Neutrino Masses

How to add them to the Standard Model

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The Standard Model includes a set of particles—the quarks and leptons—and their interactions. The quarks and leptons are spin-1/2 particles, or fermions. They fall into three families that differ only in the masses of the member particles. The origin of those masses is one of the greatest unsolved mysteries of particle physics. The greatest success of the Standard Model is the description of the forces of nature in terms of local symmetries. The three families of quarks and leptons transform identically under these local symmetries, and thus they have identical strong, weak, and electromagnetic interactions.

In the Standard Model, quarks and leptons are assumed to obtain their masses in the same way that the W and Z^0 bosons obtain theirs: through interactions with the mysterious Higgs boson (named the "God Particle" by Leon Lederman). But before we write down some simple formulas that describe the interactions of quarks and leptons with the Higgs boson, let us define some notation.

Defining the Lepton Fields. For every elementary particle, we associate a field residing in space and time. Ripples in these fields describe the motions of these particles. A quantum mechanical description of the fields, which allows one to describe multiparticle systems, makes each field a quantum mechanical operator that can create particles out of the ground state—called the *vacuum.* The act of creating one or more particles in the vacuum is equivalent to describing a system in which one or more ripples in the fabric of the field move through space-time.

Let us now discuss the simple system of one family of leptons. To be specific, we will call the particles in this family the electron and the electron neutrino. The electron field describes four types of ripples (or particles). We label these four types by two quantum charges called fermion number N and handedness, or chirality, N_x. For the electron field, the particle state with fermion number $N = +1$ is the electron, and the particle state with $N = -1$ is the antielectron (or positron). Each of these states comes as right-handed, $N_x = \mathrm{R}$, and left-handed, $N_x = \mathrm{L}$. Handedness is a Lorentz invariant quantity that is related in a nontrivial way to helicity, the projection of the spin $\boldsymbol{s}$ in the direction of the momentum $\boldsymbol{p}$. (For a discussion of handedness versus helicity, see "The Oscillating Neutrino" on page 28.)

In relativistic quantum field theory, the right-handed and left-handed electron and the right-handed and left-handed antielectron can be defined in terms of two fields denoted by e and e^c, where each field is a Weyl two-component left-handed spinor. The compositions of the fields are such that

> e annihilates a left-handed electron e_L or creates a right-handed positron $\bar{e}_\mathrm{R}$, and
>
> e^c annihilates a left-handed positron $\bar{e}_\mathrm{L}$ or creates a right-handed electron e_R.

These fields are complex, and for the action of the Hermitian conjugate fields $e^\dagger$ and $e^{c\dagger}$, just interchange the words annihilate and create above. For example, $e^\dagger$ creates a left-handed electron or annihilates a right-handed positron. Hence, the fields e and e^c and their complex conjugates can create or annihilate all the possible excitations of the physical electron. Note that parity (defined as the inversion of spatial coordinates) has the property of interchanging the two states e_R and e_L.

What about the neutrino? The right-handed neutrino has never been observed, and it is not known whether that particle state and the left-handed antineutrino exist. In the Standard Model, the field $\nu_e^{\,c}$, which would create those states, is not included. Instead, the neutrino is associated with only two types of ripples (particle states) and is defined by a single field ν_e:

> ν_e annihilates a left-handed electron neutrino $\nu_{e\mathrm{L}}$ or creates a right-handed electron antineutrino $\bar{\nu}_{e\mathrm{R}}$.

The left-handed electron neutrino has fermion number $N = +1$, and the right-handed electron antineutrino has fermion number $N = -1$. This description of the neutrino is not invariant under the parity operation. Parity interchanges left-handed and right-handed particles, but we just said that, in the Standard Model, the right-handed neutrino does not exist. The left-handedness of the neutrino mimics the left-handedness of the charged-current weak interactions. In other words, the W gauge boson, which mediates all weak charge-changing processes, acts only on the fields e and ν_e. The interaction with the W transforms the left-handed neutrino into the left-handed electron and vice versa ($e_\mathrm{L} \leftrightarrow \nu_{e\mathrm{L}}$) or the right-handed antineutrino into the right-handed positron and vice versa ($\bar{\nu}_{e\mathrm{R}} \leftrightarrow \bar{e}_\mathrm{R}$). Thus, we say that the fields e and ν_e, or the particles e_L and $\nu_{e\mathrm{L}}$, are a weak isospin doublet under the weak interactions.

