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THE RANGE OF DEUTERONS IN HEAVY WATER AND OF PROTONS IN HYDROGEN

J. Ashkin and H. A. Bethe

ABSTRACT

The stopping power of heavy water for deuterons, and the range-energy relation for deuterons in heavy water, have been calculated for deuteron energies from 0 to 750 kv. The range of protons in hydrogen for proton energies from 0 to 1 MV has also been obtained.

For the evaluation of various experiments it is necessary to know the range-energy relation for deuterons in heavy water, for deuteron energies from 0 to about 1 MV. An attempt has been made to calculate this relation theoretically. Since, in the course of the calculation, the stopping power of hydrogen (or deuterium) for deuterons or protons, was required, the range-energy for deuterons and protons in hydrogen was also obtained.

METHOD OF CALCULATION

The range R of a charged particle in a given medium is obtained from the loss in energy per cm. of path of the particle, $-dE/dx$, by performing the integration

$$R(E) = \int_0^E \frac{dE}{-\left(\frac{dE}{dx}\right)} \qquad (1)$$

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When the stopping medium is heavy water, the energy loss per cm. will be a sum of two terms, one arising from the inelastic collisions of the charged particle with the electrons of the deuterium and the other from the inelastic collisions with the electrons of the oxygen. These contributions are treated separately.

If the velocity of the incident deuteron is large compared to the velocities of the electrons in the atoms of the medium, application of the Born approximation to the collisions is valid, and the energy loss per cm. has the form

$$-\frac{dE}{dx} = \frac{4\pi e^4}{mv^2} NB \qquad (2)$$

with

$$B = Z \log\left(\frac{2mv^2}{I}\right) \qquad (2a)$$

In this expression m is the mass of the electron, v is the velocity of the incident deuteron, N is the number of atoms per cm^3 of the medium, Ze is nuclear charge and I some average ionization potential of the atom. For deuterons being stopped in oxygen, the form (2a) for B is valid only for deuteron energies large compared to 2 MV. This expression, therefore, cannot be used in the range in which we are interested.

It has been demonstrated, however, that the Born treatment gives correct results all the way down to velocities of the incident heavy particle which are small compared with that of the atomic electron if the incident particle has a smaller charge than that of the atomic nucleus.(1) The difference from the high velocity case arises from the breakdown of a simplifying approximation which makes the summation over the electronic transitions relatively easy to carry out. More exact methods for the evaluation of this sum are therefore resorted to, and the final result expressed numerically instead of analytically.

(1) Mott, Proc. Camb. Phil. Soc., 27, 553 (1931).

the calculation of the energy loss due to collisions with the K electrons. This has been done by Bethe[2]. For the contribution B_K of the two K electrons to the stopping number B, defined in (2), he finds

$$B_K = 1.81 \log (3.63\eta) - C_K(\eta) \tag{3}$$

where C_K is given graphically and

$$\eta = \frac{mv^2}{2Z_{eff}^2 Ry} = \frac{mE}{MZ_{eff}^2 Ry} \tag{4}$$

Here $E = \frac{1}{2} Mv^2$ is the kinetic energy of the incident deuteron, Ry is the Rydberg energy, and Z_{eff} is the effective nuclear charge in the K shell. The quantity η is a measure of the square of the ratio of the velocity of the incident particle to the velocity of the K electron. The number 1.81 is the "effective number of K electrons" or the total oscillator strength for all transitions from the K shell to the unoccupied discrete levels and to the continuous spectrum, and is about right for the elements from carbon to aluminum. If the average excitation potential of the electrons outside the K shell is denoted by I', the total stopping number can be written

$$B = (Z - 1.81) \log \frac{2mv^2}{I'} + B_K \tag{5}$$

OXYGEN

For oxygen, the effective nuclear charge in the K shell was taken as $8 - 0.3 = 7.7$, and the average excitation potential, I', of the L shell, guessed as about 50 volts on the basis of the empirical value of 40.3 volts for air and an estimate (from the Slater screening constants) of the effective nuclear charges in the L shells of air and oxygen. The "stopping cross section" per atom of oxygen, $\sigma_{ox.}$, or the energy loss per cm. divided by the number of atoms per cm^3

(2) Bethe and Livingston, Rev. Mod. Phys., 9, 1937, pp. 264-265.

