September 13, 1943.

This document contains 17 pages

ONE-DIMENSIONAL MOTION OF A DETONATED EXPLOSIVE

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WD - Principles + Testing

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ABSTRACT

The equations of the one-dimensional motion of a detonated explosive and projectile are given. It is assumed that the detonated explosive behaves as a perfect gas and that it is initially homogeneous and at rest.

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ONE-DIMENSIONAL MOTION OF A DETONATED EXPLOSIVE

INTRODUCTION

It is desired to estimate roughly the motions that occur if a spherical metal shell is surrounded by a spherical shell of high explosive, and the latter detonated. It is assumed that the strength of the metal is negligible, and that only its inertia is important.

To avoid the mathematical difficulties of the spherical problems, corresponding one-dimensional situations will be considered. We shall consider the motions of high explosive and projectiles in a tube and since in the spherical case there is no effect corresponding to a frictional force between the wall of the tube and the bodies of the tube, this force will be ignored. The results of the one-dimensional calculation should give some information on the motion in the spherical case.

It will be assumed that the detonation is instantaneous and leaves the gaseous products homogeneous and at rest. Heat transfer and viscosity effects are neglected. For simplicity the equations of state of a thermally and calorically perfect gas will be assumed to describe the behavior of the detonated explosive.

COORDINATE SYSTEM

A particle of gas is identified by its index x; that is, the coordinate of its position in a one-dimensional coordinate system when the gas has its initial distribution. The displacement of a particle of the gas from its initial position is denoted by y.

SHOCK WAVE

If there exists a discontinuity of the pressure, p, in a body of gas homogeneous in composition, and if in their relative motion the bodies of gas on the two sides of the discontinuity are approaching one another, then the relative velocity of the gas on the two sides of the discontinuity is:

$$U = c_1\left(\frac{p_2}{p_1} - 1\right)\sqrt{\frac{2/\gamma}{(\gamma - 1) + (\gamma + 1)\frac{p_2}{p_1}}} = c_2\left(1 - \frac{p_1}{p_2}\right)\sqrt{\frac{2/\gamma}{(\gamma - 1) + (\gamma + 1)\frac{p_1}{p_2}}}\ .$$

The discontinuity moves into the rarer body of gas with velocities relative to the gas adjacent to the discontinuity

$$V_1 = c_1\sqrt{\frac{1}{2\gamma}\left[(\gamma - 1) + (\gamma + 1)\frac{p_2}{p_1}\right]} > c_1 \quad \text{and} \quad V_2 = c_2\sqrt{\frac{1}{2\gamma}\left[(\gamma - 1) + (\gamma + 1)\frac{p}{p_2}\right]} < c_2$$

where $p_2 > p_1$ and $c = \sqrt{\gamma(p/\rho)}$ is the speed of sound. (Fig. 1)

The reverse motion is impossible since it violates the Second Law of Thermodynamics.

THE DIFFERENTIAL EQUATION OF MOTION

If there are no discontinuities of p, the motion of an initially homogeneous body of gas is described by:

$$\left(1 + \frac{\partial y}{\partial x}\right)^{\gamma+1} \frac{\partial^2 y}{\partial t^2} = c_o^2\,\frac{\partial^2 y}{\partial x^2} \qquad \text{where} \qquad c_o = \sqrt{\gamma\,\frac{p_o}{\rho_o}}$$

MOTION FROM A SIMPLE INITIAL STATE

If the system consists of two homogeneous, semi-infinite bodies of gas, initially at rest, the origin of x being at the common boundary, it follows from dimensional analysis that the solution of the problem is of the form $y/c_o t = \eta(x/c_o t)$. That is a graph of y as a function of x retains its shape, but increases in size in proportion to t, the origin

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remaining fixed. The function $\eta(\xi)$ satisfies the differential equation

$$\frac{d^2\eta}{d\xi^2}\left[\left(1+\frac{d\eta}{d\xi}\right)^{\gamma+1}\xi^2-1\right] = 0 .$$

