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EQUATION OF STATE DATA
FROM SHOCK WAVE OBSERVATIONS

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Abstract

Extremely short duration bursts of x-rays have been used at this project to obtain sharp images of detonation waves in explosives and induced shocks in metal blocks placed in contact with exploding high explosive charges. This report contains an analysis of these experiments, with suggestions for future experiments in which the charge assemblies will be arranged in such a way that optimum accuracy of experimental data may be obtained.

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Equation of State Data
From Shock Wave Observations

I. Introduction

By use of flash radiographic observations, that is photographs by x-rays of extremely short duration, it has proved possible to obtain very sharp images of shocks in blocks of metal placed beside an exploding charge. The position of the detonation front in the charge is similarly sharply visible in the photographs. It is interesting to investigate how feasible it would be to use these photographs to obtain data on the equation of state, or more properly on the Hugoniot curve of the metallic material.

II. The Pertinent Equations

We consider the general problem of a semi-infinite block of metal in contact with a semi-infinite explosive material. The plane of contact between the two materials is the XZ plane, normal to the y axis at $y = 0$. A plane detonation wave moves normal to a line in the xy plane, the line making an angle $0 \leqslant \psi \leqslant 90^\circ$ with the positive X axis, the detonation wave moving in such a direction that the angle between the wave front and the explosive metal interface is $90^\circ - \psi$, equal or less than 90°. Behind the detonation front the metal explosive interface recedes from its initial position in the XZ plane the interface lying in a plane making an angle ϕ with the XZ plane.

All conditions of the materials are independent of Z. A sketch of the situation in the XY plane is given in Fig. 1.

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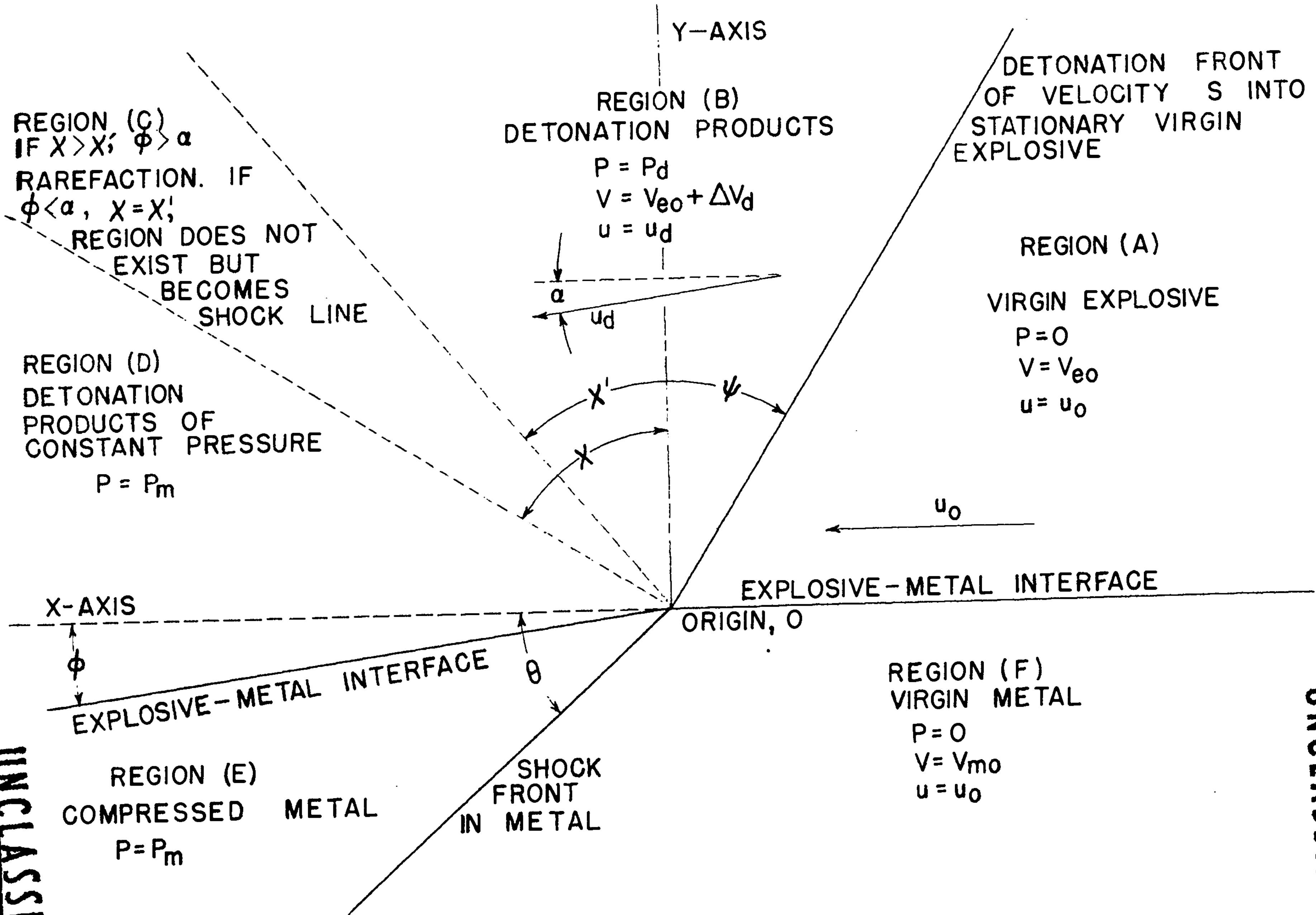

FIGURE I

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There are six general regions of material bounded by straight lines originating from the point 0 where the detonation front cuts the original metal-explosive interface on the X -axis. These are:

a) Between the X -axis and the line of angle $90^\circ - \psi$ making the detonation front: Virgin explosive.

b) Between the detonation front at $90^\circ - \psi$ and a line at $90^\circ + \chi'$: Detonation products at the constant pressure, density, and material velocity characteristic of the Chapman-Jouget conditions.

c) Between the lines of angle $90^\circ + \chi'$ and $90^\circ + \chi$: Detonation products, the pressure, density, and material velocities depending on the angle but not on the distance from the origin. This is a rarefaction region under some conditions, the pressure being lower at $90^\circ + \chi$ than at $90^\circ + \chi'$, in which case $\chi > \chi'$ and the region is of finite extent, or $\chi = \chi'$, and the region is of zero extent, the pressure being greater for angles greater than $90^\circ + \chi$ than for smaller angles.

d) Between the line at $90^\circ + \chi$ and that at $180^\circ + \phi$: Detonation products of constant pressure, density, and material velocity.

e) Between $180^\circ + \phi$ and $180^\circ + \theta$: Compressed metal of constant pressure, density, and material velocity.

f) Between $180^\circ + \theta$ and 360°: Virgin uncompressed metal.

