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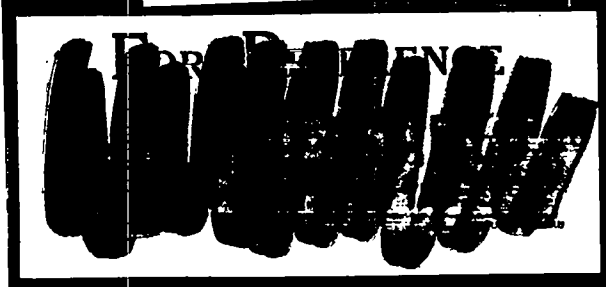

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FREE SURFACE ACCELERATION BY SHOCKS OF MODERATE STRENGTH

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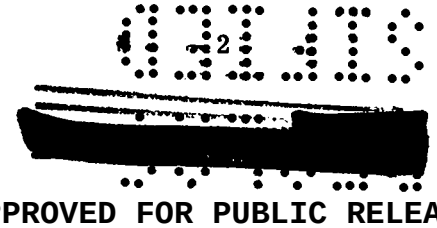

ABSTRACT

In obtaining shock pressure and compression from experimental measurements of shock and free-surface velocity in metal plates it is usually assumed that the mass velocity behind the shock (U_p) is one-half the free surface velocity (U_{fs}):

$$U_p = U_{fs}/2$$

This approximation was investigated and detailed calculations were carried out for aluminum and tuballoy using the LA-385 equations of state. It was found that for these metals U_p is less than $U_{fs}/2$ by about 1% for plane-wave shocks from Composition B. In addition, a simple method has been worked out for estimating U_{fs}/U_p from the experimental shock and free-surface velocities, provided the velocities have been determined for a variety of shock strengths. It has been assumed that the shock produces no permanent deformation of the metal. A permanent increase in the density of the material of even 1 or 2% would change the above conclusions completely and result in U_p being greater than $U_{fs}/2$.

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Chapter 1

INTRODUCTION

A program has been undertaken by GMX-4 and GMX-6[1] to determine shock Hugoniots of metals in the compression range reached by plane detonations in solid explosives. The quantities that are measured experimentally are the shock velocity (D) in the metal plate and the velocity imparted to the free surface of the plate when it is struck by a shock at normal incidence. Provided the relation between the free-surface velocity (U_{fs}) and the particle velocity (U_p) behind the shock is known, these measurements enable one to calculate the Hugoniot (shock) curve for the metal from the relations*

$$\frac{\rho}{\rho_o} = \frac{D}{D - U_p} \tag{1.1}$$

$$p - p_o = \rho_o D U_p \tag{1.2}$$

where ρ_o is the normal density of the metal (at pressure p_o) and p is the pressure developed when the metal is shocked to a density ρ.

For very weak shocks, the relation

$$U_{fs}/U_p = 2 \tag{1.3}$$

can be shown to hold. This approximation is commonly used also for shocks of moderate intensity (p up to half a megabar or so); the error thereby encountered is thought to be no greater than 2 or 3%. However, experimental techniques have recently been improved to the point where D and U_{fs} can be measured with errors of less than 1%, so that it is desirable to know U_{fs}/U_p with greater precision than is given by the approximation (1.3).

Some theoretical calculations of the ratio U_{fs}/U_p have been made by Fuchs and Stark.[2] However, they were primarily interested in the effects of the free-surface evaporation which might result from strong shocks. Thus the equation of state which they used was devised primarily to handle solid-vapor phase changes and is not particularly accurate for dealing with the moderate-strength shocks of interest to us here. An analytical treatment has also been given by Mayer[3] based on a hypothetical equation of state fitted to experimental data for the material under normal conditions. Application of his equations is a somewhat tedious process and he has made numerical calculations only for aluminum. (An error in decimal points was made on p. 29 of his report and the value $U_{fs}/U_p = 2.0046$ quoted for $p - p_o = 0.332$ Mb should read $U_{fs}/U_p = 2.046$.)

*These merely express the laws of conservation of mass and momentum, respectively, across the shock front.

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We have made detailed calculations for aluminum and uranium based on the equations of state of LA-385, and have also derived a theoretical expression by means of which a value for U_{fs}/U_p can be estimated for any substance from the specific heat and coefficient of expansion of the material under normal conditions and from the curvature of the (experimental or theoretical) shock curve.

