The New Los Alamos Center for Nonlinear Studies

By David K. Campbell and Basil Nichols

Linear analysis as a mathematical discipline began in the nineteenth century and in the intervening years has achieved many spectacular successes throughout science. But in fact, most problems posed by nature are nonlinear, and the linear approximations we use to describe them are often a tacit admission that the underlying equations cannot be solved.

Recently, however, a confluence of ideas from diverse branches of science has led to solutions of problems once thought to be intractable. These exciting developments have kindled interest in studying the generic features of nonlinear phenomena.

Of course, when we speak of a given physical system, we must realize that it can behave sometimes in a linear and sometimes in a nonlinear manner. The ripples that distort the surface of a flag as it waves in a gentle, steady breeze are smooth, predictable motions that can be described mathematically by a linear model. When the flag convulses in response to strong, gusty winds, the sharp, intermittent snaps are not uniform; the flag's energy is released in spurts as it adjusts its shape violently to the changes in its environment. The flag has changed from a linear to a nonlinear system at the bidding of the wind.

Similar transitions occur when a steady stream breaks up into turbulent vortices, or when magnetically confined plasmas break up and splatter on the containment walls. These examples suggest the tremendous qualitative differences between the two types of behavior. Linear systems are typically smooth, regular, and predictable, whereas nonlinear systems are often characterized by sharp and unstable boundaries, erratic or chaotic behavior, and dramatic responses to very small changes. Such properties make quantitative description a very difficult task.

Mathematically, what makes linear problems easier to solve than nonlinear ones? Perhaps the most crucial property is that two solutions of a linear equation may be added together to give a new solution to the equation. As a consequence, there exist established methods for solving completely any linear system, even very complicated ones. The methods amount to breaking up the complicated system into many simple pieces and, from solutions to the pieces, "patching together" a solution for the full system. In contrast, two solutions to a nonlinear system do not add together to give a new solution; hence, patching together simple pieces to understand the whole simply does not work. In a sense, nonlinear systems must be treated ***in toto*** and in their full complexity, and so it is not surprising that there exists no general method for solving them. And yet some methods must be found because for many systems the point at which they enter the nonlinear regime is precisely the point at which they become of interest for technological application.

To link the recent progress in basic research with vital applications to technology, a Center for Nonlinear Studies was founded at Los Alamos last fall. The Center will coordinate common themes, stimulate interchange among those in diverse fields who have similar problems, encourage collaboration between other nonlinear research centers, and bridge the gap between fundamental research and applied technology. Researchers will no longer have to reinvent the wheel independently for each new nonlinear problem.