We assume that the detonation front velocity S is accurately known, as is the angle ψ , and that the angles ϕ and θ can be measured from flash radiographs. We refer everything to coordinates in which the origin O is at rest, so that the pattern is stationary, and the materials

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in front of the two shocks, (that is between $180^\circ + \theta$ and 360°, and between 0° and $90^\circ - \psi$, the virgin explosive and metal) are streaming towards the shocks with a velocity, U_o, parallel to the X axis in the direction of negative X.

This velocity, U_o, is given by the equation:

$$U_o = S \cos^{-1} \psi \tag{1}$$

The conditions for the flow of the metal are now those of the flow of the material of (supersonic) velocity U_o around a compressing corner of angle ϕ. A shock is formed at angle θ with the original direction of flow. If V_{mo} is the original specific volume of the uncompressed metal, and

$$-\Delta V_m = V_{mo} - V_m , \tag{2}$$

is the negative of the specific volume change in the metal at the shock we then have, $-\Delta V_m = V_{mo} [\sin^2 \theta]^{-1} [1 + \cot \phi \cot \theta]^{-1}$

$$\tag{3}$$

and, $P_m = V_{mo}^{-1} \; u_o^2 \, [1 + \cot \phi \cot \theta]^{-1}$

$$= V_{mo}^{-1} \, s^2 \, [\cos^2 \psi]^{-2} \, [1 + \cot \phi \cot \theta]^{-1} \tag{4}$$

for the increase in pressure at the shock.

Of course this change in pressure and the specific volume obey the Hugoniot condition that: $\Delta E_m = \frac{1}{2} (-\Delta V_m) \, P_m$

$$= S^2 \, [\sin^2 \theta]^{-1} \, [\cos^2 \psi]^{-2} \, [1 + \cot \phi \cot \theta]^{-2} \tag{5}$$

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In order to evaluate the state at all places in the detonation products one must actually know the equation of state of the detonation products. However, certain features are of interest even if one wishes only to assume an imperfect knowledge of the gases. Let P_d be the detonation pressure, $-\Delta V_e$ be the decrease in specific volume in the detonation products, and v be the material velocity of the detonation products behind the detonation front, relative to the stationary material before the front. The equations:

$$S^2 = V_{eo}^{\,2} P_d \, (-\Delta V_e)^{-1} \tag{6}$$

$$v^2 = (-\Delta V_e) \, P_d \tag{7}$$

hold between these quantities and the detonation velocity S, and specific volume, V_{eo}, of the virgin explosive. The normal sound velocity, C_d, in the detonation products at the detonation pressure, relative to stationary material, is

$$C_d = S - v \tag{8}$$

In our coordinate system the velocity of the detonation products behind the detonation front will be U_d, of components:

$$U_{dy} = -v \sin \psi$$

$$U_{dx} = -U_o + v \cos \psi = -s \cos^{-1} \psi + v \cos \psi$$

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The square of the velocity is $U_d^2 = U_{dy}^2 + U_{dx}^2$ or:

$$U_d^2 = S^2 \cos^{-2}\psi - 2sv + v^2 = S^2[\cos^{-2}\psi - 1] + (s - v)^2$$

$$= S^2 \tan^2\psi + C_d^2 \qquad (9)$$

The flow is along the negative direction of a line making an angle α with the X axis where α is determined by the equation:

$$\tan \alpha = \frac{U_y}{U_x} = v \sin\psi [S \cos^{-1}\psi - v\cos\psi]^{-1}$$

$$= (S - C_d)\tan\psi\, [S\tan^2\psi + C_d]^{-1} \qquad (10)$$

We may now profitably resketch the figure as it applies to the detonation products. After passing the detonation front the products are moving (in the negative direction) along a line making the angle α with the X axis. They must then change their direction to flow along the interface making the angle ϕ with the X axis. If $\phi > \alpha$ they flow with

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rarefaction around a corner convex inwards. If $\phi > \alpha$ they flow with shock compression around a corner concave inwards.

If we now shift our coordinate system by the angle α so that the detonation products, immediately following the detonation front, flow along the negative X axis, the sketch becomes that of Fig. 2. There are three cases:

A) That of rarefaction behind the detonation front, $\phi > \alpha$

B) The unique case of no change behind the detonation front, $\phi = \alpha$

C) That of shock compression behind the detonation front, $\phi < \alpha$

In the case of rarefaction, if $\phi > \alpha$, the angle $\chi' - \alpha$ will be determined by the condition that, to the material in front of it, the wave appears to move with sound velocity towards the material. But the detonation front, of angle $\psi + \alpha$ from the y axis appears to recede from this material with sound velocity. Consequently:

$$\chi' - \alpha = \psi + \alpha, \quad \chi' = \psi + 2\alpha, \quad (\phi > \alpha)$$

In the case that $\phi = \alpha$ the angle χ has approached χ' as α increases to equal ϕ, and both lines disappear, since there is no change in conditions. As α becomes greater than ϕ a single shock line $\chi = \chi'$ is present, and its apparent velocity becomes supersonic, so that:

$$\chi' - \alpha < \psi + \alpha, \quad \chi' < \psi + 2\alpha \quad (\alpha > \phi)$$

