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TITLE: Dynamic Compaction of Granular Materials in a Tube with Wall Friction, Applied to Deflagration-to-Detonation Transition

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Dynamic Compaction of Granular Materials in a Tube with Wall Friction, Applied to Deflagration-to-Detonation Transition

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Abstract: A theoretical problem is considered in which a granular material is pushed through a tube of arbitrary cross-section by a constant velocity piston against the resistance of compaction work and wall friction. The crushing of the material is dictated by a simple yet physically reasonable compaction law. By considering two special cases – the limit of vanishing friction and the quasi-static limit – we identify the two basic compaction wave structures. We then consider the general case in which the two waves interact. Estimates suggest that for typical deflagration-to-detonation tests explosive at the wall melts on time scales short compared to the experiment.

Key words: compaction, DDT, friction, granular explosive, ignition, melting

1. Introduction

Many investigators have examined deflagration-to-detonation transition (DDT) of granular explosives using shock-tube like experiments (e.g. McAfee, Asay, & Campbell 1989). Often a piston is driven explosively into the bed, and the resulting compaction work is thought to initiate combustion which ultimately leads to detonation. We examine an aspect of the DDT tube problem that has received little attention, namely the resistance and resulting energy dissipation due to wall friction. Wall friction has a large effect in ram-pressed charges (Elban & Chiarito 1986) - essentially the quasi-static limit of the DDT tube test – so that one expects even greater effects for the longer aspect-ratio DDT tubes. The resistance mechanism is that axial stress applied by the piston is transmitted to the walls, giving rise to a proportional frictional drag. Since particles are interlocked, resistance at the wall is transmitted throughout the interior. The entire bed resists motion in proportion to how hard it is pushed upon - much like the ubiquitous "chinese finger" but acting in compression.

2. Rheology

Because a confined granular material exhibits solid-like properties it is appropriate to adopt the generalized definition of pressure used in elasticity, i.e., $p = -(\sigma_x + \sigma_y + \sigma_z)/3 = -(\sigma_r + \sigma_\theta + \sigma_z)/3$, where all stresses are force per unit *total* area, normal stress components are positive in tension, and pressure is positive in compression. We define an ideal granular material whose loading state is dictated by a *compaction law* (Herrmann 1969) of the form $\phi = \rho/\rho_s = f(p)$, where ϕ is the solid volume fraction, ρ is the mixture density, and the subscript s means "solid". To determine analytic solutions we choose a simple but physically realistic form for $f(p)$, where S_0 and ϕ_0 are the zero-pressure slope and volume fraction, respectively:

$$\phi = 1 - \frac{(1-\phi_0)^2}{S_0 p + (1-\phi_0)} \tag{1}$$

We consider Class-A granular HMX explosive as an example, and infer its compaction law from pressing data (Elban & Chiarito 1986) using the fact that the experimental geometry enforced nearly uniaxial strain. Both this task and the formulation of the equations of motion in Sec. 3 are greatly simplified by the inference that Poisson's ratio, ν, remains constant and equal to its solid value during crushing. This follows from pressing data (e.g., Campbell, Elban, & Coyne 1988) for which the ratio of normal to axial stress, which depends only on ν (specifically, $\sigma_n/\sigma_z = \nu/(1-\nu)$), remains constant. The inferred compaction law is shown in Fig. 1 together with a least-squares fit to Eqn. 1.

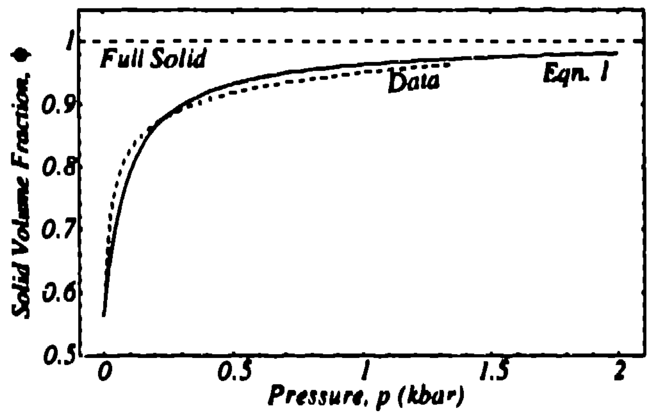

Figure 1. Compaction law for Class A HMX, $\phi_0 = 0.563$: data vs. fit to Eqn. 1.

The wall drag is assumed to obey a standard friction law except that, because the solid area fraction in contact with the wall increases as the material is crushed, the friction coefficient is not constant. Intuitively one expects that the coefficient should be weighted by the solid area ratio, $\mu/\mu_s = A_s/A = \phi$. Equating the solid area and volume fractions is a common practice following from the statistical argument that for many randomly arranged particles, each plane within a volume element intersects the same solid area. This behavior for μ can be more rigorously justified, but here we merely note that the limits $\phi \to 0$ (no material) and $\phi \to 1$ (solid material) are both sensible.