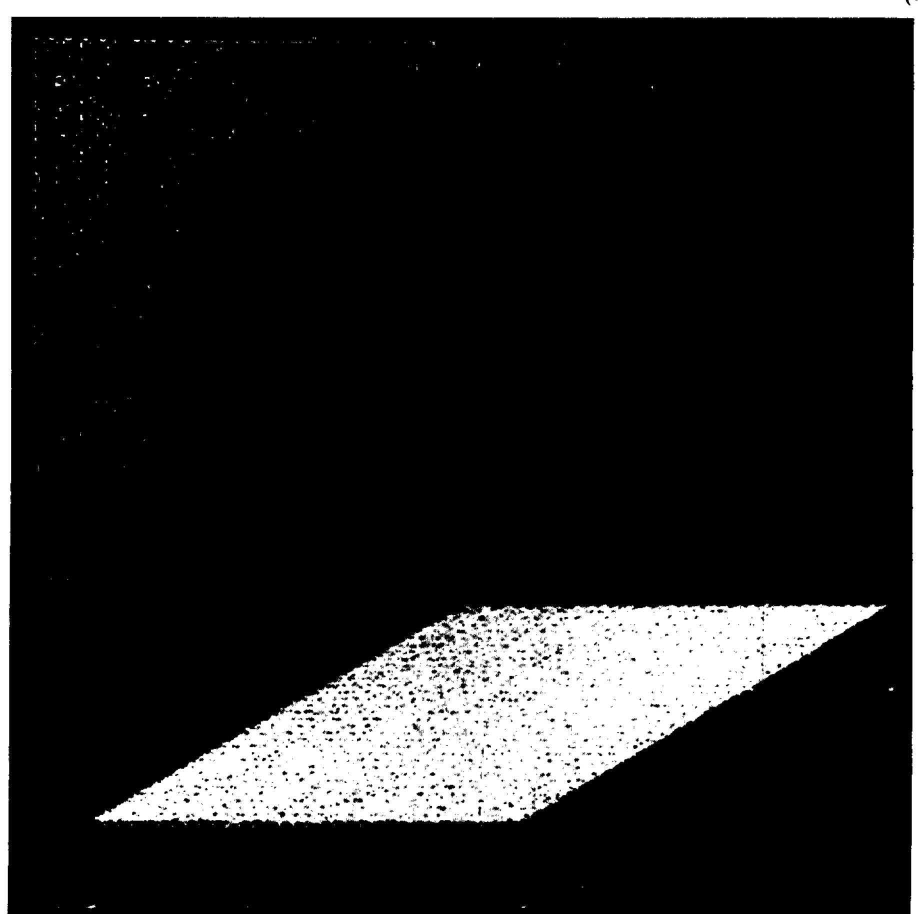
At Los Alamos, important nonlinear problems can be found in nearly every area of research, from fundamental studies of materials properties through combustion models designed to improve engine efficiency to vital questions of reactor safety. Two prime examples of nonlinear systems that are extremely important to our long-term energy goals are lasers and plasmas. Each system can exhibit complicated nonlinear behavior, and when, as in the case of inertial confinement fusion, lasers and plasmas are combined, the problems become even more complex,

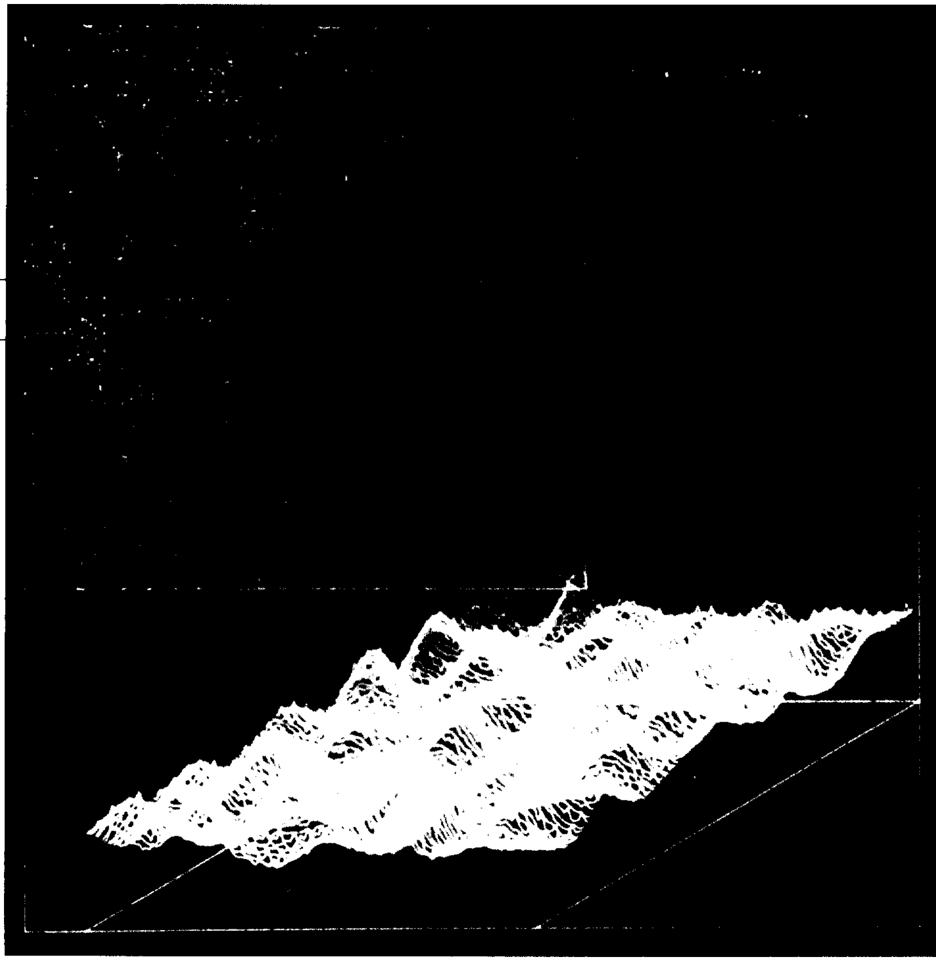
At moderate intensities, laser light passes through a medium—a gas or a solid—without major change; its transit is well characterized by a linear equation. As the intensity of the light increases, a nonlinear phenomenon called self-focusing occurs. The intense laser beam changes the index of refraction of the medium through which it moves, and thus causes the light to bend back upon itself and become more intense. As the cycle repeats, the laser beam can break up into self-focused filaments of extreme intensity; these not only disrupt the uniformity of the laser pulse, but can also destroy the laser system optics.