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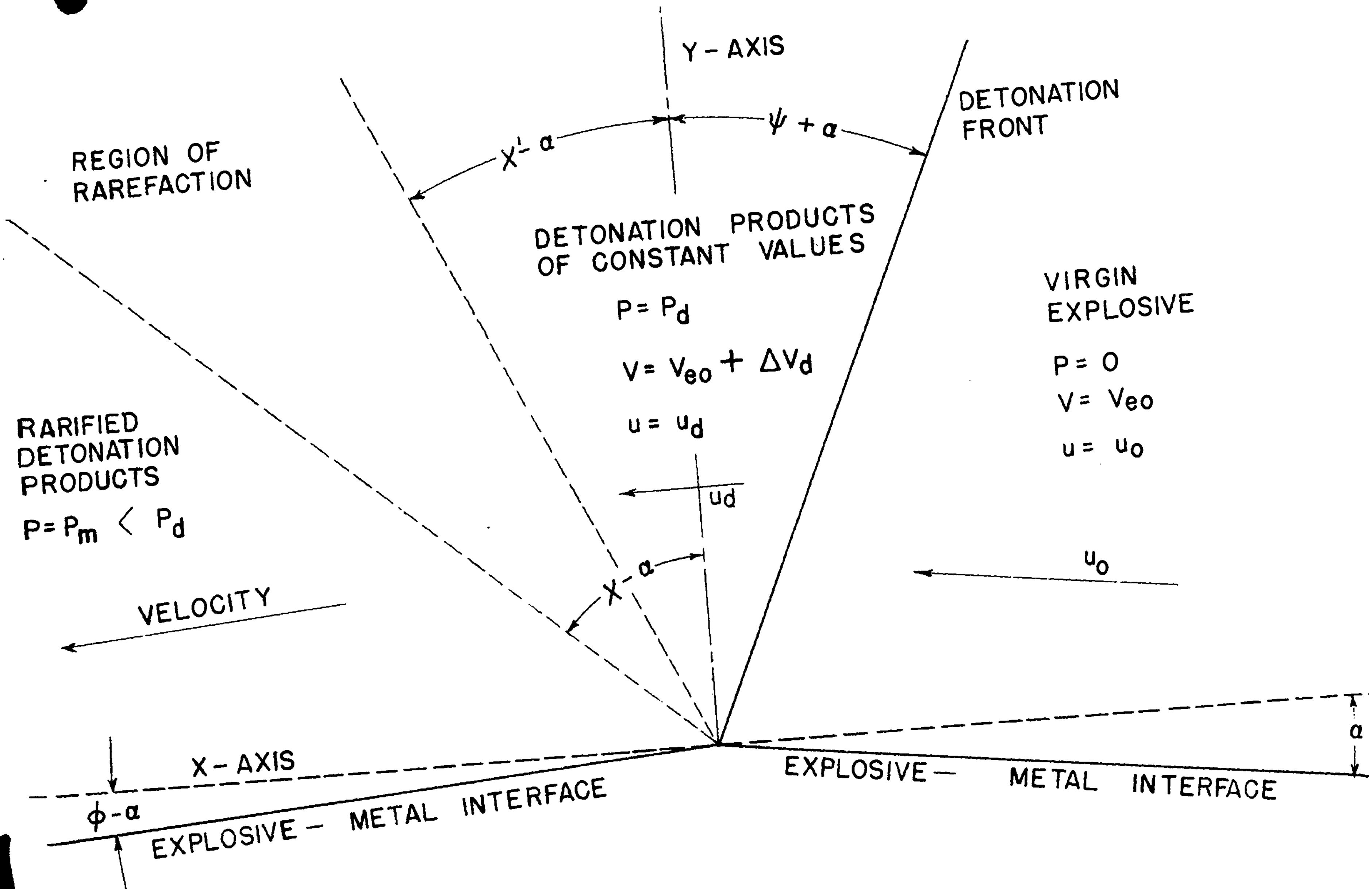

FIGURE 2A($\phi > \alpha$)

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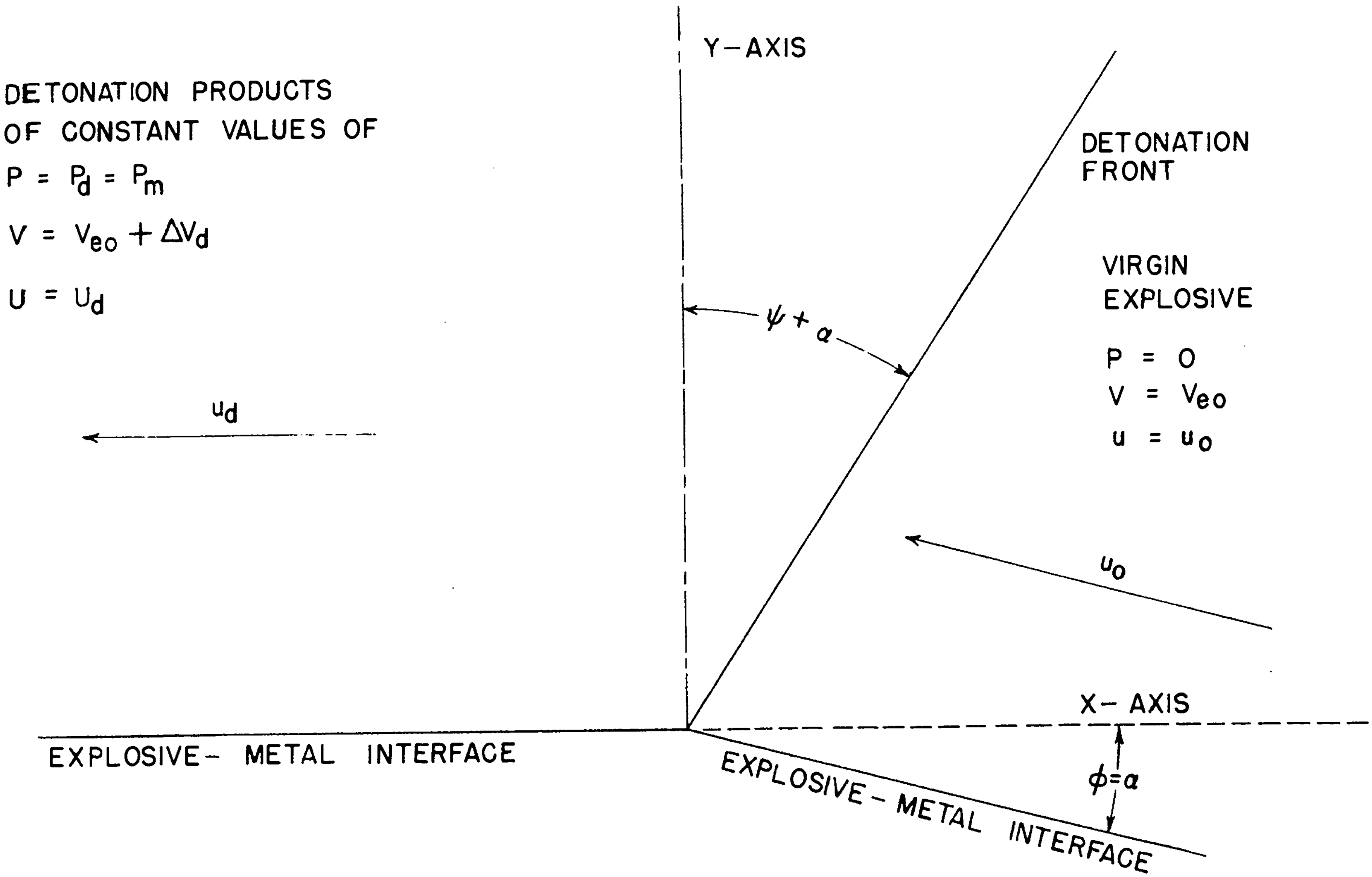

