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THE THERMAL CONDUCTIVITY OF STABILIZED δ-PHASE PLUTONIUM

REF. LAB. CMR. 5

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Abstract

The thermal conductivity of stabilized δ-phase plutonium (3.4 atomic percent Gallium-Plutonium Alloy) has been determined as 0.0204 ± 0.0005 cal/cm^{2}sec/$^\circ$C/cm in the range 0-60°. The conductivity was determined from the steady state measurement of the temperature difference between the center and various points on the diametral plane of a 2 1/2 inch diameter sphere by means of the equation

$$k = Qr^2/\,6\Delta T$$

where Q is the heat source strength per unit volume, and r is the sphere radius for the ΔT observed.

The Thermal Conductivity of Stabilized δ-Phase Plutonium

Introduction

For homogeneous isotropic materials in which there is a constant volume source of heat, e.g. conversion of the kinetic energy of α particles into heat energy, the equation of conduction is:

$$k\,\Delta^2 T + Q = c\,\rho\,\partial T/\partial t$$

where

- k is the thermal conductivity in cal/(cm^2) (sec) ($^\circ$C/cm)
- T is the temperature in $^\circ$C
- Q is the heat source strength per unit volume in cal/(cm^3) (sec)
- c is the specific heat per gram in cal/($^\circ$C) (g)
- ρ is the density in g/cm^3

and t is the time in seconds.

For steady state conditions

$$k\,\Delta^2 T + Q = 0$$

or $\Delta^2 T = -Q/k$ (1)

For spherical symmetry, the flow of heat is a function only of r and T, hence equation (1) becomes

$$1/r\; d^2(rT)/dr^2 = -Q/k \qquad (2)$$

the solution of which is

$$T = -Qr^2/6k + a + b/r$$

Since $T = T_1$ at $r = 0$, b must be zero and $a = T_1$. The equation of conduction is therefore given by