In plasmas, the central nonlinear problems are connected with instabilities. At low particle density and effective temperature, a plasma discharge can be relatively well controlled and made to move in a predictable, linear fashion through a magnetic field. But to achieve fusion by magnetic con-

The development of a self-focusing instability in a laser beam passing through a plasma is shown in these three frames from a Los Alamos computer-generated movie. The frames show both a contour plot (upper left) and a projected plot of the laser intensity across the profile of the beam. When the laser beam enters the plasma (a), it is of essentially uniform intensity. As the beam moves into the plasma (b), the self-focusing instability initiates the formation of filaments of high intensity. The self-focused filaments become fully developed as the beam moves even further into the plasma (c). (Courtesy of Fred Tappert, University of Miami and Laboratory Consultant)

(a)





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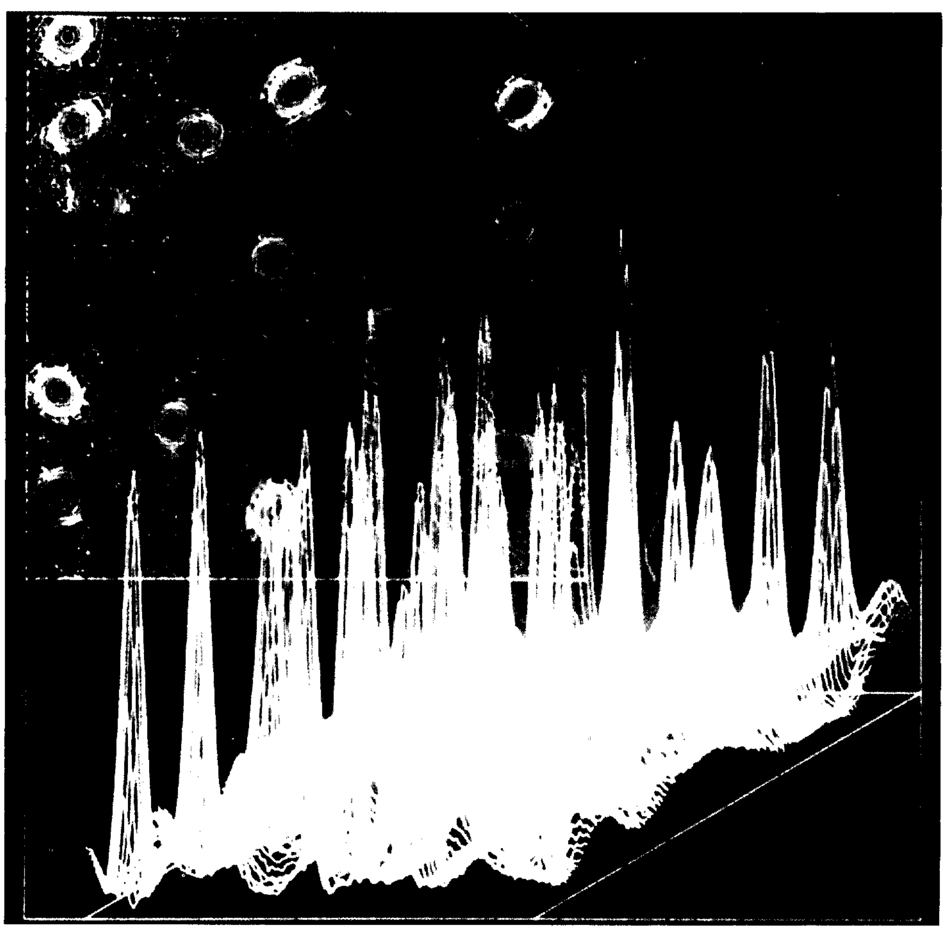
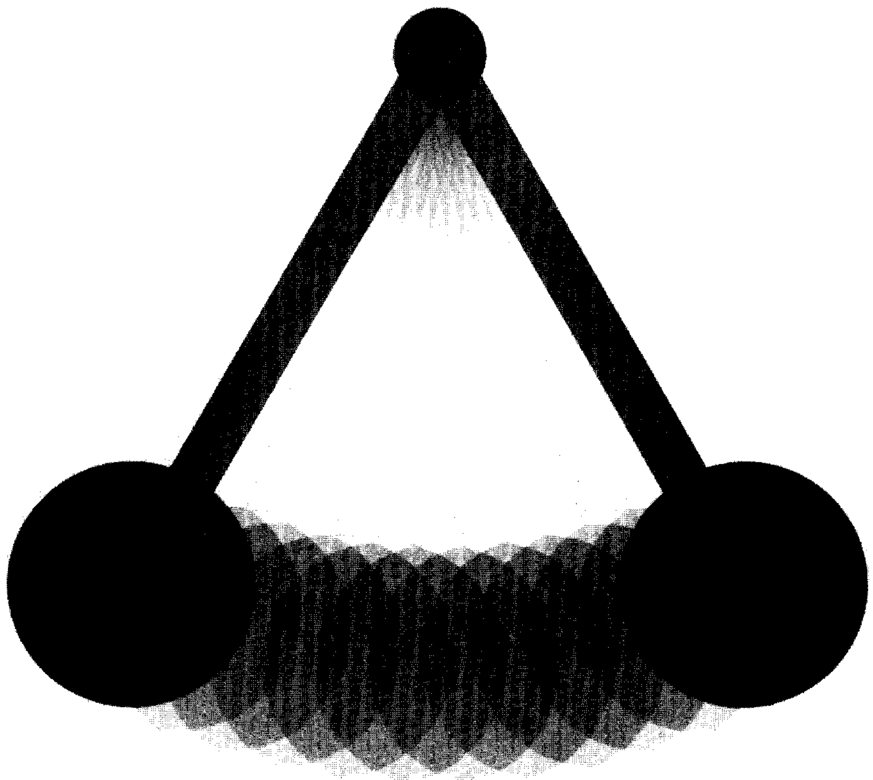


(c)

Galileo Revisited