This is solved by the solutions of either $\dfrac{d^2\eta}{d\xi^2} = 0$ or

$$\left(1+\frac{d\eta}{d\xi}\right)^{\gamma+1}\xi^2-1=0$$

which have the general solutions $\eta(\xi) = A\xi + B$ and

$$\pm\eta(\xi) = \pm C + \frac{\gamma+1}{\gamma-1}(\pm\xi)^{\frac{\gamma-1}{\gamma+1}} - (\pm\xi)$$

that is, $y = Ax + Bt$ or

$$\pm\frac{y}{c_o t} = \pm C + \frac{\gamma+1}{\gamma-1}\left(\pm\frac{x}{c_o t}\right)^{\frac{\gamma-1}{\gamma+1}} - \left(\pm\frac{x}{c_o t}\right);$$

here A, B, C, are any constants and the upper or lower sign is taken according as $x/c_o t > 0$ or $x/c_o t < 0$. The first solution for $\eta(\xi)$ is an arbitrary straight line; the nature of the second solution is shown graphically in Fig. 2. In the second solution, the velocity relative to the gas of the point corresponding to a particular value of $\xi = x/c_o t$ is the local velocity of sound.

These solutions may be combined to solve the following problems:

COMPRESSION UNDER CONSTANT PRESSURE

The gas is in the region $x > 0$, initially at rest, and is compressed by a piston at $x = 0$ exerting a constant pressure $P > p_o$:

$$y = Ax + Bt \quad (0 \leq x \leq Vt), \qquad y = 0 \quad (Vt \leq x)$$

where $V = c_o\sqrt{\dfrac{1}{2\gamma}\left[(\gamma-1)+(\gamma+1)\dfrac{P}{p_o}\right]} > c_o$,

$$A = -\frac{2(P/p_o - 1)}{(\gamma-1)+(\gamma+1)P/p_o}, \qquad \text{and}$$

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$$B = c_o\left(\frac{P}{p_o} - 1\right)\sqrt{\frac{2/\gamma}{(\gamma - 1) + (\gamma + 1)\,P/p_o}}$$

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The gas is in two bodies; one is in its initial state; the other, adjacent to the piston, is at pressure P and moving with constant velocity B; the discontinuity between the two originates at the piston and moves with a velocity V. (Fig. 3)

EXPANSION UNDER CONSTANT PRESSURE

The gas is in the region $x \geqslant 0$, initially at rest, and expands against a piston at $x = 0$ exerting a constant pressure $P < p_o$:

$$y = Ax + Bt \qquad (0 \leqslant x \leqslant Vt),$$

$$y = -\frac{2}{\gamma - 1}\,c_o t + \frac{\gamma + 1}{\gamma - 1}\left(\frac{x}{c_o t}\right)^{\frac{\gamma-1}{\gamma+1}} c_o t - x \qquad (Vt \leqslant x \leqslant c_o t),$$

$$y = 0 \qquad (c_o t \leqslant x)$$

where $V = (p_o/P)^{-(\gamma + 1)/2\gamma}\,c_o,$

$$A = (p_o/P)^{1/\gamma} - 1, \quad \text{and}$$

$$B = -\frac{2}{\gamma - 1}\left[1 - (p_o/P)^{-(\gamma - 1)/2\gamma}\right] c_o\,.$$

One homogeneous body of gas adjacent to the piston is at pressure P and moving with the constant velocity B; one body of gas is in its initial state; between these two is a third body in which the gas is accelerating and expanding reversibly. The boundary between the last two is moving with the initial speed of sound. (Fig. 4)

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SIMULTANEOUS EXPANSION AND COMPRESSION

Two semi-infinite bodies of gas are initially homogeneous and motionless and separated by a common boundary. The one under higher pressure expands, the one under lower pressure is compressed, under the constant pressure P given by equating the two corresponding expressions for the velocity of a piston moving with constant pressure:

$$c_1\left(\frac{P}{p_1}-1\right)\sqrt{\frac{2/\gamma_1}{(\gamma_1-1)+(\gamma_1+1)P/p_1}} = \frac{2}{\gamma_2-1}\,c_2\left[1-(p_2/P)^{-(\gamma_2-1)/2\gamma_2}\right]$$

where $p_2 > p$. If $p_2 \gg p_1$ and $T_2 \gg T_1$, we may assume $p_2 \gg P \gg p_1$ and solve the equation approximately, obtaining

$$P = 2\,\frac{\gamma_2}{(\gamma_2-1)^2}\,(\gamma_1+1)\,\frac{\rho_1}{\rho_2}\,p_2 \qquad \text{(Fig. 5)}$$