$$T_1 - T = \Delta T = Qr^2/6k$$

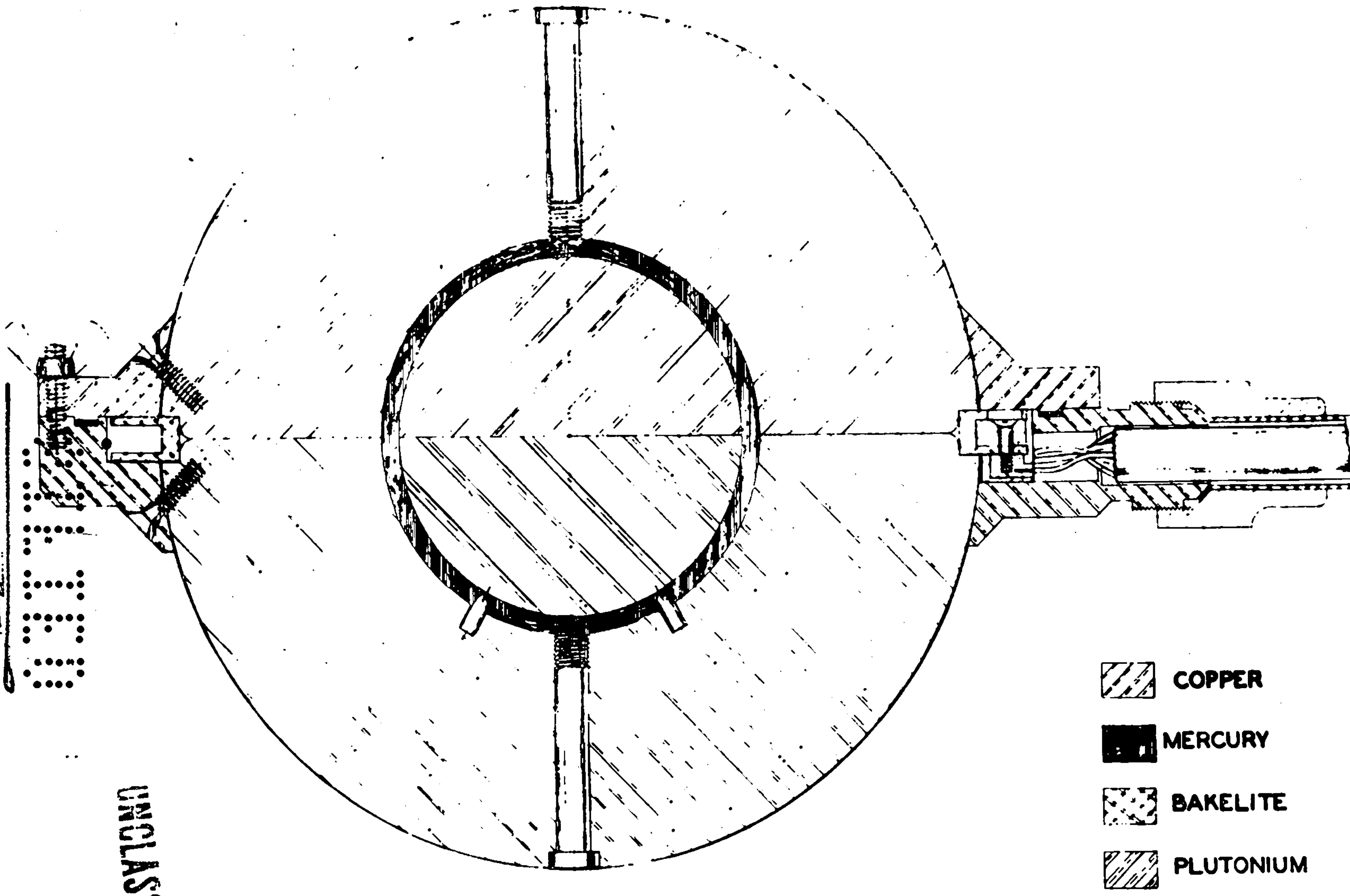
Experimental

The ΔT between the center and the points near the surface of the sphere was obtained by inserting a network of differential thermocouples between two plutonium hemispheres. In order to insure uniform surface temperature for the plutonium, the sphere was encased in a larger spherical shell of copper. A gap of approximately 1/8 inch thickness was left between the plutonium sphere and the copper shell, and this was filled with mercury to insure uniform radial heat transfer between the plutonium and the copper. A layer of Apiezon grease between the copper shells sealed the mercury in place. The cold junctions of the thermocouples were located in a hollow copper ring on the surface of the copper sphere, from which a cable led to the potentiometer. A cross section of the assembled apparatus is shown in Fig. 1, and a horizontal section is shown in Fig. 2.

1. Plutonium Sphere: The sphere was fabricated as hemispheres in order to permit the placement of a network of differential thermocouples upon a diametral plane. The polar height of one of the hemispheres was made 11 mils greater than the other. The plane surface of the higher hemisphere was then grooved as shown in Fig. 3 and the thermocouple network placed in these grooves. The hemispheres were fabricated as δ-phase stabilized gallium alloy. A table of specifications is given below, and the chemical analysis of the hemispheres is given in Table II.

TABLE I

Specifications of hemispheres	Lower Hemisphere C-17	Upper Hemisphere C-18
Equatorial diam. (inches)	2.502	2.502
Polar height (inches)	1.256	1.245
Weight of alloy (grams)	1069.45	1056.85
Atomic percent gallium	3.57 calc 3.41 found	3.60 calc 3.41 found
Density (g/cc	15.80	15.91