One of the simplest examples of a nonlinear phenomenon arises in a system that was deeply involved in the birth of modern physics: the simple plane pendulum. When Galileo observed the swinging chandelier in the cathedral at Pisa and drew the conclusion that the period of its motion was independent of amplitude, he did not realize that his considerations contained a crucial linearization approximation, With the hindsight of 300 years, understanding this approximation has become an elementary application of Newton's Laws.

The force acting cm the plane pendulum is that of gravity. The pendulum's motion is completely described by giving 0, the angle that it makes with the vertical, as a function of time. The component of gravitational force that drives the pendulum is, by elementary trigonometry, $-mg \sin \theta$. Thus Newton's law, $\vec{F} = m\vec{a}$, becomes

$$-mg \sin \theta(t) = m\ell \frac{d^2\theta(t)}{dt^2}$$

or

$$\frac{d^2\theta(t)}{dt^2} + \frac{g}{\ell} \sin \theta(t) = 0 \,.$$

This is a nonlinear equation because the sum of two given solutions, $\theta_1(t)$ and $\theta_2(t)$, is *not* a solution. That is,

$$\frac{d^2}{dt^2}[\theta_1(t) + \theta_2(t)] = \frac{d^2\theta_1(t)}{dt^2} + \frac{d^2\theta_2(t)}{dt^2}$$

but

$$\begin{aligned}\sin[\theta_1(t) + \theta_2(t)] &= \sin \theta_1(t) \cos \theta_2(t) + \sin \theta_2(t) \cos \theta_1(t) \\ &\neq \sin \theta_1(t) + \sin \theta_2(t) \,.\end{aligned}$$

