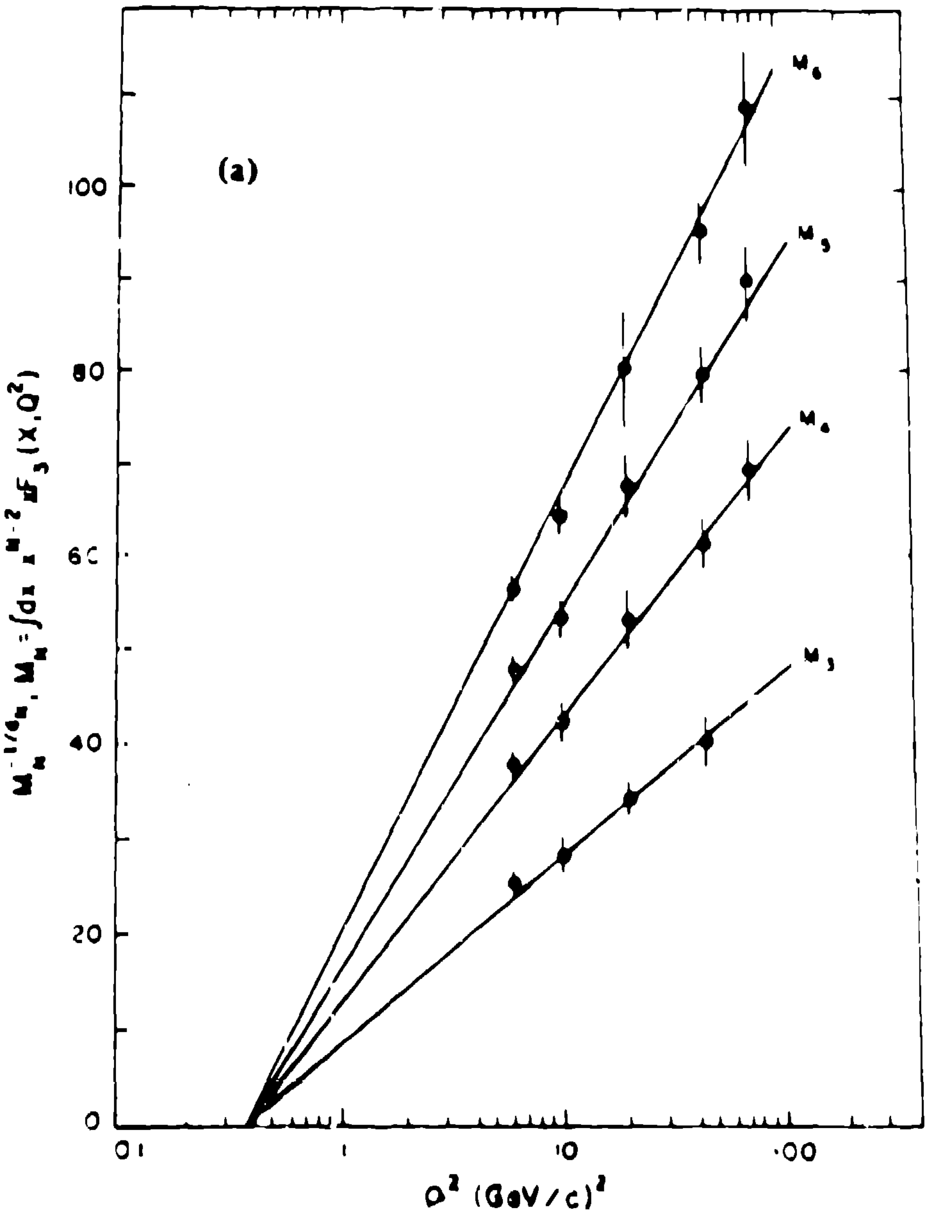

Figure 6: Structure function moments vs. q^2 showing agreement with predictions from the light-cone expansion and asymptotic freedom

