LA-712 c.3

[redacted]

UNCLASSIFIED

LOS ALAMOS SCIENTIFIC LABORATORY

OF THE

UNIVERSITY OF CALIFORNIA

CONTRACT W-7405-ENG. 36 WITH

U. S. ATOMIC ENERGY COMMISSION

[redacted]

UNCLASSIFIED

[redacted]

[redacted]

UNCLASSIFIED

LA-712

Series A

[redacted]

October 8, 1948 This document contains 11 pages.

A CLASSICAL TREATMENT OF ELECTROSTATIC EXCITATION OF NUCLEAR P_2 VIBRATIONS

Work done by:

M. N. Rosenbluth

Report written by:

M. N. Rosenbluth

UNCLASSIFIED

[redacted]

APPROVED FOR PUBLIC RELEASE

[redacted]

Abstract

The excitation of a nucleus by the electrostatic force exerted on it by another highly energetic nucleus is treated classically. It is concluded that a fission fragment can not produce fission.

(Ed.)

[redacted]

 [redacted]

A Classical Treatment of Electrostatic Excitation of Nuclear P_2 Vibrations

The purpose of this paper is to give a classical treatment to the problem of excitation of a nucleus by the electrostatic force exerted on it from another highly energetic nucleus. An example of such an effect would be a fission fragment causing fission in a nearby nucleus.

The mechanism of the effect is shown below:

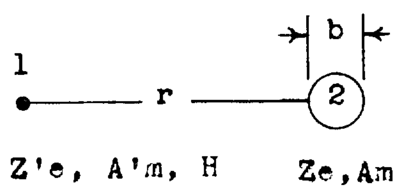


Fig. 1

Vibrational mode P_2

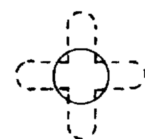


Fig. 2

The energetic particle 1 (charge Z'e, mass A'm, energy H) approaches particle 2 (charge Ze, mass Am). It exerts a force $Z\,Z'e^2/r^2$ on the near side of nucleus 2, a force $Z\,Z'e^2/(r+b)^2$ on the far side. Thus there is a force proportional to $1/r^3$ tending to excite the vibrational mode shown in Fig. 2 (the so-called P_2 mode). We are interested in seeing whether this mode can be excited to a high-enough energy to permit a nuclear reaction to take place. For example, we know from the liquid-drop model that a 6 Mev excitation of a uranium nucleus would probably lead to fission.

[redacted]

[redacted]

Since the vibrational quantum of the P_2 mode is about .8 Mev for heavy nucleii, we should be able safely to treat excitation energies of greater than several Mev by a classical method. It should also be noted that the energies involved are low enough to use non-relativistic equations throughout, and that the distance of closest approach in the cases of interest is about 10^{-4} cm so that electronic shielding may be disregarded and Coulomb's Law used. We shall also assume a head-on collision so that conditions for excitation are as favorable as possible.

Our task then will be to solve for the relative motion of the particles, and thus obtain the deforming force $F = Z\,Z'e^2\, b/r^3$. We then Fourier-analyze the force to find its Fourier component corresponding to the excitation energy in which we are interested. The size of this component will give us the probability that the nucleus will be excited sufficiently to undergo the reaction in which we may be interested.

The equation of energy conservation gives us:

$$\frac{1}{2}\, m \frac{A\,A'}{A+A'}\, \dot{r}^2 + \frac{Z\,Z'e^2}{r} = \frac{H\,A}{A+A'} \qquad (1)$$

Here H is the energy of the incident particle in its rest system; $HA/(A+A')$ is the energy in the c-of-g system; $m\,\frac{AA'}{A+A'}$ is the reduced mass.

[redacted]

 [redacted]

If we let

$$r' = \frac{H\ A}{(A + A')Z\ Z'e^2}\ r\ ; \quad t' = \left(\frac{H\ A}{A + A'}\right)^{3/2} \frac{1}{Z\ Z'e^2} \sqrt{\frac{2(A + A')}{m\ A\ A'}}\ t$$

then (1) reduces to

$$\dot{r}'^2 + \frac{1}{r'} = 1$$

where the dot now represents differentiation with respect to t'.

We now substitute $u = \frac{1}{r'}$, and $\dot{u} = -u^2\dot{r}'$ which yields

$$\dot{u} = u^2 \sqrt{1 - u} \tag{1'}$$

which may be integrated to yield

$$|t'| = \frac{\sqrt{1 - u}}{u} + \tanh^{-1} \sqrt{1 - u} \tag{2}$$

Equation (2) now gives us u as an implicit function of t'. We can thus plot $u^3 \propto F$ (Fig. 3) as a function of T' and Fourier analyze u^3 to find the magnitude of the excitation that we are interested in.