[redacted]

$$\sigma_{ox} = -\frac{1}{N_{ox}}\left(\frac{dE}{dx}\right)_{ox} = \frac{4\pi e^4}{mv^2} B_{ox} \qquad (6)$$

is plotted in Fig. 1 as a function of the deuteron energy in the region above 0.1 MV. However, it can be seen that for very low deuteron energies, the formula (5) cannot be used without modification. For v sufficiently small, the logarithmic term becomes negatively infinite, while B_K tends to zero. The formula (5) is not to be trusted in the case of oxygen for deuteron energies below 80 or 90 kv. To be complete, a special calculation analagous to that of B_K would have to be made for the stopping due to the L electrons for velocities of the incident deuteron comparable to that of an L electron. It is not considered wotth while to perform this calculation at this time. What has been done is to guess very roughly that the stopping cross section per atom of oxygen for low energy deuterons is about four times the stopping cross section per atom of hydrogen or deuterium, and then to extend the curve, in a reasonable way, down to about 20 kv, the lower limit for which the hydrogen stopping cross section has been calculated. For the higher energies the calculated values of σ_{ox} are probably good to 3 or 4%. Thus, for the distance in oxygen (at 15° C and 760 mm pressure) required to reduce the deuteron energy from 1 to 0.5 MV, we find .87 cm., on the basis of the computed values of σ_{ox}, and when multiplied by the stopping power of oxygen relative to air, which is 1.07, this gives as the distance in air .93 cm. The experimental value obtained from the curve of Parkinson, Morb, Bellamy and Hudson is .90 cm.

HYDROGEN

The energy loss in deuterium (or hydrogen) is due to the inelastic collisions of the deuteron with the single K electron of the atom. Since the total oscillator strength for all optical transitions from the K shell to higher states in both the discrete and continuous spectrum is unity, the best values for the stopping number B that could be obtained without making a special calculation for hydrogen, were $(1/1.81)B_K$, where B_K is given by (3). The term $\log(3.63\eta)$ becomes

asymptotically correct as the deuteron energy increases. However, the correction $(1/1.81)C_K(\eta)$ is subject to an error which is perhaps of the order of 10%, since it was calculated for elements between carbon and aluminum. This would mean an error in the stopping cross section of deuterium for deuterons of about 5% for 75kv, 2.5% at 125 kv., 1% at 250 kv., and less than 1% above 300 kv. For Z^2_{eff} the value 1.173 was used, corresponding to the ionization potential of something like 1.173 Rydbergs for the removal of one of the electrons in the hydrogen molecule. The stopping cross section resulting from these approximations is shown in Fig. 2.

RANGE

With the stopping cross section of hydrogen or deuterium for deuterons, and hence also for protons, at hand, the range-energy relation for protons in hydrogen was calculated by numerical integration. However, it was only possible to calculate the range of the protons down to about 10 kv. since the theoretical curve for the stopping cross section tends to zero so rapidly as to make the range of low energy particles very large (actually infinite). Below about 10 kv., furthermore, the protons capture and lose electrons and there is at present no satisfactory theoretical method for handling these processes. For these reasons we have plotted in Fig. 3 the distance required to slow the protons down to 10 kv., instead of the range for slowing to zero energy. If the curve is extrapolated linearly to zero energy, a range of about 0.05 cm. is found for the slowing from 10 kv. to zero.

HEAVY WATER

Finally the stopping cross section per atom of oxygen was added to twice the stopping cross section per atom of deuterium to obtain the stopping cross section per molecule of D_2O, expressed in units of 10^{-15} cm^2 volt. The results as shown in Fig. 4 may be in error by about 5% between 200 and 300 kv. and by somewhat less for the higher energies. The range as a function of the energy, extrapolated to zero energy, is plotted in Fig. 5. The units used are molecules of D_2O per cm^2, one molecule of D_2O per cm^2 being equal to 3.325×10^{-23} gm/cm^2.

