Early Work in Numerical Hydrodynamics

by Francis H. Harlow

I met Stan Ulam shortly after coming to Los Alamos in 1953. As an eager youngster chasing new dreams, I was inspired and encouraged (and sometimes properly chastised) by the older resident scientists and Laboratory consultants. Several stand out especially for their powerful encouragement; one of these is Stan.

Some of my associates, especially during the first six years, didn't like many of my wild ideas about fluid dynamics and the techniques for solving such problems by high-speed computers. Stan continually took the time to see what was going on and had the faith (not always justified) to tell others positive things about my explorations. I shall always be grateful for Stan's, as well as Conrad Longmire's, crucial influence in establishing our fluid dynamics group in the Theoretical Division.

Stan and I had many talks, especially on the stochastic behavior of complex systems. He seemed to *feel* how these systems worked: their collective properties were very real to him. He was intrigued by the almost-cyclic properties they sometimes could exhibit and participated in pioneering numerical experiments on fluid-like, many-particle dynamics.

His early work with John Pasta* created the grandaddy of the free-Lagrangian method of modeling turbulence and, in the sixties, led ultimately to the Particle-and-Force technique for the calculation of shock formation and interaction problems. Although couched in terms of hydrodynamics, the pioneering work has had significant impact on many branches of numerical analysis, especially in terms of the interpretation and meaning of results. The main thrust of their thinking is captured in the following excerpts.

"Our approach to the problem of dynamics of continua can be called perhaps "kinetic' '-the continuum is treated, in an approximation. as a collection of a finite number of elements of "points;" these "points" can represent actual points of the fluid, or centers of mass of zones, i.e., globules of the fluid, or, more abstractly, *coefficients* of functions, representing the fluid, developed into series. "

One of the motivations behind the free-Lagrangian approach was the computational difficulties for fluid flow with large internal shears in which elements that were initially close later found themselves widely separated.

"It was found impractical to use a "classical" method of calculation for this hydrodynamical problem, involving two independent spatial variables in an essential way . . . . This "classical" procedure, correct for infinitesimal steps in time and space, breaks down for any reasonable (i.e., practical) finite length of step in time. The reason is, of course, that the computation assumes that "neighboring" points, determining a "small" area—stay as neighbors for a considerable number of cycles. It is clear that in problems which involve mixing specifically this is not true . . the classical way of computing by referring to initial (at time t = O) ordering of points becomes meaningless. "

The next point is one that Stan emphasized repeatedly, illustrating what he felt to be a potential power of their approach.

"The meaningful results of the calculations are not so much the precise positions of our elements themselves as the behavior, in time, of a *few functional* of the motion of the continuum.

"Thus in the problem relating to the *mixing* of two fluids, it is not the exact position of each globule that is of interest but quantities such as the *degree* of mixing (suitably defined); in problems of turbulence, not the shapes of each portion of the fluid, but the *overall* rate at which energy goes from simple *modes* of motion to higher frequencies."

As it turned out, the behavior, in time, of the functional of the motion that they calculated was very smooth despite the complicated, turbulent nature of the fluid's motion. Thus, an important perspective on the modeling of complex phenomenon had been established. Indeed, turbulence transport theory, the subject of the following article, *depends* upon the strong tendencies in nature towards universal behavior that are the basis for the observed smoothness in their functional. This theory is an excellent example of Stan's idea that wonderful numerical results can emerge from averaging discrete-representations over a set of possible scale sizes. But the theory goes further in providing an analytic formulation of turbulence transport.

Stan lived to see the realization of some of his ideas—others are still being investigated—but I always had confidence that if Stan had a *feeling* for something, it was sure to be significant. He was a friend I shall long remember. ■

"John Pasta and Stan Ulam. 1953. Heuristic studies in problems of mathematical physics on high speed computing machines, Los Alamos report LA-1557.