FIGURE 2 B ($\phi = \alpha$)

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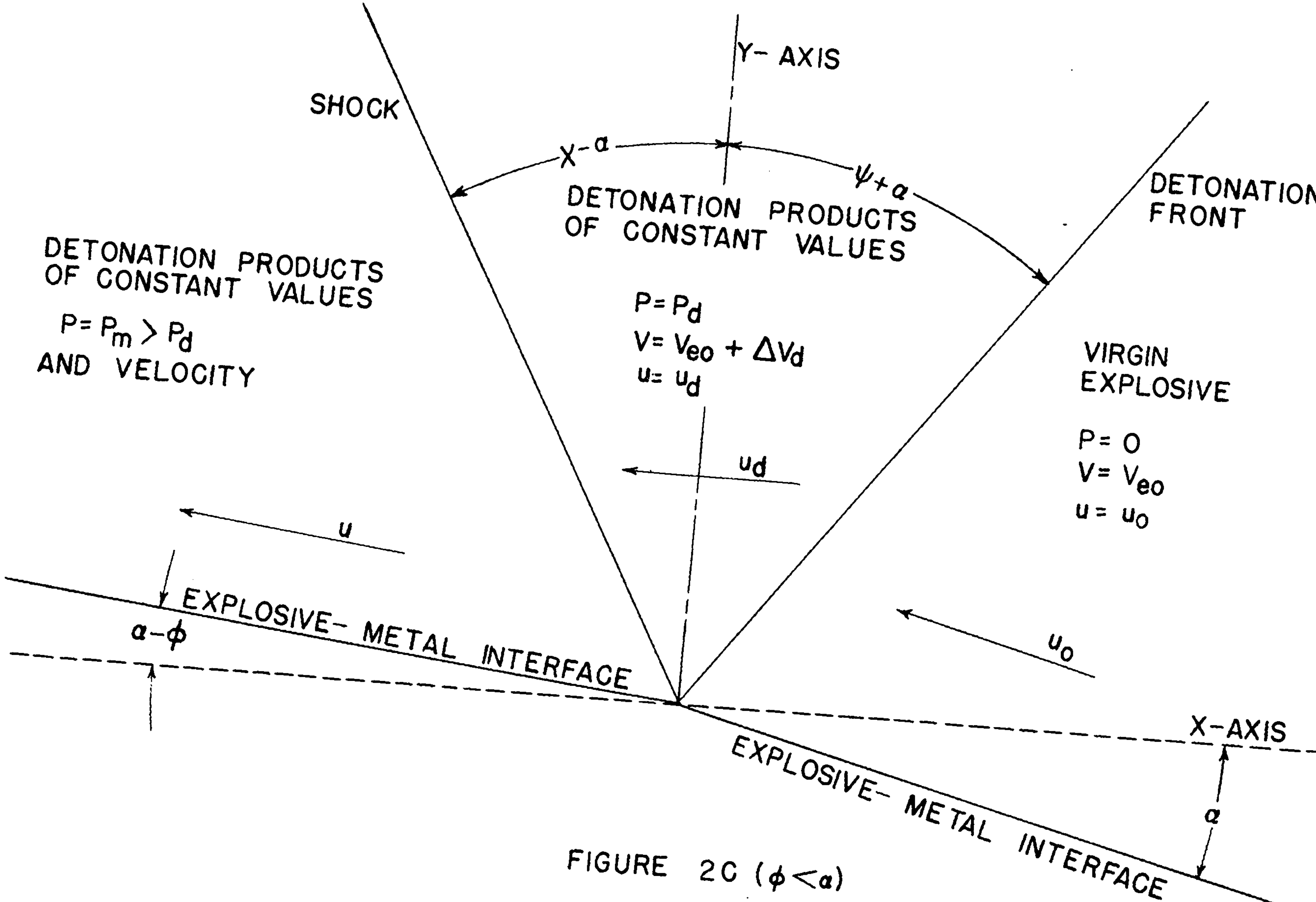

FIGURE 2C ($\phi < \alpha$)

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It is impossible to write the equations for α or the conditions that $\phi = \alpha$ without some more knowledge of the properties of the detonation than merely a knowledge of the detonation velocity S. It is convenient to assume the pressure, P_d, to be known, and to use the dimensionless fraction,

$$\lambda = \frac{P_d \, V_{eo}}{S^2} < 1 \tag{11}$$

Equations (6) and (7) show that,

$$\lambda = \frac{v}{S} = -\frac{\Delta V_e}{V_{eo}} \tag{12}$$

and from (8),

$$\frac{C_d}{S} = 1 - \lambda \tag{13}$$

Equation (10) for $\tan \alpha$ may now be written as:

$$\tan \alpha = \lambda \tan \phi \, [\tan^2 \phi + 1 - \lambda]^{-1} \tag{14}$$

It is interesting to note that the angle $\alpha + \phi$ which is the angle between the direction of motion of the detonation products behind the front and the direction of propagation of the front is given by:

$$\tan (\alpha + \phi) = \left(\frac{S}{C_d}\right) \tan \phi = \left[\frac{1}{(1-\lambda)}\right] \tan \phi \tag{15}$$

which follows from (14) by manipulation with the usual trigonometric formula for $\tan (x + y)$

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For Composition B we may estimate α by using Brinckley's caloulated values for pure cyclonite. These are:

$$S = 7.60 \times 10^5 \ cm/sec$$
$$v = 1.78 \times 10^5 \ cm/sec$$
$$\lambda = \frac{v}{S} = 0.234$$

for $\frac{1}{V_{eo}} = 1.6 \ gm/cm^3$

A numerical caloulation gives the results in Table I, which are plotted in Fig. 3.

