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EFFECT OF RIGIDITY ON SHOCK WAVES IN SOLIDS

WORK DONE BY:

R. E. Peierls

P. R. Stein

REPORT WRITTEN BY:

R. E. Peierls

[redacted]

[redacted]

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ABSTRACT

Passage of a plane shock wave in a solid causes a change in shape as well as a compression of each mass element. The deformation produces a shear stress with a component normal to the plane of the wave front; this acts in addition to the hydrostatic pressure which is usually assumed as the only important force. For small displacements this has the same effect as increasing the bulk modulus by an amount of the order of 15 to 30 percent depending on Poisson's ratio. When the plastic yield strength is reached, the shear stress ceases to increase and from there on the total force differs from the hydrostatic pressure by a constant amount. As a result the curve representing this force as a function of specific volume will have a point of inflection. Hence the velocity of a weak shock (pressures of the order of 0.1 to 0.4 megabars) can be less than the longitudinal sound velocity. Such a shock starting from normal density would not be stable and would not, in fact, be formed. Instead, the material will be compressed in two stages, an initial compression from zero pressure to about the yield limit, and a second compression from then to the shock pressure. The first compression is continuous, and travels with the longitudinal sound velocity. If the compression is caused by impact, this first compression wave has a very steep front. The second part is by shock and travels with a velocity depending on this final pressure. When the pressure is high enough to make this velocity equal to or greater than longitudinal sound velocity the two compression waves coalesce into a single shock. Figures are given to show this effect for Al, Cd, Cu, Fe, and tuballoy. During expansion the strength reduces, rather than increases, the effective compression modulus on compression and expansion, and one obtains a hysteresis loop in a pressure-volume diagram, the area of the loop being equal to the energy lost irreversibly.

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EFFECT OF RIGIDITY ON SHOCK WAVES IN SOLIDS

The theory of shock waves is usually worked out for the case of a liquid or gas in which the only force that matters is the hydrostatic pressure. Viscosity, which gives rise to a shear stress, is of importance only for the detailed structure of the shock, and can be neglected if we are concerned with dimensions large compared to the thickness of the shock front. (This is, for gases, of the order of the mean free path).

For solids, small deformations may give rise to shear stresses of considerable magnitude, and we shall discuss their effect below.

Consider a case of one-dimensional motion, for which the velocity is everywhere in the x direction, and depends only on x and on the time. Then the only component of the stress tensor which gives rise to a force is σ_{xx}. This can be written as

$$\sigma_{xx} = p + \sigma'_{xx} \tag{1}$$

where p is the hydrostatic pressure and σ'_{xx} a component of the shear stress. The pressure p is a function of the specific volume, and it may be assumed to be the same as for all-sided compression. This we take as a known function given by the curves of LA-208.

σ'_{xx} depends on the reduced shear strain τ'_{xx}

$$\tau'_{xx} = \frac{\partial \xi}{\partial x} - \frac{1}{3}\left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z}\right) \tag{2}$$

where ξ, η, ζ are the displacements. In our case $\eta = \zeta = 0$ and hence

$$\tau'_{xx} = \frac{2}{3}\frac{\partial \xi}{\partial x} = \frac{2}{3}\left(\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z}\right)$$

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which, for small displacements becomes

$$\tau'_{xx} = \frac{2}{3}\left(\frac{v}{v_0} - 1\right) \qquad (3)$$

In any event since, in our one-dimensional case, the deformation of each volume element depends only on one parameter, τ'_{xx} is a unique function of v.

Within the elastic limits,

$$\sigma'_{xx} = 2G\,\tau'_{xx} \qquad (4)$$

where G is the rigidity modulus, and for higher strains the stress becomes constant. We shall assume the stress-strain relation to be given by

$$\sigma'_{xx} = \begin{cases} 2G\,\tau'_{xx} & \tau'_{xx} \leq Y/2G \\ Y & \tau'_{xx} > Y/2G \end{cases} \qquad (5)$$

where Y is the yield stress. This relation is not far from the truth.

The stress above the yield point cannot depend on the strain, since, once plastic flow sets in, the initial shape is no longer relevant. It may however depend on the rate of strain. The effect of this dependence is rather like that of viscosity in a gaseous shock and we will not investigate it here.

We can now plot the stress σ_{xx} against the specific volume v and the curve we obtain is shown schematically in Fig. 1. v_0-A is the hydrostatic pressure only, v_0-B is the shear stress, and v_0-C is the sum of the two, and hence σ_{xx}.