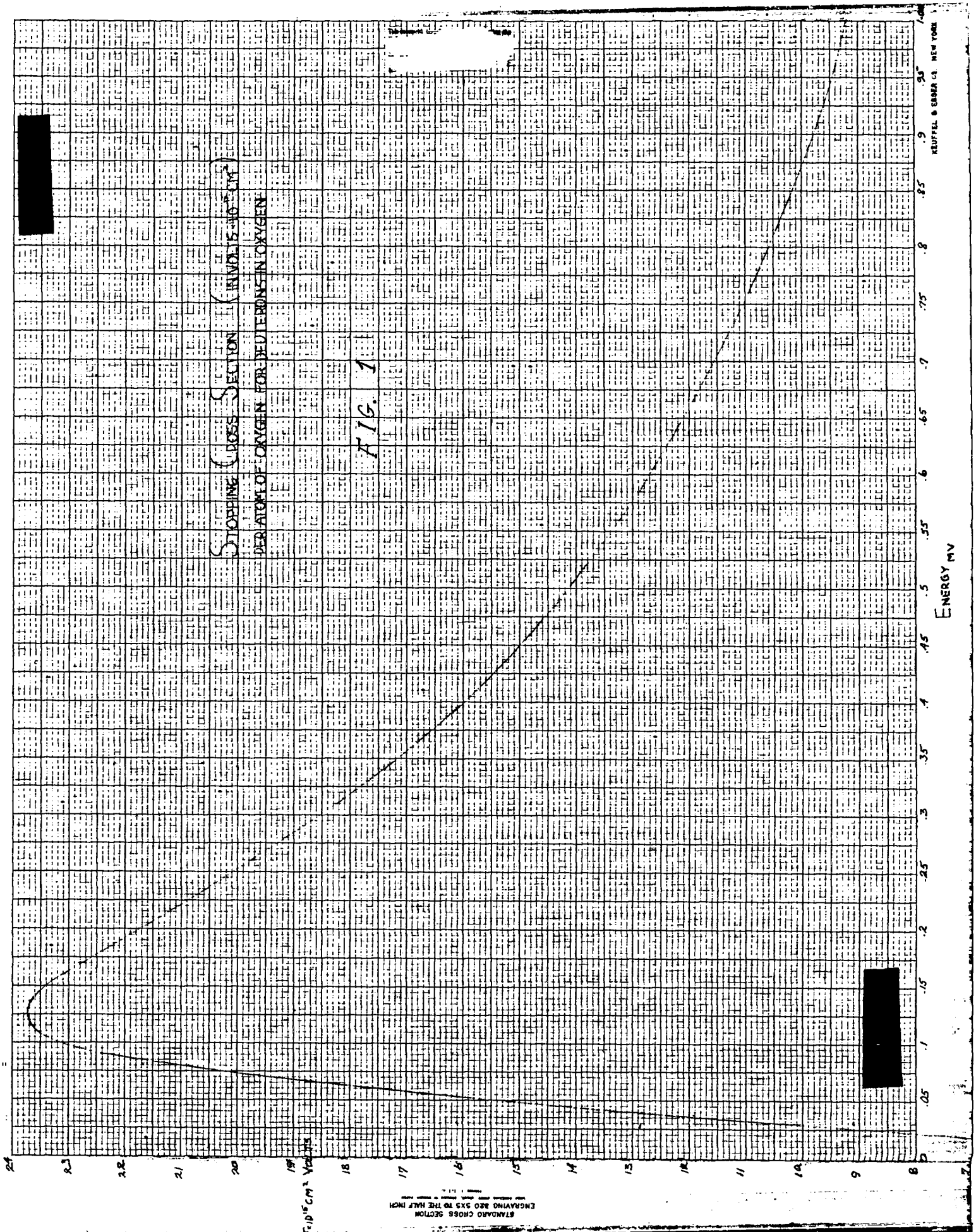
