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Informal Report

The Normal Ionizing Shock as a Theta Pinch Preionizer

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THE NORMAL IONIZING SHOCK AS A THETA PINCH PREIONIZER

by

A. DeSilva

ABSTRACT

The normal ionizing shock wave is considered as a potential preionizer for high density ($\gtrsim$15 mtorr), high bias field ($\gtrsim$ 1 kG) theta pinches. Available experimental data compare well with the theory of Kunkel and Gross. Limitations to the validity of the theory due to finite ionization rate, and to radial plasma drift are determined.

I. INTRODUCTION

A normal ionizing shock wave is a hydromagnetic discontinuity that propagates parallel to a steady bias magnetic field in a cold ionized gas. Figure 1 shows the idealized geometry for a plane wave of infinite transverse extent. Here it is assumed the wave propagates between two plane conductors that lie parallel to the initial magnetic field. A current flows in the front, where the gas is ionized, and the j x B impulse due to this current sets the plasma behind the shock in motion in a direction parallel to the plates but transverse to $\underline{B}_x$. The plasma is also set in motion in the direction of shock propagation. The adaptation of this shock to a cylindrical geometry is shown in Fig. 2. The vacuum vessel may be an insulator, with the electrodes being a coaxial structure at one end, as the current flows easily along field lines in the absence of a metal wall.

II. THEORY AND EXPERIMENT

The theory of these shocks has been treated by several authors,[1-4] and experiments comparing with the theory have been performed in several laboratories.[5-9] In general, the experiments support the theory of Kunkel and Gross[1] (hereafter referred to as K-G) in the few parameters that have been measured--the parametric variation of shock speed with driving current and bias magnetic field, fill density and gas type. The only reported measurements of temperature were performed averaging over the full length of the tube, and show temperatures in good agreement with the predicted temperature immediately behind the shock.[10] The theory also predicts axial flow behind the shock at speeds of 40-60% of shock speed. This leads to a nonuniform axial density, which has been observed by Cooper[10] and by Brennan.[6]

III. LIMITATIONS TO THEORY

The K-G theory is written for a plane geometry, and makes several assumptions that limit its validity. In cylindrical geometry, the current density in the front is no longer uniform, leading to variations in shock conditions with radius. The shock front is, as a result, no longer plane. In addition, a j_θ current appears in the rotating plasma behind the front. It is this current, interacting with B_z, that provides the centripetal force required to hold the plasma against radially outward drift. However, as this current is damped by finite resistivity, the plasma drifts radially to the tube wall. An estimate of the magnitude of this effect is made as follows:

The θ current is obtained by taking the radial component of the momentum equation:

$$\rho \frac{Dv}{Dt} = \underline{j} \times \underline{B} \quad , \tag{1}$$

which, for steady state conditions yields:

$$j_\theta = \frac{V^2}{rB_z} \quad . \tag{2}$$

Now, from Ohm's Law

$$E_\theta - V_r B_z = \eta j_\theta \quad , \tag{3}$$

we find V_r, assuming $E_\theta \simeq 0$:

$$V_r = \frac{V^2}{r B_z^2} \quad . \tag{4}$$

The condition that the radial drift be negligible is that the time R/V_r for the plasma to drift radially through one radius be very much greater than the time L/V for the ionizing shock to propagate at velocity V the length L of the tube:

$$L << \frac{RV}{V_r} \quad . \tag{5}$$

A second assumption of the theory is that ionization occurs in the front instantaneously. To test this assumption, we must have a model for the ionization process. Looking at the front in its rest frame, cold gas is streaming in at speed V on the upstream side. As the gas nears the front, it feels the radial electric field E_r. We assume that some charged particles diffuse upstream from the plasma behind the front, and that these electrons are then accelerated by the electric field. Since they are bound to the bias magnetic field, the electrons may only gain energy through collisions. We assume an electron's guiding center shifts by one gyroradius in the direction of $\underline{E}$, in each collision, thus picking up energy.

We consider first the diffusion process. Electrons can diffuse only by ambipolar diffusion, with diffusion coefficient $2\,D_i = 2\,V_i^2/3\ \nu_{mi}$, where V_i is the ion thermal velocity and ν_{mi} the ion-neutral collision rate. This rate is nearly constant,[11] and we take $\nu_m = 1.5 \times 10^9$ P (Sec^{-1}) with P the fill pressure in torr. The thermal velocity is that of the post-shock gas, as determined from the K-G theory. Assuming a linear density profile we have

$$\Gamma = nV = - D \frac{dn}{dx} \simeq - D \frac{n}{\Delta} \quad , \tag{6}$$

where V is the diffusion speed.

In order that the electron can move against the inflowing gas, we must set V = U, whereupon

$$\Delta \simeq \frac{D}{V} \quad . \tag{7}$$

Δ has been calculated for a variety of conditions (see Tables I-X), and it is always small ($\lesssim 1$ cm), for conditions of interest. (P 15 mtorr.)

We turn now to the ionization process. The rate of gain of energy by an electron in the crossed E_r and B_z fields is

$$\frac{dw}{dt} = eE_r R_{Le} / \tau_e \quad , \tag{8}$$

where τ_e is the collision time for an electron. Since $R_{Le} = V_e M_e / eB_z$ and $V_e = \sqrt{2W/m_e}$, we find

$$\frac{dw}{dt} = v_D \sqrt{2 m_e} \; W^{1/2} / \tau_e \quad , \tag{9}$$

where $v_D \equiv E_r/B_z$.