Consider now a discontinuous transition by shock from the normal state (volume v_0) to a state given by the point P. From the relations of Hugoniot we can see as usual that the slope of the line Pv_0 is equal to U^2/v_0^2 where U is the shock velocity. For a liquid with $Y = 0$, $G = 0$ the line would lead to P' instead, and it is seen that, in the case shown, the difference is small if the

1) LA-165 p.74

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shock strength is large compared to Y.

However, for a somewhat weaker shock, as shown by Q in the figure, the line v_oQ intersects the curve v_oc at an intermediate point s. Since the line v_oQ now lies partly below the curve, such a shock would be unstable according to the criterion of von Neumann.[1] What actually happens then is that the shock will extend only from T to Q, where T is the point of contact with the curves from Q.

The initial compression from v_o to T will in general also be discontinuous. This follows if we take our definition (6) as exactly valid, since the function $\sigma'(v)$ is linear, whereas p(v) is always convex towards the v-axis, so that the resultant is also slightly convex and a stable shock from v_o to T is possible.

Actually even if the line from v_o to T were exactly straight, the compression would be discontinuous, since we would then have the acoustic case in which the pressure distribution propagates without change of shape. If, therefore, the shock is caused, for example, by an impact, it will have a discontinuous rise initially and then be propagated without change.

We find, therefore, that a shock which is not too strong will consist of an initial small compression to the yield limit, and a final shock. The velocity of the first part is given by the slope of the line v_o-T, which evidently is

$$-(dp/dv)_o + (4/3)G$$

or since

$$-(dp/dv)_o = \mu$$

where μ is the compression modulus, it is

$$\mu + (4/3)G$$

This is equal to the square of velocity times density, hence the velocity of the initial shock is

$$\sqrt{\frac{\mu + (4/3)G}{\rho_o}}$$

[redacted]

1) LA-165 p. 74.

This is the velocity of a longitudinal sound signal.

It is evident from the figure that conditions will be as exemplified by the point P or Q respectively according to whether the velocity of the main shock is greater or less than longitudinal sound velocity.

Figs. 2 to 6 show the quantitative pictures for a number of metals. The hydrostatic pressures are taken from the report by Metropolis (LA-208). For the elastic constants values were collected from the literature. In general the value of Poisson's ratio and Young's modulus given in Kaye and Laby's tables, were most nearly consistent with Bridgman's values for the compressibility on which LA-208 is based. In each case two slopes are shown for the longitudinal sound velocity, using either the published value of Poisson's ratio or of Young's modulus, to give an idea of the consistency. In the case of Al only one curve is shown, since in this case the data are exactly consistent.

Elastic constants for tuballoy were taken from measurements by Birch. The data are summarized in the following table. The first column gives the hydrodynamic sound velocity, the second the longitudinal velocity, which is also equal to the critical shock velocity, for which the shock just ceases to be preceded by a sound signal. The next column gives the pressure corresponding to the pressure belonging to this shock if strength is negligible. Actually since the strength raises the curve v_oP of Fig. 1 somewhat above the curve v_oP', the actual critical shock pressure is slightly less than the figure given in the table.

The fourth column gives the corresponding compression, again neglecting strength.

The last column gives, approximately, the shock pressure expected to be caused in each material by a detonation wave in Comp. B striking the metal normally.

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It is seen that this exceeds the critical pressure for Al, Cd, Cu, that it is below p_{crit} in Fe, and about equal to it for Tu. Hence the effect described here ought to exist in steel, might exist to a very small extent in tuballoy, and should be absent in normal conditions in all other metals included in the table.

Experiments by Group G-8 have shown strong indications of such an effect in steel and none in the other metals; this agrees therefore with our expectation. In all cases, the effect might be expected in thick plates and with very small amounts of H.E., since the shock attenuation will reduce the shock strength so that the sound signal will detach itself from the shock after some initial run as a single shock. It would, however, be necessary to use very high weight ratios of metal to H.E. to make the effect observable.

Material	hydro. velocity m/sec.	longit. velocity m/sec.	p_{crit} megabars	$\frac{\rho_{crit}}{\rho_0}$	P_{shock} Megabars
Al	5210	6320	.17	1.19	.28
Cd	2270	2880 2790	.101 .097	1.18 1.15	.35
Cu	3920	4780 4580	.216 .184	1.12 1.11	.37
Fe	4630	6020 5290	.438 .388	1.18 1.16	.36
Tu	2440	3240	.40	1.26	.39

The part of the curve v_0P in Fig. 1 which lies above the yield point T is irreversible, and on expansion the material will follow a curve which instead

[redacted]

[redacted]

of lying above the hydrostatic curve will lie below it since the plastic stress will again appose deformation and will now act in a direction opposite to the hydrostatic pressure.