THERMAL CONDUCTIVITY SPHERE ASSEMBLY

FIG. 1

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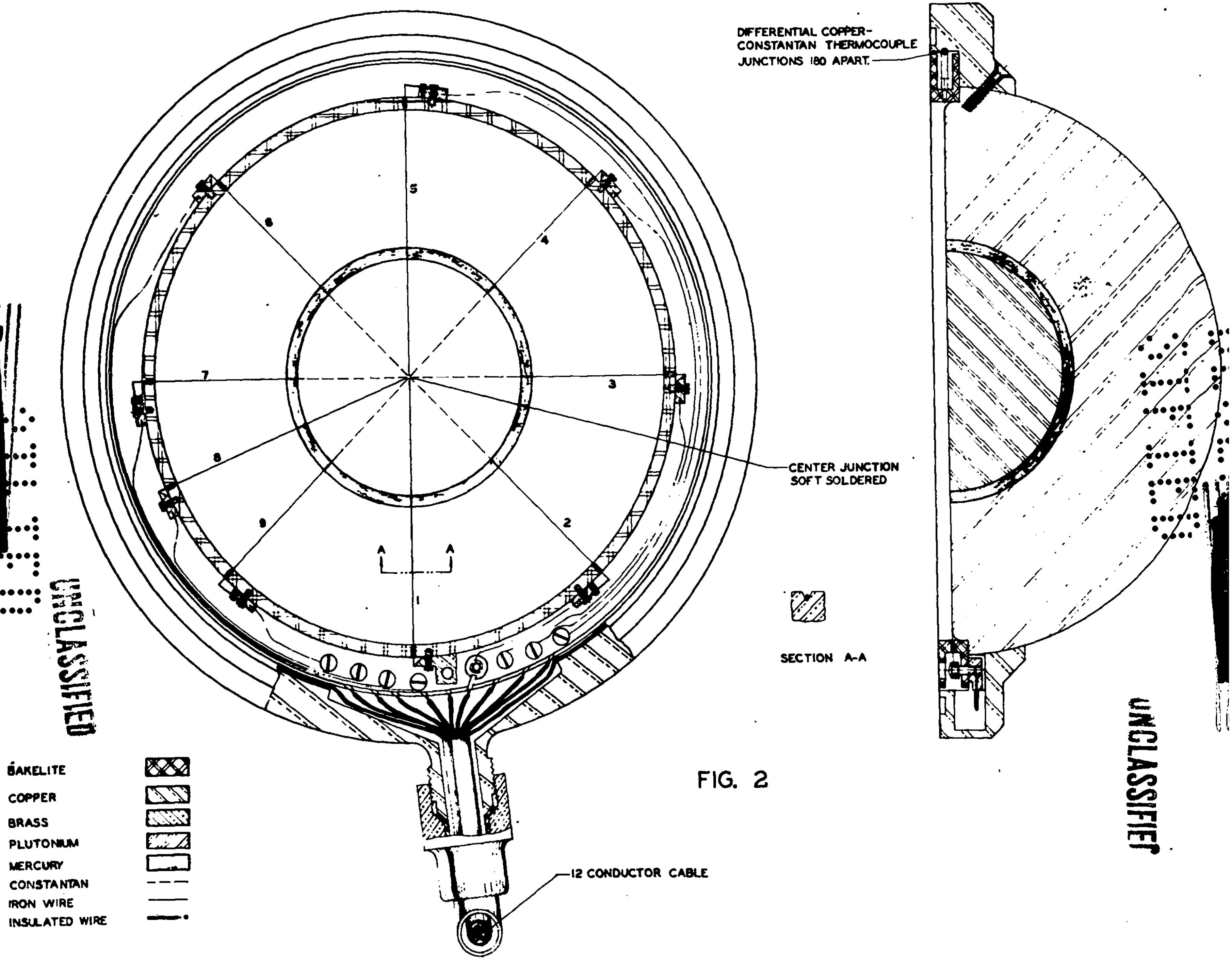
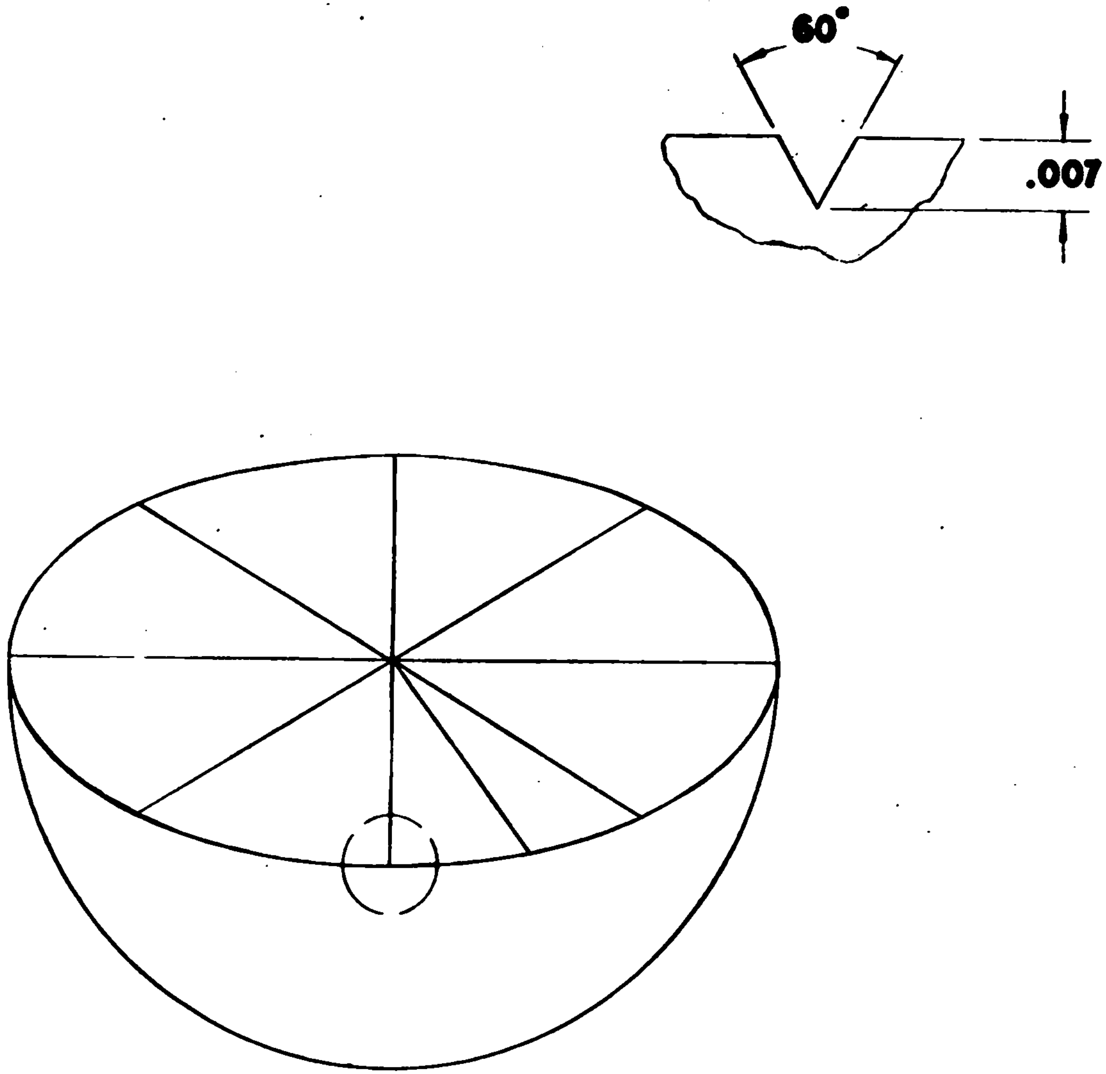

DIFFERENTIAL COPPER-CONSTANTAN THERMOCOUPLE JUNCTIONS 180 APART.
CENTER JUNCTION SOFT SOLDERED
SECTION A-A
12 CONDUCTOR CABLE
BAKELITE
COPPER
BRASS
PLUTONIUM
MERCURY
CONSTANTAN
IRON WIRE
INSULATED WIRE

FIG. 2



C-17 LOWER PLUTONIUM HEMISPHERE

FIG. 3

TABLE II

Analysis of Pu Hemispheres in ppm

Impurity	Sample C-17	Sample C-18
B	< 0.5	0.5
Ga	10,200	10,100
S	20,25	< 5
Al	10	9.0
Ba	ND<2.1	ND<1.8
Be	ND<0.2	ND<0.18
Ca	2.1	3.6
Cd	ND<21	ND<1.8
Cd	ND<210	ND<180
Co	ND<210	ND<180
Cr	ND<2.1	ND<1.8
K	ND<21	ND<18
La	ND<2.1	ND<1.8
Li	ND<1.0	ND<0.9
Mg	2.1	9.0
Mn	ND<2.1	ND<1.8
Na	6.3	5.4
Ni	ND<21	ND<18
Pb	ND<21	ND<18
Sr	ND<2.1	ND<1.8
Hg	ND<60	ND<60
Fe	ND	ND<500
Bi	ND<1500	ND<1500
Cu	40	20
Ge		ND<60
In	ND<60	ND<60

TABLE II Cont'd.