Table I

Values of α for Different Values of ψ

ψ	α	ψ	α	ψ	α
0°	0°	20°	5.4°	40°	7.6°
5°	1.5°	25°	6.3°	45°	7.5°
10°	3.0°	30°	7.0°	50°	7.2°
15°	4.3°	35°	7.4°		

Equations (3) and (4) for the metal were written without derivation. It is convenient to introduce the ratios:

$$\sigma = \frac{S_m}{S} \tag{16}$$

of shock velocity in the metal to the detonation velocity, and,

$$\rho = \frac{V_{eo}}{V_{mo}} \tag{17}$$

of density in uncompressed metal to that in the explosive. We also use

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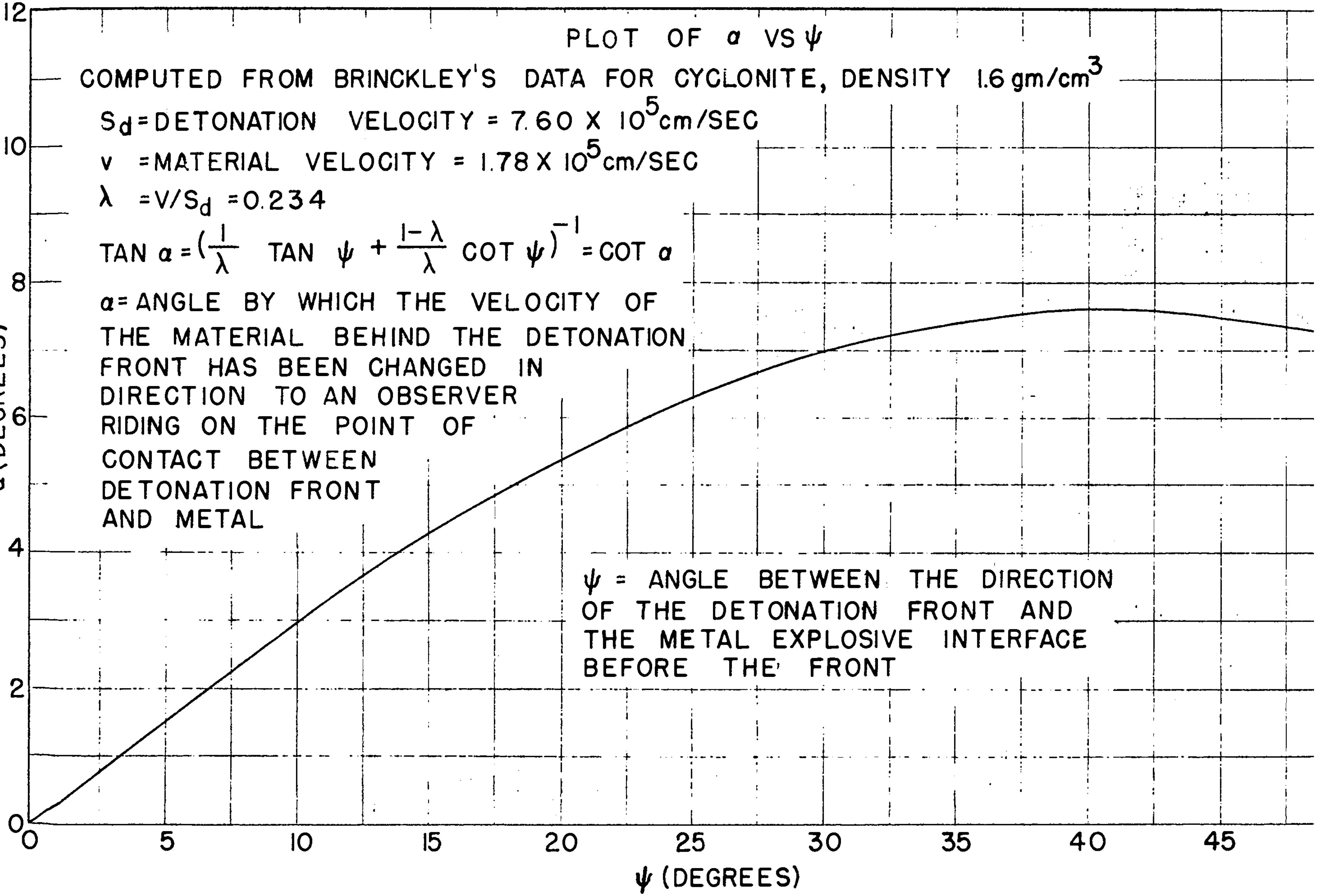

FIGURE 3

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a quantity,

$$K \frac{P_m V_{mo}}{S_m^2} = \frac{-\Delta V_m}{V_{mo}} \tag{18}$$

which is analagous to λ for the explosive.

The geometry of Fig. 1 requires that,

$$\sigma = \frac{\sin\theta}{\cos\psi} \tag{19}$$

The material velocity in the metal behind the shock, relative to metal at rest in front of the shock, is $S_m K$, and if this is added vectorially to the initial velocity $U_o = S\cos^{-1}\psi$ (in the coordinates of Fig. 1) and in a direction normal to the shock at angle $180^\circ + \theta$, one finds:

$$\tan\phi = \frac{K S_m \cos\theta\cos\psi}{S - K S_m \sin\theta\cos\psi} \tag{20}$$

or with (16) and (19),

$$\tan\phi = \frac{K\cos\theta\sin\theta}{1 - K\sin^2\theta} \tag{21}$$

If this is solved for K and (16) and (18) are used one finds (3) and (4).

It is interesting to reverse the proceeding, and to estimate ϕ in advance of the measurement, from the approximately known properties of the metal. In particular we wish to estimate the value of ψ for which $\phi = \alpha$, the condition that the pressure in the metal shock is just the detonation pressure. In this case,

$$K = \frac{\lambda}{\rho\sigma^2} \quad (\phi = \alpha,\ P_m = P_d) \tag{22}$$

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from (11), (16), (17) and (18). If the relatively small $K \sin^2\theta$ is omitted in the denominator of (21), and (19) is used to replace θ by ψ, one finds as an approximate equation, for small angles when $\tan\phi = \phi$, that

$$\phi \cong \left(\frac{\lambda}{\rho\sigma}\right) \cos\psi \sqrt{1-\sigma^2\cos^2\psi} \qquad (23)$$

The shock velocity in the metal will be approximately the sound velocity, so if σ in (23) is replaced by the ratio of the sound velocity in the metal to the detonation velocity, 7.6×10^5 cm/sec in cyclonite one obtains:

$$\phi\,(\text{Al degrees}) \cong 12 \cos\psi \sqrt{1-0.455\cos^2\psi}$$

$$\phi\,(\text{Mg degrees}) \cong 20 \cos\psi \sqrt{1-0.37\cos^2\psi}$$

$$\phi\,(\text{Fe degrees}) \cong 4.2 \cos\psi \sqrt{1-0.43\cos^2\psi}$$

$$\phi\,(\text{Pb degrees}) \cong 12 \cos\psi \sqrt{1-0.026\cos^2\psi}$$

The values used were those of Table II:

Table II

Metal	Sound velocity meters/sec	density gms/cm3	ρ	σ
Al	5100	2.7	1.7	0.67
Mg	4600	1.74	1.09	0.61
Fe	5000	7.85	4.9	0.66
Pb	1230	11.35	7.1	0.16

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The approximate expressions given for ϕ are only valid when $\phi = \alpha$. One finds that for iron with $\psi = 15^\circ$ the angle ϕ is about 307°, so that, comparing with Table I one sees that $\phi = \alpha$ when iron metal is placed on cyclonite with the interface inclined about 13° to the direction of propagation of the detonation front.