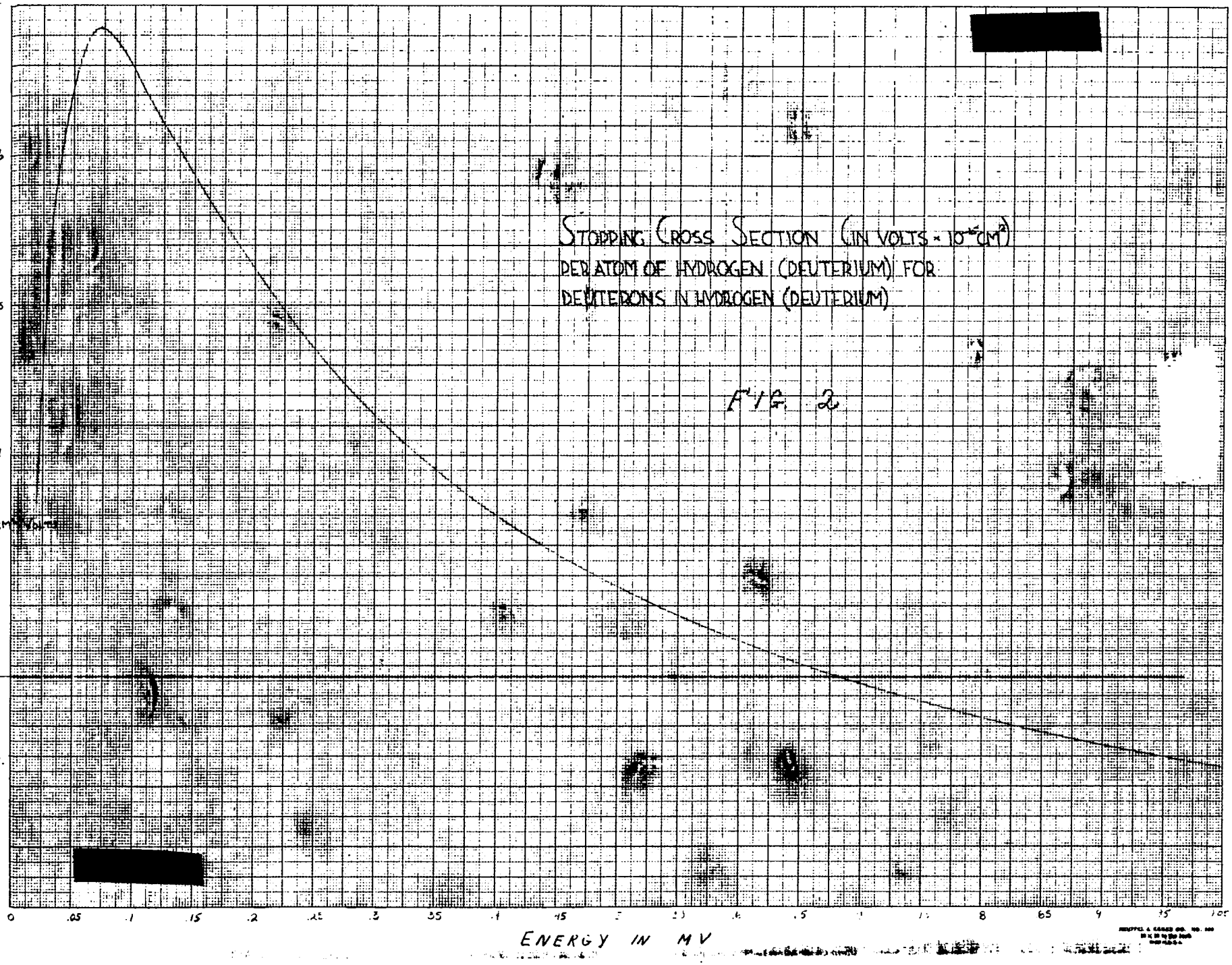