Impurity	Sample C-17	Sample C-18
Sb	ND<300	ND<300
Si	ND<1500	ND<1500
Sn	ND<200	ND<200
Tl	ND<60	
V	ND	ND
Zn	ND<1000	ND<1000

ND - Not detected.

The hemispheres were then brushed free of adherent oxide with a wire brush. A 0.3 mil coating was then applied in the usual manner[1]. After coating, the hemispheres were buffed by a soft wire brush.

2. Thermocouple Assembly: Iron-constantan thermocouples, previously tested for thermoelectric inhomogeneities, were employed since these yield a high and reproducible EMF per degree. In order to minimize heat leakage effects, the diameter of the wires was made as small as practicable, i.e., 3 mils. Since the measurement is fairly sensitive to the position of the junctions, they were made by butt-welding[2] with the exception of the center junction which was soft soldered. Photomicrographs of three junctions are shown in Fig. 4. The wires were insulated by spraying the assembled network with diluted glyptol and baking after each spraying. Six coatings, approximately 0.16 mils thick were applied. The network of thermocouples was supported on a micarta ring as shown in Fig. 2. It was felt that calibration of the thermocouples after assembling the network was unwise in

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1 Lipkin; LA-378

2 Hammel; LADC-393

[redacted]

Fig. 4

Photo micrographs under polarized light, magnification 75x of three iron-constantan junctions. The upper junction shows the effect of too high a current, resulting in the partial melting of the constantan. The exact position of the junction is seen clearly in the upper and middle photographs, and somewhat less definitely in the lower photograph. This is however due to difficulties in illumination for purposes of photography. To the eye under an ordinary microscope, the position of the junction is unambiguous.

view of its fragile character. Hence several similar junctions were made (one junction being butt-welded, the other soldered) from the same lot of wire used in the assembly. Each junction was insulated by a baked glyptol coating, then placed in a hole in a large copper block, thermal contact between thermocouple and block being made by kerosene, and the assembly was then placed in a dewar flask. The copper blocks also contained calibrated Pt-resistance thermometers. The block containing the soldered junction was heated approximately 1°C above the other, and after the temperature had stabilized the temperature difference obtained from the Pt-resistance thermometer was compared with the EMF observed. It was found that the junctions produced an EMF of 49.0 ± 0.5 µv/°C.

3. Miscellaneous: The assembled sphere was placed in a vigorously stirred water bath which was variable in temperature from 10° to 60°C and was constant at any temperature to $\pm$ 0.01°C. The temperature of the bath was read with a calibrated Pt-resistance thermometer.

Electrical measurements were made with a Wenner potentiometer.

The temperature of the cold junctions was uniform to $\sim$.002°C.

Discussion of Errors

Unavoidable deviations from the sphere model treated theoretically are present in the experimental setup. Since these deviations introduce errors in the measurement, it was necessary to minimize them or correct for them in the final results.

1. Coating of the Hemispheres: In order to minimize the spread of contamination, each hemisphere was nickel coated. Obviously, the thinner the coating the closer ideal conditions are approached. An investigation was therefore made of the probable error for a coating of 0.3 mils of nickel, the thinnest practical coating thickness. Setting up and solving the heat equation for the composite system actually used is a relatively difficult mathematical exercise. However, a satisfactory approximation can be obtained by considering

[redacted]

the results of two simpler models, model II more closely resembling the actual situation than model I. These models are discussed fully in Appendix I. The results are summarized in Table III.

TABLE III

Model	% Error for 0.3 mil nickel coat
I (infinite cylinder)	0.38
II (finite cylinder)	0.36

Although no extrapolation is possible due to the discontinuous nature of the models, it appears unlikely that the error for the sphere could be greater than 0.3 ± 0.2 %. Since this error is negative, in that it causes the observed thermal conductivity to be less than the true value, a correction of approximately 0.0001 in thermal conductivity units should be added to the observed value.

2. Thermocouple Errors: The existence of the thermocouples themselves as well as the insulating material covering the thermocouple wires introduces additional errors in the measurement. Heat may be conducted from the interior of the sphere by the thermocouple wires, thus distorting slightly the otherwise spherically symmetric radial heat flow; and as the temperature of the wire depends upon the steady state attained between heat flow into the wire through the insulation and heat flow along the wire, the observed temperature at a junction cannot represent the temperature of the material adjacent to the insulation at that point. In Appendix II it is shown that these errors are quite negligible. The difference in temperature between an isothermal spherical shell in a perfect sphere and the corresponding point on the thermocouple wire cutting a similar shell in the actual assembly, is of the order of 10^{-5} °C. Hence no appreciable distortion of the radial heat flow occurs. The amount of heat lost from the sphere through the thermocouple wires is less than $10^{-[illegible]}$ of the total heat generated by the sphere. The effect of the insulation is also

[redacted]

negligible. Details of this calculation are given in Appendix III. It was shown that the difference in temperature between thermocouple and adjacent nickel coating is of the order of 10^{-4} °C in the interior of the sphere, rising rather abruptly to about 5×10^{-3} °C at the surface.