EXPANSION AGAINST A PROJECTILE

The solution for expansion of a gas initially at rest in the region $x > 0$ into a vacuum in the region $x < 0$ is

$$y = -\frac{2}{\gamma-1}\,c_0t\,\frac{\gamma+1}{\gamma-1}\left(\frac{x}{c_0t}\right)^{\frac{\gamma-1}{\gamma+1}}c_0t - x \qquad (0 \le x \le c_0t),$$

$$y = 0 \qquad (c_0t \le x).$$

This gives $M\ddot{y}(x_M, t) = -p(x_M, t)$ $\left(t \ge \frac{x_M}{c_0}\right)$ where $M = \frac{\gamma+1}{2\gamma}\rho_0 x_M$. This is the boundary condition for the following problem: At time $t_M = \frac{x_M}{c_0}$ the region $x > x_M$ is occupied by a motionless homogeneous body of gas bounded at $x = x_M$ by a projectile of cross-sectional density M. The displacement and velocity of the projectile are respectively:

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$$\left.\begin{aligned} Y(t) &= -\frac{2}{\gamma-1} c_0 t + \frac{\gamma+1}{\gamma-1}\left(\frac{x_M}{c_0 t}\right)^{\frac{\gamma-1}{\gamma+1}} c_0 t - x_M \\ \dot{Y}(t) &= -\frac{2}{\gamma-1} c_0 \left[1 - \left(\frac{x_M}{c_0 t}\right)^{\frac{\gamma-1}{\gamma+1}}\right] \end{aligned}\right\} \qquad \left(t \geq \frac{x_M}{c_0}\right)$$

where $x_M = \frac{2\gamma}{\gamma+1} \frac{M}{\rho_0}$.

This method of treating the projectile problem is successful only if the pressure-density relation for the expanding gas is of the form here assumed, namely $p = K\rho^{\gamma}$ where K and γ are constants.

PROJECTILE PROPELLED BY A FINITE BODY OF GAS

An initially homogeneous finite body of gas is bounded on one end at $x = x_M$ by a projectile; at the other end it is limited by a stationary wall (closed breech) at $x = x_M - \ell$ or permitted to expand into a region of lower pressure (open breech) beginning at $x = x_M + 2\ell$.

In the interval of time $x_M/c_0 \leq t \leq (x_M + \ell)/c_0$, the motion of the projectile and the gas in the region $x_M < x < x_M \ell$ is described by the solution for the semi-infinite body of gas expanding against a projectile. At the time $t = (x_M + \ell)/c_0$ the rarefaction from the projectile meets the closed breech or the rarefaction from the open breech, and a new solution is required. The junction between the new solution and the original solution moves toward the projectile with a velocity relative to the gas equal to the local velocity of sound. The junction arrives at the projectile at the time

$$t_c = (x_M/c_0)\,(1 + \ell/x_M)^2 = x_M/c_0\,\left(1 + \frac{\gamma+1}{2\gamma}\frac{m}{M}\right)^2$$

where $m = \rho_0 \ell$ is the cross-sectional density of the gas in the length ℓ.

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The motion of the projectile is described by the solution for the semi-infinite body of gas in the interval of time $x_M/c_0 \leq t \leq t_c$, namely its displacement and velocity are, respectively,

$$\left.\begin{aligned} Y(t) &= -\frac{2}{\gamma - 1} c_0 t + \frac{\gamma + 1}{\gamma - 1}\left(\frac{x_M}{c_0 t}\right)^{\frac{\gamma-1}{\gamma+1}} c_0 t - x_M \\ \dot{Y}(t) &= -\frac{2}{\gamma - 1} c_0 \left[1 - \left(\frac{x_M}{c_0 t}\right)^{\frac{\gamma-1}{\gamma+1}}\right] \end{aligned}\right\} \quad \left(\frac{x_M}{c_0} \leq t \leq t_c\right)$$

Fig. 7 gives the projectile velocity as a function of position, and the velocity at the time t_c as a function of the ratio of the mass of gas to the mass of projectile.