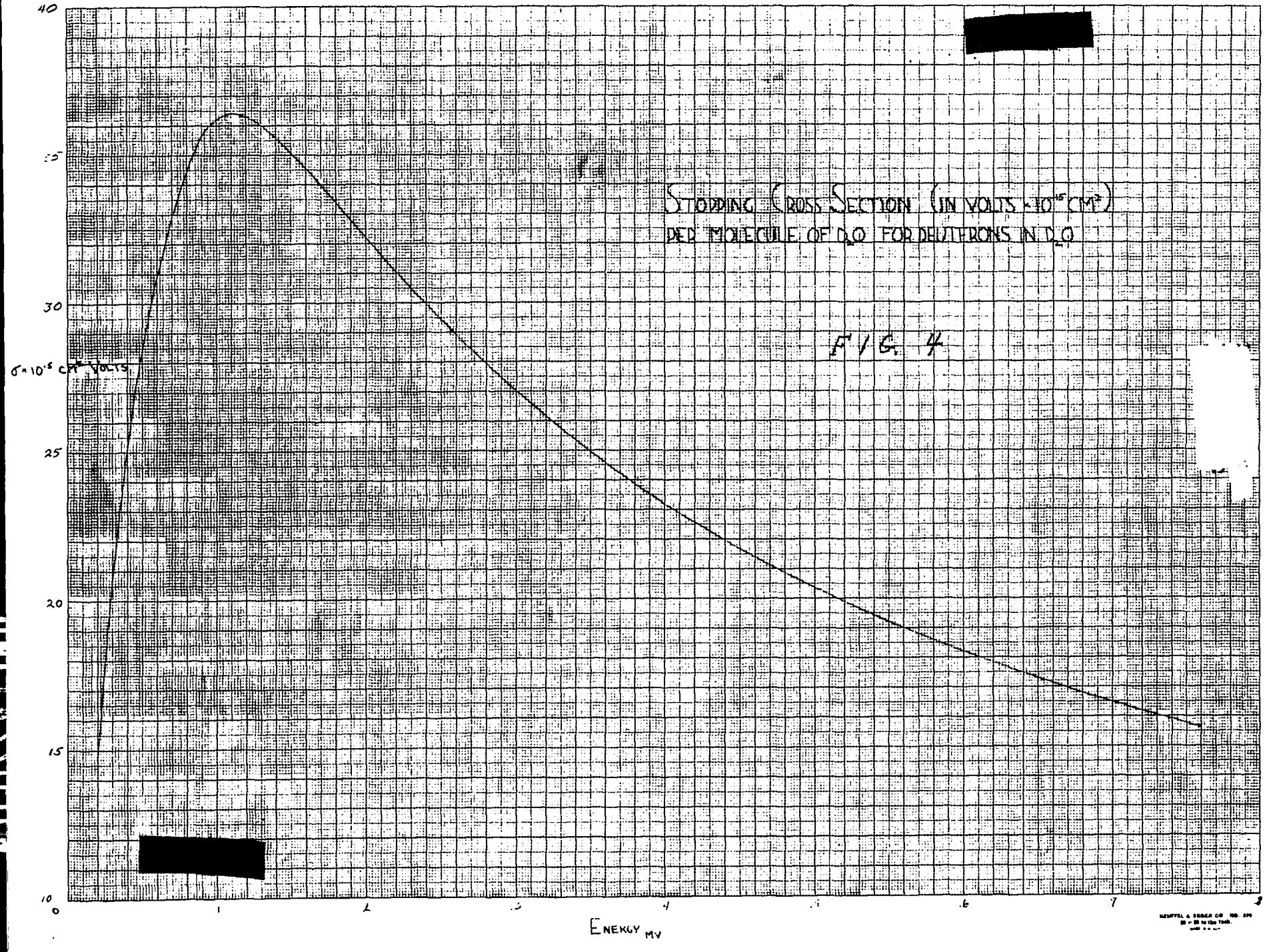