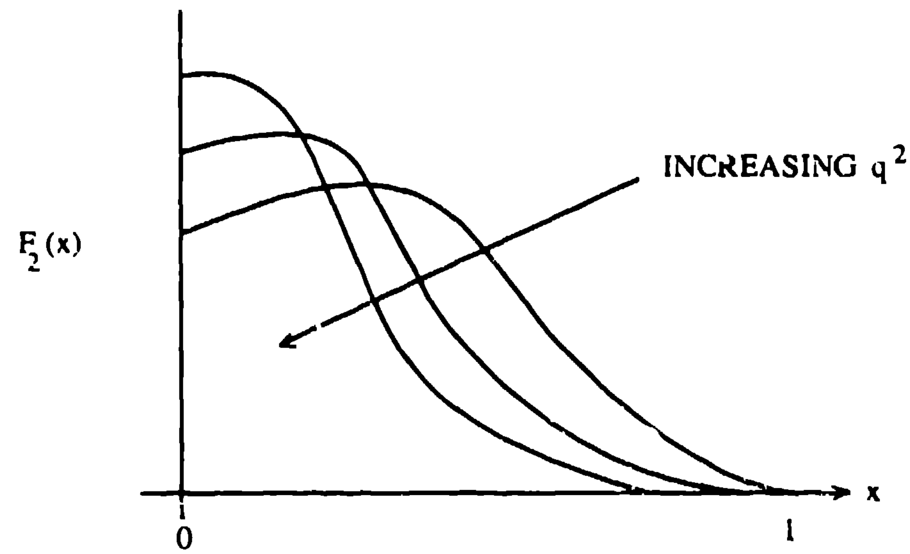

Figure 7: Pattern of scale-violations in QCD

$$= \left(\frac{\sum Q_f^2}{N_f}\right)\left(\frac{N_q}{N_q + N_g}\right) \tag{39}$$

where f means flavour. This sum rule can be thought of as measuring the fraction of momentum carried by the quarks. For $SU(3)$ this reduces to

$$\int_0^1 F_2(q^2, x)dx \approx \frac{5N}{6(3N+8)} \tag{40}$$

where N is the number of quark generations. Thus, for $N = 4$, this gives 5/42 whereas for $N = 3$, 5/34. The data are shown in fig. 8. These indicate that $M(q^2, 2)$ is approaching a constant which appears to be consistent with 3 generations. Note, incidentally, that the operator $O^{\mu_1\mu_2}$ contains another operator beyond the energy-momentum tensor and that this is not conserved and so has a non-vanishing value for its γ_2. This means that there are corrections to the sum rule, eq. (40), which are of the form $a(ln\, q^2)^{-n}$. Remarkably, a can be shown to be positive so that the approach to scaling must be from above which is in agreement with the data. Further corrections are given by the higher twist operators containing more than just two quark and gluon fields. These are down by $O(1/q^2)$ and so are presumably not of importance for high values of q^2.

An Aside - Application to the EMC Effect

A remarkable property of the sum rule, eq.(40) beyond the fact that its right-hand-side is independent of q^2 (i.e. of the probe) is that it is also independent of the target! Thus, if one introduces the difference

$$\Delta(q^2, x) \equiv \frac{F_A(q^2, x)}{A} - F_N(q^2, x) \tag{41}$$

[A denoting a nucleus and N the nucleon], then

$$\Delta M(q^2, 2) \equiv \int_0^A \Delta(q^2, x)dx \approx \frac{(C_A - C_N)}{(ln\, q^2)^n} \tag{42}$$

In fact **all** moments of Δ vanish asymptotically so ultimately Δ itself must vanish, with increasing q^2, albeit very slowly. Thus at very large q^2, the EMC effect must eventually disappear. Notice also, incidentally, that $|\Delta M(q^2, 2)|$ must decrease monotically with q^2 which is, in fact, violated when the original EMC data is