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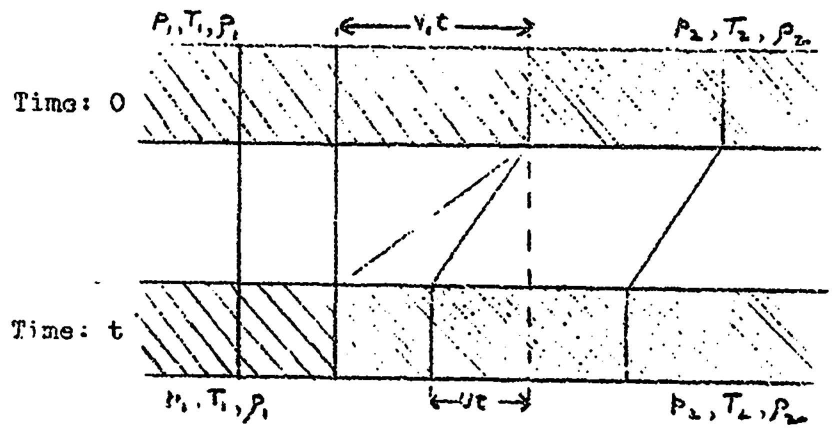

Fig. 1 Shock Wave

The diagram shows the positions of typical elements of gas and of the discontinuity of pressure at two instants. The coordinate system is chosen so that the rarer body of gas is at rest.