Table I. Lepton Charges

$$Q = I_3^{\,\mathrm{w}} + \frac{Y^\mathrm{w}}{2}$$

N	N_x	Particle States	$I_3^{\,\mathrm{w}}$	Y^w	Q
+1	L	e_L	−1/2	−1	−1
+1	L	ν_L	+1/2	−1	0
−1	R	$\bar{e}_\mathrm{R}$	+1/2	+1	+1
−1	R	$\bar{\nu}_\mathrm{R}$	−1/2	+1	0
+1	R	e_R	0	−2	−1
−1	L	$\bar{e}_\mathrm{L}$	0	+2	+1
+1	R	ν_R	0	0	0
−1	L	$\bar{\nu}_\mathrm{L}$	0	0	0

These lepton fields carry two types of weak charge: The weak isotopic charge $I_3^{\,\mathrm{w}}$ couples them to the W and the Z^0, and the weak hypercharge Y^w couples them to the Z^0. (The Z^0 is the neutral gauge boson that mediates neutral-current weak interactions.) Electric charge Q is related to the two weak charges through the equation $Q = I_3^{\,\mathrm{w}} + Y^\mathrm{w}/2$. Table I lists the weak charges for the particle states defined by the three fields e, ν_e, and e^c. Note that the particle states e_R and $\bar{e}_\mathrm{L}$ defined by the field e^c do not couple to the W and have no weak isotopic charge. The field and the particle states are thus called weak isotopic singlets. However, e_R and $\bar{e}_\mathrm{L}$ do carry weak hypercharge and electric charge and therefore couple to the Z^0 and the photon.

Likewise, the field $\nu_e^{\,c}$ and its neutrino states ν_R and $\bar{\nu}_\mathrm{L}$ would be isotopic singlets with no coupling to the W. But unlike their electron counterparts, they must be electrically neutral ($Q = I_3^{\,\mathrm{w}} + Y^\mathrm{w}/2 = 0$), which implies they cannot have weak hypercharge. Thus, they would not couple to the W, the Z^0, or the photon. Having no interactions and, therefore, not being measurable, they are called *sterile* neutrinos and are not included in the Standard Model. However, if the left-handed neutrino has mass, it may oscillate into a sterile right-handed neutrino, a possibility that could be invoked in trying to give consistency to all the data on neutrino oscillations.

The Origin of Electron Mass in the Standard Model. What is mass? Mass is the inertial energy of a particle. It is the energy a particle has when at rest and the measure of the resistance to an applied force according to Newton's law $F = ma$. A massless particle cannot exist at rest; it must always move at the speed of light.

A fermion (spin-1/2 particle) with mass has an additional constraint. It must exist in both right-handed and left-handed states because the only field operators that yield a nonzero mass for fermions are bilinear products of fields that flip the particle's handedness. For example, in the two-component notation introduced above, the standard, or Dirac, mass term in the Lagrangian for free electrons is given by*

$$m_e e^c e \quad . \tag{1}$$

This fermion mass operator annihilates a left-handed electron and creates a right-handed electron in its place. The mass term does not change the charge of the particle, so we say that it conserves electric charge. Also, because this mass term does not change a particle into an antiparticle, we say that it conserves fermion number N. However, the weak isospin symmetry forbids such a mass operator because it is not an invariant under that symmetry. (The field e is a member of a weak isotopic doublet, whereas the field e^c is a weak isotopic singlet, so that the product of the two is not a singlet as it should be to preserve the weak isospin symmetry.) But the electron does have mass. We seem to be in a bind.

The Standard Model solves this problem: the electron and electron neutrino fields are postulated to interact with the spin-zero Higgs field h^0 (the God particle). The field h^0 is one member of a weak isospin doublet whose second member is h^+. The superscripts denote the electric charge of the state annihilated by each field (see Table III on page 57 for the other quantum number of the two fields). The field h^0 plays a special role in the Standard Model. Its ground state is not a vacuum state empty of particles, but it has a nonzero mean value, much like a Bose-Einstein condensate. This nonzero value, written as the vacuum expectation value $<0|h^0|0> \equiv <h^0> = \mathrm{v}/\sqrt{2}$ is the putative "origin of mass." (The "mystery" of mass then becomes the origin of the Higgs boson and its nonzero vacuum value.)

