This document consists of 26 pages

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SPECIAL RE-REVIEW

FINAL DETERMINATION

UNCLASSIFIED, DATE: 4/7/82

THE OPTICAL BRIGHTNESS OF RADIATION

AND SHOCK FRONTS AT HIGH TEMPERATURE

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Per EMS 6-18-79

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ABSTRACT

The visible radiation observed from the neighborhood of a high temperature shock or radiation front is discussed in this paper. If this front is at sufficiently low temperature (below about 30 volts in air or about 50 volts in helium) the visible light is radiated from just behind the front itself and has an intensity and spectral distribution corresponding to a high temperature (about 10 volts in air, higher in helium). If the front is at a higher temperature, however, the visible light does not come from the front itself. In this case, the material ahead of the shock or radiation front is sufficiently preionized by high energy photons to become very opaque to visible radiation. The observed visible light comes from this preionized layer with an intensity and spectrum characteristic of a relatively low temperature (about 1/2 volt for air, 1 to 3 volts for helium). The mechanism discussed in this paper is a possible explanation of the observations by Felt et al. on luminosities in the visible of such fronts.
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1. Introduction

It has been found experimentally by optical observation of high temperature (5 to 500 ev) radiation and shock fronts in various gases that the brightness (in the visible spectrum) of the radiating surface does not bear a simple relation to the temperature which exists well behind the front. In fact, it appears that high temperatures (above roughly 30 volts in air, for example) are usually associated with very low intensity in the visible spectrum, whereas lower temperatures give an intensity 2 or 3 orders of magnitude as great. This type of phenomenon is of particular interest in photographic observation of such radiation and shock fronts since the anomalously low intensity associated with high temperatures can lead to the failure of such observations. It is the purpose of this paper to discuss a possible mechanism for the production of these anomalies and to consider means for alleviating the resulting difficulties of observation.

We shall consider first the characteristic radiation that can be expected to come from radiation and shock fronts; in this connection we shall make some further remarks on the velocities and configurations of such fronts. We shall then consider the non-equilibrium processes occurring at a radiating surface which alter the emission of visible light, and we shall discuss a method for determining the light intensity when such non-equilibrium processes are important. Usually we shall relate in part the conclusions of this analysis to experiment.
2. Configurations of Radiation and Shock Fronts

Under the assumption that the flow of radiant energy can be approximately described as a diffusion process, and neglecting hydrodynamic effects, we use the well known diffusion equation for radiation

\[
\frac{\partial E}{\partial t} = - \frac{\partial F}{\partial x}
\]

(1)

where \( E = (c_v \theta + a_0^4) \rho \) is the internal energy per gram, \( x \) is the mass coordinate (density times position), and

\[
F = - \frac{c}{3K} \frac{\partial a \theta^4}{\partial x}
\]

(2)
is the radiation flux, and \( K \) the opacity. We shall not discuss this equation in detail, but only write down the properties in which we are interested. Under the assumption of constant flux, the velocity of the radiation front as a function of time is

\[
v_R = \frac{1}{2} \left[ \frac{2ac}{3Kc_v} \frac{\theta_0^3 t^{-1}}{1 + a_\theta^3/c_v} \right]^{1/2}
\]

(3)

where \( \theta_0 \) is the temperature of the source of radiation driving the front. The configuration of the front is

\[
\frac{\theta}{\theta_0} = \left(1 - \frac{x}{x_R}\right)^{\frac{1}{n+4}}
\]

(4)

where \( x_R \) is the position of the radiation front and \( n \) is the coefficient of \( \theta \) in the opacity law, if the opacity is assumed to depend on \( \theta \).
according to

$$K = K_o e^{-n}. \quad (5)$$

These considerations are no longer valid when hydrodynamic effects start to predominate in the transport of energy. In this case, if radiation transport is ignored, the shock velocity is given by