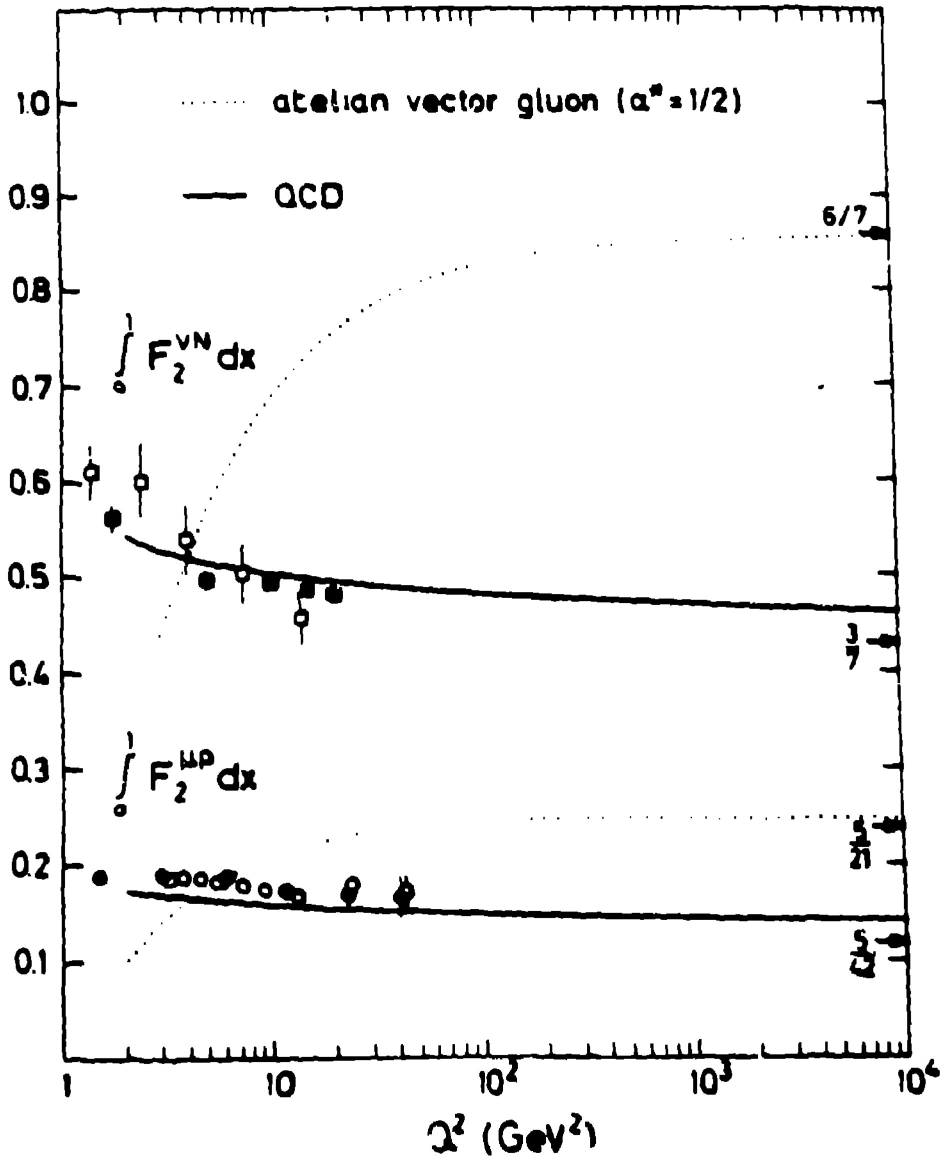

Figure 8: $M(q^2, 2)$ vs. q^2 showing asymptote to a constant from above.

compared to the later SLAC data! [7] Since that time[8] the EMC points near $x \approx O$ which were the largest deviations of Δ from zero $x \approx O$ have been amended so that the data is now consistent with this requirement on $|\Delta M(q^2, 2)|$.

Correlations, Higher Twist and Shadowing

We have seen that the operator product expansion on the light cone leads to sum rules with the structure:

$$\begin{aligned} M(q^2, 2) &\equiv \int_0^1 F_2(q^2, x)\, dx \\ &\approx \frac{< Q^2 >}{(1 + 16/3N_f)} + \frac{C}{(\ln q^2)^{\gamma_2}} + O\left(\frac{1}{q^2}\right) + \cdots\cdots \end{aligned} \quad (43)$$

The first two terms represent the lowest twist contribution arising from quark and gluon bilinears. These can be represented by graphs of the kind shown generically in fig. 5. These incorporate the naive parton model, modulated with leading logarithmic gluon radiative corrections which give rise to the second term in eq. (43). The leading corrections to these asymptotic estimates come from higher twist terms: the four-quark operator, as illustrated in fig. 5, gives rise to $O(1/q^2)$ corrections. Notice that these leading graphs are identical in structure to those that arose in the $1/q^2$ expansion for the structure function in non-relativistic many-body theory.