3. Formulation

Consider a cylindrical semi-infinite tube of arbitrary cross-section, filled with powder and sealed at the origin by a piston, as shown in Fig. 2. The initial

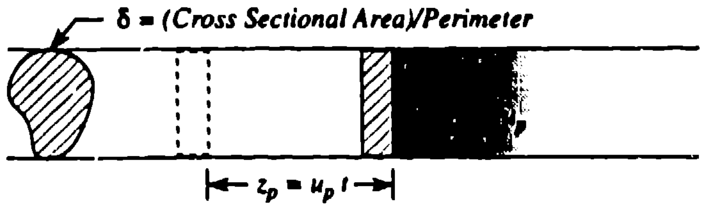

Figure 2. Schematic of the problem.

conditions are uniform mixture density and zero pressure. At $t = 0^+$ the piston is moved impulsively to the right at a constant velocity u_p. The resulting material motion is assumed to be always and everywhere one dimensional. To simplify the equations we transform to a Lagrangian coordinate system, whereby the spatial coordinate z is replaced by the mass-weighted spatial coordinate h. Then, incorporating the above results and assumptions, the dimensionless equations for mass and momentum conservation become:

$$\frac{\partial \phi}{\partial \tilde{t}} + \phi^2 \frac{\partial M}{\partial \tilde{h}} = 0 \tag{2}$$

$$\frac{\partial M}{\partial \tilde{t}} = -\frac{\partial \tilde{p}}{\partial \tilde{h}} - \tilde{p} \tag{3}$$

Here, $M = u/a_0$ is the Mach number, and the "tilded" quantities have been scaled as follows: $\tilde{p} = S_0 p$, $\tilde{h} = h/l_f$, and $\tilde{t} = a_0 t/l_f$. The reference quantities l_f (the "friction" length) and a_0 (the longitudinal sound speed) are given by:

$$l_f = \frac{\nu}{1-\nu}\frac{\delta}{\mu_s}, \quad a_0 = \sqrt{\frac{3(1-\nu)}{1+\nu}\frac{1}{\rho_s S_0}} \tag{4}$$

with δ the area-to-perimeter ratio of the cross section.

4. Results

4.1. Zero Friction Limit

In the absence of friction the second term on the right-hand side of Eqn. 3 does not appear. The piston drives a steady compaction shock (sw) into the powder, whose speed M_{sw} and amplitude p_{sw} are given by:

$$M_{sw} = \frac{\phi_0 M_p}{2(1-\phi_0)}\left(1+\sqrt{1+\frac{4(1-\phi_0)^2}{M_p^2}}\right), \qquad p_{sw} = M_p\, M_{sw} \tag{5}$$

where $M_p = u_p/a_0$ is the piston Mach number. In reality the shock has a finite thickness equal to several grains (e.g.. McAfee, Asay, & Campbell 1989).

4.2. Quasi-Static Limit

This case corresponds to a vanishingly small piston speed. Over correspondingly long time intervals Eqn. 2 survives intact but the left-hand side of Eqn. 3 is negligible. Subject to the requirements that the initial piston pressure is zero and that the powder far upstream is undisturbed, an analytic solution emerges:

$$\bar{p}(\bar{t},\bar{h}) = \bar{p}(\bar{t},0)e^{-\bar{h}}, \quad \phi(\bar{t},\bar{h}) = 1 - \frac{(1-\phi_0)^2}{\bar{p}(\bar{t},\bar{h}) + (1-\phi_0)} \tag{6}$$

$$\bar{p}(\bar{t},0) = \phi_0(1-\phi_0)\left[\exp\left(\frac{\phi_0}{1-\phi_0}M_p\bar{t}\right) - 1\right] \tag{7}$$

The pressure and compaction fields, mapped back to the Eulerian frame, are shown in Fig. 3. The wave is not strictly steady as the pressure is always rising. But, due to the shape of the compaction law, the compaction field assumes a steady profile of width $\approx 5\,l_f$ following a start-up transient. The downstream state of the developed frictional wave (fw) is fully compacted, so its speed is:

$$M_{fw} = \left(\frac{1}{1-\phi_0}\right)M_p \tag{8}$$

4.3. General Case

One expects the general solution, corresponding to nonzero friction and order unity piston speeds, to involve a friction-attenuated shock. One may further surmise that to a good approximation the problem can be divided into two distinct regions: the leading shock for which inertial forces dominate, and the downstream flow in which frictional forces dominate. A perturbation solution valid for short times, or equivalently, small friction coefficient (not presented

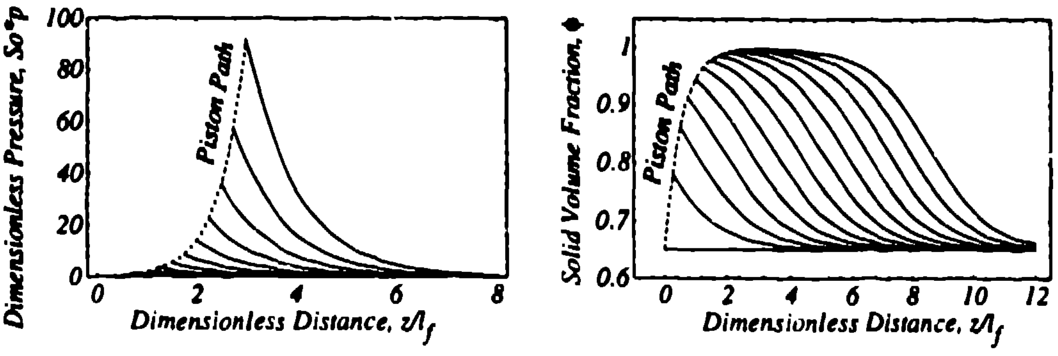

Figure 3. Pressure, compaction fields in quasi-static limit; $\phi_0 = 0.65$, $\Delta\bar{t} = 0.25$.