For almost any conditions of interest, the appropriate collision time will be the electron-ion time, which we take to be twice the Spitzer electron-electron time:

$$\tau_e = AW^{3/2}/n_i \quad , \tag{10}$$

with A = 1.8 x 10^{39} (for MKS units). We have approximated n by 11.4. Plugging (10) into (9) and integrating we find

$$W^2 = 2\sqrt{2\, m_e}\, V_D\, n_i t/A \quad . \tag{11}$$

From Eq. (11) we can find the time required for an electron to gain energy $W_o \simeq 30$ eV, which is about the peak of the ionization cross section. We obtain

$$\tau = \gamma/n_i \quad , \tag{12}$$

with

$$\gamma \equiv AW_o^2/(2\sqrt{2\, me}\ \ V_D) \tag{13}$$

The time to an ionization event is $\tau_I = \tau + (n_n \sigma V)^{-1}$, the sum of the time to gain energy and the ionization time. We use = 0.9 x 10^{-20} m^2 as an average ionization cross section,[11] and $V = \sqrt{2\, W_o/m.}$ The electron density increases according to

$$\frac{dn}{dt} = \frac{n}{\tau_I} \quad . \tag{14}$$

Writing $n_n = n_o - n$ and integrating this equation, we find the solution for n(t) to be very insensitive to the initial density, with a characteristic time to $\sim$ 85% ionization

$$\tau^* = \frac{\gamma}{n_o} \quad , \tag{15}$$

where n_o is the initial atom density.

The front thickness implied by this time is

$$\Delta^* = V\tau^* = \frac{U\gamma}{n_o} \quad . \tag{16}$$

This thickness should be small compared to the tube dimensions for the theory to be valid.

Figure 3 shows the two limits plotted as contours in the P-B_T plane where P is fill pressure and B_T is the transverse magnetic field behind the shock, and is thus proportional to driving currents, for three different axial bias fields B_z. The limits shown are $\Delta^* = 1$ cm (Eq. 16) and $L = 2$ m (Eq. 5). The use of 1 cm for Δ^* follows from the observation that this is the value of Δ^* for which Brennan[6] sees departures from the K-G theory. When B_T approaches B_z, the condition of K-G that the plasma flow speed behind the shock be just the slow wave speed (in the shock frame), no longer admits a solution. This limit is designated by an "S" on the figures.

For smaller B_z, the radial drift limit closes in on the ionization limit and the "S" limit, until near $B_z = 1$ kG the acceptable region shrinks to near zero.

Figure 4 shows the low density region for $B_z = 5$ kG on an expanded scale, with post-shock temperatures and shock speeds from the K-G theory also indicated. The axial flow speed of the gas in the lab frame ranges from 40-60% of the shock speed.

Tables I-X give results for post-shock conditions from the K-G theory for $B_z = 5$ kG (Tables I-IV), 3 kG (Tables V-VIII), and 1 kG (Tables IX-X); for various fill pressures, for deuterium, calculated using the BASIC code of Fig. 5. The column labels are: B1 = transverse magnetic field (T), U = shock speed (cm/μs), T2 = temperature (eV), V2 = axial flow speed (cm/μs), W2 = transverse flow speed (cm/μs), E2 = electric field (volt/mm), L1 = logarithm of L (m) from Eq. (5), L2 = Δ^* from Eq. (16) (cm).

IV. APPLICATION AS PREIONIZER

Normal θ-pinch preionization fails for high densities and high bias magnetic fields, which are just the conditions for which the ExB ionization process discussed here becomes useful. For densities that are too low, below around 20 mtorr, it becomes difficult to get breakdown at all in the crossed-field process. Experiments reported in the literature have operated at fill densities down to about 15 mtorr[6] at bias fields of about 7 kG. This corresponds to $\Delta^* \simeq 1$ cm from K-G(6). It should be possible to go to lower pressures at lower bias.

The electrode structure used in all previous experiments has consisted of a central electrode having diameter about 1/3 the tube diameter, with an outer ring electrode that may,[6,7] or may not[12] extend along the tube length. All experiments have operated in nearly uniform magnetic fields.

For application as a theta pinch preionizer, it is desirable to eliminate the central electrode. If the magnetic bias field is generated by the compression coil (see Fig. 6) then field lines will naturally bend out to intercept the wall. With an added cusp coil, the field lines may be packed with any desired density at the tube ends. This makes possible the electrode structure shown in Fig. 6, consisting of two ring electrodes separated axially. They will, of course have to be split to allow penetration of the compression field. So long as the field strength at the wall is about the same as that at the coil midplane, the ionizing shock should follow field lines.

The axial motion of plasma that follows the shock may be important. If the asymmetry so induced is undesirable, it is possible to drive the shock from both ends symmetrically.

The plasma rotation accompanying the shock may be halted by crowbarring the drive, or by providing a set of axial shorting wires at the far end where the field lines intersect the wall. The latter would, of course, preclude the symmetrical drive option.

Finally it is amusing to consider the possibility of applying a controlled "bias" rotation to the plasma. If shocks were driven with the same polarity from both ends, and when they meet the banks are crowbarred through resistances, the rotation will come down slowly, controlled by the external resistance, and the pinch could be initiated at any desired rotation speed.

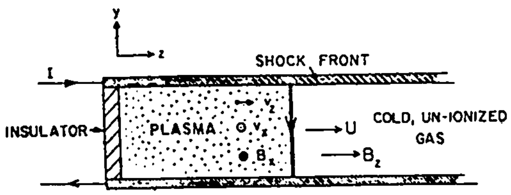

Fig. 1
Geometry used in the theory of Kunkel and Gross.[1]

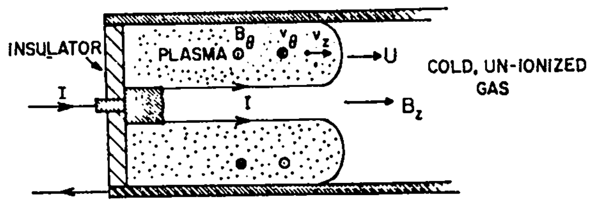

Fig. 2
Ionizing shock wave in cylindrical geometry.[6,7,10]