Thus the motion of even a simple plane pendulum is nonlinear and, in fact, the period of its motion ***does*** depend on the amplitude. What Galileo really observed was the behavior of the pendulum for very small values of θ. In this regime, we can use the approximation

$$\sin \theta(t) = \theta(t) - \frac{\theta^3(t)}{3!} + \frac{\theta^5(t)}{5!} + \ldots ,$$

and notice that if θ is very small, then θ^3 is even smaller. Hence keeping only the first term in the expansion of $\sin \theta$ gives the ***approximate*** equation for the simple pendulum

$$\frac{d^2\theta(t)}{dt^2} + \frac{g}{l}\theta(t) = 0 .$$

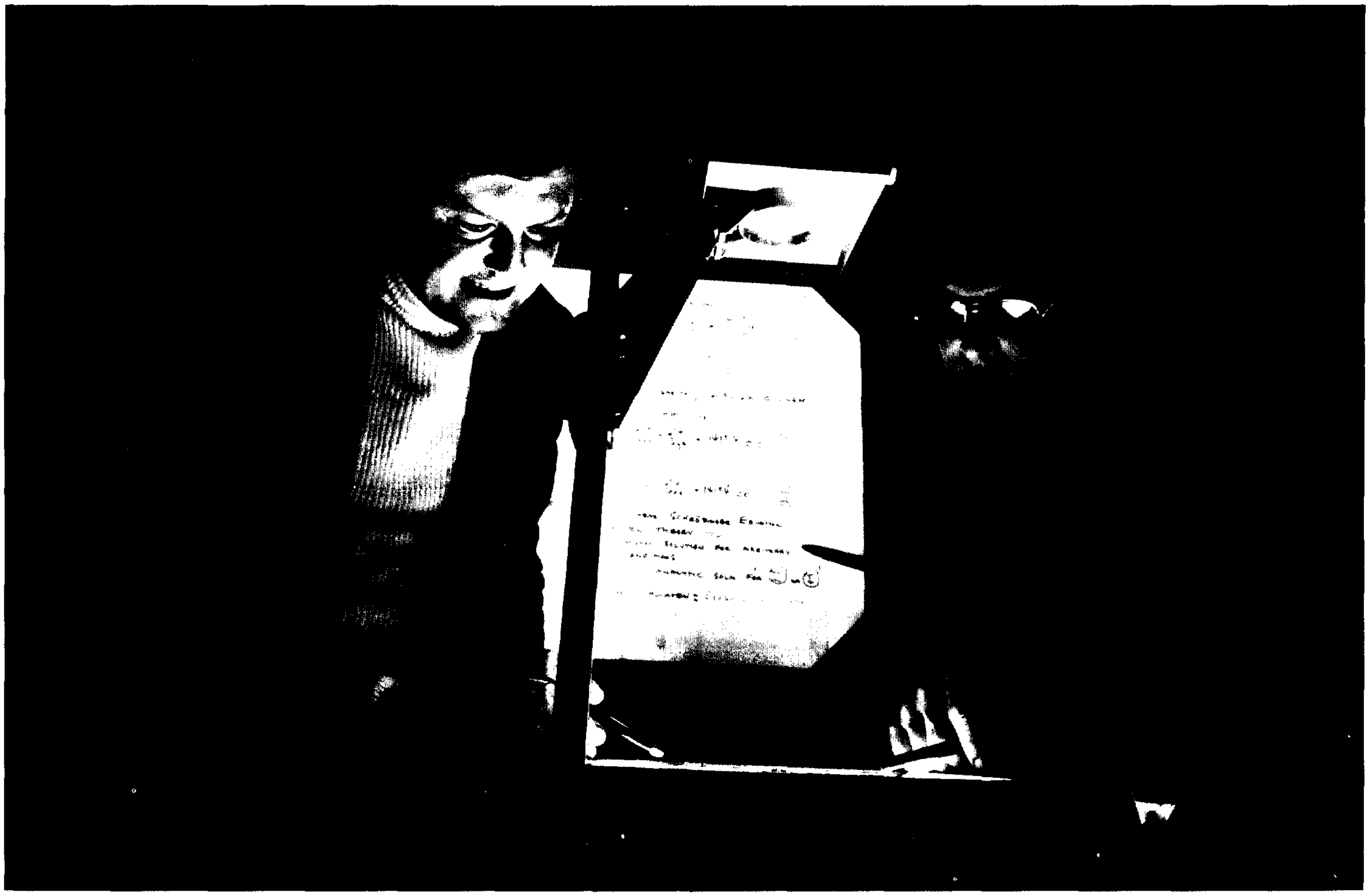
This equation is manifestly linear and its solution demonstrates that the period of the motion is independent of its amplitude. Hence the "familiar" result for the simple pendulum is the consequence of a linearizing approximation applied to a fundamentally nonlinear problem.

