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PENETRATION OF RADIATION INTO COLD MATERIAL, II

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Washington Document Room	1 thru 4
J. R. Oppenheimer	5
Los Alamos Document Room	6 thru 20

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ABSTRACT

If density can be treated as constant, and if hydrodynamics can be neglected, the penetration of radiation into cold material is described approximately by the equations

$$T(x,t) \cong T_0(t)\left(1 - \frac{x}{x_0(t)}\right)^{\frac{1}{n+4-\beta}}$$

$$x_0(t)^2 \cong \frac{8}{n+4-\beta} \; \frac{a\,c}{3bK_0\rho^2} \int_0^t T_0(t')^{\,n+4-\beta}\, dt'$$

The condition for validity of these equations is that $T_0(t)$, the driving temperature at $x = 0$, is a smoothly increasing function of time. Here energy per unit mass (including radiant energy) has been written as bT^{β}, and opacity as K_0T^{-n}. Other symbols have familiar meanings.

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INTRODUCTION

The problem of the penetration of radiation into cold material is one that is difficult to handle, and also one that arises with disagreeable frequency. In LA-230, Marshak solved this problem to a high degree of approximation under the following special assumptions

1. All hydrodynamics may be neglected. Density is assumed constant.
2. Opacity is assumed to vary with temperature as T^{-n}, where $n > 0$. For most materials, $n \sim 3$.
3. Energy per unit volume is proportioned to the absolute temperature T.
4. Radiation is driven from some arbitrary surface. Along this surface, the driving temperature varies with time either according to some power of the time, or exponentially with time.

The present paper contains two generalizations of Marshak's work, which, though trivial in content, considerably extend the range of applicability of Marshak's results. One of these generalizations is the observation that Marshak's method should be applicable to cases where the driving temperature is any smoothly rising function of time. The other is the extension to cases

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where energy can be approximated as proportion to some power of the temperature (not necessarily the first). Marshak considered radiation in a plane geometry, and also radiation from a point source. We treat only the former case.

The differential equation for flow of radiation, where hydrodynamics can be neglected, and in the absence of energy sources, is

$$\frac{\partial}{\partial t}(E\rho) = \operatorname{div}\left(\frac{\Lambda c}{3}\ \operatorname{grad}\ (a\ T^4)\right), \tag{1}$$

where E is energy (including radiant energy) per unit mass

ρ is density (assumed constant)

Λ is radiation mean free path

a is the Stefan - Boltzmann constant

c is velocity of light

T is temperature

t is time

We will assume that

$$E = B\ T^{\beta}$$

and

$$\Lambda = \frac{T^n}{\rho K_o}$$

B, β, K_o and n are all to be treated as constants. $K_o T^{-n}$ is the opacity of the material. Then Eq. (1) takes the form

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$$\frac{\partial (T^{\beta})}{\partial t} = \frac{ac}{3\,B\,K_o \rho^2} \quad \frac{4}{n+4} \quad \frac{\partial^2 (T^{\,n+4})}{\partial x^2} = D\,\frac{\partial^2 (T^{\,n+4})}{\partial x^2}\ . \tag{2}$$

SOLUTION OF THE EQUATION BY SERIES.

We first study the equation

$$\frac{\partial T\,(x,t)}{\partial t} = D\,\frac{\partial^2 \left(T(x,t)^k \right)}{\partial x^2} \tag{3}$$

with boundary conditions

$T(x,0) = 0$ for $x > 0$

$T(0,t) = T_o\ (t)$ for $t > 0$

T_o is a smoothly varying function of time. Marshak has pointed out that the solution to Eq. (3) has a sharp front, which constitutes a singular point in the solution. If $x_o(t)$ represents the location of the front, as a function of time, the behavior of the solution in the neighborhood of the singularity is as $(x_o - x)^{\frac{1}{k-1}}$. Thus we seek a solution of the form

$$T(x,t) = Y^{\frac{1}{k-1}} \left[A_o(t) + A_1(t)\,Y + A_2(t)\,Y^2 + \ldots \right], \tag{4}$$

where we have set

$$Y = 1 - \frac{x}{x_o\,(t)}\ . \tag{5}$$

$$T^k = Y^{\frac{k}{k-1}} \left\{ A_o^{\,k} + kA_o^{\,k-1} A_1 Y + \left(kA_o^{\,k-1} A_2 + \frac{k(k-1)}{2} A_o^{\,k-2} A_1^{\,2} \right) Y^2 + \ldots \right\}$$

$$\frac{\partial Y}{\partial x} = -\frac{1}{x_o}$$

$$\frac{\partial Y}{\partial t} = \frac{x}{x_o^2}\,\frac{dx_o}{dt} = (1 - Y)\ \frac{1}{x_o}\ \frac{dx_o}{dt}$$

