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OPACITY OF Air, BeO, C, Fe, and U AT HIGH TEMPERATURES

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ABSTRACT

The opacities of air, BeO, C, Fe, and U at different densities and temperatures have been calculated taking into account the bound-free and free-free transitions and Compton scattering, but neglecting bound-bound transitions. The results are probably fairly accurate at the higher temperatures but fall below the true values at the lower temperatures. Calculations on the bound-bound transitions are being carried out elsewhere.

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For some problems connected with the gadget, it is necessary to investigate the role played by radiation in modifying the hydrodynamics, e. g. the structure of the shock wave travelling into the tamper during the explosion is changed by radiation. The quantity which measures the effect of radiation is the opacity K, defined as:

$$\frac{1}{K} = \int_0^\infty \frac{1}{a_\nu (1-e^{-h\nu/kT})} \left(\frac{\partial I_\nu}{\partial T}\right) d\nu \Big/ \int_0^\infty \left(\frac{\partial I_\nu}{\partial T}\right) d\nu$$

where a_ν is the mass absorption coefficient corresponding to radiation of frequency ν, $I_\nu = B\nu^3/(e^{h\nu/kT}-1)$ is the Planck distribution function corresponding to temperature T, k the Boltzmann constant and B another constant. The factor $(1-e^{-h\nu/kT})$ takes accounts of the fact that only absorptions not balanced by stimulated emissions affect the net flux of energy. The dimension of K is cm^2/gm so that the mean free path, ℓ, is $1/K\rho$ (ρ is the density); it is ℓ which enters into the energy balance equation when radiation is taken into account.

The contributions to the opacity come from photo-electric ionization of the ionized atom (bound-free and free-free transitions), from bound-bound transitions, and from Compton scattering. At low temperatures the bound-bound transitions are important, at high temperatures the Compton scattering dominates. The bound-bound transitions are very laborious to calculate [1] so that we have restricted

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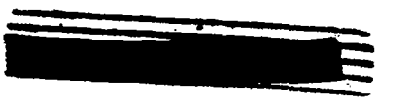
1) The bound-bound transitions are being calculated by Mayer and Mayer at Columbia for air and U under the supervision of Teller.



our computations to fairly high temperatures at which the bound-bound transitions do not contribute appreciably. Our results may therefore be somewhat in error - possibly by a factor two - but it seems worthwhile to present these rough values of the opacity for the more pressing applications which are needed.

The bound-free and free-free transitions were, with some slight modifications, calculated in the standard fashion [2]. For purposes of reference, we outline briefly the procedure; we write for the opacity K_a, due to the bound-free and free-free transitions:

$$K_a = 7.23 \cdot 10^3 \, (\bar{Z}^2/M) \, \rho T^{-3.5} \, \bar{g}/t \qquad (1)$$

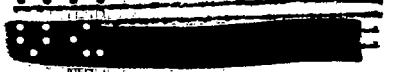
where $\bar{Z}$ is the effective charge (defined as the effective charge of the nucleus at an electron binding energy equal to kT), M is the atomic weight, ρ is the density in gm/cm^3, T is the temperature in millions of degrees, $\bar{g}$ is the Gaunt factor (given in Table 2 of Morse's article as a function of temperature and density) and t is the "guillotine factor". The definition of t is:

$$t = \frac{S(W_1/T_a)}{F_o} + \sum_{s=1}^{N} \frac{\left[S(W_{s+1}/T_a) - S(W_s/T_a)\right]}{(F_o + F_s)} \qquad (2)$$

where W_1, W_2, ...W_N are the energies of the states of the atom in the state of ionization under consideration, arranged in order of increasing magnitude and expressed in units of the ionization energy of hydrogen, T_a is the temperature in units of 157,000°C, S(x) is the Strömgren function (tabulated in Table 1 of Morse's article; $S(W_{N+1}/T_a) \equiv 196.5$), $F_o = 1/\eta \, \log\left[(1+A)/(1+Ae^{-7})\right]$

and $$F_s = \frac{2}{\bar{Z}^2 \eta T_a} \left[\sum_{\sigma=1}^{s} \frac{\bar{Z}_\sigma^4 \, P(W_\sigma/T_a)}{n_\sigma^2} \right].$$

2) Cf. P. M. Morse, Astroph. J. 92, 27 (1940) and R. E. Marshak, Annals of N.Y. Acad. of Sci. 41, 49 (1941); Morse's notation is adhered to as much as possible.