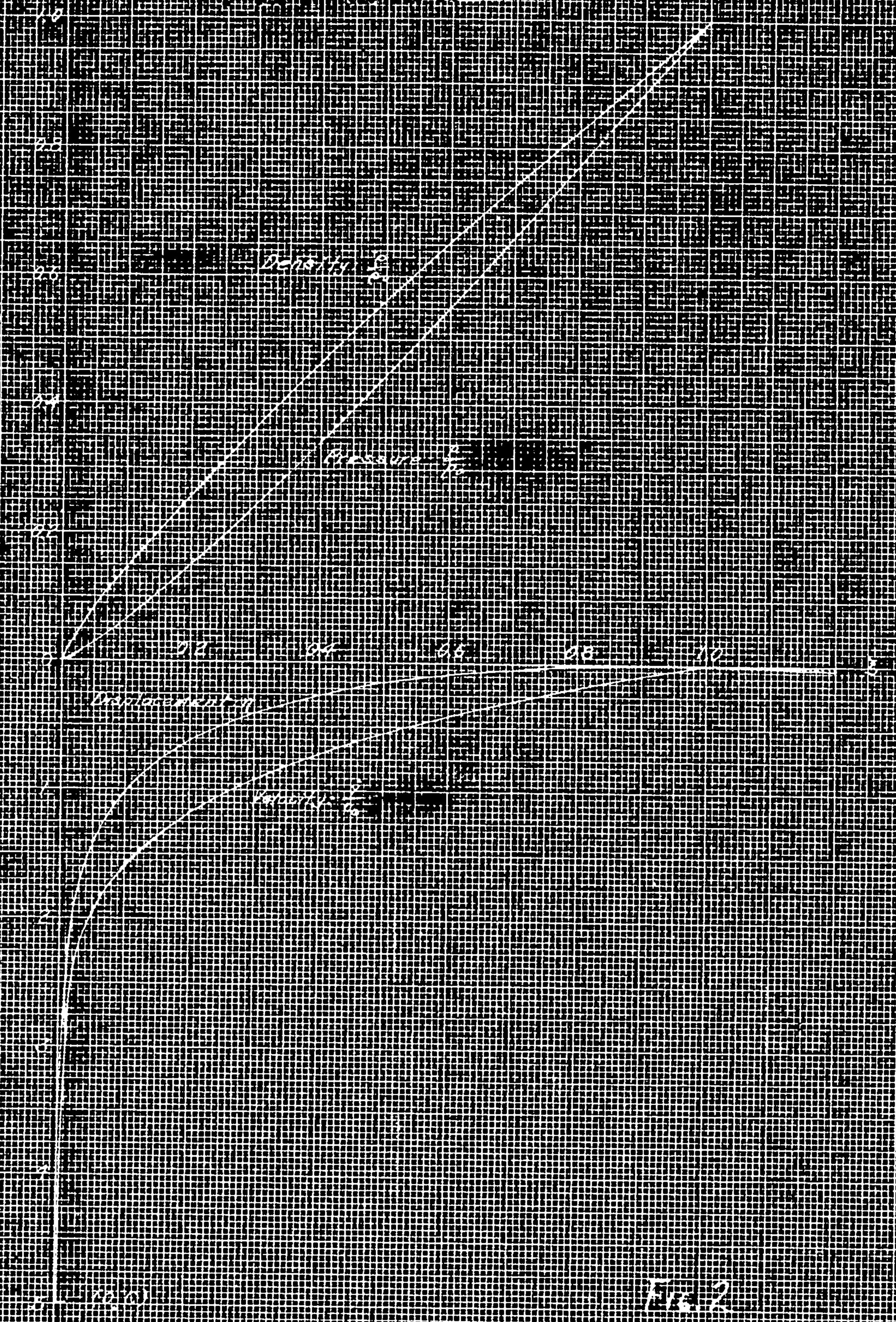

Fig. 2

Caption to Fig. 2 Expansion of the Gas

The abscissa $\xi = x/c_0t$ is the initial position of the gas particle in units of the distance which the rarefaction has penetrated into the body of gas.

The displacement of the particle, in this same unit, is:

$$\eta = \frac{y}{c_0t} = C + \frac{\gamma+1}{\gamma-1}\xi^{\frac{\gamma-1}{\gamma+1}} - \xi$$

The value of C used in the solution, $-2/(\gamma-1)$, is inserted; varying C shifts this curve vertically. The curve is tangent to the axis of ordinates at $(0,C)$. In this region $\xi > 1$, η is negative, the curvature is negative and the slope approaches -1. This graph indicates only positive values of ξ; the complete solution is symmetric about the point $(0,C)$. In the problems mentioned here only that part of the function in the region $0 \leq \xi \leq 1$ is used.

The velocity of the particle is given in units of the initial speed of sound: $\dot{y}/c_0 = C\,(\xi^{\frac{\gamma-1}{\gamma+1}} - 1)$. The curve is tangent to the axis of ordinates at $(0,C)$ and intersects the axis of abscissa at $(1,0)$.

The density and pressure are given in units of the initial density and pressure:

$$\rho/\rho_0 = (1+\partial y/\partial x)^{-1} = (1+d\eta/d\xi)^{-1} = \xi^{2/(\gamma+1)}, \qquad p/p_0 = (\rho/\rho_0).$$

The numerical value of γ used in plotting these curves is 7/5. Note the difference of scales on the positive and negative axes of ordinates.

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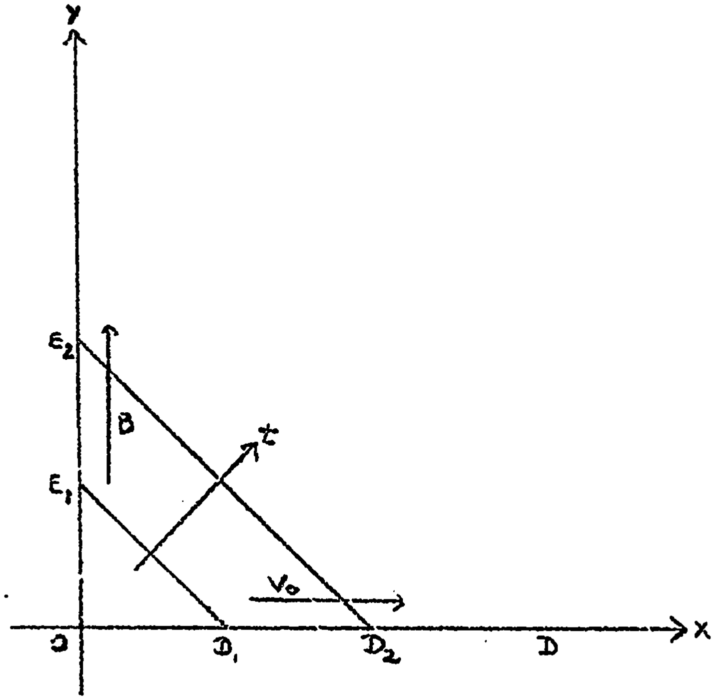

Fig. 3 Compression Under Constant Pressure

At times 0, $t_1 > 0$ and $2t_1$, the displacement y is given as a function of the index x by the curves OD, E_1D_1D, and E_2D_2D respectively. This graph is drawn for the case $P \gg p_o$.