Suppose that we are considering an excitation energy E. It corresponds to a vibration frequency $E/\hbar$ or, in our units of t', to a frequency:

$$\omega = \frac{E}{\hbar} \left(\frac{A\ A'}{HA}\right)^{3/2} Z\ Z'e^2 \sqrt{\frac{m\ A\ A'}{2(A + A')}} \tag{3}$$

To a first approximation we can represent the force function F by a Gaussian:

[redacted]

 [redacted]

$$u^3 = e^{-at'^2}$$

(where a is determined from Fig. 3 to equal about .6) which if we Fourier-analyze we obtain

$$u^3 = \int C(\omega)\, e^{i\omega t'}\, d\omega \qquad (4)$$

where

$$C(\omega) \approx e^{-\omega^2/4a} \approx e^{-(\omega/\omega_0)^2}$$

If we use for Z, Z', A, A', H, typical values for fission fragments we find that ω_0 corresponds to an energy of about .25 Mev. In fact, it is typically so that any energy in which we might be interested is considerably larger than the energy of the most probable induced vibration.

It is, however, quite clear that our Gaussian approximation can give us no reliable information on the Fourier coefficient of frequencies very much higher than ω_0.

In order to find these high-frequency components we therefore use the following trick:

If $$u^3 = \int C(\omega)\, e^{i\omega t'}\, d\omega$$

then $$\frac{d^n(u^3)}{d\,t'^n} = \int C'(\omega)\, e^{i\omega t'}\, d\omega = \int \omega^n\, C(\omega)\, e^{i\omega t'}\, d\omega$$

In other words, the nth derivative of u^3 gives us a function in which the high-energy components are "boosted"

[redacted]

 [redacted]

by a factor ω^n. If we calculate and plot the nth derivative, and then once again make an approximate Fourier analysis, we can find:

$$C(\omega) = \frac{C'(\omega)}{\omega^n} \tag{5}$$

and thus obtain a value for any Fourier component desired.

The differentiation may be performed analytically using (1'):

$$\frac{d(u^3)}{dt'} = 3u^2 \dot{u} = 3u^4 \sqrt{1 - u}$$

The process can be continued indefinitely and we finally obtain

$$\frac{d^8(u^3)}{dt'^8} = 1{,}814{,}400\, u^{11} - 11{,}840{,}760\, u^{12} + 27{,}204{,}984\, u^{13} - 26{,}474{,}227.5\, u^{14} + 9{,}300{,}217.5\, u^{15} \tag{6}$$

This function is plotted in Fig. 4, along with an approximate representation:

$$\frac{d^8(u^3)}{dt'^8} = 4614 \times e^{-1.98t'^2} \cos(2\pi t' - .156\pi t'^2)$$

We see that for the region near $t' = 0$ (which is the region where high frequencies will be present) this is a very good approximation.

If we neglect the t'^2 term (unimportant for high frequencies), we obtain:

$$C^1(\omega) = \frac{1}{2\pi} \int e^{i\omega t'}\, 4614\, e^{-1.98t'^2} \cos 6.28\, t'\, dt'$$

$$= 450 \left[e^{-\left[(\omega+6.28)^2/4\cdot 1.98\right]} + e^{-\left[(\omega-6.28)^2/4\cdot 1.98\right]} \right] \tag{7}$$

[redacted]

[redacted]

and finally using (5):

$$C(\omega) = \frac{450}{\omega^8} \, e^{-\left[\frac{(\omega-6.28)^2}{7.92}\right]} \qquad (8)$$

where ω is given by (3), and we note that the 2nd term in brackets in (7) is larger than the first for high frequencies.

It should be remarked that here, as in the case of our original Gaussian approximation for $\dot{u}^3$, the absolute value of the $C(\omega)$ should not be trusted unless the negative exponent is of the order of magnitude -1, and that to properly obtain its magnitude, one should continue differentiation until one does obtain an exponent about equal to -1. However, (8) does provide a useful limit for $C(\omega)$, since if the exponent is much more negative than -1 we may say with assurance that $C(\omega) << \frac{360}{\omega^8}$.