The interaction between the Higgs fields and the electron and electron neutrino is given by

$$\lambda_e e^c\left(\nu_e (h^+)^\dagger + e(h^0)^\dagger\right) \quad , \tag{2}$$

where λ_e is called a Yukawa coupling constant and describes the strength of the coupling between the Higgs field and the electron. The Higgs field is a weak isospin doublet, so the term in parentheses is an inner product of two doublets, making an invariant quantity under the weak isospin symmetry. Since it also conserves weak hypercharge, it preserves the symmetries of the Standard Model.

Because the mean value of h^0 in the vacuum is $<h^0> = \mathrm{v}/\sqrt{2}$, the operator in (2) contributes a term to the Standard Model of the form

$$\lambda_e <h^0> e^c e = (\lambda_e \mathrm{v}/\sqrt{2}) e^c e \quad . \tag{3}$$

In other words, as the electron moves through the vacuum, it constantly feels the interaction with the Higgs field in the vacuum. But (3) is a fermion mass operator exactly analogous to the Dirac mass operator in (1), except that here the electron rest mass is given by

$$m_e = \lambda_e \mathrm{v}/\sqrt{2} \quad . \tag{4}$$

We see that, in the Standard Model, electron mass comes from the Yukawa interaction of the electron with the Higgs background.

Why Neutrinos Are Massless in the Minimal Standard Model. What about the neutrino? Because the neutrino has spin 1/2, its mass operator must also change handedness if it is to yield a nonzero value. We could introduce a Dirac mass term for the neutrino that would mirror the mass term for the electron. It would have the form

$$m_\nu \nu_e{}^c \nu_e \quad . \tag{5}$$

But, as we said above, the field $\nu_e{}^c$ is not included in the Standard Model because, so far, weak-interaction experiments have not required it. The neutrino, though, has no electric charge, which makes it possible to write down a mass term from the existing neutrino field ν_e with the form

$$\frac{1}{2}\, \mu_\nu \nu_e \nu_e \quad . \tag{6}$$

(Note that m_ν and μ_ν refer just to the electron neutrinos, but similar masses can be defined for the μ and τ neutrinos.) The mass operator in (6) annihilates a left-handed neutrino and creates a right-handed antineutrino, which means that it is a Majorana mass term. *Any mass term that changes a particle to an antiparticle is called a Majorana mass term.* In changing a neutrino to an antineutrino, this term violates fermion number N, changing it by two units. It is a legitimate mass term in that it changes handedness in the right way to yield a nonzero rest mass, and it conserves electric charge because the neutrino is electrically neutral. Nevertheless, it is not included in the Standard Model because it violates the weak symmetry in two ways: It is not invariant under the weak isospin symmetry, and it changes the weak hypercharge by two units. We conclude that, in the minimal Standard Model, which does not include $\nu_e{}^c$ and contains only the Higgs doublet mentioned above, there is no way to give mass to the neutrinos if fermion number is conserved.

Two consequences follow directly from the result that neutrino masses are identically zero in the minimal Standard Model. First, the weak eigenstates and the mass eigenstates of the leptons are equivalent, and therefore individual-lepton-family number (electron number, muon number, and tau number) are conserved (for the proof, see "Family Mixing and the Origin of Mass" on page 72). Thus, the Standard Model forbids such processes as

$$\mu^+ \rightarrow e^+ + \gamma \quad , \text{or} \tag{7}$$

$$\mu^+ \rightarrow e^+ + e^+ + e^- \quad . \tag{8}$$

Similarly, the proposed process of neutrino oscillation, which may recently have been observed, is forbidden. Second, total lepton number, equal to the sum of individual-family-lepton numbers, is also conserved, and the process of neutrinoless double beta decay is forbidden.

The converse is also true: If individual-lepton-number violation is observed, or if the LSND results on neutrino oscillation are confirmed, then either of those experiments could claim the discovery of nonzero neutrino masses and thus of *new physics beyond the Standard Model*.

Adding Neutrino Masses to the Standard Model. What could this new physics be? There are several simple extensions to the Standard Model that could yield nonzero neutrino masses without changing the local symmetry of the weak interactions.

The simplest extension would be to add no new fields but just a new "effective" interaction with the Higgs field:

$$\frac{1}{M_{\text{effective}}}(h^0 \nu_e - h^+ e)^2 \quad . \tag{9}$$

The addition of the Hermitian conjugate is assumed in l equations if the operator is not explicitly Hermitian.