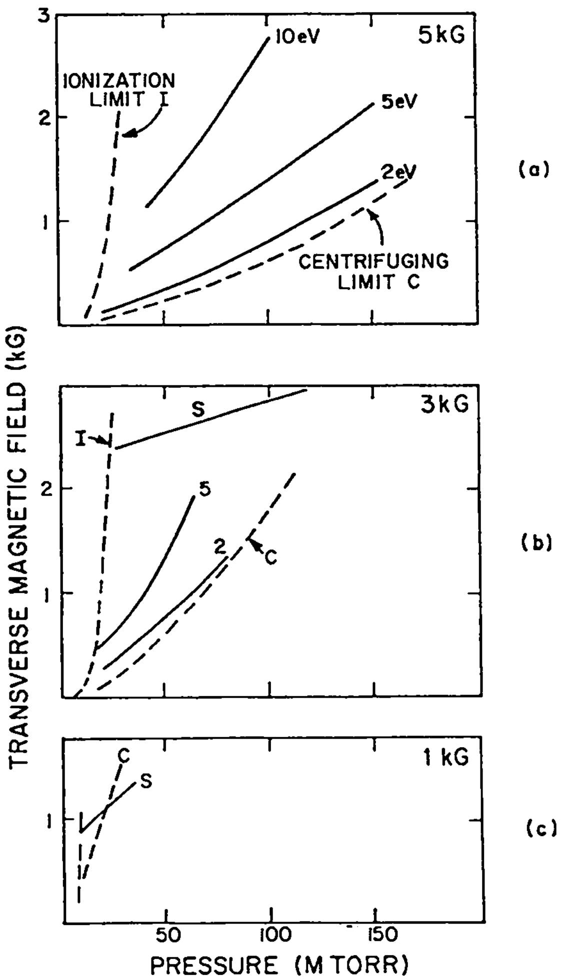

Fig. 3
Post-shock conditions from K-G theory for three axial fields. The ionization limit is from Eq. (16), with U and V_D from theory. The centrifuging limit is from Eq. (5), with 6 set equal to 2 m. The curve "S" is the limit above which the equations of K-G do not admit solution.

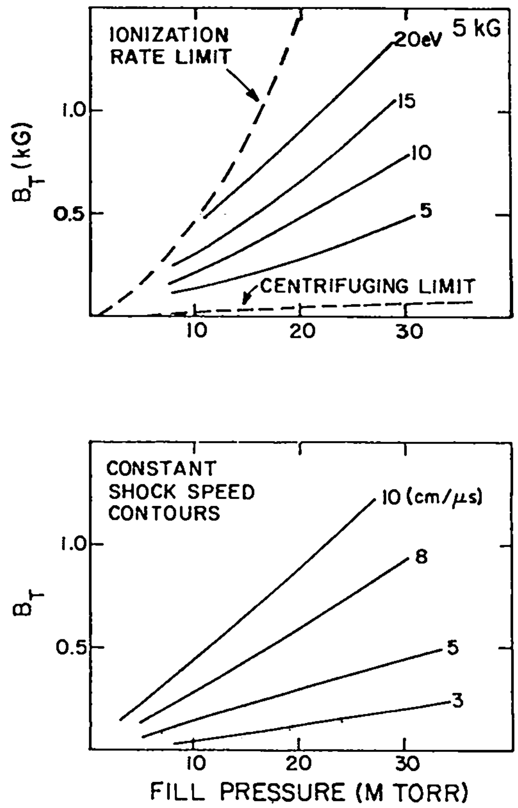

Fig. 4
Expanded scale version of Fig. 3a.

LIMITS

```
10E9=30
20N1=2
30M1=2
40M2=9.1E-31
50PRINT"IONIZ ENRG/MOLEC="E9;"   ATOMS/MOLEC="N1;"   ATOM MASS="M1
60M=1.6E-27
70K1=1.6E-19
80R=.1
90PRINT"TUBE RADIUS="R;"METERS"
100M=M*M1
110P1=3.14159
120M0=4*P1*1E-7
130B0=.5
140PRINT"BIAS FIELD="B0;"TESLA"
150FORZ=1TO7
160N0=2**(Z-1)
170PRINT"DENSITY="N0;"MTORR"
180PRINT
190PRINT" B1";"      U";"       T2";"      V2";"      W2";"      E2";
200PRINT"      L1";"      L2"
210PRINT
220FORJ=1TO20
230B1=.01*J
240IFB1>.8*B0THEN690
250K9=0
260D=.1
270I=P1*R*B1/M0
280E0=E9*K1/(M*N1)
290R1=3.55E19*N0*N1*M
300V5=SQR(B0**2/(M0*R1))
310E=R1*E0*M0/B1**2
320A=B0/B1
330B=(4*E-1)/3
340C=(-4*E+1-12*A**2)/48
350X=(-B+SQR(B**2-4*C))/2
360X=.5*X
370Y=(8*X**2-X+2*A**2)/(6*X+4*E-1)
380S1=Y-X
390S2=8*X/3-Y-5/6
400S2=S2*(A**2+X-Y)
410IFS1<S2THEN440
420X=X+D
430GOTO370
440IFK9>3THEN500
450X=X-D
460K9=K9+1
470D=D/10
480X=X+D
490GOTO370
```

```
500U=SQR(Y*B1**2/(M0*R1))
510V2=U*X/Y
520E2=(A**2+X)/Y*U*B1
530W2=-A*U/Y
540R2=R1*Y/(Y-X)
550P2=X-.5
560P3=P2*B1**2/M0
570T2=M*P3/(K1*R2*2)
580S3=1.59E3*T2**1.5
590L1=U*R**2*B0**2*S3/(R2*W2**2)
600L1=LOG10(L1)
610G=1.54E19*B0/E2
620L2=U*G/(N0*N1*3.5E19)
630U=U*1E-4
640V2=V2*1E-4
650W2=W2*1E-4
660E2=E2*1E-3
670L2=100*L2
680PRINTUSING740,B1,U,T2,V2,W2,E2,L1,L2
690NEXTJ
700PRINT
710PRINT
720PRINT
730NEXTZ
740:##.##  ###.#  ###.#  ###.#  ###.#  ###.#  ###.#  ###.#
750END
```

Fig. 5
BASIC code used to compute shock conditions in Tables I-X.