Thus for $\omega > 20$

$$C(\omega) << 2 \times 10^{-8}$$

which means that for all practical purposes the reaction is impossible. Using (3) this may be put into a somewhat more useful form by eliminating the constant factors: If

$$\xi = \frac{E}{(H)^{3/2}} \cdot \frac{(A + A')}{A} \sqrt{A'} \; Z \, Z' > 135 \qquad (9)$$

[redacted]

 [redacted]

then

$$C(\omega) << 10^{-8}$$

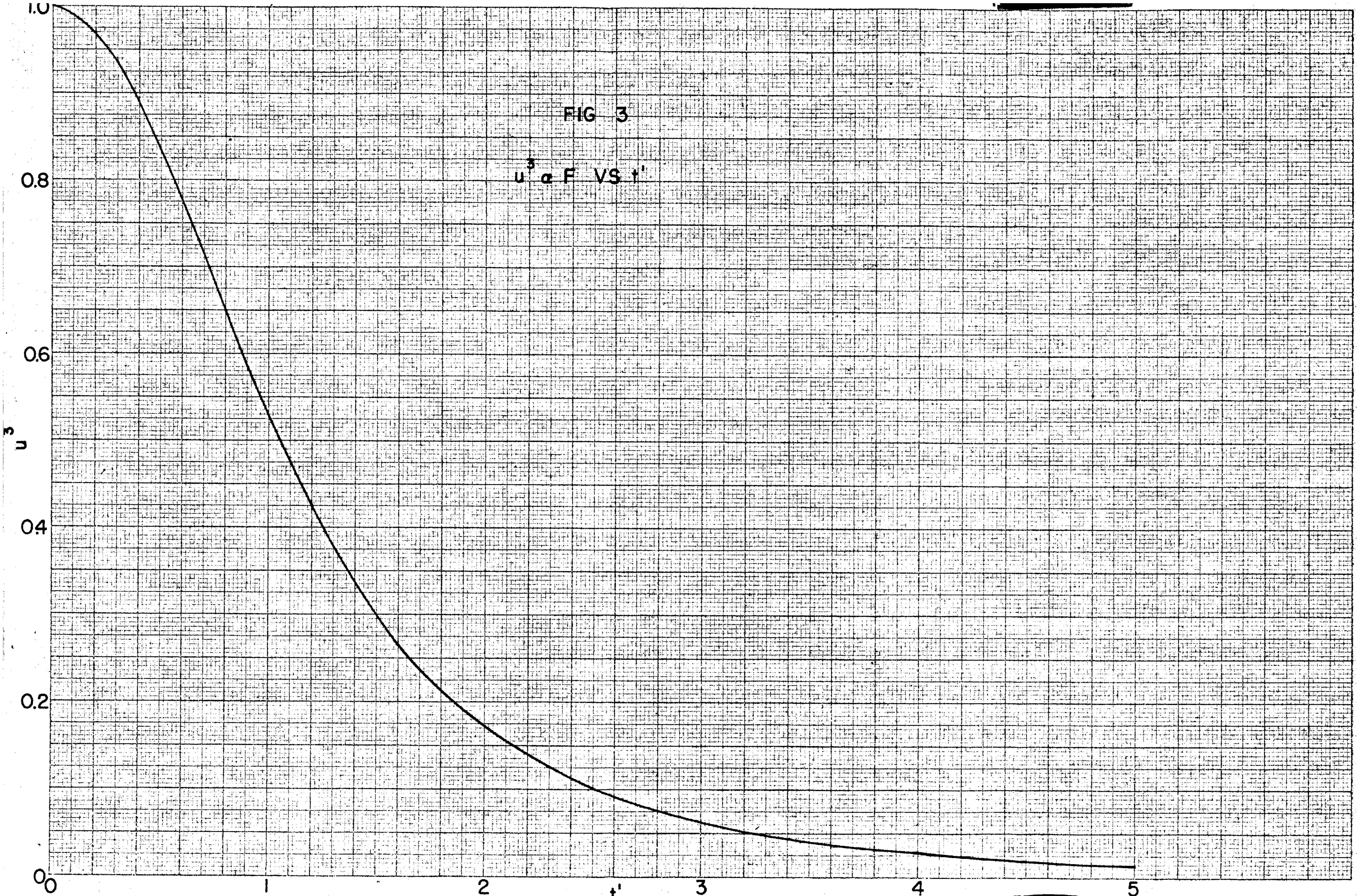
and the reaction is "impossible". E, the desired excitation energy, and H, the incident particle energy, here are in Mev.

In the case of fission fragments producing fission, the example we have been considering, we find $\xi = 400$. So we may be quite certain that this reaction will never be observed.

I would like to thank Professor Edward Teller, who suggested this problem.