Results

After assembling the apparatus, it was accidentally jarred. This resulted in several short circuits between the plutonium and the thermocouple wires. Upon investigation it was seen that the upper plutonium hemisphere had shifted slightly, and in doing so had ruptured the glyptol insulation at various points between the hemispheres. While remedying this, the thermocouple wires outside the plutonium were stretched, and upon reassembly it was not possible to mount the center of the thermocouple network exactly at the center of the plutonium hemisphere. The network was off center by about 1/4 mm. Consequently, the measured distances of junctions from the center are uncertain by this amount, and an uncertainty of about 5 percent is introduced into the values of k. Although the system was free from short circuits at the beginning of the experiment (after the reassembly), a short soon developed between the center junction and the plutonium. Then one of the thermocouple wires shorted to the copper. The final values of k therefore constitute an average taken from independent measurements on the five remaining thermocouples. Upon dismantling the assembly, it was found that the weight of the upper plutonium hemisphere had flattened the center junction slightly (a depression to accommodate this junction had been made in the lower hemisphere but apparently was not deep enough).

The distances of the junctions from the center as originally determined with a Gaertner comparator are given in Table IV.

[redacted]

TABLE IV.

Junction	Distance - Center to Junction
2	28.084 ± 0.006 mm
3	27.723 ± 0.004
4	27.906 ± 0.004
6	23.598 ± 0.005
7	26.299 ± 0.005
9	25.140 ± 0.004

Note: The position of the butt welds could be determined to < 0.002 mm. The large uncertainty in the above measurements is due to uncertainty of the center of the center junction.

The steady state EMF values with their corresponding k values for the various junctions, except #4 which was discarded, are given in Table V for various temperatures. The values of k are summarized in Table VI.

[redacted]

[redacted]

TABLE V

Temp. of Sphere Center	0.85°C		8.53°C		10.14°C		19.93°C		35.80°C		60.07°C	
Junction	EMF μv	k	EMF μv	k	EMF μv	k	EMF μv	k	EMF μv	k	EMF μv	k
6	17.7	0.0195	17.4	0.0198	17.4	0.0198	17.2	0.0201	17.8	0.0194	18.1	0.0191
7	21.8	0.0195	21.2	0.0201	21.2	0.0201	20.9	0.0203	21.4	0.0199	21.4	0.0199
3	23.1	0.0204	22.5	0.0211	22.4	0.0211	22.2	0.0213	22.1	0.0214	23.0	0.0205
2	23.8	0.0204	23.1	0.0210	23.0	0.0211	22.7	0.0214	22.7	0.0214	23.0	0.0211
9	24.4	0.0200	23.7	0.0205	23.7	0.0205	23.4	0.0208	23.9	0.0204	24.6	0.0205

TABLE VI

Average Values of the Thermal Conductivity

of δ-Phase Stabilized Plutonium

Temperature in °C	k in cal/(cm^2) (sec) (°C/cm)
0.85	0.0200 ± 0.0004
8.53	0.0205 ± 0.0004
10.14	0.0205 ± 0.0004
19.93	0.0207 ± 0.0004
35.80	0.0205 ± 0.0007
60.07	0.0202 ± 0.0005

Note: The above limits of k represent the precision of the results. In view of the experimental difficulties the absolute error is of the order of 5 percent.

[redacted]

[redacted]

The determinations at 0.85°C and at 60°C requires special mention. The low temperature value was obtained by immersing the sphere in a slush of ice and water. Since adequate circulation was quite impossible, the low value of k may be attributable to a possible non-uniformity in temperature of the copper shell. At 60°C the measurements were made immediately upon reaching the desired temperature. Shortly thereafter, the mercury forced the melted grease from between the spheres and shorted all thermocouples. While the measurement was being made, therefore, it is highly probable that the copper-mercury spherical interface was partially covered with melted grease. Hence the radial heat flow was probably markedly distorted.

Discussion

In view of the difficulties which were encountered during the experiment, the agreement among the values of k calculated from the five different thermocouples emphasizes the merit of this method and leads one to expect that if carried out properly the technique would yield results as satisfactory as those obtained by the usual, rod-type methods.

With respect to the results as reported in Table VI, it should be pointed out that only the first two significant figures have any validity in view of the non-centering of the thermocouple network. The values were reported to three figures simply to show the agreement among thermocouples.

In view of the lack of other data on physical properties of δ-phase plutonium, no further discussion is warranted at this time.

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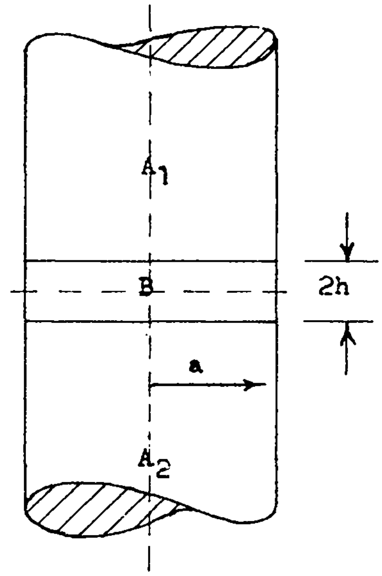
APPENDIX

Model I



Consider a cylinder of radius a, infinitely long. Let the cylinder be composed of material A except for a disc of thickness 2h in the center which is material B. Let material A generate Q cal/sec cm^3 and have a thermal conductivity k_A. The thermal conductivity of material B is k_B, and the temperature of the cylinder surface is maintained at zero.