This effective interaction is invariant under the local symmetries and yields a Majorana mass term equal to

$$\frac{1}{M_{\text{effective}}} \langle h^0 \rangle^2 \nu_e \nu_e \ , \tag{10}$$

and a value for the neutrino mass

$$\mu_\nu = \frac{2\langle h^0 \rangle^2}{M_{\text{effective}}} = \frac{\mathrm{v}^2}{M_{\text{effective}}} \ . \tag{11}$$

This mass term, as all fermion mass terms, changes handedness from left to right, but it violates the fermion number N listed in Table I. The term $M_{\text{effective}}$ must be large so that the mass of the neutrino be small. The new term in (9) is called "effective" because it can only be used to compute the physics at energies well below $M_{\text{effective}}c^2$, just as Fermi's "effective" theory of beta decay yields valid approximations to weak processes only at energies well below $M_W c^2$, where M_W is the mass of the W. (Outside their specified energy ranges, "effective" theories are, in technical language, nonrenormalizable and yield infinite values for finite quantities.) Thus, the mass term in (9) implicitly introduces a new scale of physics, in which new particles with masses on the order of $M_{\text{effective}}$ presumably play a role. Below that energy scale, (9) describes the effects of the seesaw mechanism for generating small neutrino masses (see below as well as the box "The Seesaw Mechanism at Low Energies" on page 71).

A Dirac Mass Term. Another extension would be to introduce a right-handed neutrino field ν_i^{c}, one for each neutrino flavor i ($i = e, \mu, \tau$), where, for example, the right-handed field for the electron neutrino is defined such that

> ν_e^{c} annihilates a left-handed electron antineutrino $\bar{\nu}_{eL}$ and creates a right-handed electron neutrino ν_{eR}.

We could then define an interaction with the Higgs field exactly analogous to the interaction in (3) that gives electrons their mass:

$$\lambda_\nu \ \nu_e^{\text{c}}(\nu h^0 - e h^+) \ . \tag{12}$$

Again, because the Higgs field h^0 has a nonzero vacuum expectation value, the interaction in (12) would give the neutrino a Dirac mass

$$m_\nu = \frac{\lambda_\nu \mathrm{v}}{\sqrt{2}} \ . \tag{13}$$

But why are neutrino masses much smaller than the masses of their charged lepton weak partners? Specifically, why is $m_\nu << m_e$? The electron mass is 500,000 eV, whereas from experiment, the electron neutrino mass is known to be less than 10 eV. The only explanation within the context of the interaction above is that the strength of the Yukawa coupling to the Higgs field is much greater for the electron than for the electron neutrino, that is, $\lambda_e > 5 \times 10^4 \lambda_\nu$. But this is not an explanation; it just parametrizes the obvious.

The Seesaw Mechanism and Majorana Neutrinos. The first real model of why neutrino masses are very much smaller than the masses of their lepton partners was provided by Murray Gell-Mann, Pierre Ramond, and Richard Slansky. Motivated by a class of theories that attempt to unify the interactions of the Standard Model, including the strong interactions, they observed that, if one introduced the right-handed neutrino field ν_e^{c} into the Standard Model to form a Dirac mass term, one could also add a Majorana mass term of the form

$$\frac{1}{2} M \nu_e^{\text{c}} \nu_e^{\text{c}} \tag{14}$$

without violating the local symmetries of the Standard Model (as stated above, ν_e^{c} has no weak charge and is thus an invariant under the local symmetry). Further, if M were large enough, the mass of the left-handed neutrino would be small enough to satisfy the experimental bounds.

To see how this reduction occurs, we write the operators for both the Dirac mass term and the Majorana mass term:

$$\mathscr{L}_{\text{mass}} = \lambda_\nu (h^0 \nu_e - h^+ e)\nu_e^{\text{c}} + \frac{1}{2} M \nu_e^{\text{c}} \nu_e^{\text{c}} + \text{other terms} \ . \tag{15}$$

Here we are assuming that $\lambda_\nu \cong \lambda_e$. These additions to the Lagrangian yield the following mass terms:

$$\mathscr{L}_{\nu_e \text{ mass}} = m_{\nu_e} \nu_e \nu_e^{\text{c}} + \frac{1}{2} M \nu_e^{\text{c}} \nu_e^{\text{c}} \ , \tag{16}$$

where m_{ν_e} is the Dirac mass defined in (13), except that now we assume $\lambda_\nu \cong \lambda_e$, in which case $m_{\nu_e} \cong \lambda_e \mathrm{v}/\sqrt{2}$. In other words, the Dirac neutrino mass is about equal to the electron mass (or some other fermion mass in the first family).
The two neutrino mass terms may be rewritten as a matrix, frequently referred to as the mass matrix:

$$1/2(\nu_e \quad \nu_e^{\text{c}}) \begin{pmatrix} 0 & m_{\nu_e} \\ m_{\nu_e} & M \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_e^{\text{c}} \end{pmatrix} \ . \tag{17}$$