In F_o and F_s, the quantities $\bar{Z}_\sigma$, n_σ, η, A, and $P(W_\sigma/T_a)$ are defined by:

Z_σ is the effective nuclear charge for the state σ.

n_σ is the principal quantum number of state σ

$\eta = (5/28)\,\rho(\pi/T_a)^{3/2}$ (free electrons alone are considered in the determination of A)

$$A = \eta + 0.35\eta^2 \qquad (\eta < 0.1)$$

$$= \exp\left[(1.21\eta)^{2/3}\,(1-0.56\,\eta^{-4/3})\right] \qquad (\eta > 10)$$

(in the range of η between 0.1 and 10, log A - which is really the ratio of the Fermi energy to the temperature - is given in Table 1 of Morse's article)

$$P(W_\sigma/T_a) = \left[1 + A^{-1}\,e^{-W_\sigma/T_a}\right]^{-1}$$ (Probability of finding the state W_σ occupied)

The successive steps were then as follows:

1) $P(w_\sigma^{(o)}/T_a)$ was computed, taking $W_\sigma^{(o)} = Z_\sigma^2/N_\sigma^2$ (for a highly ionized atom) or the $W_\sigma^{(o)}$'s for the normal atom (if the temperature-density relation was such that ionization was small).

2) Using these P's, the Z_σ's were computed for the various states on the basis of Slater's screening constants (distinguishing if necessary among s, p, d, ... electrons).

3) The W_σ's were recalculated taking account if necessary of their reduction at high mass densities through the pushing of the highest bound states into the continuum. The latter correction was made by regarding the bound state, associated with the orbit whose radius was larger than that allowed to the atom, as the bottom of the continuum; the correction was unnecessary for most of the cases considered here

4) New P_o's were recomputed and finally F_o, F_s and t were computed.

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Table I contains the results. Column 1 lists the substance, columns 2 and 3 the density and temperature respectively, column 4 the guillotine factor, column 5, K_a . Column 6 lists the total opacity K_{tot} which includes the Compton opacity, K_s, and is obtained by the empirical rule:

$$K_{tot} = K_a + 1.5\, K_s \qquad (K_s < K_a)$$

$$= K_s + 1.5\, K_a \qquad (K_a < K_s)$$

For the light elements K_s was taken as 0.20 while for U it was taken as 0.15.

In Fig. 1 we have plotted on a log-log scale K_{tot} as a function of T for each substance and each density; furthermore we have drawn straight lines through the data so that the opacity K_{tot} can be represented by a simpler power law, i.e. $K_o\, \rho/T^n$ where K_o and n are constants for each density. In Table II, columns 2 and 3 give n and K_o (K_o is in units of $cm^5(kv)^n/gm^2$) for each substance. Column 4 of Table II contains the critical temperature, T_{crit} , for each substance and each density, i.e. the temperature at which $K_a = K_s$; above T_{crit} , the opacity is essentially constant and equal to K_s .

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TABLE I

Substance	$\rho\left(\frac{gm}{cm^3}\right)$	T	t	$K_{abs.}$	K_{tot}
Air	$1.3 \cdot 10^{-3}$	100 e.v.	6.33	2.96	3.26
	$1.3 \cdot 10^{-3}$	150 e.v.	3.55	1.25	1.55
	$1.3 \cdot 10^{-3}$	300 e.v.	9.23	.039	.26
BeO	3	0.67 k.v.	22.9	1.83	2.13
	3	1.01 k.v.	44	.21	.51
	3	1.35 k.v.	62.9	.056	.28
C	3	0.67 k.v.	47.8	.81	1.21
	3	1.01 k.v.	73.8	.124	.39
	3	1.35 k.v.	106.1	.032	.24
	30	1.35 k.v.	95.9	.36	.66
	30	2.03 k.v.	120.3	.066	.30
Fe	7.6	1.35 k.v.	7.15	4.73	5.03
	7.6	3.38 k.v.	3.21	.43	.73
	7.6	6.75 k.v.	16.3	$7.2 \cdot 10^{-3}$	.20
U	18.7	5.63 k.v.	2.70	.58	.80
	18.7	8.10 k.v.	2.38	.16	.38
	18.7	12.2 k.v.	5.36	.019	.18
	187	2.81 k.v.	5.89	27.1	27.3
	187	5.63 k.v.	3.52	4.48	4.70
	187	12.2 k.v.	5.71	.19	.41

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TABLE II

Substance	n in $(K_o\rho/T^n)$	K_o	$T_{crit.}$
Air: $\rho = 1.3 \cdot 10^{-3}$	5/2	9.8	230 e.v.
BeO: $\rho = 3$	3	.21	1.07 k.v.
C: $\rho = 3$	7/3	.14	.94 k.v.
$\rho = 30$	2	.04	1.55 k.v.
Fe: $\rho = 7.6$	2	1.2	4.2 k.v.
U: $\rho = 18.7$	2	1.3	8.2 k.v.
$\rho = 187$	3	4.4	13.0 k.v.