Chapter 2

THEORY

Consider a medium at rest and at normal density (ρ_o), pressure (p_o = 1 atm), temperature ($T_o \cong 300^oK$), and entropy (S_o), and through which a shock front travels with velocity D. Behind the shock front the density, pressure, temperature, and entropy have been raised to values ρ_1, p_1, T_1, and S_1, respectively, and the material is moving (in the same direction as the shock front) with a velocity U_p. In addition to the relations (1.1) and (1.2) the law of conservation of energy applied to changes across the shock front leads to the Hugoniot equation

$$H(p,v) \equiv E(p,v) - E(p_o, v_o) + (p + p_o)(v - v_o)/2 = 0, \qquad (2.1)$$

where

$$v = 1/\rho \qquad (2.2)$$

is the specific volume of the material and $E(p,v)$ is its specific internal energy (given as a function of p and v by the equation of state). Equation (2.1) thus defines a curve (the Hugoniot curve) in the (p,v) plane which passes through the point (p_o, v_o); the action of the shock is to carry the state of the material from the point (p_o, v_o) to the point (p_1, v_1) which also lies on this curve.

When the shock reaches the "free" surface of the material (at normal incidence), a shock is transmitted into the adjoining medium (air at atmospheric pressure) and a rarefaction wave is reflected back through the first medium. As this reflected wave (traveling with velocity U_r) passes a given point, the material expands adiabatically (and we shall assume also reversibly) along the isentropic curve $S = S_1$ which passes through (p_1, v_1) (cf. Fig. 2.1). The expansion proceeds until the pressure has dropped to the pressure of the air shock. The air shock may be 100 atm or more, but is negligible compared with p_1 and so may be taken as p_o for our purposes. Thus the material ends up at a point (p_o, v_o') on the adiabat $S = S_1$, v_o' being greater than v_o since S_1 is greater than S_o and since $(\partial S/\partial v)_p > 0$ (assuming there has been no permanent compression of the material).

The free-surface velocity which results is given by[4]

$$\left.\begin{aligned} U_{fs} &= U_p + U_r \\ \text{or} \qquad & \\ \frac{U_{fs}}{U_p} &= 1 + \frac{U_r}{U_p} \end{aligned}\right\} \qquad (2.3)$$

where from (1.1) and (1.2)