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Substitution of these forms into Eq. (3) results in the equation

$$Y^{\frac{1}{k-1}}\left\{\frac{A_o}{k-1}\,\frac{1}{Y}\,\frac{1}{x_o}\,\frac{dx_o}{dt} + \sum_{r=0}^{\infty}\left[\frac{dA_r}{dt} + \left[\left(r+1+\frac{1}{k-1}\right)A_{r+1} - \left(r+\frac{1}{k-1}\right)A_r\right]\frac{1}{x_o}\,\frac{dx_o}{dt}\right]Y^r\right\} = \frac{D}{x_o^2}\,\frac{1}{(k-1)^2}\,Y^{\frac{1}{k-1}}\left\{k\cdot 1\,\frac{A_o^{k}}{Y} + (2k-1)\,k\cdot kA_o^{k-1}A_1 + (3k-2)(2k-1)\left(kA_o^{k-1}A_2 + \frac{k(k-1)}{2}A_o^{k-2}A_1^{2}\right)Y + \dots\right\} \qquad (6)$$

We have expanded both sides of Eq. (3) according to powers of Y, and now must equate coefficients of like powers. From the terms proportional to $Y^{\frac{1}{k-1}-1}$, we find that

$$\frac{1}{x_o}\,\frac{dx_o}{dt} = \frac{-D}{x_o^2}\,\frac{k}{k-1}\,A_o^{k-1} \qquad (7)$$

or

$$x_o^2 = \frac{2k}{k-1}\,D\int_0^t A_o(t')^{k-1}\,dt' = \frac{2k}{k-1}\,D\,Z(t) \qquad (8)$$

The succeeding A's can be determined in terms of A_o by equating coefficients of successively higher powers of Y in Eq. (6). For example (making use of Eqs. (7) and (8)),

$$\frac{2Z}{A_o^{k-1}}\,\frac{dA_o}{dt} + \frac{kA_1 - A_o}{k-1} = \frac{(2k-1)k}{k-1}\,A_1\,,$$

$$\frac{2Z}{A_o^{k-1}}\,\frac{dA_1}{dt} + \frac{(2k-1)A_2 - kA_1}{k-1} = \frac{(3k-2)(2k-1)}{k-1}\left(A_2 + \frac{k-1}{2}\,\frac{A_1^2}{A_o}\right),$$

etc. Finally, the boundary condition at $x = 0$ ($Y = 1$) requires that

$$T_o(t) = A_o(t) + A_1(t) + A_2(t) + \dots \qquad (9)$$

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APPROXIMATE SOLUTION

The proposed approximate solution to Eq. (3) consists of dropping all terms involving powers of Y greater than $\frac{1}{k-1}$, and identifying A_o with T_o. Then

$$\left.\begin{aligned} T(x,t) &\cong T_o(t)\left(1-\frac{x}{x_o(t)}\right)^{\frac{1}{k-1}} ; \\ x_o(t)^2 &\cong \frac{2k}{k-1}\, D \int_o^t T_o(t')^{k-1}\, dt' . \end{aligned}\right\} \qquad (10)$$

Eq. (10) with $k = n + 4$, contains both of Marshak's solutions as special cases. It should be equally valid, however, if $T_o(t)$ is any smoothly increasing function of time.

EXTENSION FOR $E \propto T^{\beta}$

Eq. (2) can be reduced to Eq. (3) by letting $T^{\beta} = V$, and $\frac{n+4}{\beta} = k$. The new variable V staisfies Eq. (3). Thus to the same approximation as Eq. (10),

$$\left.\begin{aligned} T(x,t) &\cong T_o(t)\left(1-\frac{x}{x_o(t)}\right)^{\frac{1}{n+4-\beta}} ; \\ x_o(t)^2 &\cong \frac{2(n+4)}{n+4-\beta}\, D \int_o^t T_o(t')^{n+4-\beta}\, dt' \end{aligned}\right\} \qquad (11)$$

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