Then for source free region B

$$\Delta^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0$$

and for region A

$$\Delta^2 T + \frac{Q}{k_A} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{Q}{k_A} = 0$$

The solution for region B is

$$T_B = \sum_k B_k \cosh (kz)\, J_o (kr) \qquad \text{where } J_o (ka) = 0$$

and for region A

$$T_A = \frac{Q}{4 k_A} (a^2 - r^2) + \Sigma_k A_k \exp (-kz)\, J_o (kr) \qquad \text{where } J_o (ka) = 0$$

Let $\quad \Sigma_k G_k J_o (kr) = \dfrac{Q}{4 k_A} (a^2 - r^2) = C (a^2 - r^2)$

[redacted]

Then

$$G_k = \frac{2C}{a^2 J_1^2(ka)} \int_0^a (a^2 - r^2) \, r \, J_0 \; (kr) \; dr$$

$$= \frac{2C}{a^2 J_1^2 \; (ka)} \left[a^2 \int_0^a r \; J_0 \; (kr) \; dr - \int_0^a r^3 J_0 \; (kr) \; dr \right]$$

but: $\int_0^x x \; J_0 \; (x) \; dx = x J_1(x)$ and $\int_0^x x^3 J_0(x) dx = (x^3 - 4x) \; J_1(x) + 2x^2 J_0(x)$

Therefore

$$G_k = \frac{8a^2 C}{(ka)^3 J_1(ka)} = \frac{2a^2 Q}{k_A (ka)^3 J_1(ka)}$$

Applying the boundary conditions:

$$\left[T_A\right]_{z=h} = \left[T_B\right]_{z=h}$$

and

$$k_A \left(\frac{\partial \, T_A}{\partial z} \right)_{z=h} = k_B \left(\frac{\partial \, T_B}{\partial z} \right)_{z=h}$$

$$B_k \cosh(kh) = G_k + A_k \; \exp \; (-kh)$$

$$k_B B_k \sinh(kh) = -k_A A_k \exp \; (-kh)$$

Solving

$$B_k = \frac{G_k}{\cosh(kh) + \dfrac{k_B}{k_A} \; \sinh(kh)}$$

The temperature at the origin is given by $T_{po} = \Sigma \; B_k$. *Therefore*

 [redacted]

$$\frac{\Delta T}{T_{oo}(\text{ideal})} = \frac{\Sigma G_k - \Sigma g_k}{\Sigma G_k}$$

To a first approximation since $\frac{h}{a}$ is small

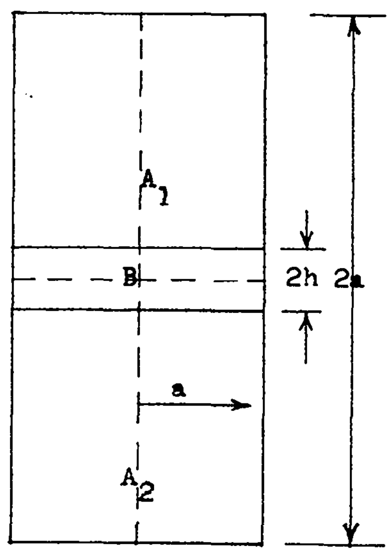
$$2.07 \frac{k_B}{k_A} \cdot \frac{h}{a} < \frac{\Delta T}{T_{oo}(\text{ideal})} < 2.22 \frac{k_B}{k_A} \cdot \frac{h}{a}$$

Using the proper values of k_B/k_A and a, and for a coating thickness of 0.3 mils,

$$\frac{\Delta T}{T_{oo}\ (\text{ideal})} \approx 0.38\ \%$$

Model II

This model is similar to Model I except that the cylinder is finite of length 2 a as shown in the drawing. It is instructive, however, to first consider the problem of temperature distribution in a finite cylinder of length 2 a composed entirely of material A, the boundaries of which are held at zero.



The equation of conduction for the latter problem is

$$\Delta^2 T + \frac{Q}{k_A} = 0$$

the solution of which is

$$T = \frac{Q}{4 k_A} (a^2 - r^2) + \Sigma_k A_k J_0(kr) \cosh (kz)$$

Using the result obtained in the solution for Model I this may be written

$$T = \Sigma \left(G_k + A_k \cosh (kz) \right) J_0(kr) \qquad \text{where } J_0(ka) = 0$$