$$U_p = \left\{(p_1 - p_o)(v_o - v_1)\right\}^{1/2} \qquad (2.4)$$

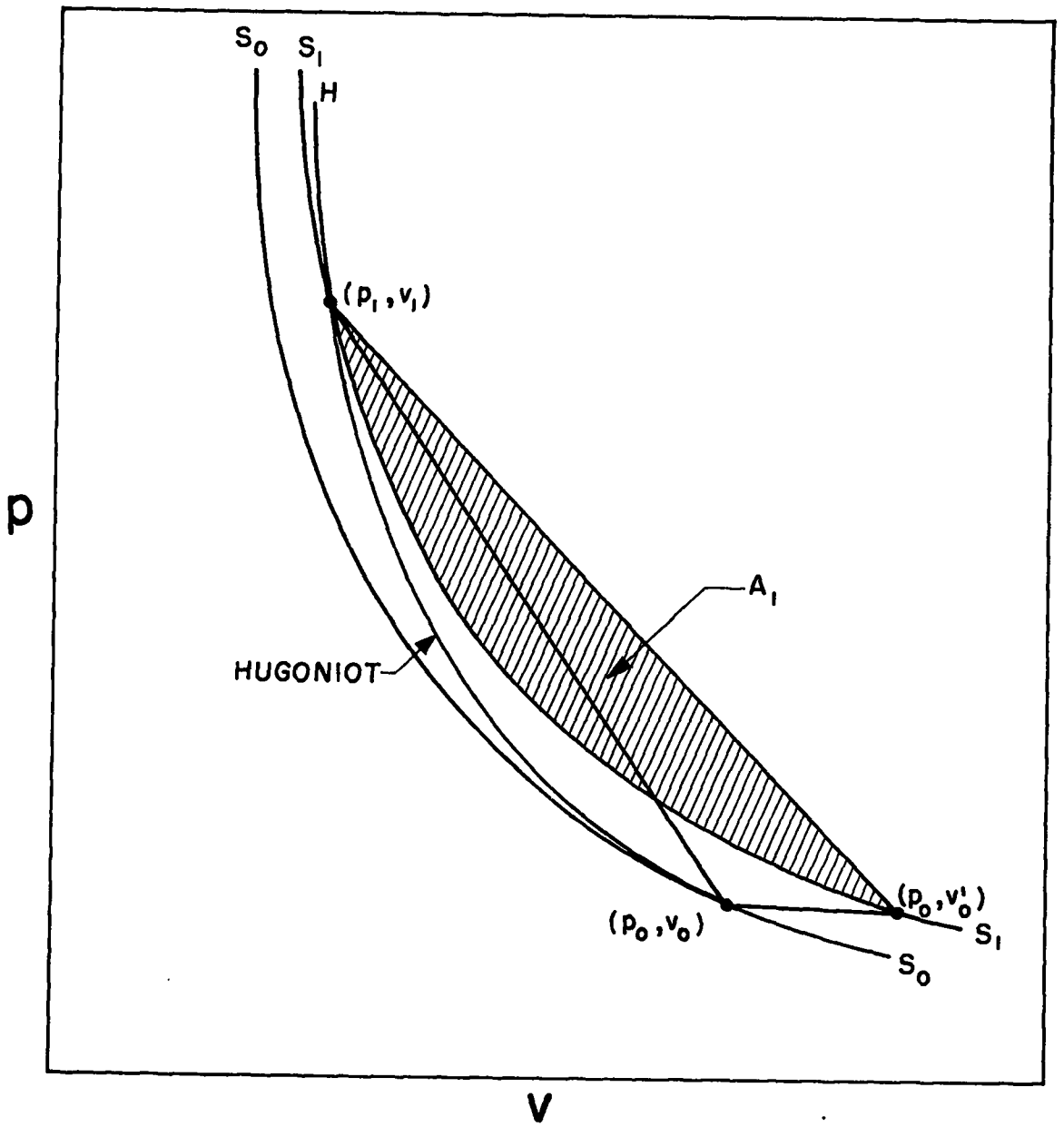

Fig. 2.1 Hugoniot and adiabats

and U_r is given by the Riemann integral

$$U_r = \int_{v_1}^{v_o'} \left(-\frac{\partial p}{\partial v}\right)_{S_1}^{1/2} dv = \int_{p_o}^{p_1} \left(-\frac{\partial v}{\partial p}\right)_{S_1}^{1/2} dp \qquad (2.5)$$

Our problem is to calculate the value of U_r/U_p as a function of p_1 or v_1. If the equation of state is known (i.e., if the Hugoniot curve and the adiabats through each point (p_1, v_1) thereon are known), this calculation is perfectly straightforward. However, the equation of state of some material of interest may not be known to this degree of completeness, and an approximate expression for U_r/U_p involving less extensive equation of state data would be useful. In addition, it is instructive to continue a theoretical analysis further to indicate which properties of the equation of state are of importance.

Toward these ends, we may introduce a quantity

$$U_p' = \left\{(p_1 - p_o)(v_o' - v_1)\right\}^{1/2} \qquad (2.6)$$

in terms of which

$$\frac{U_r}{U_p} = \frac{U_p'}{U_p}\,\frac{U_r}{U_p'} = \left(1 + \frac{v_o' - v_o}{v_o - v_1}\right)^{1/2} \frac{U_r}{U_p'}. \qquad (2.7)$$

The quantity U_r/U_p may thus be considered as the product of two factors: the first factor depends on the volume difference $v_o' - v_o$ and is somewhat greater than unity; the second factor, U_r/U_p', would be unity if the adiabat $S = S_1$ were a straight line (as may be readily seen by evaluating (2.5) in this case), but is otherwise less than unity. (It has been pointed out by Walsh[5] that application of the calculus of variations shows the integral (2.5) to have a maximum value when the path of integration is a straight line.) Thus it may be said that the value of U_r/U_p depends on a volume effect $(v_o' - v_o)$ and a curvature effect (that of $S = S_1$), and that these two effects tend to cancel one another. Actually, the volume effect is a direct consequence of the curvature as will be seen from the following analysis.[6]

Differentiating the Hugoniot function (2.1) and using the equilibrium thermodynamic relation

$$dE = TdS - pdv \qquad (2.8)$$

we obtain

$$dH = TdS + \frac{1}{2}\left\{(v - v_o)\,dp - (p - p_o)\,dv\right\}$$

$$= TdS + dA \qquad (2.9)$$

[redacted]