$$v_s = \sqrt{\frac{n+1}{2} \frac{p^1}{\rho_o}} \quad (6)$$

where $p^1$ is the pressure driving the shock and $\rho_o$ is the density of the unshocked material. The temperature at the shock front, again ignoring radiation, rises almost discontinuously (in a few mean free paths for intermolecular collisions) to its maximum value. This is, however, no longer true when radiation transport is included. The hot shocked front tends to drive a radiation front ahead of it to an equilibrium configuration determined by the temperature and the velocity of the shock. This effect has been discussed by Hirschfelder and Magee\(^1\) who show that in this case the configuration of the front is

$$\frac{\theta - \theta_o}{\theta_o} = \left(1 - \frac{x_s}{x} \right)^{\frac{1}{n+3}} \quad (7)$$

where $\theta_o$ is the temperature in the shocked material, $x_s$ is the position of the front, and $n$ is again the exponent of $\theta^{-1}$ in an assumed power law for the opacity.

We see from Eqs. 3, 4, 6, and 7 that although the velocities of

\(^1\)Hirschfelder and Magee, LA-1020 (1947).
radiation and shock propagation are governed by entirely different laws, the temperature configurations of the fronts are nearly the same. We shall make use of the approximate identity of the temperature distribution in the fronts in what follows, but use the correct velocity relations.

3. Effective Temperatures of Radiation Emerging from Fronts

Let us now consider the properties of the radiation emerging from a temperature distribution of the form of Eq. 4 or 7. We shall assume, as is done in the derivation of the diffusion equation, that the material is everywhere in equilibrium at the indicated temperature and ignore until the next section non-equilibrium effects at the front itself. To determine an effective temperature for radiation of a given wavelength, we shall adopt the concept of optical depth under which we assume that the effective temperature is that which exists one mean free path (for the given wavelength) into the hot material. Let us consider first radiation which is of sufficiently high energy to ionize the gas, since this process will interest us in detail later. In air this radiation is absorbed primarily through photoelectric processes (bound-free transition) in the K and L shells with a rather short mean free path. As the temperature of the gas rises back of the radiation front, however, the L-shell is ionized over a rather narrow temperature range and in the region of higher temperature behind this transition region, the radiation below the K edge (with the K shell assumed to be yet unionized) has a quite long mean free path with ab-
sorption and attenuation only due to free-free transitions and Compton scattering. Since the temperature rises very rapidly in the front, we can safely assume that the temperature of the radiation between the \( l \) and \( K \) edges in the region where the \( l \) shell is ionized is approximately the maximum temperature \( \theta_0 \). Similarly, when the material is sufficiently hot to ionize the \( K \) shell, the radiation above the \( K \) edge no longer has a short mean free path and takes on approximately the temperature \( \theta_0 \).

We indicate this distribution of temperatures of radiation by Fig. 1.

\[
\begin{align*}
\theta_{(l-K)} & \approx \theta_0 \\
\theta_{(>K)} & \approx \theta_0 \\
\theta_{(>l)} & = \theta(x)
\end{align*}
\]

Fig. 1. Qualitative temperatures of radiation as a function of position in radiation or shock front.

To make this discussion more quantitative, let us consider the case of air. The ratio of the occupations of the \( l, K \) shells to the continuum is given approximately by

\[
N_K: N_l: N_c = 2e^{-\frac{\hbar \nu_K}{kT}}: 8e^{-\frac{\hbar \nu_l}{kT}}: \int N_c(E)e^{-\frac{E}{kT}}dE
\]

(8)
where for simplicity we have replaced the K and \( l \) energies by a value averaged for each shell. \( N_c(E) \) is the number of continuum states available per atom at the energy \( E \) in the interval \( dE \). Making use of the relation

\[
N_c(E) = 2x \frac{\hbar^2 \varphi}{\hbar^3 N_a dE} = \frac{8\pi \varphi M}{\hbar^3 N_a}
\]

we obtain

\[
N_K: N_L: N_C = 2e \frac{\hbar \nu_K}{kT} : 8e \frac{\hbar \nu_L}{kT} : 2 \left( \frac{2\pi M kT}{\hbar^2} \right)^{3/2} N_a
\]

Taking \( \hbar \nu_K \) as 600 ev, \( \hbar \nu_L \) as 60 ev, and \( N_a = 5.40 \times 10^{19} \), we obtain the results of Table 1. From these it is apparent that the \( l \) shell ionizes