It is natural to ask whether the erratic, "unpredictable" behavior often associated with nonlinear systems occurs in the simple pendulum for large amplitude, The answer is a firm "almost": that is, the pendulum by itself, even for large amplitude, is too simple a system to reveal the true complexities arising from nonlinearity. But, if the pendulum is driven—as it would be in a clock—and damped-as it is slightly by air resistance—then, for certain ranges of the driving and damping forces, the pendulum exhibits the full complexity of a nonlinear system, In particular, in a manner that can be analyzed quantitatively by a universality theory [see Los Alamos Science 1, No. 1, 4 (1980)], the pendulum proceeds through a sequence of subharmonic bifurcations leading to essentially turbulent motion ■

finement, the densities and temperatures must be increased until the plasma enters a highly nonlinear regime, one in which every slight change in circumstances tends to drive the system to instability.

New problems arise when plasmas interact with intense laser beams. The laser must compress the plasma to very high densities before fusion will occur, but the interface between the laser light and the plasma becomes highly unstable and fragments into jagged, fingerlike protrusions. This "Rayleigh-Taylor" instability plays havoc with the careful estimates of correct plasma density and time factor needed to initiate a fusion chain reaction. Such instabilities stand between us and the goal of controlled fusion—the hoped-for energy technology of the future—and they are not confined to lasers and plasmas. Essentially the same Rayleigh-Taylor phenomenon bedevils attempts at secondary oil recovery: when water is used to force oil to the surface, the interface between the two is unstable. To recover the maximum amount of oil from a given field, this instability must be understood and. if possible, transformed by clever technology from a vice into a virtue.

The occurrence of similar phenomena in such apparently different problems as laser-plasma interactions and oil recovery is one source of great optimism



Basil Nichols (left) and David K. Campbell of the Laboratory's Theoretical Division have charge of the administrative details of the new Los Alamos Center for Nonlinear Studies.

