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REMARKS ON THE FISSION-CAPTURE RATIO OF 25

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The capture-fission ratio $\alpha = \sigma_{capture}/\sigma_{fission}$ of 25 can be measured at thermal energies but this measurement cannot be extended to energies occuring in the fission spectrum. Therefore theoretical considerations to estimate the energy dependence of α are relevant.

The ratio α is given by

$$\alpha = \frac{\Gamma_r}{\Gamma_f}$$

where Γ_r and Γ_f are the partial widths in respect to radiation and to fission respectively. We are interested in the ratio K of the values of α at low energies to the values around 1 MeV:

$$K = \frac{(\Gamma_f)_{1\ \mathrm{MeV}}}{(\Gamma_f)_{1\ \mathrm{eV}}} \quad \frac{(\Gamma_r)_{1\ \mathrm{eV}}}{(\Gamma_r)_{1\ \mathrm{MeV}}} \qquad (1)$$

The expression for σ_f at 1 MeV is given by:

$$\sigma_f = \xi\, \sigma_o \frac{\Gamma_f}{\Gamma_f + \Gamma_n + \Gamma_r} \qquad (2)$$

σ_o is taken as πR^2 (R nuclear radius), which defines the sticking probability ξ. Since for 1 MeV: $R/\lambda\!\!\!^{-} = 2.3$, ξ won't be

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larger than unity. Γ_n is the neutron width, which consists of several terms: $\Gamma_n = \Gamma_{no} + \Gamma_{n1} + \Gamma_{n2} + \ldots$, corresponding to the elastic reemission (Γ_{no}) and to the inelastic emissions Γ_{ni}. If $R > \lambda\!\!\!^{-}$, the following relation holds:[1)]

$$\Gamma_{no} = \xi \sum_{\ell=0}^{L} (2\ell + 1) \frac{D_\ell}{\pi}$$

L is the maximum ℓ which can get into the nucleus ($L = R/\lambda\!\!\!^{-}$) and D_ℓ is the average distance between the states which can be formed by neutrons with the angular momentum ℓ , by collision with a nucleus in a state with definite quantum numbers. Let us call Δ the distance between degenerate levels of given angular momentum, J, and assume Δ independent of J, we get $D_\ell \doteq \frac{\Delta}{2(2\ell + 1)}$, the factor 2 coming from the spin. One then gets

$$\Gamma_{no} = \xi \frac{L + 1}{\pi} \frac{\Delta}{2}$$

We may neglect Γ_r in (2) for 1 MeV and get for Γ_f:

$$\Gamma_f = \frac{\Gamma_n}{\xi \frac{\sigma_o}{\sigma_f} - 1}.$$

We then write: $\Gamma_n = N\,\Gamma_{no}$ where $N \geqslant 1$ is related to the number of levels which can be reached via inelastic scattering of 1 MeV neutrons. It is somewhat smaller than this number because Γ_{ni} is

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1) See Bohr and Wheeler Phys. Rev. 56, 426 (1939) and also LA-24 (33 and 34) page 8, formula 17.

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smaller than Γ_{no}. We then get for Γ_f:

$$\Gamma_f = \left[\xi N \Big/ \left(\frac{\sigma_o}{\sigma_f} \xi - 1 \right) \right] \frac{L+1}{\pi} \frac{\Delta}{2}$$

After inserting $L = R/\lambda = 2.2$ and $\sigma_o/\sigma_f = 2$ on the basis of the experimental value $\sigma_f = 1.6$ barns[1] and $\sigma_o = \pi R^2 = 3$ barns, one obtains:

$$\Gamma_f = \frac{N\xi}{2\xi - 1} \frac{\Delta}{2}$$

Δ can be estimated in the following way: At thermal energies $\Delta/2$ is just equal to the level distance D_o between the levels observed by McDaniel.[2] (The factor two comes from the fact that the neutrons excite levels with two values of J.) The decrease of the level distance from thermal energies to 1 MeV can be estimated by using a dependence $e^{-\sqrt{aE}}$ for the level distance (E is the excitation of the compound nucleus) and by adjusting the constant a so that the level distance decreases from 100 - 300 kilovolts at $E = 0$, to 2 eV at $E = 6$ MeV. We then obtain $a = 22\ (\text{MeV})^{-1}$ and a decrease of Δ by a factor of 2.5 if E is raised from 6 to 7 MeV.

In order to get a lower limit on Γ_f, we put $\xi = 1$, $N = 2$, $\Delta/2 = 1/3\ D_o$, and $D_o = 1$ eV.

This gives:

$$\Gamma_f > 0.66 \text{ eV.}$$

1) A. O. Hanson, CF-618

2) LA Report in preparation.

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The most plausible value for Γ_f may be obtained by putting $N = 2$, $\xi = 0.75$, $D_o = 2$:

$$(\Gamma_f)_{1\ \mathrm{MeV}} \sim 2\ \mathrm{eV}.$$

We used Γ_f at low energies in order to estimate K which is defined in (1). $(\Gamma_f)_{low}$ is smaller or equal to the total width Γ of the levels observed by McDaniel. According to his curves one may put:

$$\Gamma \cong 0.25\ \mathrm{eV}.$$

One obtains for the fission width at low energies:

$$\Gamma_f = \frac{\Gamma}{1+\alpha}$$

which gives 0.2 eV with a value α of 0.25. Thus the ratio K obeys the relation:

$$K \geqslant 2.6\,(1+\alpha)$$

With the most probable value of Γ_f, we obtain

$$K \sim 8.$$

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