<table>
<thead>
<tr>
<th>( \theta ) (volts)</th>
<th>( N_K )</th>
<th>( N_L )</th>
<th>( N_{\text{continuum}} )</th>
</tr>
</thead>
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<tr>
<td>5</td>
<td>2</td>
<td>5.00</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>3.98</td>
<td>1.02</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
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<tr>
<td>12</td>
<td>2</td>
<td>0.66</td>
<td>4.34</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>0.00</td>
<td>5.00</td>
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<tr>
<td>50</td>
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<td>0.00</td>
<td>5.28</td>
</tr>
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<td>55</td>
<td>1.39</td>
<td>0.00</td>
<td>5.61</td>
</tr>
<tr>
<td>60</td>
<td>0.91</td>
<td>0.00</td>
<td>6.09</td>
</tr>
<tr>
<td>67</td>
<td>0.47</td>
<td>0.00</td>
<td>6.53</td>
</tr>
<tr>
<td>75</td>
<td>0.21</td>
<td>0.00</td>
<td>6.79</td>
</tr>
</tbody>
</table>

Table 1

Region of ionization of K and \( l \) shells in air as a fraction of temperature.
over a range of approximately 5 to 15 volts and the K shell ionizes at about 60 volts. This fixes approximately the boundaries of the various regions of Fig. 1.

Let us now consider the visible radiation which is emitted by a region with this distribution of temperature and ionization. We shall use the criterion that the effective temperature of the radiation is that which exists at one mean free path into the radiative region. To determine the temperature, we therefore must first discuss the absorption of visible light. The dominant process which attenuates visible light in these regions is that of free-free transitions of the ionized electrons. We can obtain a qualitatively correct result for this process by the following considerations based on some previous results of R. Garwin². He gives for the mean free path of light

\[ \lambda = \frac{m \omega^2 c}{6 \pi n e^2} \tau \]  

where \( m \) = electron mass, \( \frac{\omega}{c} \) = frequency of the light, \( n \) = number of electrons per cubic centimeter, and \( \tau \) is the scattering mean free time of the electrons. This classical result is based on the assumption that the coherent polarization energy which resides in the electrons under action of the electric field of the radiation is transformed into incoherent heat energy when the electrons collide with the molecules. To determine \( \tau \), we note that

\[ \tau = \frac{\lambda_c}{v} \]  

where \( \lambda_c \) is the mean free path for collisions. The collisions are both

with the neutral molecules and the ionized gas; these give

$$\frac{1}{\lambda_c} = N_n \sigma_n + N_i \frac{4\pi}{v} \frac{1}{\theta_{\text{min}}^2} \left( \frac{e^2}{mc^2} \right)^2$$  \hspace{1cm} (13)

where $N_n$ is the number of neutral atoms, $\sigma_n$ is the cross section for scattering on these, $N_i$ is the number of (singly) ionized atoms, $v$ is the velocity, and $\theta_{\text{min}}$ is the minimum angle of coulomb scattering which induces incoherence in the motion of the electrons. Taking $\theta_{\text{min}} \approx 1$ radian, we find for visible light

$$\lambda(\text{cm}) = 0.810 \left( \frac{5.40 \times 10^{19}}{N_a} \right)^2 \frac{E^{3/2}}{148s^2 + s(1-s)E^2}$$  \hspace{1cm} (14)

where $N_a$ is the number of atoms per cubic centimeter, $s$ is the fraction of ionization, and $E$ is the electron energy. For temperatures of less than a volt and for ionization greater than roughly 1%, this formula can be approximated by

$$\lambda = 0.810 \left( \frac{5.40 \times 10^{19}}{N_a} \right)^2 \frac{E^{3/2}}{148s^2} \text{ cm}$$  \hspace{1cm} (15)

Considering now the radiative region, we notice that the electron density builds up very rapidly at about 8-10 volts so that while $\lambda$ is very large at 5 volts, it has dropped to 1.14 cm at 8 volts, and to .310 cm at 10 volts. This implies that one mean free path for visible radiation extends to the depth where the ionization sets in strongly at about 10 volts and that visible radiation cannot emerge from appreciably behind
this depth. It seems reasonable to conclude from this analysis that
the origin of the visible light is approximately the point at which the
l-shell starts to ionize so that our picture of the front shows the
visible radiation coming from nearly the depth behind which the higher
energy radiation from the l to K edges has very long mean free paths
and approximately the temperature \( \theta_0 \) deep in the radiating region.
The radiation above the K edge in this region, however, where the K
shell is still unionized, still has a very short mean free path and so
is at the local temperature (roughly 10 volts) which will usually be
much less than \( \theta_0 \).