It is clear that the fields ν_e and ν_e^{c} do not describe states of definite mass, or mass eigenstates, but rather the two fields are mixed by the interaction with the Higgs field. Diagonalizing this matrix yields the masses of the physical neutrinos.
[The expressions in (16) and Equation (17) are equivalent. The proof requires more detail than is presented here.] One mass is very small:

$$\mu_{\text{light}} \approx \frac{m_{\nu_e}^2}{M} \ . \tag{18}$$

It is the Dirac mass reduced by ratio m_{ν_e}/M that gave this mechanism its name—the "seesaw." The second mass is very large:

$$\mu_{\text{heavy}} \approx M \ . \tag{19}$$

The fields corresponding to these masses are given by

$$\nu_{\text{light}} \approx \nu_e + \left(\frac{m_{\nu_e}}{M}\right) \nu_e^{\text{c}} \approx \nu_e \ , \tag{20}$$

and

$$\nu_{\text{heavy}} \approx \nu_e^{\text{c}} - \left(\frac{m_{\nu_e}}{M}\right) \nu_e \approx \nu_e^{\text{c}} \ . \tag{21}$$

Both fields define Majorana particles, that is, particles that are their own antiparticles, and total-lepton-number conservation can be violated in processes involving these neutrinos. The light neutrino would correspond to the neutrino we see in the weak processes observed so far, and is essentially the left-handed neutrino field ν_e. The right-handed neutrino field ν_e^{c} would not be observed directly at low energies. Its effect in the low-energy theory would only be visible as an effective neutrino mass operator, like the operator in (9), which would give the neutrino a very small mass and would signal the presence of a new scale of physics on the order of $M_{\text{effective}} = 2M/\lambda_\nu^2$ (see the box "The Seesaw Mechanism at Low Energies" on the facing page).

A New Higgs Isospin Triplet. Another possibility is that there are no right-handed neutrinos, but there is, instead, a new set of Higgs-type bosons ϕ that come in three varieties —ϕ^0, ϕ^+, ϕ^{++}— and transform as a triplet under the local weak isospin symmetry. The superscript denotes the electric charge of each boson. Using this Higgs triplet, we can introduce the interaction

$$\lambda_m(\nu\nu\phi^0 + \nu e\phi^+ + ee\phi^{++}) \ , \tag{22}$$

which is consistent with all Standard Model symmetries. If, in analogy with h^0, the Higgs field ϕ^0 has a nonzero vacuum expectation value $\langle\phi^0\rangle = \mathrm{v}_m$, the neutrino would also have a Majorana mass given by

$$\mu_\nu = \lambda_m\langle\phi^0\rangle = \lambda_m \mathrm{v}_m \ , \tag{23}$$

where this fermion mass is a Majorana mass. In a theory with a Higgs triplet, the Higgs doublet is still necessary. In fact, in order to preserve the observed ratio of strengths of neutral- to charged-current interactions (equal to 1 ± .01), the vacuum expectation value v_m must be much smaller than in (3). Also, such a theory has a massless Nambu-Goldstone boson ϕ due to the spontaneous breaking of total lepton number, and it allows the process

$$\nu_\mu \rightarrow \nu_e + \phi \ . \tag{24}$$

Apart from the effective interaction in Equation (9), the other extensions we discussed introduce new states. Each makes predictions that can be tested. The Higgs triplet extension is the largest departure from the Standard Model. The seesaw mechanism is less intrusive than the Higgs triplet. In general, its only low-energy consequence is an arbitrary Majorana mass term for the three neutrino species given by

$$\mu_{ij}\nu_i\nu_j \ , \text{ where } i, j = e, \mu, \tau \ . \tag{25}$$

A general mass matrix such as the one in (25) would lead to lepton-family-number violating processes, CP (charge-conjugation/parity) violation, and neutrino oscillations. This simple hypothesis will be tested by present or proposed experiments.