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where dA is the area of the triangle defined by the vectors $(p_o, v_o) \rightarrow (p, v)$ and $(p, v) \rightarrow (p + dp, v + dv)$ and is negative or positive according as the vector $(p_o, v_o) \rightarrow (p, v)$ must be rotated clockwise or counterclockwise to be made parallel to $(p_o, v_o) \rightarrow (p + dp, v + dv)$. Now if v_o' were equal to v_o for all shock strengths (all values of p_1), this would imply $S_1 = S_o$ for all p_1 and hence the Hugoniot would be identical with the adiabat $S = S_o$. But then along the Hugoniot curve both dH and dS would be zero and hence dA would be zero also; i.e., the Hugoniot (and the adiabat which is identical to it) would be a straight line through (p_o, v_o). Conversely, if the adiabat $S = S_o$ were a straight line, then along this line $dS = dA = 0$ and hence $dH = 0$ from (2.9); i.e., the Hugoniot would be a straight line identical with $S = S_o$, there would be no entropy change along the Hugoniot, and v_o' would be equal to v_o.

This intimate connection between $v_o' - v_o$ and the curvature of the adiabat (which is always greater than zero in the cases of interest here) may be made quantitative. Consider Fig. 2.1, in which are shown the Hugoniot curve H, the adiabats $S = S_o$ and $S = S_1$, and the straight lines $(p_o, v_o) \rightarrow (p_o, v_o')$, $(p_o, v_o') \rightarrow (p_1, v_1)$, and $(p_o, v_o) \rightarrow (p_1, v_1)$. The area between S_1 and its chord $(p_o, v_o') \rightarrow (p_1, v_1)$--shown shaded in Fig. 2.1--will be denoted by A_1 (and considered positive). Integrating (2.8) clockwise around the area A_1 we obtain

$$\left(\int_{v_1}^{v_o'} T dS \right)_{\text{chord to } S_1} = A_1 \tag{2.10}$$

since the integral of dE around a closed path is zero and since $dS \equiv 0$ along the adiabat $S = S_1$. Integrating (2.8) from (p_o, v_o) to (p_o, v_o') along the straight lines $(p_o, v_o) \rightarrow (p_1, v_1)$ and $(p_1, v_1) \rightarrow (p_o, v_o')$ gives

$$E(p_o, v_o') - E(p_o, v_o) = \left(\int_{v_o}^{v_1} T dS \right)_{\text{chord to H}} + \left(\int_{v_1}^{v_o'} T dS \right)_{\text{chord to } S_1} - \left(\frac{p_1 + p_o}{2} \right) \left(v_o' - v_o \right) \tag{2.11}$$

(The final term on the right is the area of the triangle $(p_o, v_o) \rightarrow (p_1, v_1) \rightarrow (p_o, v_o')$ plus the area under the straight line $(p_o, v_o) \rightarrow (p_o, v_o')$. But integrating (2.9) along the Hugoniot chord gives

$$H(p_1, v_1) - H(p_o, v_o) = 0 = \left(\int_{v_o}^{v_1} T dS \right)_{\text{chord to H}} \tag{2.12}$$

[redacted]

[redacted]

since dA is zero along this straight line. Substituting (2.10) and (2.12) into (2.11) we obtain

$$E(p_o, v_o') - E(p_o, v_o) = A_1 - \left(\frac{p_1 + p_o}{2}\right)(v_o' - v_o) \tag{2.13}$$

(Equation (2.12) shows, incidentally, that dS cannot be positive along the entire Hugoniot chord, so that the chord must intersect the adiabat $S = S_1$, as shown in Fig. 2.1. Moreover, since $E(p_o, v_o') - E(p_o, v_o)$ is positive, (2.13) shows that the shaded area between the chord and $S = S_1$ in Fig. 2.1 is greater than the unshaded area between the chord and adiabat.)

Letting $\overline{(\partial E/\partial v)}_{p_o}$ be the average value of $(\partial E/\partial v)_p$ at pressure p_o over the range v_o to v_o', then by definition

$$E(p_o, v_o') - E(p_o, v_o) = \overline{\left(\frac{\partial E}{\partial v}\right)}_{p_o}(v_o' - v_o), \tag{2.14}$$

which with (2.13) and (2.8) gives

$$v_o' - v_o = \frac{A_1}{\overline{\left(\frac{\partial E}{\partial v}\right)}_{p_o} + \frac{p_1 + p_o}{2}} = \frac{A_1}{\overline{T\left(\frac{\partial S}{\partial v}\right)}_{p_o} + \frac{p_1 - p_o}{2}} \tag{2.15}$$

(Since $v_o' - v_o$ is usually small, the mean value $\overline{(\partial E/\partial v)}_{p_o}$ is not greatly different from the value of the derivative at v_o.) This expression shows that for a given value of $\overline{(\partial E/\partial v)}_{p_o}$ and a given shock strength $p_1 - p_o$, the volume increase $v_o' - v_o$ is directly proportional to the area A_1; this area, in turn, will be shown to be essentially proportional to the (average) curvature of the adiabat.