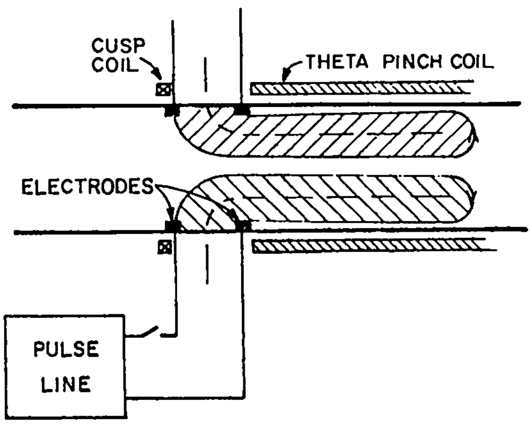

Fig. 6
Proposed scheme for preionizing a theta pinch.

TABLE I

IONIZ ENRG/MOLEC= 30   ATOMS/MOLEC= 2   ATOM MASS= 2
TUBE RADIUS= 0.1 METERS
BIAS FIELD= 0.5 TESLA
DENSITY= 1 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	14.9	51.1	5.7	-11.8	59.4	4.6	27.6
0.02	21.4	104.1	8.2	-16.4	83.5	5.0	29.2
0.03	26.4	156.0	10.3	-19.9	102.6	5.1	28.3
0.04	30.6	207.0	12.1	-22.9	119.3	5.3	28.2
0.05	34.4	257.2	13.8	-25.5	134.3	5.4	28.1
0.06	37.8	306.5	15.3	-27.8	148.3	5.4	28.0
0.07	40.9	355.2	16.9	-30.0	161.6	5.5	27.9
0.08	43.9	403.2	18.3	-31.9	174.3	5.6	27.7
0.09	46.7	450.8	19.8	-33.8	186.6	5.6	27.5
0.10	49.4	498.0	21.2	-35.5	198.5	5.6	27.4
0.11	52.0	544.9	22.7	-37.1	210.3	5.7	27.2
0.12	54.5	591.6	24.1	-38.6	221.9	5.7	27.0
0.13	56.9	638.3	25.6	-40.0	233.3	5.7	26.8
0.14	59.3	685.0	27.1	-41.4	244.7	5.8	26.6
0.15	61.6	732.0	28.5	-42.6	256.1	5.8	26.5
0.16	63.9	779.4	30.1	-43.9	267.4	5.8	26.3
0.17	66.1	327.3	31.6	-45.0	278.8	5.9	26.1
0.18	68.4	875.9	33.2	-46.1	290.2	5.9	25.9
0.19	70.6	925.3	34.8	-47.1	301.8	5.9	25.7
0.20	72.8	975.7	36.4	-48.1	313.4	5.9	25.6

DENSITY= 2 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	10.2	24.1	3.9	-8.6	43.2	4.0	13.0
0.02	14.9	50.5	5.7	-11.7	59.9	4.3	13.7
0.03	18.5	76.5	7.2	-14.2	73.2	4.5	13.9
0.04	21.5	102.0	8.5	-16.3	84.9	4.6	13.9
0.05	24.2	127.0	9.7	-18.1	95.5	4.7	13.9
0.06	26.6	151.7	10.8	-19.8	105.3	4.8	13.9
0.07	28.8	176.0	11.9	-21.3	114.7	4.9	13.8
0.08	30.9	200.0	12.9	-22.7	123.6	4.9	13.8
0.09	32.9	223.8	14.0	-23.9	132.3	5.0	13.7
0.10	34.8	247.4	15.0	-25.1	140.7	5.0	13.6
0.11	36.7	270.8	16.0	-26.3	149.0	5.1	13.5
0.12	38.4	294.1	17.0	-27.3	157.2	5.1	13.4
0.13	40.1	317.5	18.1	-28.4	165.3	5.1	13.4
0.14	41.8	340.8	19.1	-29.3	173.3	5.2	13.3
0.15	43.5	364.3	20.2	-30.2	181.3	5.2	13.2
0.16	45.1	388.0	21.2	-31.1	189.3	5.2	13.1
0.17	46.7	411.9	22.3	-31.9	197.4	5.2	13.0
0.18	48.3	436.1	23.4	-32.6	205.4	5.3	12.9
0.19	49.8	460.8	24.6	-33.4	213.6	5.3	12.8
0.20	51.4	486.0	25.7	-34.1	221.8	5.3	12.9

TABLE II

DENSITY= 4 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	6.8	10.8	2.6	-6.4	32.3	3.2	5.8
0.02	10.3	23.8	4.0	-8.5	43.5	3.6	6.5
0.03	12.8	36.7	5.0	-10.2	52.7	3.9	6.7
0.04	15.0	49.5	5.9	-11.7	60.8	4.0	6.8
0.05	16.9	62.0	6,9	-13.0	68.2	4.1	6.8
0.06	18.6	74.3	7.6	-14.1	75.1	4.2	6.8
0.07	20.2	86.4	8.3	-15.2	81.7	4.3	6.8
0.08	21.7	98.4	9.1	-16.1	87.9	4.3	6.8
0.09	23.1	110.3	9.8	-17.0	94.0	4.4	6.8
0.10	24.5	122.1	10.5	-17.9	100.0	4.4	6.7
0.11	25.8	133.8	11.3	-18.7	105,8	4.4	6.7
0.12	27.0	145.4	12.0	-19.4	111.5	4.5	6.7
0.13	28.3	157.1	12.7	-20.1	117.2	4.5	6.6
0.14	29.5	168.7	13.5	-20.8	122.9	4.5	6.6
0.15	30.6	180.4	14.2	-21.4	128.5	4.6	6.6
0.16	31.8	192.3	15.0	-22.0	134.2	4.6	6.5
0.17	32.9	204.2	15.7	-22.6	139.9	4.6	6.5
0.18	34.0	216.3	16.5	-23.2	145.6	4.7	6.4
0.19	35.1	228.6	17.3	-23.7	151.3	4.7	6.4
0.20	36.3	241.2	18.2	-24.2	157.1	4.7	6.3