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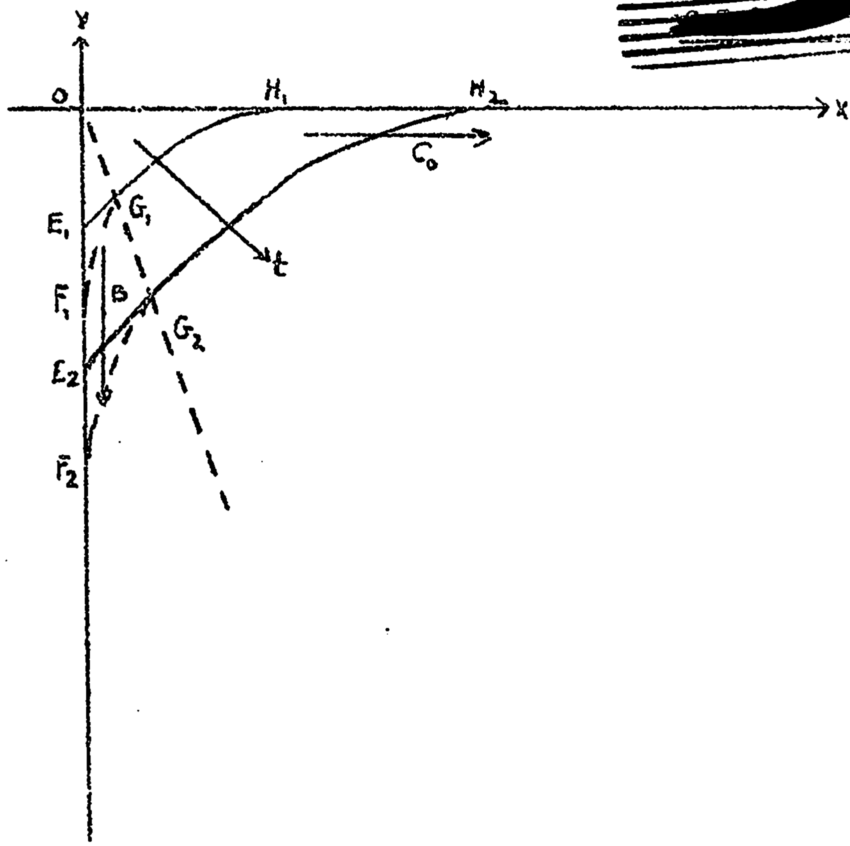

Fig. 4 Expansion Under Constant Pressure

At times 0, $t_1 > 0$ and $2t_1$, the displacement y is given as a function of the index x by the curves OH, $E_1G_1H_1H$, and $E_2G_2H_2H$, respectively. The curves $F_1G_1H_1$ and $F_2G_2H_2$ are repetitions of the displacement curve of Fig. 2. The curves E_1G_1 and E_2G_2 are straight lines tangent to the curves $F_1G_1H_1$ and $F_2G_2H_2$ respectively.

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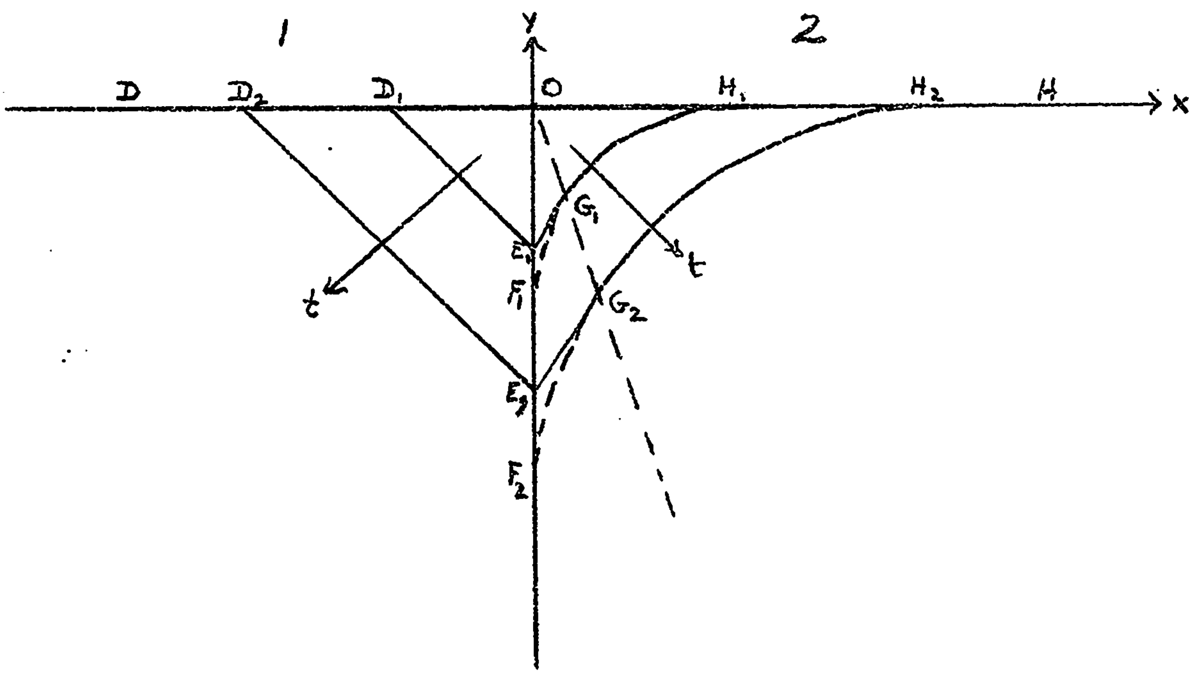