We shall therefore idealize the radiating region for the visible
spectrum by assuming the temperatures indicated in Table 2.

Table 2

<table>
<thead>
<tr>
<th>Spectral region</th>
<th>Temperature</th>
</tr>
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<tbody>
<tr>
<td>visible</td>
<td>10 volts</td>
</tr>
<tr>
<td>l to K edges (50 to 600 volts)</td>
<td>( \theta_0 )</td>
</tr>
<tr>
<td>above K edge (600 volts)</td>
<td>10 volts</td>
</tr>
</tbody>
</table>
4. Perturbation of Radiating Front by Preionization

We have obtained the effective temperature (and therefore intensity) of visible radiation by assuming that the solution of the diffusion equation adequately describes the situation at regions close to the radiation front. That this is not in general true can easily be seen if we consider the conclusions of the last section. Emerging from the radiating region together with the visible radiation is ionizing radiation in the K-δ band at a temperature T_0 determined by the conditions driving the front. This ionizing radiation will build up a non-equilibrium electron density ahead of the radiating region which density will depend on the velocity of the front and on the intensity of ionizing radiation. Since visible radiation is strongly attenuated in regions of only moderate electron density, we see that such preionization may completely alter the character of the surface radiating the visible and may in particular change the effective temperature. These effects we shall now consider in detail.

To determine the preionization, we use the photoelectric cross section per electron as approximated by Harris Mayer\(^3\), i.e.,

\[ \sigma_n = \frac{\alpha n}{3} \frac{2}{\sqrt{3}} \alpha^2 z \frac{n^4}{n^3} \left( \frac{mc^2}{h\nu_n} \right)^3 \]  

(16)

where \( n \) is the principal quantum number of the level in question and \( z \) is the effective charge of the nucleus. Using for the absorption edge

\[ \nu_n = \frac{1}{2} mc^2 \left( \frac{\alpha z}{n} \right)^2 \]  

(17)

\(^3\)Harris Mayer, LAMS-1117 (1950).
For the K shell in nitrogen (z=7), the absorption coefficient then is

\[ \tau_K = 31.5 \left( \frac{\nu_K}{\nu} \right)^3 (1-s) \text{ cm}^{-1} \]  \hspace{0.5cm} (19)

and for the L shell (z=3.4, correcting roughly for screening)

\[ \tau_L = 2670 \left( \frac{\nu_L}{\nu} \right)^3 (1-s) \text{ cm}^{-1} \]  \hspace{0.5cm} (20)

where s is the fraction of the electrons in the shells that is ionized.

According to these formulae, then, the radiation between the L and K edges emerging from the radiation region is attenuated by a law

\[ N(\nu) = N_0(\nu)e^{-\tau_\nu x} \]  \hspace{0.5cm} (21)

where \( N_0(\nu) \) is the number of quanta of frequency \( \nu \) leaving the surface.

The total energy deposited in ionization per unit volume at a distance \( x \) from the surface per unit time then is

\[ E(x) = \int_{\nu_L}^{\nu_K} h\nu N_0(\nu)e^{-\tau_\nu x} \tau_\nu d\nu \]  \hspace{0.5cm} (22)

For a somewhat approximate answer, if we neglect the angular distribution of the photons leaving the surface, we can use for \( N_0(\nu) \) the black-body law.
\[ N_0(v) = \frac{2\pi v^2}{c^2} \frac{1}{e^{\hbar v/kT} - 1} \]  

where \( T \) is the temperature of the radiation, assumed to be \( \theta_o \), the driving temperature behind the radiation front. The integral then is

\[ E(x) = \frac{2\pi \hbar}{c^2} \left( \frac{k\theta_o}{\hbar} \right)^4 \int_{u\ell}^{u_K} \frac{u^3}{e^{u/\ell} - 1} \mathcal{T}(u)e^{-\mathcal{T}(u)x} du \]

where we have introduced the variable \( u = \frac{\hbar v}{kT} \).