The shape of the descending part of this curve is of great importance for the theory of spalling.

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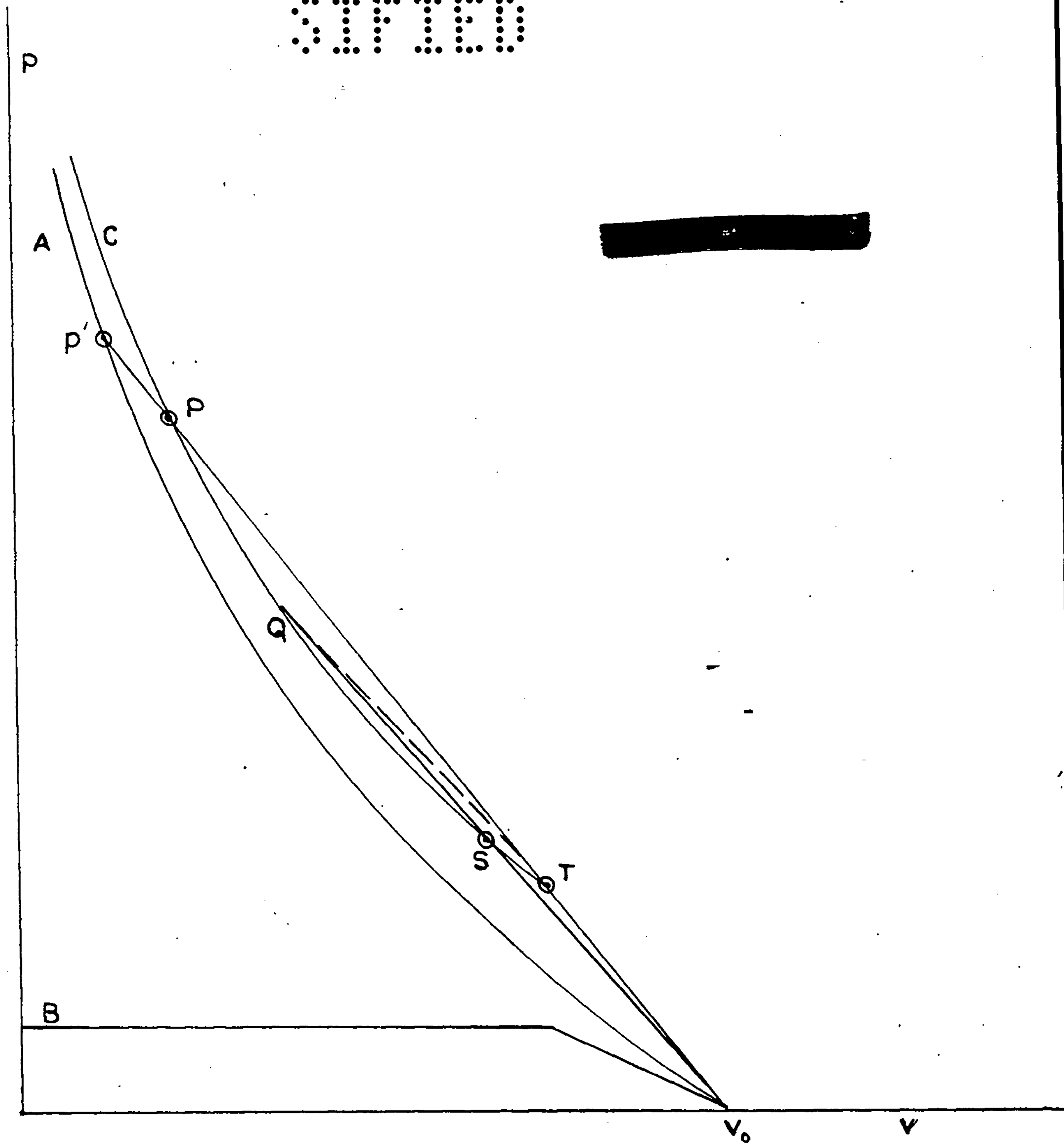