III. Experimental Possibilities

The general theoretical equations presented in the previous sections suffice to show that if, in a semi-infinite explosive charge, a plane wave detonation can be made to impact on a plane metal surface contiguous to the charge, the plane of the detonation front making an angle $90^\circ - \psi$ with that of the metal-explosive interface, then the ΔP-ΔV relations in the metal on the Hugoniot curve can be determined. It is necessary to know the angle ψ, the speed, S, of the detonation front, the original specific volume V_{om} of the metal, and to observe the two angles θ and ϕ that the shock front in the metal and the explosive metal interface behind the shock, respectively, make with the explosive-metal interface plane before the shock. The equations for ΔP and $-\Delta V$ in the metal are given as equations (3) and (4) of the previous section.

In practice observations cannot be made on a semi-infinite explosive charge, but there exist other bounding planes to the explosive. If these other boundaries of the explosive are planes parallel to the direction of propagation of the detonation front, rarefaction waves from them will intersect the regions described in the previous section, and will

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intersect them at all places immediately behind the detonation front. This means that the solutions of the previous section would be invalid, the shock front in the metal would be curved, and the angles θ and ϕ at the point of intersection would be difficult to measure with accuracy.

If, however, the boundaries of the explosive are planes which slope in towards the center of the charge, making an angle ψ with the direction of propagation of the detonation, the rarefaction waves follow behind the front at an angle $2(\psi + \alpha)$, where α is an angle given numerically (approximately) for cyclonite as a function of ψ in Table I and Fig. 3. These will disturb the shock in the metal only well behind the intersection with the detonation front if ψ is reasonably large, say 10°.

Actually if the explosive charge is surrounded by a heavy material, a metal, on the surface planes, and the angle ψ is correctly chosen for the particular metal, (namely so that $\phi = \alpha$) then no disturbance at all will be propagated inward from these planes. An estimate shows that this unique value of ψ is about 13° for iron. The angle will be smaller if a very dense metal with high sound velocity is used. Tungsten, uranium, or platinum would undoubtedly permit lower ψ values than iron, but it is questionable whether this advantage is great enough to warrant their use.

The assumption of a semi-infinite charge is also contrary-to-fact in that the origin of the detonation is not at infinite distance. Even if no rarefaction from the sides disturb the solutions there will be a rarefaction following the detonation from the point of initiation. It would seem desirable to minimize this disturbance by making the origin of rarefaction

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far from the point of measurement.

For these reasons an experimental set-up of about the proportions of Figs. 4 and 5 are suggested.

The comparatively long "run-up" in the iron-encased chamber should ensure a steady plane wave front without appreciable rarefaction behind the front. It may well be that the iron "casing" might better start with parallel sides, and only have the convergent 13° slope in the region where the tested metal block occupies one side.

Preliminary experiments with the complete iron casing might indicate whether the 13° angle estimated here is really that at which no detectable disturbance follows the detonation front. In any case neither this angle, nor one selected after experimentation, will be a perfect match such that absolutely no disturbance follows the detonation. However the disturbance from the opposite side should only propagate inward with an approximately 31° angle to the detonation front, thus not influencing the metal explosive interface, nor metal shock for a considerable distance behind the detonation front.

In addition to disturbances entering via the detonation product gases there will be reflections where the shock in the metal intersects a boundary surface, and from the "corners" of the rectangular set-up. It is difficult to estimate their effect. One principle might, however, be borne in mind in attempting to ameliorate any unpleasant observed phenomena. If a shock in a material impinges on a bounding surface with the shock plane and surface making an angle considerably less than 90° then the reflected