DENSITY= 8 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	4.3	4.3	1.7	-5.0	25.4	2.3	2.3
0.02	6.9	10.6	2.7	-6.4	32.5	2.9	2.9
0.03	8.7	17.0	3.4	-7.5	38.6	3.2	3.1
0.04	10.3	23.3	4.1	-8.5	44.2	3.3	3.2
0.05	11.7	29.5	4.7	-9.4	49.2	3.4	3.3
0.06	12.9	35.6	5.3	-10.2	54.0	3.5	3.3
0.07	14.1	41.7	5.8	-10.9	58.6	3.6	3.3
0.08	15.1	47.7	6.4	-11.6	62.9	3.7	3.3
0.09	16.1	53.6	6.9	-12.2	67.2	3.7	3.3
0.10	17.1	59.4	7.4	-12.8	71.3	3.8	3.3
0.11	18.0	65.3	7.9	-13.3	75.4	3.8	3.3
0.12	18.9	71.1	8.4	-13.9	79.4	3.9	3.3
0.13	19.8	76.9	8.9	-14.4	83.4	3.9	3.3
0.14	20.7	82.7	9.5	-14.8	87.4	3.9	3.2
0.15	21.5	88.5	10.0	-15.3	91.4	4.0	3.2
0.16	22.3	94.4	10.5	-15.7	95.4	4.0	3.2
0.17	23.1	100.4	11.1	-16.1	99.3	4.0	3.2
0.18	23.9	106.4	11.6	-16.5	103.3	4.0	3.2
0.19	24.7	112.5	12.2	-16.9	107.4	4.1	3.2
0.20	25.5	118.8	12.8	-17.2	111.5	4.1	3.1

TABLE III

DENSITY= 16 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	2.5	1.5	1.0	-4.3	21.6	1.2	0.8
0.02	4.3	4.2	1.7	-5.0	25.5	2.0	1.2
0.03	5.7	7.2	2.3	-5.7	29.3	2.4	1.3
0.04	6.9	10.3	2.8	-6.4	32.9	2.6	1.4
0.05	7.9	13.4	3.2	-6.9	36.3	2.7	1.5
0.06	8.8	16.4	3.6	-7.5	39.5	2.8	1.5
0.07	9.6	19.4	4.0	-8.0	42.6	2.9	1.6
0.08	10.4	22.3	4.4	-8.4	45.6	3.0	1.6
0.09	11.1	25.3	4.8	-8.8	48.5	3.1	1.6
0.10	11.8	28.2	5.1	-9.3	51.4	3.1	1.6
0.11	12.5	31.1	5.5	-9.6	54.2	3.2	1.6
0.12	13.1	34.0	5.9	-10.0	57.0	3.2	1.6
0.13	13.8	36.8	6.2	-10.3	59.8	3.2	1.6
0.14	14.4	39.7	6.6	-10.7	62.6	3.3	1.6
0.15	15.0	42.6	7.0	-11.0	65.3	3.3	1.6
0.16	15.6	45.5	7.4	-11.3	68.1	3.3	1.6
0.17	16.1	48.5	7.8	-11.5	70.9	3.4	1.6
0.18	16.7	51.5	8.1	-11.8	73.7	3.4	1.5
0.19	17.3	54.5	8.6	-12.1	76.5	3.4	1.6
0.20	17.8	57.6	9.0	-12.3	79.4	3.4	1.5

DENSITY= 32 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	1.4	0.4	0.5	-4.0	20.1	-0.1	0.2
0.02	2.5	1.4	1.0	-4.3	21.7	0.9	0.4
0.03	3.5	2.7	1.4	-4.7	23.7	1.4	0.5
0.04	4.4	4.1	1.8	-5.0	25.8	1.7	0.6
0.05	5.1	5.5	2.1	-5.4	27.8	1.9	0.6
0.06	5.8	6.9	2.4	-5.7	29.9	2.0	0.7
0.07	6.4	8.4	2.7	-6.0	31.9	2.1	0.7
0.08	7.0	9.8	3.0	-6.3	33.9	2.2	0.7
0.09	7.5	11.2	3.2	-6.6	35.8	2.3	0.7
0.10	8.0	12.7	3.5	-6.9	37.7	2.4	0.7
0.11	8.5	14.1	3.8	-7.1	39.6	2.4	0.7
0.12	8.9	15.5	4.0	-7.3	41.5	2.5	0.7
0.13	9.4	16.9	4.3	-7.6	43.4	2.5	0.7
0.14	9.8	18.3	4.6	-7.8	45.3	2.6	0.7
0.15	10.3	19.7	4.8	-8.0	47.2	2.6	0.7
0.16	10.7	21.2	5.1	-8.2	49.1	2.6	0.7
0.17	11.1	22.6	5.4	-8.4	51.0	2.7	0.7
0.18	11.5	24.1	5.7	-8.6	53.0	2.7	0.7
0.19	11.9	25.6	5.9	-8.7	54.9	2.7	0.7
0.20	12.3	27.1	6.2	-8.9	56.9	2.8	0.7