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FIG. 1

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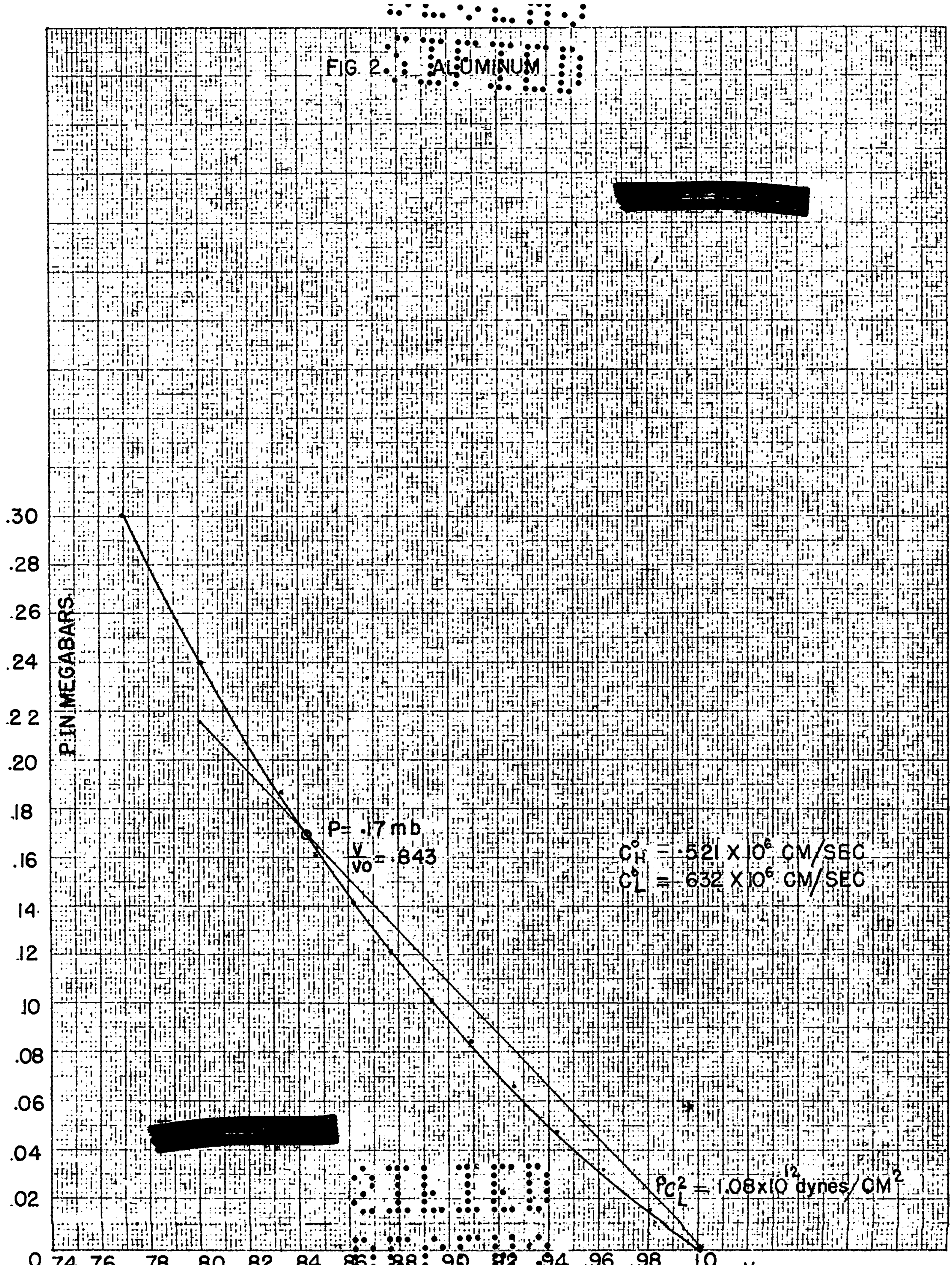
FIG. 2. ALUMINUM
[redacted]
P IN MEGABARS
.30
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
P = .17 mb
V/Vo = .843
C_H^o = .521 X 10^6 CM/SEC
C_L^o = .632 X 10^6 CM/SEC
ρC_L^2 = 1.08 x 10^{12} dynes/CM^2
[redacted]
.74 .76 .78 .80 .82 .84 .86 .88 .90 .92 .94 .96 .98 1.0
V/VO