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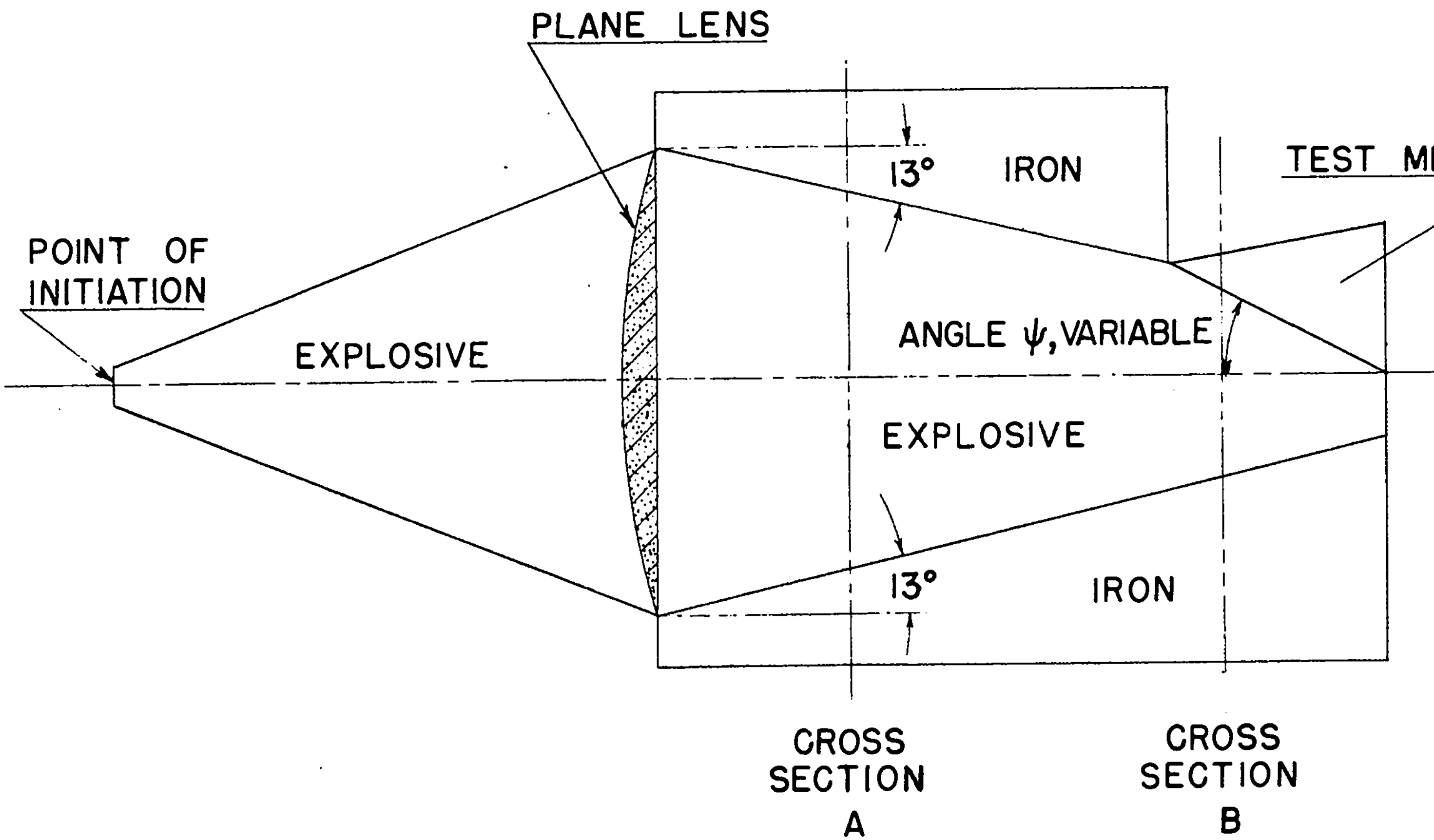

FIGURE 4

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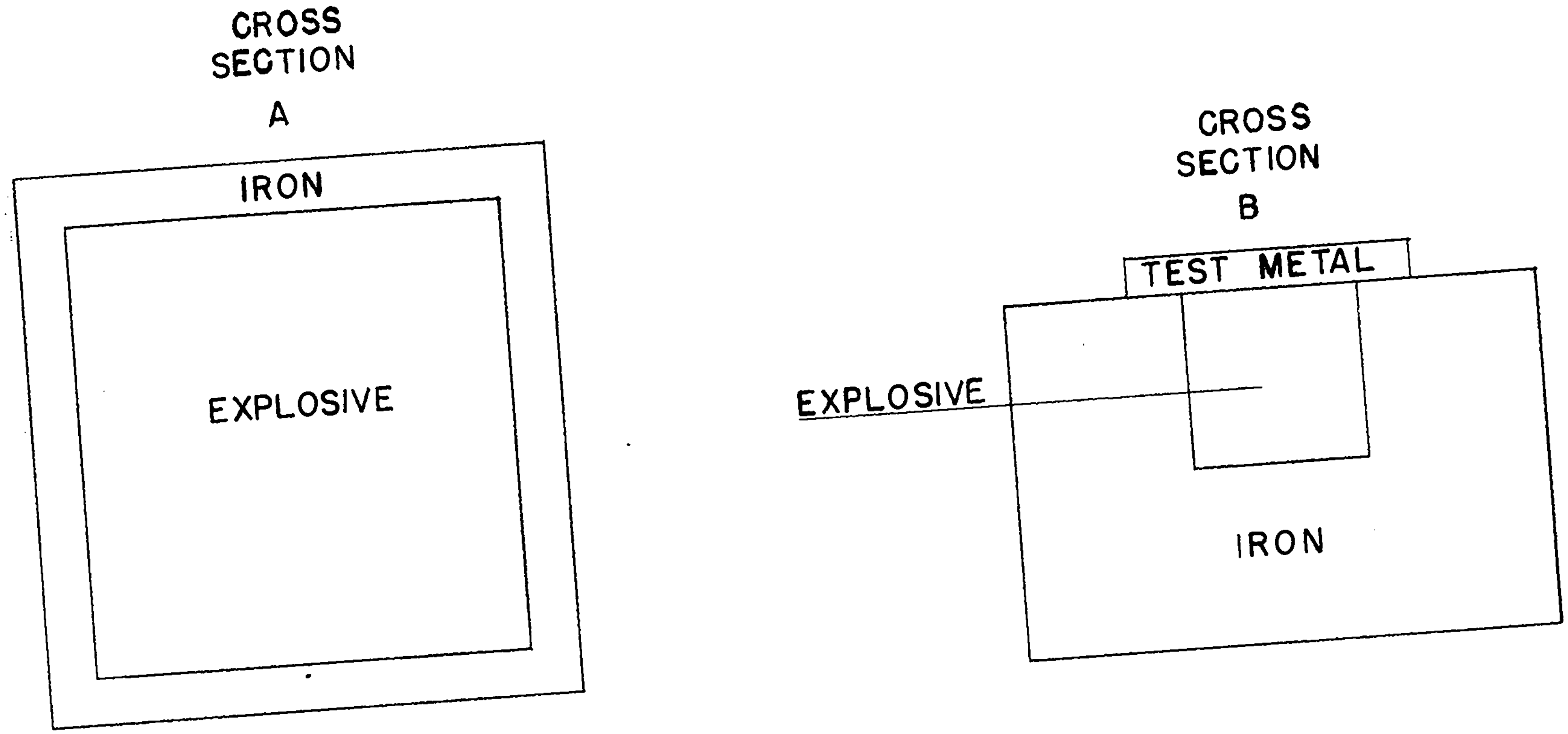

FIGURE 5

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disturbance should lag considerably behind the shock front.

IV. Accuracy

In the last section the experimental conditions which might be expected to lead to photographs showing comparatively long straight line segments of the metal shock and metal explosive interface have been discussed.

