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THE DEFINITION OF "NEUTRON MULTIPLICATION"

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ABSTRACT

The distinction is emphasized between the concepts of "net multiplication", referring to the number of neutrons escaping from a subcritical active assembly, and of "total multiplication", referring to the number of neutrons produced in the assembly. The dependence of these quantities on source distribution is discussed.

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THE DEFINITION OF "NEUTRON MULTIPLICATION"

In the description of the behavior of subcritical assemblies of active material, the "multiplication" of the system is a concept widely, but not unambiguously, used. There are at least two common usages; to avoid future confusion, we propose that these be designated as "net multiplication" and "total multiplication".

The net multiplication, N, is the number of neutrons which escape (permanently) from the active material when a single neutron is introduced as the primary source. According to this definition, N is the net number of neutrons produced in the active material, allowing for both production and capture, and including the source neutron in the count of neutrons produced. If the introduction of one neutron results in the production of Q_f fissions,

$$N = 1 + Q_f(\nu - 1 - \alpha)$$

where, as usual, ν is the number of neutrons emitted after a fission, and α is the ratio of capture cross section to fission cross section. Experiments in which the counting rates of a detector with and without active material present are compared, are designed to measure the net multiplication.

The total multiplication, T, is the total number of neutrons produced in the active material when a single neutron is introduced as a source. The source neutron is to be included in the count. In terms of the number of fissions,

$$T = 1 + Q_f \nu .$$

Thus $(N-1)/(T-1) = (\nu - 1 - \alpha)/\nu$. The total multiplication, T, is also equal to the number of fissions produced in the active material when a single

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fission serves as a source of neutrons. The source fission is to be included.

The multiplication of a subcritical assembly depends, of course, on the position in the active material at which the source of neutrons is introduced. To complete the specification, we propose that the type of source distribution be indicated by a subscript, e.g., N_o and T_o for a central source, N_n and T_n for a source with the normal mode distribution, N_u and T_u for a uniformly distributed source. In the notation of the Chicago Laboratory, $T_n = 1/(1-k)$.

For the purpose of determining a critical mass by extrapolation of multiplication measurements, the use of N_o has the considerable advantage that the plot of $1/N_o$ against sphere radius is very nearly a straight line down to quite small sphere radii. The $1/T_o$ curve is concave upwards, while the $1/N_n$ curve is concave downwards.

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In terms of one-velocity neutron diffusion theory, we can find simple expressions for N_n and T_n. The integral equation which describes the diffusion process is

$$n(r) = \int_{\text{core}} K(rr') \left[\sigma (1+f) n(r') + S(r')\right] d\tau', \qquad (1)$$

where $\sigma K(rr')$ is the probability per cm^3 that a neutron produced at r' will make its next collision in the core at r, n is the neutron flux density (neutron density times velocity in $cm^{-2}sec^{-1}$) in the core, $S(r)$ is the number of neutrons/cm^3sec produced by the source, σ is the transport cross-section in cm^{-1}, and $f = (\nu -1-\alpha)\sigma_f /\sigma$ is the net number of neutrons made per collision.

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To define the normal-mode distribution, we imagine that the properties of the active material are improved by increasing ν, or f, until the system is just critical. The values required for this will be denoted by ν_o, f_o. The integral equation satisfied by the normal-mode distribution is thus

$$S(r) = (1+f_o)\sigma \int_{core} K(rr')S(r')d\tau'. \qquad (2)$$

For a normal-mode source of $S(r)$ neutrons/cm^{3}sec, (1) has a solution of the form $n(r) = AS(r)$. Substituting this in (1), and using (2), we find $A = 1/\{\sigma(f_o - f)\}$, or

$$n(r) = S(r)/\{\sigma(f_o-f)\} .$$

The net number of neutrons produced by collisions in the core is thus

$$\int_{core} n\sigma_f(\nu-1-\alpha)d\tau = \int n\sigma f d\tau = f\int Sd\tau/(f_o-f) ,$$

and the complete net production, including the source, is

$$\left(1+f/(f_o-f)\right)\int Sd\tau = \int Sd\tau/(1-f/f_o).$$

Thus the net multiplication for the normal-mode is

$$N_n = 1/(1-f/f_o) . \qquad (3)$$

The total number of neutrons produced by fission in the core is

$$\int_{core} n\sigma_f\nu d\tau = \sigma_f\nu\int Sd\tau/\{\sigma(f_o-f)\} = \nu\int Sd\tau/(\nu_o-\nu) ,$$

since

$$\sigma(f_o-f) = \sigma_f\left[(\nu_o-1-\alpha)-(\nu-1-\alpha)\right] = \sigma_f(\nu_o-\nu)$$

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from which, adding in the source, we find

$$T_n = 1/(1-\nu/\nu_0) \; . \qquad (4)$$

For other shaped distributions, $Q(r)$, the relations are in general more complicated. However, near criticality, when the multiplication is large compared to one, only the fraction of the source which contributes to the normal-mode need be considered. This fraction is found by expanding Q in normal-modes and retaining the fundamental term; it may be written

$$N_Q/N_n = \frac{\int Qn\,d\tau \cdot \int n\,d\tau}{\int Q\,d\tau \cdot \int n^2\,d\tau} \qquad (5)$$

for $N_\tau \gg 1$. If n is taken of the form $n = \sin kr/kr$, (5) gives for the ratio of the multiplications of a central to a normal-mode source

$$N_0/N_n = 2\,\frac{((\sin ka)/ka) - \cos ka}{1 - (\sin 2ka)/2ka} \, ,$$

and for the ratio of a uniform source to a normal-mode source,

$$N_u/N_n = \frac{6}{(ka)^2}\;\frac{\left[((\sin ka)/ka) - \cos ka\right]^2}{1 - (\sin 2ka)/2ka} \; .$$

For the usual core and tamper materials

$$N_0/N_n \sim 4/3 \, ,$$

$$N_u/N_n \sim 0.975 \; .$$

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