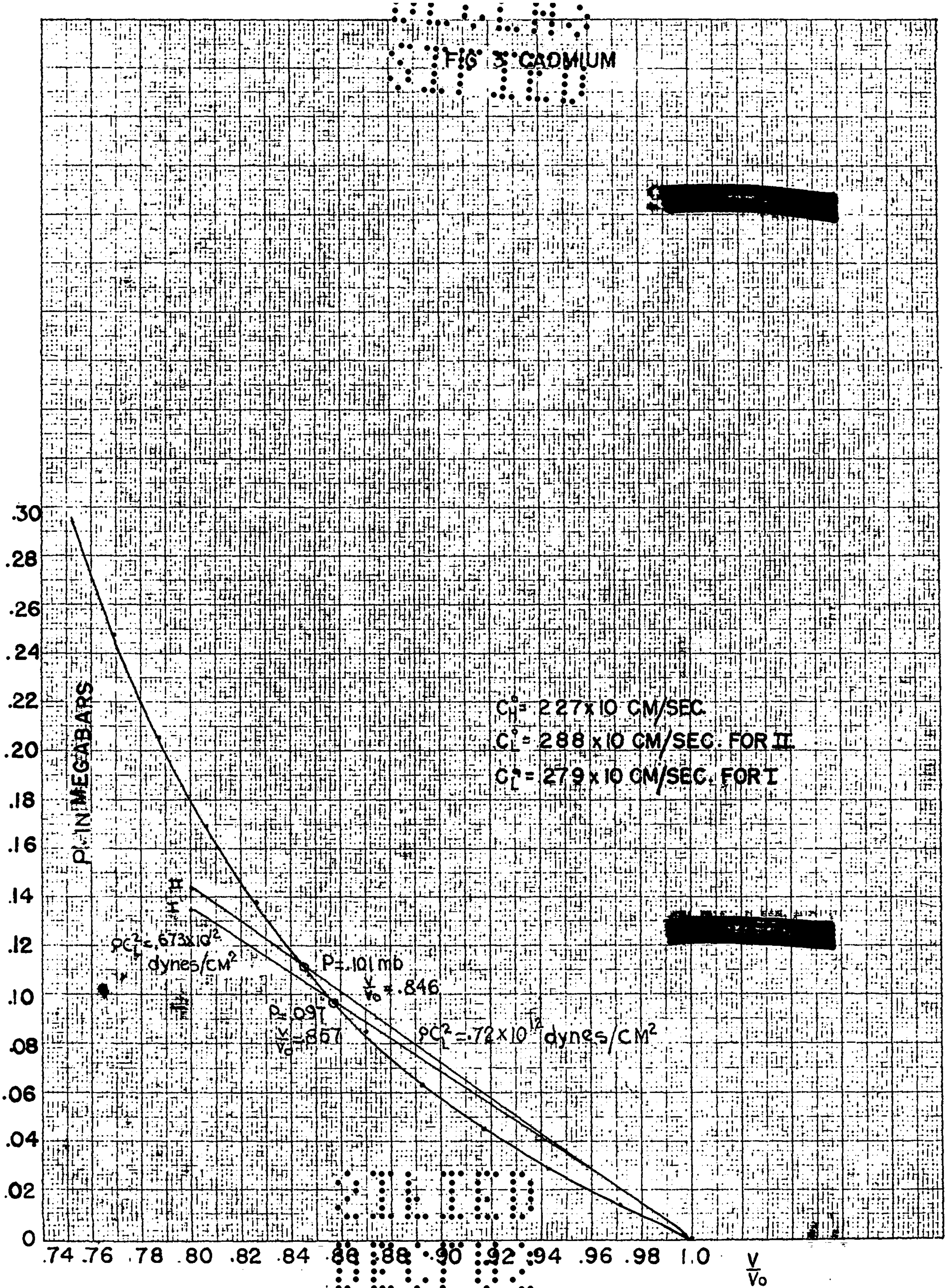
FIG 3 CADMIUM
[redacted]
[redacted]
.30
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
P IN MEGABARS
.74 .76 .78 .80 .82 .84 .86 .88 .90 .92 .94 .96 .98 1.0
V/Vo
CH° = 2.27 x 10 CM/SEC
CL° = 2.88 x 10 CM/SEC. FOR II
CL° = 2.79 x 10 CM/SEC. FOR I
H
II
I
ρCL² = .673 x 10^12 dynes/CM²
P = .101 mb
V/Vo = .846
P = .097
V/Vo = .857
ρCL² = .72 x 10^12 dynes/CM²