To evaluate this integral approximately, we note that the factor \( e^{-\mathcal{T}(u)x} \) is a very rapidly varying function of \( u \) when \( x \) is not small, in fact dropping to zero very rapidly when \( \mathcal{T}(u)x > 1 \), or

\[ u > u\ell (2670x)^{1/3} \text{ (neglecting depletion)} \]  

while for \( u \) smaller than this value, \( e^{-\mathcal{T}(u)x} \) goes very rapidly to 1.

We shall therefore approximate this factor of the integrand by

\[ e^{-\mathcal{T}(u)x} = 1 \quad u < u\ell (2670x)^{1/3} \]

\[ = 0 \quad u > u\ell (2670x)^{1/3} \]

We shall also replace the upper limit \( u_K \) by \( \infty \) which is a reasonable approximation for \( u\ell (2670x)^{1/3} \) much less than \( u_K \). Under these approximations, we obtain

\[ E(x) = \frac{2\pi \hbar}{c^2} \left( \frac{k\theta_o}{\hbar} \right)^4 2670 \ u\ell^3 \int_{u\ell}^{\infty} (e^{u/\ell} - 1)^{-1} du \quad x < \frac{1}{2670} \]

\[ = \frac{2\pi \hbar}{c^2} \left( \frac{k\theta_o}{\hbar} \right)^4 2670 \ u\ell^3 \int_{(2670x)^{1/3}u\ell}^{\infty} (e^{u/\ell} - 1)^{-1} du \quad x > \frac{1}{2670} \]
For large $x$, neither of these approximations is useful since the upper limit must then be retained explicitly. In this region, we can approximate the integral by

$$E(x) = \frac{2\pi h}{e^2} \left( \frac{k \theta}{h} \right)^4 \int_0^x 2670 \, u^3 \, e^{-u \kappa} \, e^{-2670 \left( \frac{u_x}{u} \right)^3} \, du;$$

$$= 2670 \left( \frac{u_x}{u \kappa} \right)^3 \, x > 1$$

We shall be principally interested in cases for which $u_x$ is somewhat larger than one so that we can further approximate $(e^u - 1)^{-1}$ by $e^{-u}$.

We then obtain

$$E(x) = 3.49 \times 10^{33} \, u_x^{-1} \, e^{-u \kappa} \, \text{ev/sec} \quad x < \frac{1}{2670}$$

$$= 3.49 \times 10^{33} \, u_x^{-1} \, e^{-u \kappa (2670x)^{1/3}} \, \text{ev/sec} \quad x > \frac{1}{2670} \quad (29)$$

$$= 1.85 \times 10^{22} \, e^{-u \kappa} \, e^{-2.670x} \, \text{ev/sec} \quad x > \frac{1}{2.67}$$

We can next convert this result to the number of electrons present by the following argument: the photoelectrons are produced with energies well above the $\ell$-edge but very rapidly lose energy by inelastic collisions producing additional electrons. This process continues until the electrons can no longer ionize (at about 30 volts) and the energy loss is then by excitation and elastic collision. We can therefore obtain the number of electrons produced per second by dividing $E(x)$ by the ionization energy which we take to be 30 volts. Next, the actual
number of electrons in equilibrium ahead of a radiative surface moving with velocity \( v \) is given by the interval

\[
N(x) = \int_0^{\infty} \frac{E(x+vt)}{30} \, dt = \frac{1}{30v} \int_x^{\infty} E(s) \, ds \tag{30}
\]