TABLE IV

IONIZ ENRG/MOLEC= 30   ATOMS/MOLEC= 2   ATOM MASS= 2
TUBE RADIUS= 0.1 METERS
BIAS FIELD= 0.5 TESLA
DENSITY= 64 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	0.7	0.1	0.3	-3.9	19.6	-1.6	0.1
0.02	1.4	0.4	0.6	-4.0	20.1	-0.5	0.1
0.03	2.0	0.8	0.9	-4.1	20.9	0.1	0.2
0.04	2.6	1.3	1.1	-4.3	21.9	0.5	0.2
0.05	3.1	1.9	1.3	-4.5	22.9	0.8	0.2
0.06	3.6	2.5	1.5	-4.6	24.0	1.0	0.3
0.07	4.0	3.2	1.7	-4.8	25.2	1.2	0.3
0.08	4.4	3.8	1.9	-5.0	26.4	1.3	0.3
0.09	4.8	4.5	2.1	-5.1	27.6	1.4	0.3
0.10	5.2	5.1	2.3	-5.3	28.8	1.5	0.3
0.11	5.5	5.8	2.5	-5.4	30.0	1.6	0.3
0.12	5.9	6.4	2.7	-5.6	31.2	1.7	0.3
0.13	6.2	7.1	2.9	-5.7	32.4	1.7	0.3
0.14	6.5	7.8	3.1	-5.9	33.7	1.8	0.3
0.15	6.8	8.5	3.3	-6.0	34.9	1.8	0.3
0.16	7.1	9.1	3.5	-6.1	36.2	1.9	0.3
0.17	7.4	9.8	3.7	-6.3	37.5	1.9	0.3
0.18	7.7	10.5	3.9	-6.4	38.8	2.0	0.3
0.19	8.0	11.2	4.1	-6.5	40.1	2.0	0.3
0.20	8.3	11.9	4.3	-6.6	41.4	2.0	0.3

TABLE V

IONIZ ENRG/MOLEC= 30   ATOMS/MOLEC= 2   ATOM MASS= 2
TUBE RADIUS= 0.1 METERS
BIAS FIELD= 0.3 TESLA
DENSITY= 1 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	11.3	29.3	4.3	-9.3	28.3	3.9	26.5
0.02	16.5	60.4	6.5	-12.8	39.6	4.3	27.5
0.03	20.4	90.6	8.2	-15.4	49.9	4.4	27.6
0.04	23.8	120.0	9.7	-17.7	57.0	4.6	27.5
0.05	26.7	148.8	11.2	-19.6	64.6	4.7	27.3
0.06	29.5	177.2	12.7	-21.4	71.8	4.7	27.1
0.07	32.0	205.2	14.1	-23.0	78.8	4.8	26.9
0.08	34.5	233.2	15.6	-24.4	85.6	4.9	26.6
0.09	36.8	261.3	17.1	-25.7	92.4	4.9	26.3
0.10	39.1	289.8	18.6	-26.9	99.2	4.9	26.0
0.11	41.4	318.9	20.2	-27.9	106.1	5.0	25.7
0.12	43.6	348.9	21.8	-28.9	113.0	5.0	25.5
0.13	45.8	380.1	23.5	-29.8	120.1	5.1	25.2
0.14	48.0	413.0	25.3	-30.6	127.3	5.1	24.9
0.15	50.3	447.9	27.2	-31.3	134.8	5.1	24.6
0.16	52.7	485.5	29.2	-31.9	142.5	5.2	24.4
0.17	55.1	526.4	31.3	-32.4	150.5	5.2	24.2
0.18	57.7	571.6	33.6	-32.8	158.9	5.3	24.0
0.19	60.4	622.6	36.0	-33.1	167.7	5.3	23.8
0.20	63.4	681.6	38.8	-33.2	177.0	5.4	23.6

TABLE V (CONT)

DENSITY= 2 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	7.6	13.3	2.9	-6.9	20.9	3.2	12.1
0.02	11.4	28.7	4.5	-9.2	28.6	3.6	13.1
0.03	14.2	43.8	5.7	-11.1	35.0	3.8	13.4
0.04	16.6	58.5	6.8	-12.7	40.7	3.9	13.5
0.05	18.7	72.8	7.9	-14.0	46.0	4.0	13.4
0.06	20.7	87.0	8.9	-15.2	51.1	4.1	13.4
0.07	22.5	101.0	9.9	-16.3	56.0	4.2	13.3
0.08	24.2	115.0	11.0	-17.3	60.9	4.2	13.2
0.09	25.9	129.0	12.0	-18.3	65.6	4.3	13.0
0.10	27.5	143.2	13.1	-19.1	70.4	4.3	12.9
0.11	29.1	157.7	14.2	-19.8	75.2	4.4	12.8
0.12	30.7	172.6	15.4	-20.5	80.1	4.4	12.6
0.13	32.3	188.2	16.6	-21.2	85.1	4.5	12.5
0.14	33.9	204.5	17.9	-21.7	90.2	4.5	12.4
0.15	35.5	221.9	19.2	-22.2	95.4	4.5	12.3
0.16	37.1	240.6	20.6	-22.6	100.9	4.6	12.1
0.17	38.8	260.9	22.1	-23.0	106.5	4.6	12.0
0.18	40.6	283.4	23.7	-23.3	112.4	4.7	11.9
0.19	42.6	308.6	25.4	-23.4	118.6	4.7	11.8
0.20	44.7	337.8	27.3	-23.5	125.2	4.8	11.8