On a final note, the new scale M in (15) can be very large. It may be associated with the proposed grand unification scale for strong, weak, and electromagnetic interactions, which is predicted to occur at energies on the order of 10^{16} GeV. If so, neutrino masses and mixings can give us information about the physics at this enormous energy scale. There is also the exciting possibility that, through a sequence of interactions that violate CP, lepton-number, and baryon-number conservation, the decay of the very heavy right-handed neutrino ν^c in the hot, early universe generates the observed baryon number of the universe, that is, the presence of matter as opposed to antimatter. ■

The Seesaw Mechanism at Low Energies

The seesaw mechanism for neutrino masses defines a new scale of nature given by M, the mass associated with the heavy right-handed neutrino ν_e^c. Since M is postulated to be very large, well above the energies accessible through experiment, it is interesting that the "effective" neutrino mass operator in (11) approximates the seesaw terms in (15) at energies below M. To show this, we consider the effective operator

$$\frac{1}{M_{\text{effective}}}(h^0\nu_e - h^+e)^2 \ .$$

When the Higgs vacuum expectation value is accounted for, this operator yields the nonrenormalizable mass term in diagram (a) and a Majorana mass given by

$$\mu_\nu = \frac{\mathrm{v}^2}{M_{\text{effective}}}$$

In the seesaw mechanism, the light neutrino acquires its mass through the exchange of the heavy neutrino, as shown in diagram (b). Diagram (b), which is approximated by diagram (a) at energies below Mc^2, is a renormalizable mass term that involves both Dirac and Majorana masses. It yields a neutrino mass

$$\mu_{\text{light}} = \frac{m_{\nu_e}{}^2}{M} \quad \text{with} \quad m_{\nu_e} \equiv \lambda_{\nu_e}\frac{\mathrm{v}}{\sqrt{2}} \ .$$

Equating the values for μ_ν and μ_{light}, we obtain the relation between M and $M_{\text{effective}}$:

$$\frac{1}{M_{\text{effective}}} = \frac{(\lambda_{\nu_e})^2}{2M} \ .$$

At energies below M_W, the mass of the W boson, a similar type of relationship exists between Fermi's "effective" theory shown in diagram (c) and the W-boson exchange processes shown in diagram (d). The exchange processes are defined by the gauge theory of the charged-current weak interactions. Fermi's theory is a nonrenormalizable current-current interaction of the form

$$\mathscr{L}_{\text{Fermi}} = \frac{G_{\text{F}}}{\sqrt{2}} J^{\mu}_{W}{}^{\dagger} J_{\mu}{}^{W} \ ,$$

where the weak current for the neutrino-electron doublet is given by

$$J^{\mu}_{W} = 2\nu_e{}^{\dagger}\,\overline{\sigma}^{\mu}e \ \text{ and } \overline{\sigma}^{\mu} = (1, -\sigma^i) \ ,$$

and the Fermi constant G_{F} defines the strength of the effective interaction in diagram (c), as well as a new mass/energy scale of nature. The experimentally observed value is $G_{\text{F}} = 1.66 \times 10^{-5}\ \text{GeV}^{-2}$. Equating the low-energy limit of diagram (c) with that of diagram (d) yields the formula

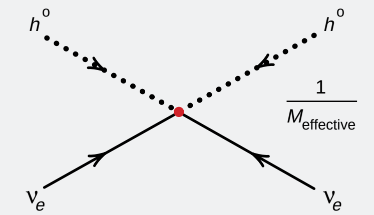
$$\frac{G_{\text{F}}}{\sqrt{2}} = \frac{g^2}{8M_W^2} \ ,$$

where g is the weak isospin coupling constant in the charged-current weak Lagrangian given by

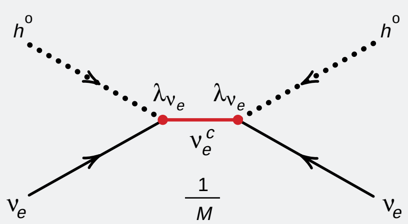
$$\mathscr{L}_{\text{weak}} = -M_W^2 W^{\mu+}W^{-}_{\mu} + \frac{g}{2\sqrt{2}}W^{+}_{\mu}J^{\mu}_{W} + \frac{g}{2\sqrt{2}}W^{-}_{\mu}J^{\mu}_{W}{}^{\dagger} \ .$$

This Lagrangian neglects the kinetic term for the W, which is a valid approximation at energies much less than the W boson mass.

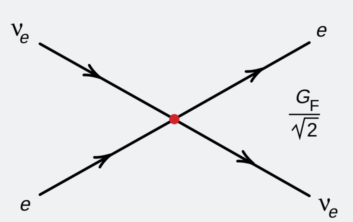
(a) Effective neutrino mass term



(b) Seesaw mass term for the light neutrino



(c) Fermi's current-current interaction



(d) Weak charged-current gauge interaction