We are now able to calculate the effect on the visible radiation coming from behind this region of preionization. To determine this, we make use of the mean free path for absorption of the light at low electron energies and moderate temperatures for air

\[
\lambda = 5.47 \times 10^{-3} \, \frac{E^{3/2}}{s^2} \tag{31}
\]

where \( E \) is the electron energy in electron volts and \( s \) is the fraction of ionization. The discussion of the next section will show that \( E \) is less than a volt so that this approximation to \( \lambda \) is valid. The attenuation of the visible light is now according to the law

\[
\frac{dq}{q} = -\frac{dx}{\lambda(x)} \tag{32}
\]

and

\[
\ln \frac{q(x)}{q_0} = -\int_0^x \frac{dx}{\lambda(x)}. \tag{33}
\]

The character of the radiative surface can be expected to change when the attenuation of the light by the preionization is a factor of somewhat greater than 1, which we shall take to be \( e \). We therefore obtain a criterion for the temperature at which a radiation front no longer radiates visible light from the equilibrium regions by specifying
\[ q(\infty) = \frac{1}{e} q_0 \]

or

\[ \int_0^\infty \frac{dx}{\Lambda(x)} = 1. \tag{34} \]

Since \( \Lambda(x) \) depends on the temperature through \( u_E \) and \( v \), this is a condition on the temperature. Taking the values of the velocity of the radiation front as given by Eq. (3) and setting \( E = 1/2 \) volts as given by the next section, we find after evaluation of this integral that the value of \( \theta_0 \) determined by this expression is approximately 30 volts. We therefore conclude that for temperatures driving the front of less than 30 volts, the preionization gives little absorption of the visible and, as discussed in the last section, the visible light has a temperature of approximately 10 volts. For driving temperatures greater than 30 volts, however, the preionization results in absorption of the light coming from the equilibrium region at 10 volts so that the visible light which is finally emitted by the front emerges from the region of preionization, of low electron densities not in equilibrium with the heavy ions and molecules. The temperature of this light then depends on the effective temperatures of this electron gas which will be discussed in the next section.
5. Temperatures of the Non-Equilibrium Electron Gas in the Preionization Region

In this discussion we shall suppose that the electrons which have been photo-produced by the high energy quanta very quickly come into approximate thermal equilibrium with each other although not with the molecules and heavy ions of the gas, since their rate of transfer of energy to the molecules is slow. The principal mechanism of loss is by inelastic collision with the gas, where a sufficiently energetic electron will raise the molecule to the first excitation level (roughly 4 volts in air). This process will drain energy from the Maxwellian distribution and therefore the temperature of the electron gas will drop. Combating this loss mechanism is, of course, the supply of energy to the electron gas by photo-ionization. To calculate the energy loss, we use

$$\frac{dE_{\text{loss}}}{dt} = I \int_{E}^{\infty} N(E) \frac{v_E}{\lambda_I} dE$$

(35)

where $I$ is the excitation energy lost per collision, $N(E)$ is the number of electrons with energy $E$, $v_E$ is the velocity of the electrons at this energy, and

$$\lambda_I = \frac{1}{N_a \sigma_I}$$

(36)

is the mean free path for excitation collision, $N_a$ being the number of molecules per cubic centimeter and $\sigma_I$ the cross section for collision.
If we use the Maxwell distribution for $N(E)$, and assume $\lambda_I$ independent of the energy, we obtain

$$\frac{dE_{\text{loss}}}{dt} = \frac{INa}{\lambda_I} (2N) \sqrt{\frac{2kT}{\pi m}} \left( \frac{I}{kT} + 1 \right) e^{-I/kT} \tag{37}$$

where $T$ is the temperature, $N$ is the total number of electrons present, and $m$ is the electron mass. For the energy gain from photo-ionization, we have from Eq. (29) in the region of large $x$

$$\frac{dE_{\text{gain}}}{dt} = 1.85 \times 10^{22} e^{-u_k} e^{-2.670x} \text{ ev/sec}$$

with $x$ the distance from the radiation front. These two processes then determine the temperature

$$\frac{d}{dt} \frac{3}{2} NkT = \frac{dE_{\text{gain}}}{dt} - \frac{dE_{\text{loss}}}{dt} \tag{38}$$

with, as before,

$$\frac{dN}{dt} = \frac{dE_{\text{gain}}}{dt} \frac{1}{30} \tag{39}$$

assuming that the energy supplied goes immediately into the production of ion pairs at 30 volts per ion pair. Measuring time from the point at which the radiation front passes the position being considered, we take $x=vt$ and obtain