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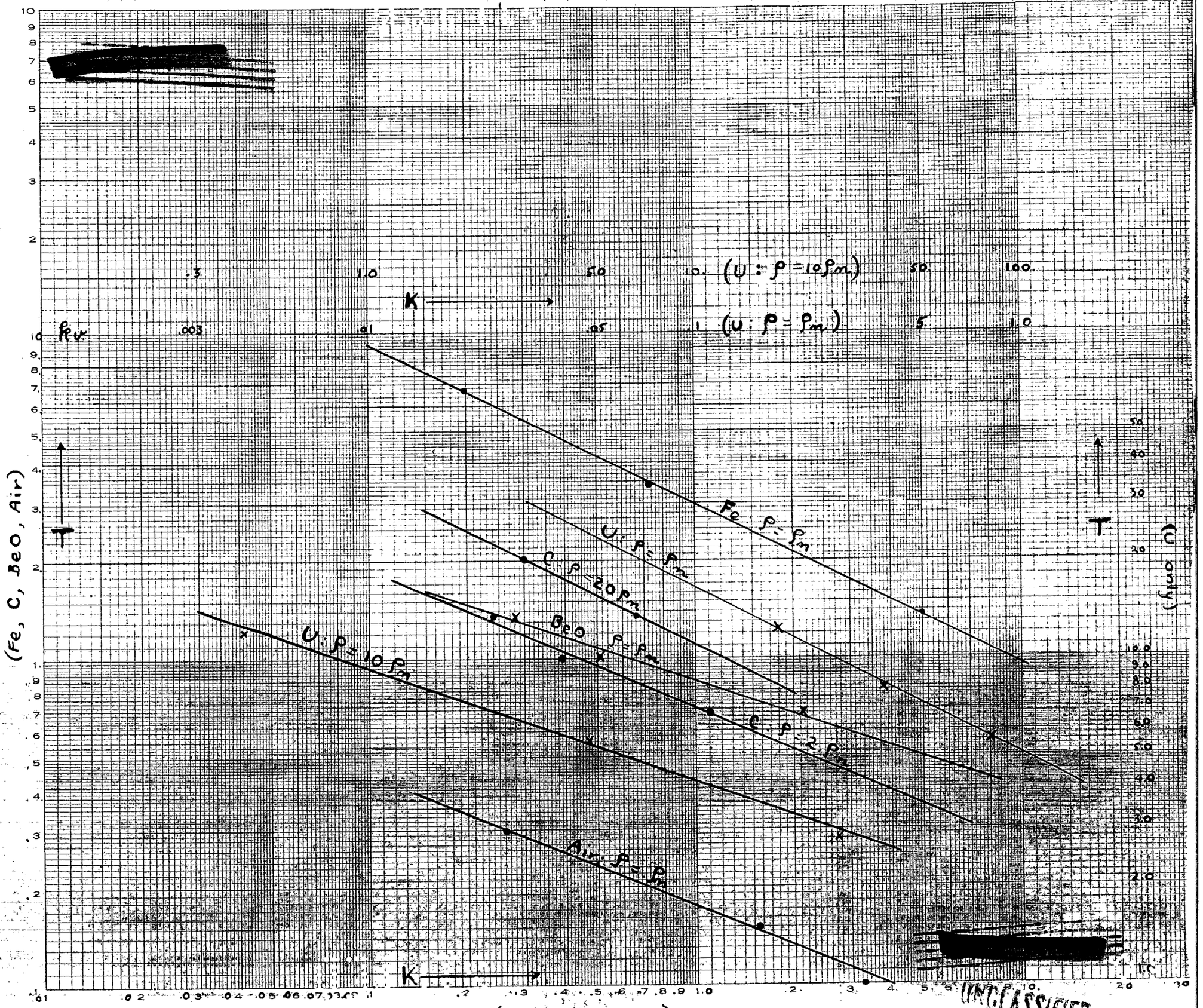

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