TABLE VI

DENSITY= 4 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	4.9	5.5	1.9	-5.3	16.2	2.3	5.0
0.02	7.7	13.0	3.0	-6.9	21.2	2.9	6.0
0.03	9.7	20.4	3.9	-8.1	25.5	3.1	6.3
0.04	11.5	27.7	4.7	-9.2	29.4	3.3	6.4
0.05	13.0	34.9	5.5	-10.1	33.1	3.4	6.5
0.06	14.4	41.9	6.2	-10.9	36.6	3.5	6.5
0.07	15.7	48.9	7.0	-11.7	40.0	3.5	6.5
0.08	16.9	55.8	7.7	-12.4	43.4	3.6	6.4
0.09	18.1	62.8	8.4	-13.0	46.7	3.6	6.4
0.10	19.3	69.9	9.2	-13.6	50.1	3.7	6.4
0.11	20.4	77.1	10.0	-14.2	53.5	3.7	6.3
0.12	21.5	84.5	10.8	-14.6	56.9	3.8	6.2
0.13	22.6	92.2	11.7	-15.1	60.4	3.8	6.2
0.14	23.8	100.3	12.6	-15.5	64.0	3.9	6.1
0.15	24.9	108.9	13.5	-15.8	67.7	3.9	6.1
0.16	26.1	118.2	14.5	-16.1	71.5	4.0	6.0
0.17	27.3	128.2	15.5	-16.4	75.5	4.0	6.0
0.18	28.6	139.3	16.6	-16.6	79.6	4.1	5.9
0.19	29.9	151.7	17.9	-16.7	84.0	4.1	5.9
0.20	31.4	166.0	19.2	-16.7	88.6	4.2	5.8

TABLE VI (CONT)

DENSITY= 8 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	3.0	1.9	1.2	-4.4	13.5	1.3	1.8
0.02	4.9	5.3	2.0	-5.3	16.3	2.0	2.5
0.03	6.5	8.9	2.6	-6.1	19.1	2.3	2.8
0.04	7.7	12.5	3.2	-6.8	21.7	2.5	2.9
0.05	8.8	16.0	3.8	-7.4	24.1	2.7	3.0
0.06	9.9	19.5	4.3	-8.0	26.5	2.8	3.1
0.07	10.8	22.9	4.8	-8.5	28.9	2.9	3.1
0.08	11.7	26.3	5.3	-9.0	31.2	2.9	3.1
0.09	12.6	29.8	5.9	-9.4	33.5	3.0	3.1
0.10	13.4	33.3	6.4	-9.8	35.9	3.0	3.1
0.11	14.2	36.8	7.0	-10.2	38.2	3.1	3.1
0.12	15.0	40.5	7.6	-10.5	40.6	3.1	3.0
0.13	15.8	44.3	8.2	-10.8	43.1	3.2	3.0
0.14	16.6	48.3	8.8	-11.1	45.6	3.2	3.0
0.15	17.4	52.5	9.4	-11.3	48.2	3.3	3.0
0.16	18.2	57.0	10.1	-11.5	50.8	3.3	3.0
0.17	19.1	61.9	10.9	-11.7	53.6	3.4	2.9
0.18	20.0	67.3	11.6	-11.8	56.5	3.4	2.9
0.19	20.9	73.3	12.5	-11.9	59.5	3.5	2.9
0.20	21.9	80.2	13.4	-12.0	62.8	3.5	2.9

TABLE VII

DENSITY= 16 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	1.6	0.6	0.7	-4.1	12.2	-0.0	0.5
0.02	3.0	1.9	1.2	-4.4	13.5	1.0	0.9
0.03	4.1	3.4	1.7	-4.9	15.1	1.4	1.1
0.04	5.0	5.1	2.1	-5.3	16.6	1.7	1.2
0.05	5.8	6.7	2.5	-5.7	18.2	1.8	1.3
0.06	6.6	8.4	2.9	-6.0	19.8	2.0	1.4
0.07	7.2	10.0	3.3	-6.3	21.3	2.1	1.4
0.08	7.9	11.7	3.6	-6.7	22.9	2.2	1.4
0.09	8.5	13.4	4.0	-6.9	24.5	2.3	1.4
0.10	9.1	15.1	4.4	-7.2	26.1	2.3	1.4
0.11	9.7	16.8	4.8	-7.5	27.7	2.4	1.4
0.12	10.3	18.6	5.2	-7.7	29.3	2.4	1.4
0.13	10.8	20.4	5.6	-7.9	31.0	2.5	1.4
0.14	11.4	22.3	6.1	-8.1	32.7	2.5	1.4
0.15	12.0	24.4	6.5	-8.2	34.5	2.6	1.4
0.16	12.6	26.5	7.0	-8.4	36.3	2.6	1.4
0.17	13.2	28.9	7.5	-8.5	38.2	2.7	1.4
0.18	13.8	31.4	8.1	-8.6	40.2	2.7	1.4
0.19	14.4	34.2	8.6	-8.6	42.3	2.8	1.4
0.20	15.1	37.4	9.3	-8.7	44.6	2.8	1.4

TABLE VII (CONT)

DENSITY= 32 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	0.8	0.1	0.4	-3.9	11.8	-1.5	0.1
0.02	1.6	0.5	0.7	-4.0	12.3	-0.4	0.3
0.03	2.3	1.1	1.0	-4.2	12.9	0.2	0.4
0.04	3.0	1.7	1.3	-4.4	13.7	0.6	0.4
0.05	3.6	2.4	1.6	-4.6	14.6	0.8	0.5
0.06	4.1	3.1	1.9	-4.8	15.5	1.0	0.5
0.07	4.6	3.9	2.1	-5.0	16.5	1.2	0.6
0.08	5.1	4.6	2.4	-5.2	17.4	1.3	0.6
0.09	5.5	5.4	2.7	-5.3	18.4	1.4	0.6
0.10	6.0	6.2	3.0	-5.5	19.5	1.5	0.6
0.11	6.4	7.0	3.2	-5.6	20.5	1.6	0.6
0.12	6.8	7.8	3.5	-5.8	21.6	1.6	0.7
0.13	7.2	8.6	3.8	-5.9	22.7	1.7	0.7
0.14	7.6	9.5	4.1	-6.0	23.9	1.7	0.7
0.15	8.0	10.5	4.4	-6.1	25.1	1.8	0.7
0.16	8.4	11.4	4.8	-6.2	26.3	1.9	0.7
0.17	8.9	12.5	5.1	-6.3	27.6	1.9	0.7
0.18	9.3	13.6	5.5	-6.4	28.9	2.0	0.7
0.19	9.7	14.9	5.9	-6.4	30.4	2.0	0.7
0.20	10.2	16.3	6.3	-6.4	31.8	2.1	0.7