Let us take this connection with the many-body result seriously - after all, the basic physics is clearly the same. In that case, as one comes down to modest values of q^2 (below a few GeV^2) correlations in the system begin to dominate. Let us therefore write

$$M(q^2, 2) = M_{\mathbf{RAD}}(q^2, 2)[1 - f(q^2)] \quad (44)$$

where $M_{\mathbf{RAD}}(q^2, 2)$ just includes the "soft-gluon" radiative corrections that we typically calculated from asymptotic freedom, i.e. the first two terms in eq. (43). This is, of course, a slowly ranging function of q^2. Writing eq.(44) in this form simply factors out the QCD radiative corrections in much the same way one removes radiative corrections in QED. What remains, i.e. $f(q^2)$, contains "dynamics". Now, suppose we mimic the non-relativistic sum rule, eq. (18), and identify f with correlations in the target (i.e. loosely with $\langle e^{i\mathbf{q}\cdot(\mathbf{r}_1-\mathbf{r}_2)}\rangle$), then below the "correlation length" (a few GeV), it becomes very rapidly varying. Of course for large q^2, it rapidly vanishes. A crude approximation for f is simply the square of the elastic form factor of the target, $G_{el}^2(q^2)$:

$$\text{i.e. } f(q^2) \approx G_{el}^2(q^2) \quad (45)$$

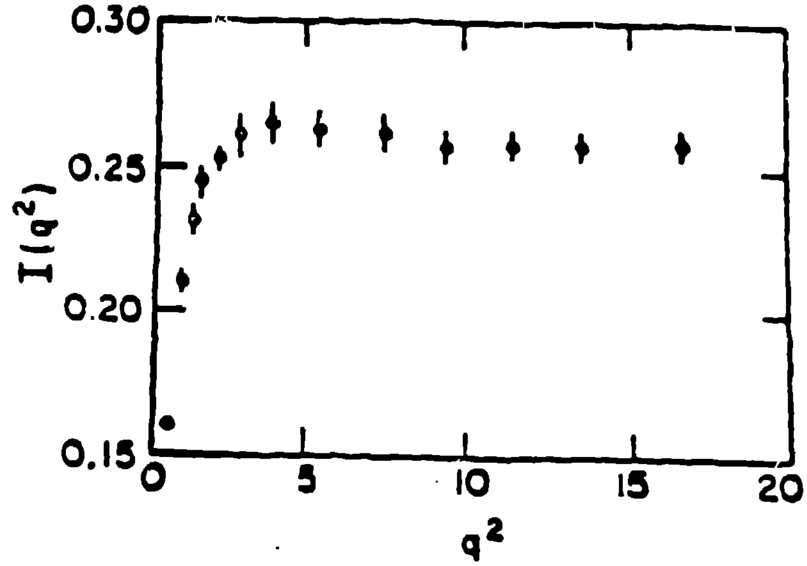

Figure 9: Approach to scaling for the sum rule

This can be "justified" by noting that diagramatically (fig. 5) f is the overlap of two triangles, each one approximately the elastic form factor. Thus, a crude approximation would have

$$M(q^2, 2) \approx M_{\mathrm{RAD}}(q^2, 2)[1 - G_{el}^2(q^2)] \tag{46}$$

For the nucleon $G_{el}(q^2)$ is a remarkably smooth function, well approximated by a dipole form:

$$G_{el}(q^2) \approx \frac{1}{(1 - q^2/M_0^2)^2} \tag{47}$$

where $M_0 \sim 0.7\,\mathrm{GeV}$. Thus the approach to the asymptotic regime governed by the light cone should, for the nucleon, be smooth - as indeed it is, as can readily be seen in fig. 9. Indeed this approach is remarkably well fit by eq. (46) On the other hand for systems such as nuclei and liquids which have spatial "edges" $G_{el}(q^2)$ is oscillatory, reflecting diffraction. In that case the approach to asymptopia should be oscillatory. For liquids this is indeed the case. Relevant data on nuclei are not yet available.

We can take this argument one step further, if we are willing to be bold: we can suppose that $f(q^2)$ dominates the approach to scaling not just for the sum rule but for the structure function itself: this suggests writing:

$$F_2(q^2, x) \approx F_2^{\mathrm{RAD}}(q^2, x)[1 - f(q^2)] \tag{48}$$

where again $F_2^{RAD}(q^2, x)$ contains only the "soft-gluon radiative corrections". In that case, it follows that

$$\tilde{F}_2(x) \equiv \frac{F_2(q^2, x)}{1 - G_{el}^2(q^2)} \tag{49}$$

should (up to logarithms) scale down to very small values of q^2 (i.e. well below a few GeV^2 and possibly even down to $q^2 = 0$!!). A fit with this formula was performed many years ago on early data and is reproduced in fig. 10. It does indeed show a remarkably good agreement.