Fig. 5 Simultaneous Expansion and Compression

The region of initially higher pressure and temperature is on the right of 0. At times 0, $t_1 > 0$, and $2t_1$, the displacement y is given as a function of the index x by the curves DOH, $DD_1E_1G_1H_1H$, and $DD_2E_2G_2H_2H$, respectively. This is a combination of the curves of Fig. 3 (reflected through the origin) and those of Fig. 4.

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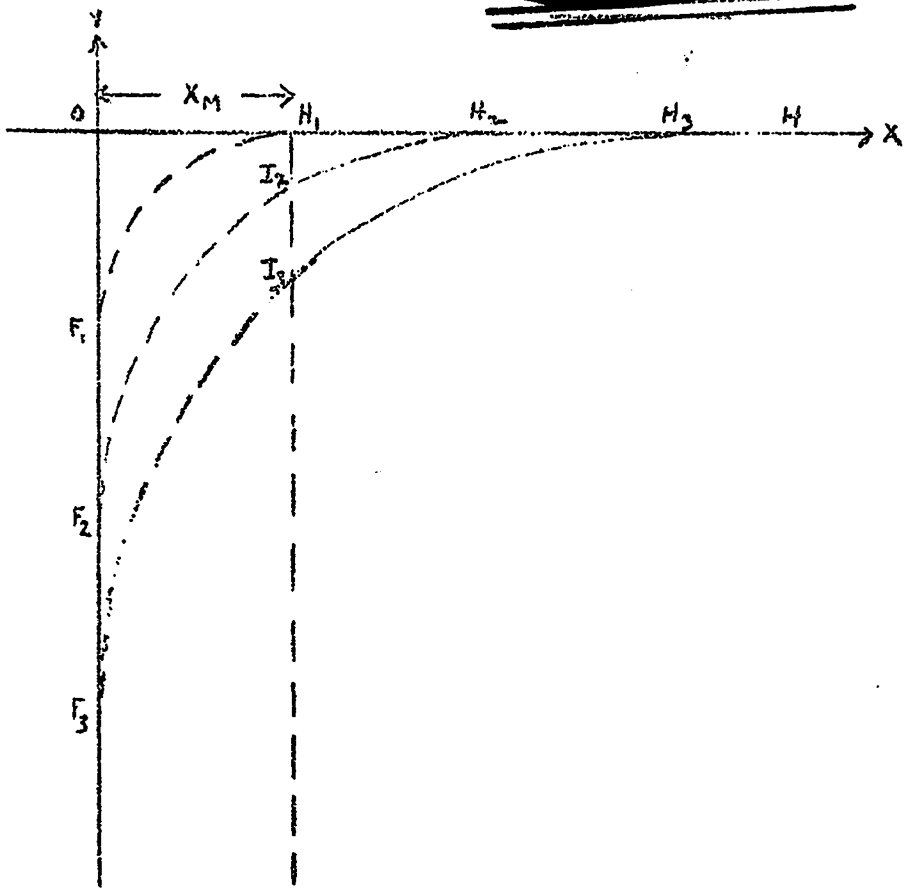

Fig. 6 Expansion Against a Projectile

At times $t_M = x_M/C_0$, $2t_M$, and $3t_M$ the displacement y of the gas is given as a function of the index x by the curves H_1H, I_2H_2H, and I_3H_3H respectively, and the displacement of the projectile y by the ordinates of the points H_1, I_2, and I_3 respectively. The motion begins at time t_M. The curves F_1H_1, $F_2I_2H_2$, and $F_3I_3H_3$ are repetitions of the displacement curve of Fig. 2.