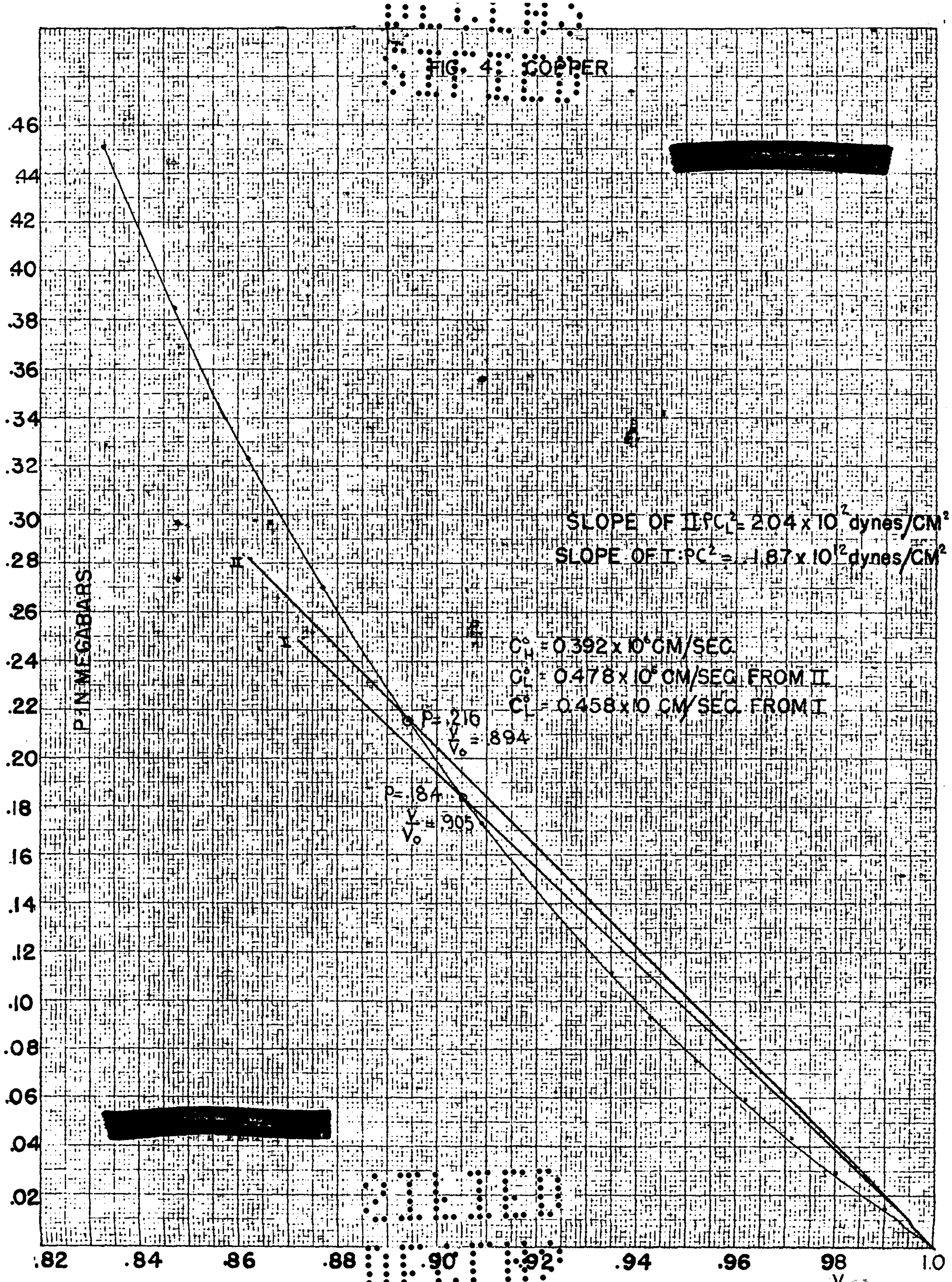

FIG. 4 COPPER
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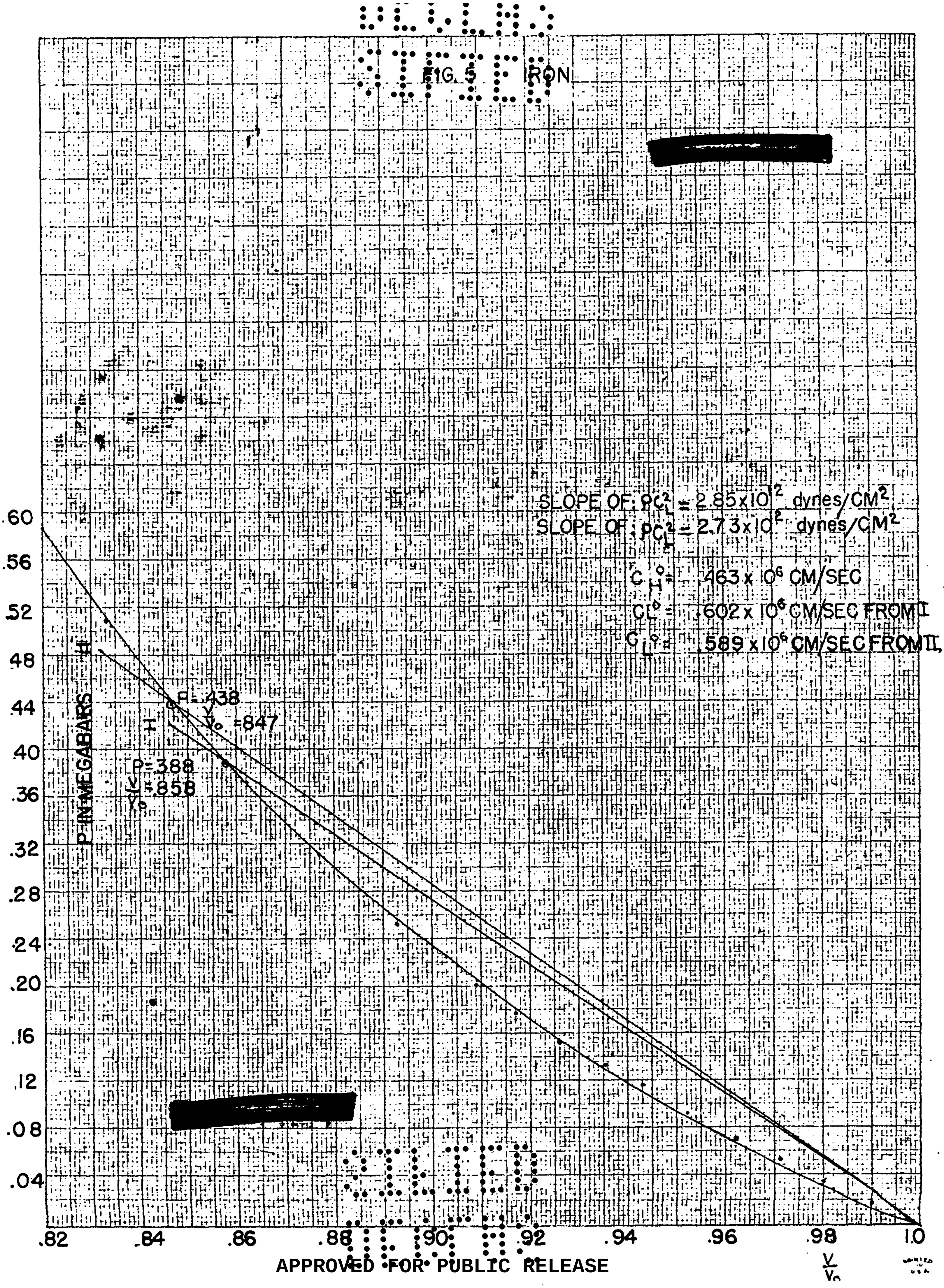
SLOPE OF II: PC_L^2 = 2.04 x 10^{12} dynes/CM²
SLOPE OF I: PC^2 = 1.87 x 10^{12} dynes/CM²
C_H^0 = 0.392 x 10^6 CM/SEC
C_L^0 = 0.478 x 10^6 CM/SEC FROM II
C_L^0 = 0.458 x 10 CM/SEC FROM I
P = .216
$\frac{V}{V_0}$ = .894
P = .184
$\frac{V}{V_0}$ = .905
II
I
P IN MEGABARS
.46
.44
.42
.40
.38
.36
.34
.32
.30
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
.82
.84
.86
.88
.90
.92
.94
.96
98
1.0
$\frac{V}{V_0}$
[redacted]



[redacted]
SLOPE OF ρC_L^2 = 2.85×10^{12} dynes/CM²
SLOPE OF ρC_L^2 = 2.73×10^{12} dynes/CM²
C_H^0 = .463 x 10⁶ CM/SEC
C_L^0 = .602 x 10⁶ CM/SEC FROM I
C_L^0 = .589 x 10⁶ CM/SEC FROM II
II
I
P=.438
V/V₀ =.847
P=.388
V/V₀ =.858
P IN MEGABARS
.60
.56
.52
.48
.44
.40
.36
.32
.28
.24
.20
.16
.12
.08
.04
[redacted]
.82
.84
.86
.88
.90
.92
.94
.96
.98
1.0
V/V₀