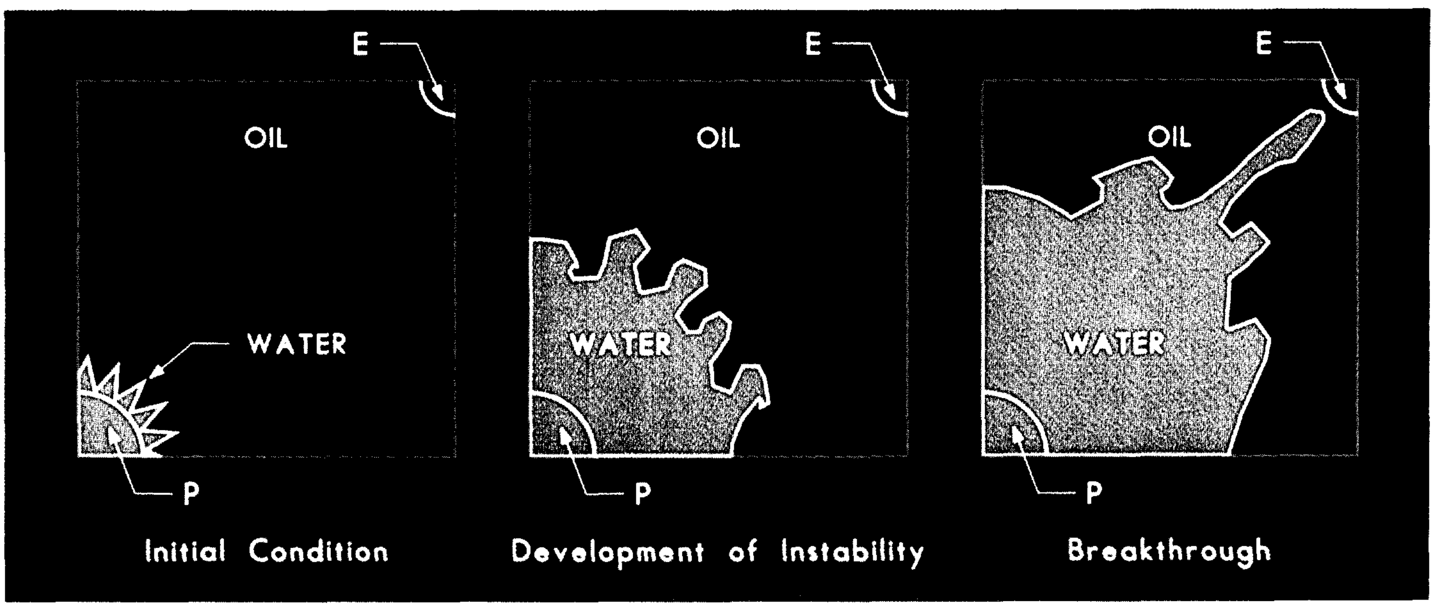
among experts in nonlinear science; it suggests that progress in one problem may quickly translate to others. Then too, today's fast, sophisticated computers are providing knowledge of the types of mathematics necessary to model nonlinear phenomena, Finally, recent progress in developing more explicit methods of solution for limited classes of nonlinear problems and, more generally. the emergence of global methods of qualitative analysis, encourage the hope that a deeper analytic understanding is close at hand.

Buoyed by this optimism and driven by the importance of the challenge, the new Los Alamos Center for Nonlinear Studies is mounting a unified attack on nonlinear problems. The Center is designed to foster the interplay of researchers in many fields and will provide a mechanism for correlating recent theories in mathematics and physics, numerical simulations, and physical experiments. The Center will also interact with other centers of nonlinear research, particularly those at University of California campuses.

To stress its interdisciplinary nature, the Center will not be associated with any specific Laboratory technical division but instead will be directed by a chairman on the staff of the Los Alamos Associate Director for Physics and Mathematics. Visiting scientists will work with Laboratory personnel on projects based on general nonlinear themes; they will be guided by a small permanent Center staff.

To strengthen ties with the academic community, a number of Senior Fellows from universities and research institutions will be appointed. Until a permanent chairman is named, the Center's efforts will be guided by a steering committee chaired by Mark Kac of Rockefeller University and including Rutherford Aris, University of Minnesota; Roger F. Dashen, Institute for Advanced Study, Princeton University; Martin D. Kruskal, Princeton University; Alwyn C. Scott, University of Wisconsin; J. Robert Schrieffer, University of California, Santa Barbara; and 1. M. Singer, University of California, Berkeley.

David K. Campbell and Basil Nichols of the Laboratory's Theoretical Division are handling the Center's administrative responsibilities ■



In some advanced methods of oil recovery, water is pumped into reservoirs to force oil to the surface. The diagrams above illustrate how instability of the oil-water interface effectively limits the amount of oil that can be recovered. The quarter circles P and E represent the water-input pump and the oil-extraction pump, respectively. The initial form of the oil-water interface shown here, symmetric and five-fingered, was chosen to model certain aspects of an actual recovery problem. At "breakthrough" the unstable finger- like protrusion of water reaches the extraction pump and no further oil can be recovered. In this modeling, the change in area occupied by water from initial conditions to breakthrough is directly related to the amount of oil recovered. Clearly, maximum oil recovery by this method requires understanding and control of the interfacial instability. (Courtesy of J. Glimm and E. Isaacson, The Rockefeller University, and D. Marchesin and O. McBryan, Courant Institute of Mathematical Sciences)