here), finds the correction to the particle speed between the piston and the shock to be time independent, which supports this notion. In this "inertialess" approximation Eqn. 3 formally reduces to that for the quasi-static limit, but the initial piston pressure is the zero-friction shock pressure corresponding to the prescribed piston speed, and the solution extends only to the shock location $\bar{h}_{sw}(\bar{t})$, at which point it is matched to shock jump conditions. A second (coupled) ODE arises from the matching, and the problem must be solved numerically. A phase plane analysis, also not presented here, shows that below a critical value of M_p, $M_{pcrit} = 1 - \phi_0$, the strength of the leading shock decays to zero asymptotically. Above M_{pcrit} its amplitude decays to a finite value.

Fig. 4 compares the results of this inertialess approximation (again mapped to the Eulerian frame) to the results of a full numerical solution. Their agreement is remarkable, with respect to both pressures and shock location.

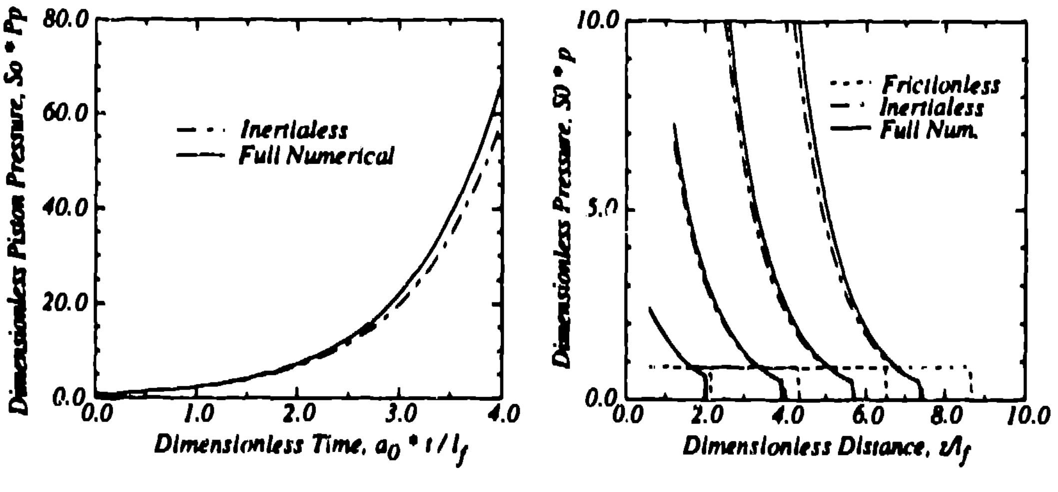

Figure 4. Pressure for full problem; $\phi_0 = 0.65$, $\Delta\bar{t} = 1$, $M_p = 0.6$, $M_{pcrit} = 0.35$.

5. Thermal Problem at the Wall

Heat generation at the wall can be equated to the frictional work there: $\dot{q_w} = \dot{w_f} = \mu_s \phi (-\sigma_n) u$. The problem is simplified by noting that most of the heat flux flows into the (metal!) wall, and that the temperature distribution is, for short times, confined to a thin boundary layer. The wall temperature is then:

$$T_w(z,t) = T_{w0} + \left(\frac{3\nu}{1+\nu}\right)\left(\frac{2\mu_s}{\sqrt{\pi} b_w}\right) \int_0^t \frac{\phi(z,\tau)\, p(z,\tau)\, u(z,\tau)\, d\tau}{\sqrt{t-\tau}} \tag{9}$$

where b_w is the wall heat penetration coefficient. After the leading shock passes one may briefly assume a constant state. Then for a steel wall and $u_p = 100$ m/s, dT_w/dt is of order 100 C/μsec. The melting temperature (247 C) is reached in order 1 μsec – the same time scale as the leading shock rise. One expects a melt layer to lubricate, decreasing further frictional resistance and energy dissipation. The question of wall ignition may therefore depend strongly on the difference between the melting and critical temperatures of the explosive.

6. Conclusions

Without wall friction an impulsively started piston drives a narrow compaction shock through the tube. In the quasi-static limit with wall friction a different kind of compaction wave occurs whose width is proportional to the tube diameter and inversely proportional to the friction coefficient. In general both wave types are present, and their interaction is such that the amplitude of the leading shock is attenuated. Neglecting inertia behind the shock yields an excellent approximation to the wave structure. In a typical DDT test the rate of work at the wall is sufficient to cause rapid melting and, possibly, ignition.

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