TABLE VIII

DENSITY= 64 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	0.4	0.0	0.2	-3.9	11.7	-3.1	0.0
0.02	0.8	0.1	0.4	-3.9	11.8	-1.9	0.1
0.03	1.2	0.3	0.6	-4.0	12.1	-1.3	0.1
0.04	1.6	0.4	0.8	-4.0	12.4	-0.8	0.1
0.05	2.0	0.7	1.0	-4.1	12.8	-0.5	0.2
0.06	2.4	0.9	1.2	-4.2	13.2	-0.2	0.2
0.07	2.7	1.2	1.4	-4.2	13.7	-0.0	0.2
0.08	3.0	1.4	1.5	-4.3	14.2	0.1	0.2
0.09	3.4	1.7	1.7	-4.4	14.8	0.3	0.2
0.10	3.7	2.1	1.9	-4.5	15.4	0.4	0.2
0.11	4.0	2.4	2.1	-4.6	16.0	0.5	0.3
0.12	4.3	2.7	2.3	-4.6	16.6	0.6	0.3
0.13	4.6	3.1	2.5	-4.7	17.3	0.7	0.3
0.14	4.8	3.4	2.7	-4.7	18.1	0.8	0.3
0.15	5.1	3.8	2.9	-4.8	18.8	0.8	0.3
0.16	5.4	4.2	3.2	-4.8	19.6	0.9	0.3
0.17	5.7	4.7	3.4	-4.9	20.4	1.0	0.3
0.18	6.0	5.1	3.6	-4.9	21.3	1.0	0.3
0.19	6.3	5.6	3.9	-4.9	22.2	1.1	0.3
0.20	6.6	6.2	4.1	-5.0	23.2	1.2	0.3

TABLE IX

IONIZ ENRG/MOLEC= 30 ATOMS/MOLEC= 2 ATOM MASS= 2
TUBE RADIUS= 0.1 METERS
BIAS FIELD= 0.1 TESLA
DENSITY= 1 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	6.0	7.7	2.4	-5.8	6.1	2.2	21.7
0.02	9.2	17.0	4.0	-7.6	8.4	2.6	24.2
0.03	11.8	26.1	5.5	-8.9	10.6	2.9	24.5
0.04	14.1	35.6	7.1	-10.0	12.8	3.0	24.2
0.05	16.3	46.2	8.9	-10.7	15.2	3.2	23.7
0.06	18.8	59.3	10.9	-11.2	17.8	3.3	23.2
0.07	21.7	77.3	13.6	-11.3	20.8	3.5	22.9

DENSITY= 2 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	3.7	2.9	1.6	-4.7	4.9	1.2	8.4
0.02	6.1	7.2	2.7	-5.8	6.3	1.8	10.6
0.03	7.9	11.6	3.8	-6.6	7.7	2.1	11.3
0.04	9.6	16.2	4.9	-7.3	9.3	2.3	11.4
0.05	11.2	21.2	6.1	-7.8	10.9	2.5	11.3
0.06	12.9	27.4	7.6	-8.1	12.7	2.6	11.2
0.07	14.9	35.9	9.3	-8.2	14.8	2.8	11.1

DENSITY= 4 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	2.1	0.9	0.9	-4.1	4.2	-0.0	2.7
0.02	3.8	2.6	1.7	-4.6	5.0	0.8	4.1
0.03	5.1	4.5	2.5	-5.1	5.9	1.2	4.8
0.04	6.3	6.6	3.3	-5.5	6.9	1.5	5.1
0.05	7.5	8.9	4.2	-5.9	7.9	1.7	5.2
0.06	8.7	11.7	5.1	-6.1	9.1	1.8	5.2
0.07	10.0	15.3	6.3	-6.1	10.5	2.0	5.2

DENSITY= 8 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	1.1	0.2	0.5	-3.9	4.0	-1.6	0.8
0.02	2.1	0.7	1.1	-4.1	4.3	-0.5	1.4
0.03	3.1	1.4	1.6	-4.3	4.8	0.1	1.8
0.04	3.9	2.2	2.1	-4.5	5.3	0.4	2.0
0.05	4.7	3.1	2.7	-4.6	6.0	0.6	2.2
0.06	5.5	4.2	3.4	-4.7	6.8	0.9	2.3
0.07	6.4	5.6	4.1	-4.8	7.6	1.1	2.3

TABLE X

DENSITY= 16 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	0.6	0.0	0.3	-3.9	3.9	-3.4	0.2
0.02	1.1	0.1	0.7	-3.9	4.0	-2.2	0.4
0.03	1.7	0.3	1.0	-3.9	4.2	-1.6	0.5
0.04	2.2	0.5	1.4	-4.0	4.5	-1.1	0.7
0.05	2.7	0.7	1.8	-4.0	4.9	-0.8	0.8
0.06	3.3	1.1	2.2	-4.0	5.3	-0.5	0.9
0.07	3.8	1.5	2.6	-4.0	5.8	-0.2	0.9

DENSITY= 32 MTORR

B1	U	T2	V2	W2	E2	L1	L2
0.01	0.3	0.0	0.2	-3.9	3.9	-6.2	0.0
0.02	0.6	0.0	0.5	-3.9	4.0	-5.0	0.1
0.03	0.9	0.0	0.7	-3.8	4.1	-4.2	0.1
0.04	1.1	0.0	0.9	-3.8	4.2	-3.7	0.2
0.05	1.4	0.1	1.2	-3.8	4.4	-3.2	0.2
0.06	1.8	0.1	1.4	-3.8	4.6	-2.9	0.3
0.07	2.1	0.1	1.7	-3.7	4.9	-2.4	0.3

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