FIG. 5 IRON

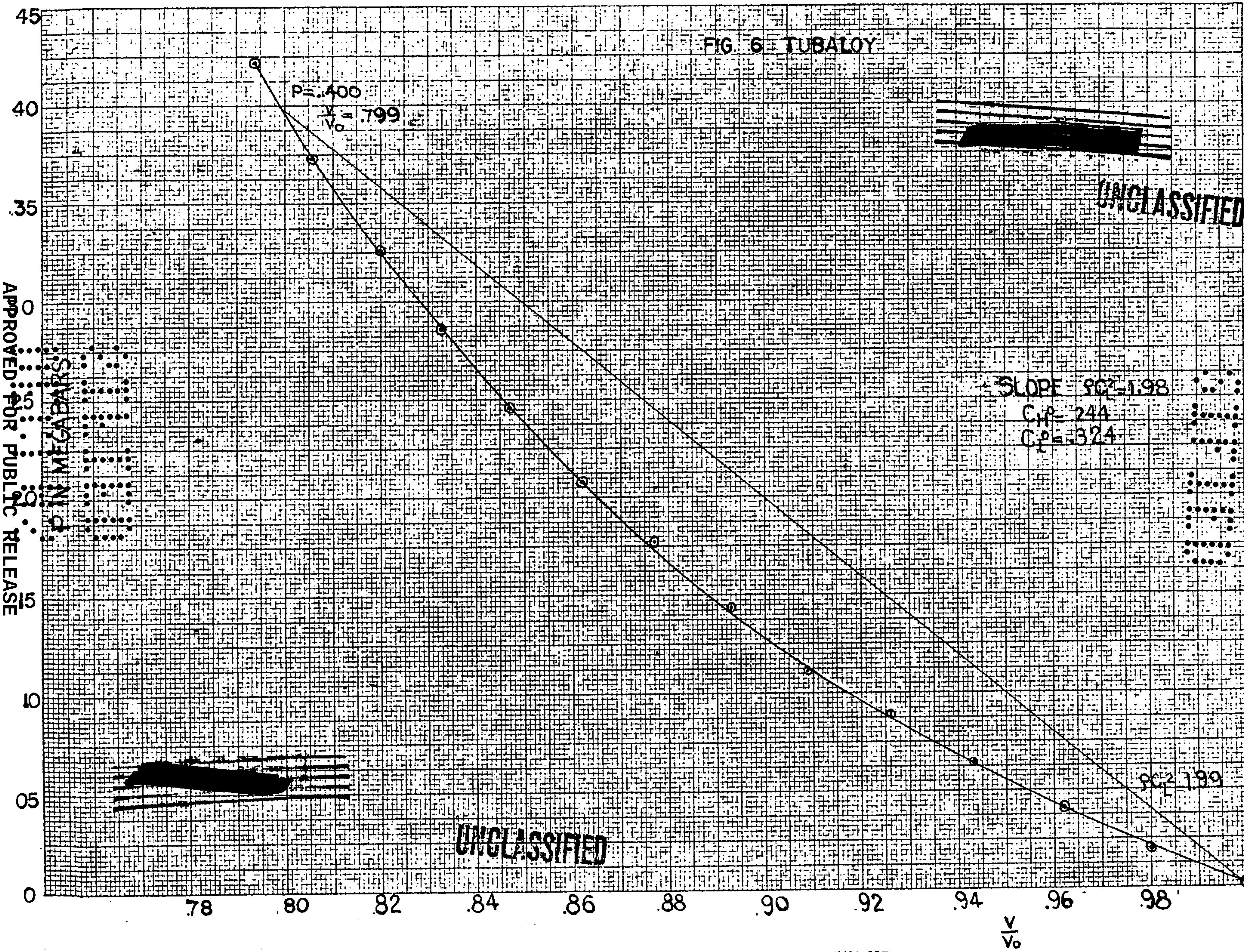


FIG 6 TUBALOY
P=.400
$\frac{V}{V_0}$=.799
SLOPE ρC_L^2=1.98
C_H^o=.244
C_L^o=.324
ρC_L^2=1.99
P IN MEGABARS
$\frac{V}{V_0}$
45
40
35
30
25
20
15
10
05
0
.78
.80
.82
.84
.86
.88
.90
.92
.94
.96
.98

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