APPROVED FOR PUBLIC RELEASE

[redacted]



[redacted]

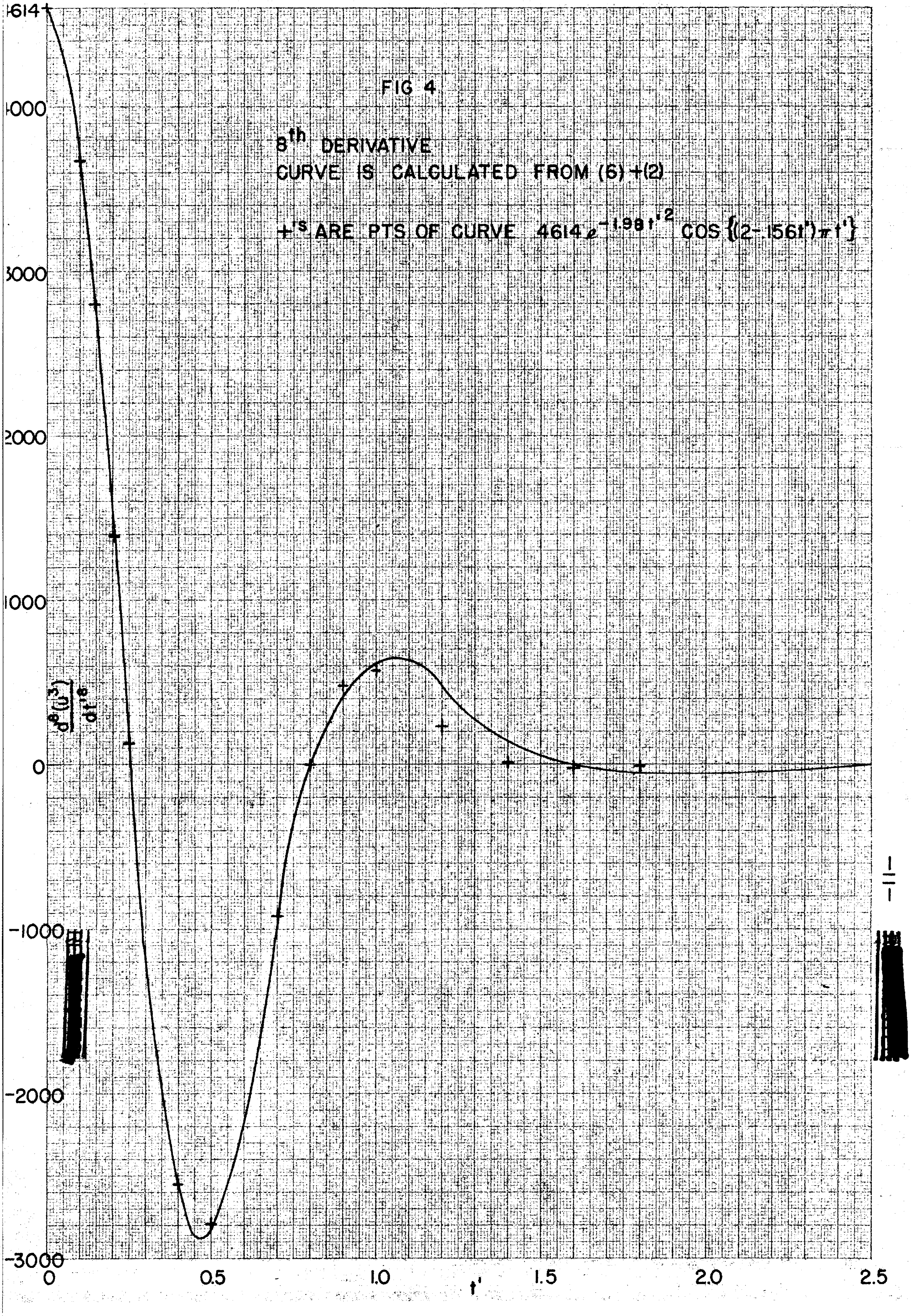


FIG 4
8th DERIVATIVE
CURVE IS CALCULATED FROM (6) + (2)
+'S ARE PTS OF CURVE $4614 e^{-1.98t'^2} \cos\{(2-.156t')\pi t'\}$
$\frac{d^8(u^3)}{dt'^8}$
t'
4614
3000
2000
1000
0
-1000
-2000
-3000
0
0.5
1.0
1.5
2.0
2.5

APPROVED FOR PUBLIC RELEASE

APPROVED FOR PUBLIC RELEASE

UNCLASSIFIED

[redacted]

UNCLASSIFIED