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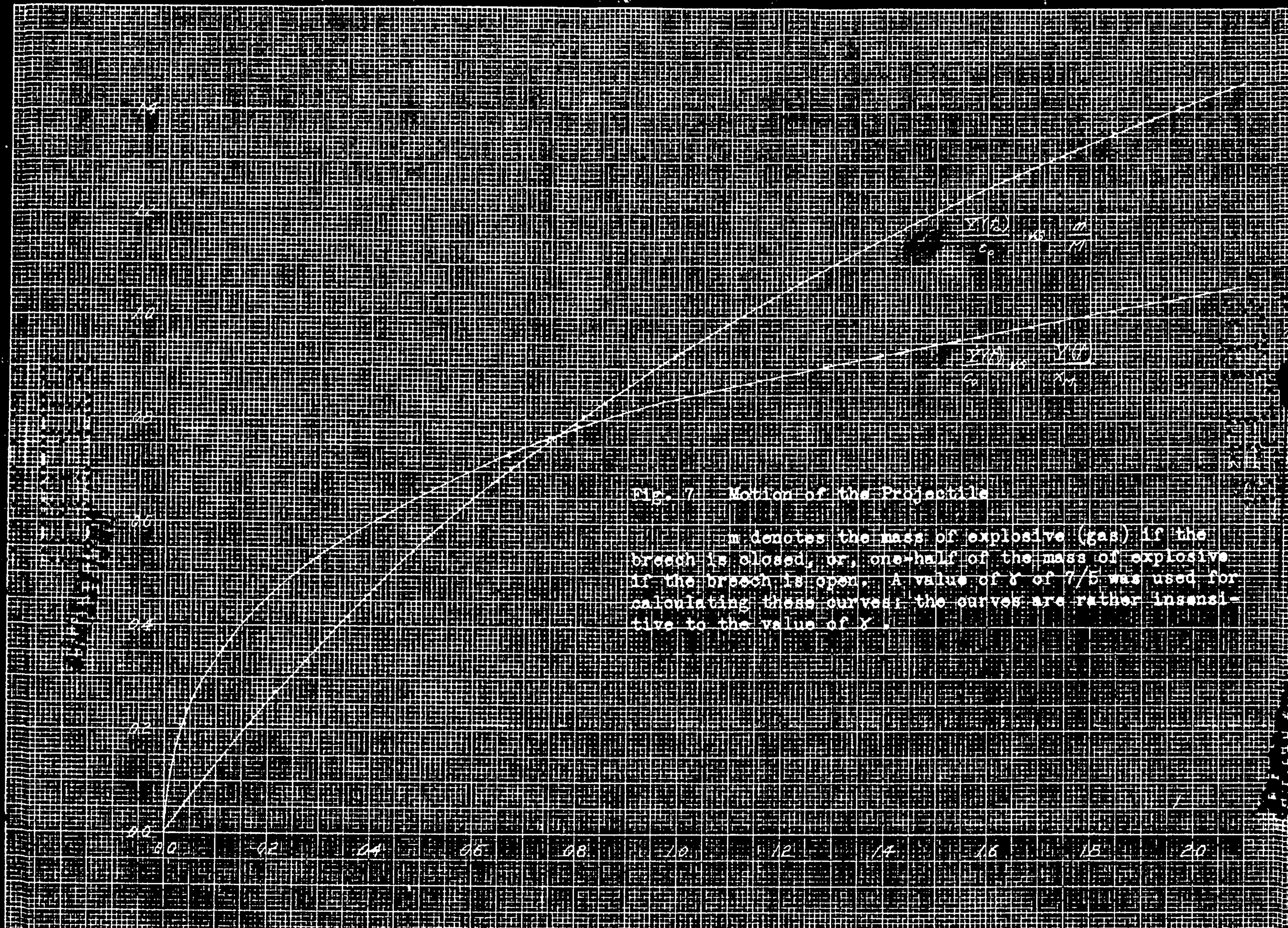

Fig. 7 Motion of the Projectile

m denotes the mass of explosive (gas) if the breech is closed, or, one-half of the mass of explosive if the breech is open. A value of γ of 7/5 was used for calculating these curves: the curves are rather insensitive to the value of γ .