Photographs already made indicate that lines as sharp as several tenths of a millimeter can be obtained. It is not unreasonable to hope that the centers of such lines at any place might be determined to 0.1 mm or less. If straight line segments of as long as 3 or 4 cm can be obtained, accuracies of 3×10^{-3} radians in the angles θ and ϕ might be hoped for. The quantity $\left(\frac{-\Delta V}{V_o}\right)$ for the metal, the relative compression, is essentially proportional to ϕ. The approximate equations for ϕ given previously indicate that values up to 15° may be obtained with some metals, although lower angles are more likely. (The equations are for $P_m = P_d$ and $\alpha = \phi$. The largest value of α is 7.5° in Fig. 3. When $\alpha < \phi$ then $P_m < P_d$ and the equations are false. The apparently large values of about 18° possible for Mg from the equations is thus an overestimate.) Since 15° is about 1/4 radian it follows that the percent error in ϕ, and therefore in $\frac{(-\Delta V)}{V_0}$ might be as little as 1.2 %. A more likely estimate is in the neighborhood of 2 or 3 %.

On the other hand the velocity, S_m, of the shock in the metal can be measured with higher accuracy. From equation (19), $S_m = \frac{S \sin\theta}{\cos\phi}$ we have a relative error in S_m, $\frac{\delta S_m}{S_m}$ equal to $\cot\theta\,\delta\theta$

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where $\delta\theta$ is the error in radians in the measurement of θ. Since θ is in the neighborhood of 45° the error would be about that in θ, or about 0.3% .

As is shown in an appendix if very thin lead foils are used in the explosive as an "indicator", and if these are at about 60° to the direction of the detonation front (about 30° to the direction of propagation), then the particle velocity in the detonation can be measured to about 2% . This leads to a measurement of the detonation pressure to about this accuracy if it is assumed that the detonation velocity is exactly known, since $vs = P_d V_{eo}$

In principle, at least, one might use the measured detonation pressure, and by calculations involving the equation of state of the detonation products one could compute the pressure in the metal, and only use the shock velocity measurement rather than the measurement of ϕ to obtain equation of state data. Although this would be a useful check, it is doubtful if the accuracy would be as high.

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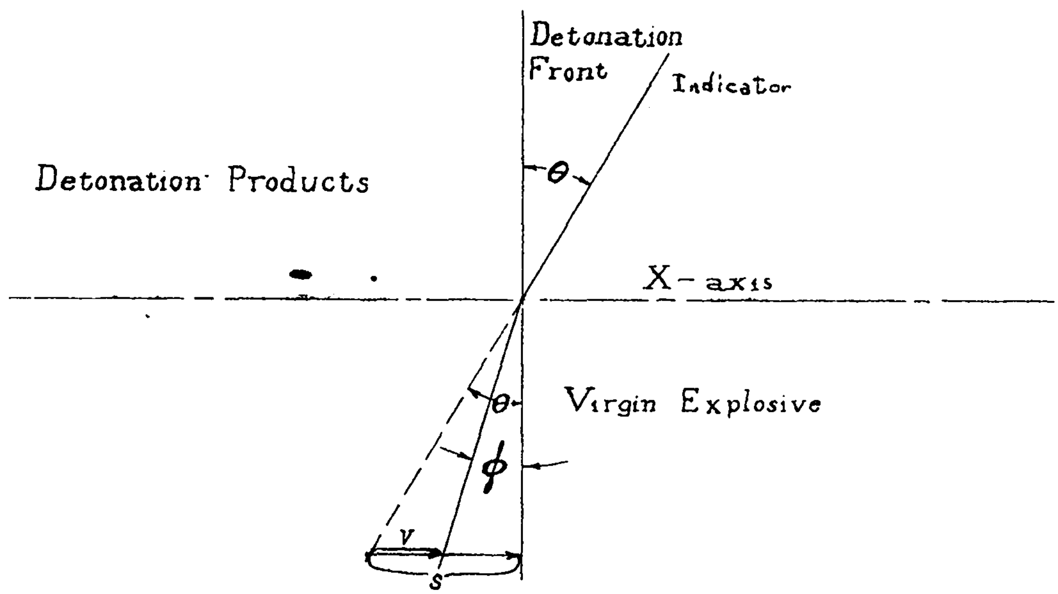
Appendix

Errors in the "indicator" measurement of material velocity

The problem of passing a detonation wave in an explosive material in the direction of the X-axis with indicators in planes normal to a line making the angle θ with the X-axis, presents the problem of which θ leads to the most accurate measurement of the particle velocity V.

The schematic diagram shows the set-up.



We call V the particle velocity after the detonation and S the detonation velocity. By knowing θ and measuring ϕ one calculates $\frac{V}{S}$ by the equation $\frac{S-V}{S} = \frac{\tan\phi}{\tan\theta} = 1 - \frac{V}{S}$

$$\left(\frac{V}{S}\right) = 1 - \left[\frac{\tan\phi}{\tan\theta}\right]$$

The value of $\frac{V}{S} = \lambda$ is about 0.234 for cyclonite. We

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find $\tan\phi = (1-\lambda)\tan\theta \cong 0.766\tan\theta$

The relative error in the measurement of λ per unit error $\delta\phi$ in the angle ϕ is

$$\frac{\delta\lambda}{\lambda} = \frac{\partial}{\partial\phi}\ln\left[1-\frac{\tan\phi}{\tan\theta}\right]\partial\phi = -\frac{1}{\left(1-\frac{\tan\phi}{\tan\theta}\right)}\frac{1}{\cos^2\phi}\frac{1}{\tan\theta}\delta\phi ,$$

$$= -\frac{1}{(\tan\theta-\tan\phi)}\frac{1}{\cos^2\phi}\delta\phi = -\frac{1+\tan^2\phi}{\tan\theta-\tan\phi}\delta\phi$$

$$= \frac{1+(1-\lambda)^2\tan^2\theta}{\lambda\tan\theta}\delta\phi$$

Let us call $\epsilon(\theta)$ the magnitude of the relative error, $|\delta\lambda/\lambda|$, in the measurement of V, per unit error of the measurement of ϕ in radians. We have

$$\epsilon(\theta) = \frac{1}{\lambda}[\tan^{-1}\theta + (1-\lambda)^2\tan\theta]$$

By differentiation

$$d\epsilon/d\theta = \lambda^{-1}\cos^{-2}\theta\,[(1-\lambda)^2 - \tan^{-2}\theta]$$

which is zero when

$$\tan\theta = (1-\lambda)^{-1} \cong 1.3$$

or

$$\theta \cong 60^\circ$$

At this angle

$$\epsilon(\theta) = \frac{2}{\lambda}(1-\lambda) \cong 7$$

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Thus a measurement of ϕ to 0.01 radians would give $\frac{V}{S}$ to within 7.%, whereas a measurement of ϕ to 0.003 radians would give $\frac{V}{S}$ to within 2%.

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