Since for $z = a$, $T = 0$

$$G_k + A_k \cosh (ka) = 0$$

$$A_k = - \frac{G_k}{\cosh(ka)}$$

and

$$T = \Sigma_k \; G_k \left[1 - \frac{\cosh(kz)}{\cosh(ka)} \right] J_0(kr)$$

For the problem of the split finite cylinder, the equation of conduction is for region A

$$\Delta^2 T + \frac{Q}{k_A} = 0$$

and for region B

$$\Delta^2 T = 0$$

the solutions of which are respectively

$$T_A = \Sigma_k \left[G_k \left(1 - \exp \left\{ -k(a-z) \right\} \right) + A_k \sinh k(a-z) \right] J_0 (kr)$$

and

$$T_B = \Sigma \; B_k \cosh (kz) \; J_0 (kr)$$

[redacted]

where

$$J_o(ka) = 0$$

The boundary conditions are

$$(T_A)_{z=h} = (T_B)_{z=h}$$

and

$$k_A \left(\frac{\partial T_A}{\partial z}\right)_{z=h} = k_B \left(\frac{\partial T_B}{\partial z}\right)_{z=h}$$

giving

$$G_k\left(1-\exp\left\{-k(a-h)\right\}\right) + A_k \sinh k(a-h) = B_k \cosh(kh)$$

and

$$-k_A\left[G_k \exp\left\{-k(a-h)\right\} + A_k \cosh k(a-h)\right] = k_B B_k \sinh(kh)$$

Solving for B_k

$$B_k = G_k \frac{\cosh\left\{k(a-h)\right\} - 1}{\cosh k(a-h)\cosh kh + k_B/k_A \sinh k(a-h)\sinh kh}$$

To determine $T_{B(o,o)}$, assume $a-h \approx a$ which is very nearly true.

Then

$$T_{B(o,o)} = \Sigma\, G_k \frac{\cosh(ka) - 1}{\cosh(ka)\cosh(kh) + k_B/k_A \sinh(ka)\sinh(kh)}$$

The value of $T_{(o,o)}$ if there were no B layer is

$$T_{A(o,o)} = \Sigma\, G_k \left(1 - \frac{1}{\cosh(ka)}\right)$$

Therefore

$$\frac{\Delta T}{T_{oo}}(\text{ideal}) = \frac{\Sigma\, G_k\left(1-\frac{1}{\cosh(ka)}\right)\left(1-\frac{1}{\cosh(kh) + k_B/k_A \frac{\sinh(ka)}{\cosh(ka)}\sinh(kh)}\right)}{\Sigma\, G_k\left(1-\frac{1}{\cosh(ka)}\right)}$$

To a first approximation this becomes

$$1.97\ k_B/k_A\ \frac{h}{a} < \frac{\Delta T}{T_{ideal}} < 2.08\ k_B/k_A\ \frac{h}{a}$$

For a coating thickness of 0.3 mils and using appropriate values for k_B/k_A and a,

$$\frac{\Delta T}{T_{ideal}} \approx 0.36\ \%$$

[redacted]

APPENDIX II

The amount of heat flowing from the sphere through a thermocouple wire is given by the expression

$$Q = k\pi r^2 \left(\frac{dT}{dx}\right)_b$$

where k is the thermal conductivity of the wire = 0.155 cal/cm sec °C

r is the radius of the wire, 0.0038 cm

and $\left(\frac{dT}{dx}\right)_b$ is the temperature gradient in the wire at the periphery of the sphere, equal in this case to -0.193 °C/cm. Q is therefore 1.33×10^{-6} cal/sec and, as may be seen from Appendix III, the amount of heat flowing into the wire per unit length is uniform except near the surface of the sphere.

An estimate of the distortion in the radial heat flow may be obtained as follows: Assume that all of the heat flowing into the wire is generated in a cylindrical shell r_3 cm from the axis of the wire.

Then $\dfrac{1.33 \times 10^{-6}}{3.175} = \dfrac{2\pi k_{Pu}(T_3 - T_2)}{\text{Ln } r_3/r_2}$ = heat flow into wire per unit length

giving:

$$3.42 \times 10^{-6} \text{ Ln } r_3/r_2 = \Delta T$$

For $r_3/r_2 = \dfrac{0.0195}{0.0065} = 3.00$ $\quad$ Ln r_3/r_2 = 1.10

$= \dfrac{0.0650}{0.0065} = 10$ $\quad$ = 2.31

$= \dfrac{0.650}{0.0065} = 100$ $\quad$ = 4.61

 [redacted]

where

r_2 is outside radius of insulation

T_3 is Temperature at r_3,

T_2 is Temperature at r_2.

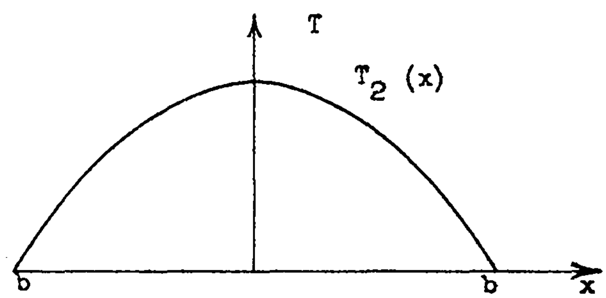
Therefore the ΔT in the body of the material caused by heat flow into a thermocouple wire is of the order of 10^{-5} °C, and hence is negligible.

[redacted]

APPENDIX III

Problem of Temperature Distribution in Thermocouple

Assume insulated wire imbedded in a medium of such a nature that the temperature distribution along the length of the insulation is $T_2 = a(b^2 - x^2)$ and $T_2 = 0$ for values of $x > b$ and that this temperature distribution is cylindrically symmetric with respect to the wire axis and symmetric with respect to x which is taken along the wire axis. This is a fairly good approximation to one of the differential thermocouples in the experimental setup.