$$\frac{3}{2} \frac{d\theta}{dt} = \left(30 - \frac{3}{2} \theta \right) \left(2.670v\right) - \frac{2NaI}{\lambda_I} \sqrt{\frac{2e}{\pi m}} \left( \frac{I}{\theta + I} \right) e^{-\frac{I}{\theta}} \tag{40}$$
where $\theta$ and $I$ are in electron volts. For an approximate solution, we can assume (as can be shown to be reasonable) that the loss and gain approximately balance each other by the time the depth in the electron gas is reached at which the visible radiation is emitted. Solving the above equation for $\theta$, we find for typical values of the radiation front velocity ($v$), taking the cross section for excitation to be the atomic area, that the equilibrium temperature is about $1/6$ or about $\frac{1}{2}$ volt. The electrons therefore lose most of their energy by excitation of the molecules; only part of this energy will be reradiated (since the times available are very short) and since it is strongly absorbed in the gas, it will not contribute appreciably to the visible light emitted.

We therefore conclude that the temperature of the electron gas created by preionization is considerably less than a volt, so that if a radiation (or shock) front in air is driven by a sufficiently high temperature to give preionization (30 volts or more), the temperature of the visible radiation is only a few tenths of a volt, independent of the driving temperature. This result is indicated qualitatively in Fig. 2.
Fig. 2. Temperature of visible radiation as a function of the maximum temperature behind the front.
6. Preionization Phenomena in Helium

The analysis carried out for air can be extended to helium with minor modifications. The velocity of a radiation front in helium cannot be described, except at low temperatures, by the diffusion approximation. We can obtain, however, an approximate result for the rate at which radiant energy penetrates helium by the following argument: when radiant energy of 10-200 volts penetrates helium, it does so with very short mean free paths until the gas is sufficiently heated (about 10 volts) to ionize, at which temperature the radiation sees only the Compton scattering absorption with accordingly very long mean free paths. The propagation of the radiation front is therefore limited only by the rate at which the gas can be heated by the radiation emitted by a surface temperature $\theta_0$. This gives

$$
\varphi \theta_0^4 = \nu \left[ c_v \theta_0 + a \theta_0^4 \right]
$$

where the first term on the right is the energy in the heated gas and the second term is the energy in radiation. Using

$$
c_v = \frac{3}{2} \frac{3R}{M}
$$

assuming that the gas is completely ionized, gives

$$
\varphi = \frac{1}{4} \frac{1}{1 + 1.66 \times 10^{-3} \theta^{-3}(\text{keV})}
$$

With these values for the velocity, the calculation for helium goes through much as in air to give a transition temperature of about 50 volts. Accordingly, above this temperature a radiation front in helium
would be expected to be much dimmer in the visible than would be expected from the equilibrium description of the front. In this temperature range, the visible radiation is emitted by the non-equilibrium electron gas ahead of the equilibrium region. The temperature in this region is again determined by the balance between supply of energy by photo-ionization and loss by ionization, excitation, and elastic collision with the helium ions and molecules. In this case the lowest excitation level is at 22.5 volts but the energy loss by elastic collision is considerably higher since the helium atoms are lighter than the air molecules. The calculation also must be modified to take into account the different dependence of electron density on distance from the radiation front which results from the absence of two shells of different ionization energy in helium. Estimates of these effects indicate that the temperature in the preionized region may be in the range of 1 to 3 volts, the higher value than air being principally due to the much higher energy of the first excited state.

7. Implications for Experiment

We have seen that radiation of shock fronts in gases emit visible radiation in a manner which is a very non-linear function of the driving temperature (Fig. 2). In particular if the temperatures lie above the approximate values of 30 volts in air (or similar gases) or 50 volts in helium, the intensity in the visible will be very much lower than would be expected on the basis of considerations of the maximum temperature alone. This implies that observations which depend on an expected light
intensity should allow for the complicated effects discussed in this paper, most simply by providing mechanisms to reduce to a reasonable range the temperatures which are observed.