FIG. 1





STOPPING CROSS SECTION (IN VOLTS × 10^{-15} CM²) PER MOLECULE OF D_2O FOR DEUTERONS IN D_2O

FIG. 4

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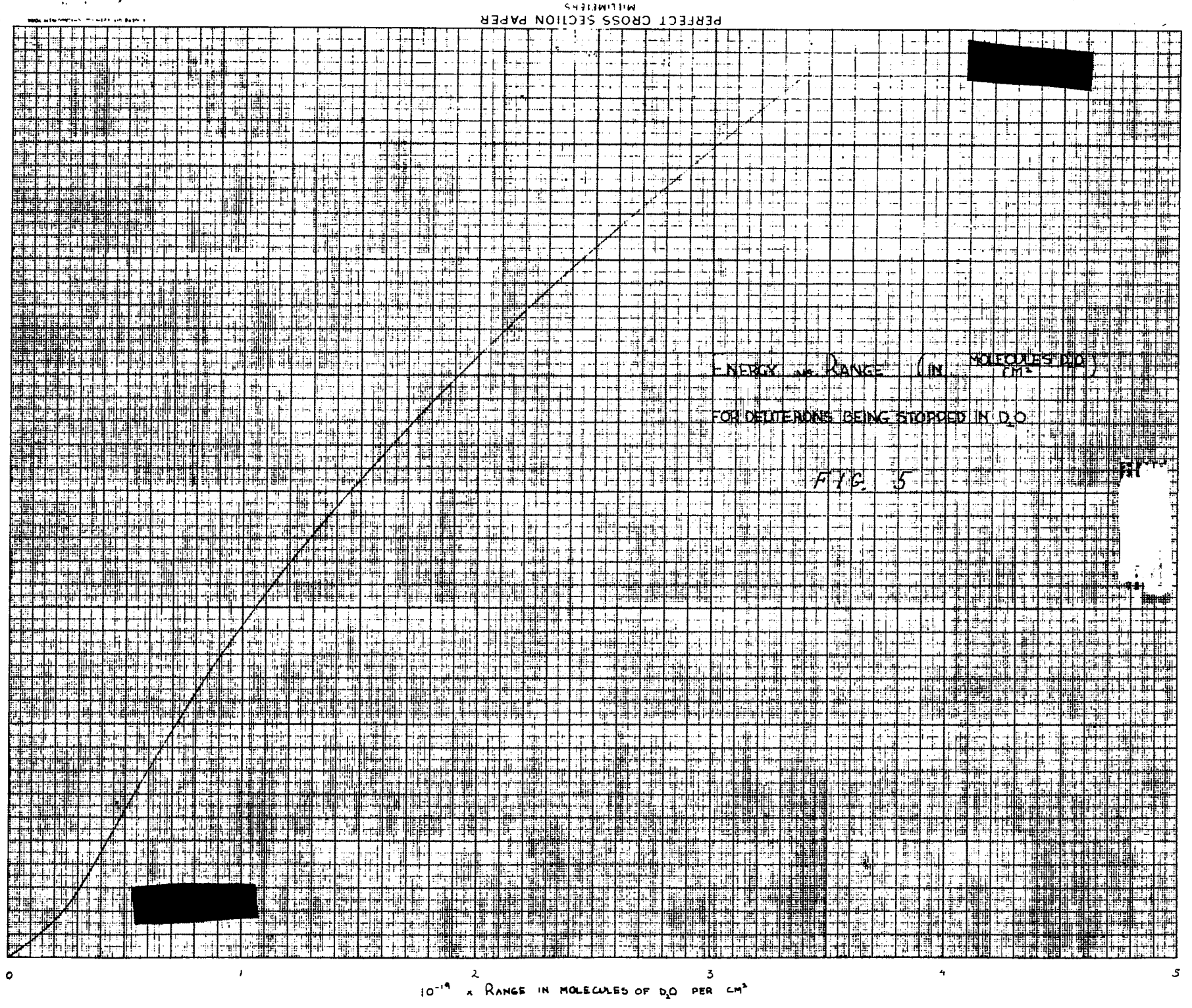

ENERGY vs. RANGE (IN MOLECULES D_2O / CM^2)

FOR DEUTERONS BEING STOPPED IN D_2O

FIG. 5

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