Considering a cross section through the cylinder, $T_2(x)$ is the temperature at the point x on the outside of the insulation or the temperature of the nickel coating in contact with the insulation at x. Since the conductivity of the wire is many times that of the insulation, it may be assumed that the temperature through any cross section of the wire is uniform and equal to $T_1(x)$. Then the heat flow into the wire, through the insulation in a length dx, is

$$F = \frac{2\pi k_1 (T_2 - T_1)}{\operatorname{Ln} r_2/r_1} \, dx$$

where r_2 is the outer radius of the insulation and r_1 is the inner radius.

or

$$(T_2 - T_1)\, dx = \frac{F \operatorname{Ln} r_2/r_1}{2\pi k_1} \qquad (1)$$

Now if we consider G(x) equal to the amount of heat flowing through the wire at a point x, then

$$F = \frac{dG}{dx}$$

[redacted]

but

$$G = -k_w \pi r_1^2 \frac{dT_1}{dx}$$

Hence

$$F = \frac{d}{dx}\left(-k_w \pi r_1^2 \frac{dT_1}{dx}\right) dx \qquad (2)$$

and substituting (2) in (1) gives

$$T_2 - T_1 = \frac{d}{dx}\left(-k_w \pi r_1^2 \frac{dT_1}{dx}\right) \frac{\mathrm{Ln}\ r_2/r_1}{2\pi k_i}$$

Letting

$$\frac{1}{C^2} = \frac{k_w}{2k_i} r_1^2 \ \mathrm{Ln} \frac{r_2}{r_1}$$

$$T_2(x) - T_1(x) = -\frac{1}{C^2}\frac{d^2T_1}{dx^2}$$

or

$$\frac{d^2T_1}{dx^2} - C^2T_1 = -C^2T_2$$

The solution of this equation is

$$T_{1(int)} = A \cosh cx + a(b^2 - x^2) - \frac{2a}{C^2}$$

since the system is symmetric in x.

Since

$T_2 = 0$ for $x > b$

for $x > b$

$T_{1(ext)} = L \exp(-cx)$

The boundary conditions are therefore

$$T_{1(int)}(b) = T_{1(ext)}(b)$$

[redacted]

and

$$\left(\frac{dT_{1\,(int)}}{dx}\right)_{x=b} = \left(\frac{dT_{1\,(ext)}}{dx}\right)_{x=b}$$

From these conditions

A is determined as $\frac{2a}{c^2}\ \exp(-cb)\ (1+bc)$

and L is $\frac{a}{c^2}\ \exp(cb)\ (bc-1)$

Since exp (-cb) is fantastically small, the "cosh term" in $T_{1(int)}$ is of little importance except near x=b, hence

$$T_{1(ext)} \approx a(b^2 - x^2) - \frac{2a}{c^2}$$

and $T_2 - T_1 \approx \frac{2a}{c^2}$ giving $\Delta T \approx \underline{1.35 \times 10^{-4}\ {}^{o}C}$

for regions where $x > b$. When $x \approx b$, the "cosh term" is of interest.

$$\left(T_2 - T_1\right)_{x=b} = \left(-T_1\right)_{x=b} = -\frac{2a}{c^2}\exp(-cb)\ (1+bc)\ \ 1/2\ \exp(cb) + \frac{2a}{c^2}$$

$$= -\frac{a}{c^2}\exp(cb)\ (bc-1)\ \exp(-cb) = -\frac{a}{c^2}\ (bc-1)$$

$$= -6.75(93.45) \times 10^{-5}$$

$$\underline{\underline{\left(T_1\right)_{x=b} = .0063^{o}\ C}}$$

Finally

$$\left(\frac{dT_2}{dx}\right)_{x=b} = -0.39$$

$$\left(\frac{dT_1}{dx}\right)_{x=b} = -0.195$$

[redacted]

APPENDIX IV

Values of constants used in this work:

Heat source strength of plutonium 3 atomic percent Ga and density 15.8 = 0.007528 cal/sec/cm^3, obtained as follows:

From LA-347 power produced by pure plutonium is 1.923 x 10^{-3} abs. watts/g, or 4.594 x 10^{-4} cal/g sec. This must be corrected for the isotope ^{240}plutonium content as follows:

Average g/T ✱ level of plutonium used in sphere = 219.

Isotope ^{240}plutonium concentration, calculated by following formula (See LA-490):

$$\frac{^{240}Pu}{^{239}Pu} = 70.9 \frac{^{239}Pu}{^{238}U},$$

is 1.68 percent by weight.

Energy produced per gram of ^{240}Pu is calculated as follows:

$$1.922 \times 10^{-4} \text{ abs. watts/g} \times \frac{2.411 \times 10^4 \times 239}{6.260 \times 10^3 \times 240}$$

$$= 7.37 \times 10^{-3} \text{ abs. watts/g}$$

Therefore the total energy produced by the plutonium used in these hemispheres is $(1.922 \times 0.983 + 7.37 \times 0.017) \times 10^{-3}$

$$= 2.014 \times 10^{-3} \text{ abs. watts/g} = 4.811 \times 10^{-4} \text{ cal/g sec.}$$

This calculation depends upon the energy of the ^{240}plutonium α's being the same as those from 49. There is no data at the present time on the energy of the α's from ^{240}plutonium, but their range is believed to lie within 3 mm of those from 49. Since the range of 49 α is 3.675 cm in air at 15° C and 760 mm, this correction is small and has not been made.

✱g/T refers to grams of plutonium produced/Ton of uranium

[redacted]

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