Suppose we go even further and try to continue this formula down to $q^2 = 0$ (with ν fixed). On the left- hand-side $x \to 0$ when $q^2 \to 0$. On the right-hand-side we have

$$F_2(x, q^2) \to \frac{q^2 \sigma_\gamma(\nu)}{4\pi^2 \alpha} \tag{50}$$

where $\sigma_\gamma(\nu)$ is the total photo-absorption cross-section. If we therefore set $q^2 = 0$ and $\nu = \infty$ in eq. (49) we obtain

$$\begin{aligned} \tilde{F}_2(0) &\approx \frac{m_0^2 \sigma_\gamma(\infty)}{8\pi^2 \alpha} \\ &\approx 0.38 \end{aligned} \tag{51}$$

which is in remarkably good agreement with experiment!

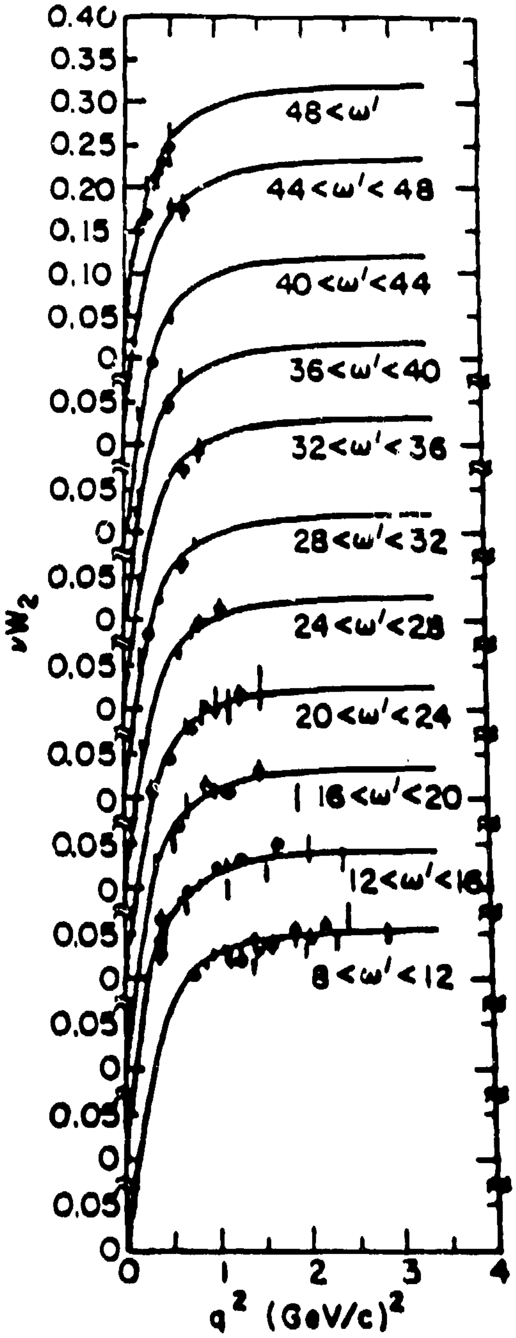

Figure 10: $F_2(x, q^2)$ vs. q^2 for fixed $x(\equiv 1/w^1)$ showing smoothness (as reflected in fig. 6). The solid lines are $[1 - G_{el}^2(q^2)]$.

References

[1] See, for example, H. D. Politzer, Phys. Rep. **14c**, 130 (1974)

[2] See, for example, S. D. Ellis, "Lectures on Perturbative QCD, Jets and the Standard Model", The Santa Fe TASI-1987 (Ed. R. Slansky and G. West), World Scientific (Singapore, 1988), Vol. I., page 174

[3] Many of these questions were discussed at this workshop and the reader is referred to other papers in these proceedings

[4] For a review, see, for example, G. B. West "The EMC Effect: Asymptotic Freedom with Nuclear Targets", Intersections Between Particle and Nuclear Physics (Ed. R. E. Mischke) American Institute of Physics (N. Y. 1984) p. 360

[5] G. B. West, Phys. Rep. **18c**, 264 (1975)

[6] For a good pedagogical introduction see, T. P. Cheng and L.-F. Li, "Gauge Theory of Elementary Particle Physics" (Oxford University Press, N. Y. 1984)

[7] G. B. West, Phys. Rev. Letts. **54**, 2576 (1985)

[8] R. Arnold, this workshop.