It will be convenient for later use to introduce the triangular area

$$A_o = \frac{1}{2}(p_1 - p_o)(v_o' - v_1) \tag{2.16}$$

of which A_1 forms a (usually small) portion, and to write (2.15) in terms of the ratio A_1/A_o:

$$v_o' - v_o = \frac{(p_1 - p_o)(v_o' - v_o + v_o - v_1)\frac{A_1}{A_o}}{2\overline{\left(\frac{\partial E}{\partial v}\right)}_{p_o} + p_1 + p_o}$$

$$= \frac{(p_1 - p_o)(v_o - v_1)\frac{A_1}{A_o}}{2\overline{\left(\frac{\partial E}{\partial v}\right)}_{p_o} + p_1 + p_o - (p_1 - p_o)\frac{A_1}{A_o}} \tag{2.17}$$

Thus for the volume factor in (2.7) we may write

$$\frac{U_p'}{U_p} = \left\{1 + \frac{v_o' - v_o}{v_o - v_1}\right\}^{1/2} = \left\{1 + \frac{(p_1 - p_o)A_1/A_o}{2\left(\frac{\partial E}{\partial v}\right)_{p_o} + p_1 + p_o - (p_1 - p_o)A_1/A_o}\right\}^{1/2} \tag{2.18}$$

Reduction of this expression to more convenient forms will be made later. First, an expression for U_r/U_p' in terms of A_1/A_o will be derived.

To calculate U_r/U_p', an analytic expression must be assumed for the form of the adiabat $S = S_1$. For small deviations from a straight line, we may use a simple quadratic expression, i.e., the two-term Taylor expansion

$$p_{S_1} - p_o = p'(v - v_o') + \frac{1}{2}p''(v - v_o')^2$$

where

$$\left.\begin{aligned} p' &= \left(\frac{\partial p}{\partial v}\right)_S < 0 \\ \text{and} \quad p'' &= \left(\frac{\partial^2 p}{\partial v^2}\right)_S > 0, \end{aligned}\right\} \tag{2.19}$$

and the partial derivatives are to be evaluated at (p_o, v_o') or, approximately, at (p_o, v_o). The pressure on the chord $(p_o, v_o') \rightarrow (p_1, v_1)$ may then be written

$$p_C - p_o = \left\{p' + \frac{1}{2}p''(v_1 - v_o')\right\}(v - v_o')$$

The area between chord and adiabat is

$$A_1 = \int_{v_1}^{v_o'} (p_C - p_{S_1})\,dv = \frac{1}{2}p'' \int_{v_1}^{v_o'} \left\{(v_1 - v_o')(v - v_o') - (v - v_o')^2\right\} dv$$

$$= \frac{p''(v_o' - v_1)^3}{12} \tag{2.20}$$

so that

$$\frac{A_1}{A_o} = \frac{p''(v_o' - v_1)^2}{6(p_1 - p_o)} = \frac{p''(v_o' - v_1)}{-6\,p' + 3\,p''(v_o' - v_1)} \tag{2.21}$$

Finally,

$$\frac{U_r}{U_p'} = \frac{\int_{v_1}^{v_o'} \left(-\frac{\partial p_{S_1}}{\partial v}\right)^{1/2} dv}{\int_{v_1}^{v_o'} \left(-\frac{\partial p_C}{\partial v}\right)^{1/2} dv}$$

$$= \frac{\left\{-p' + p''(v_o' - v_1)\right\}^{3/2} - \left\{-p'\right\}^{3/2}}{\frac{3}{2}p''\left\{-p' + \frac{1}{2}p''(v_o' - v_1)\right\}^{1/2}(v_o' - v_1)}$$

$$= \frac{A_o}{9A_1}\left\{\left(1 + 3\frac{A_1}{A_o}\right)^{3/2} - \left(1 - 3\frac{A_1}{A_o}\right)^{3/2}\right\}$$

$$= 1 - \frac{3}{8}\left(\frac{A_1}{A_o}\right)^2 - \frac{81}{128}\left(\frac{A_1}{A_o}\right)^4 - \cdots \qquad (2.22)$$

where the binomial theorem has been used in the final step. For calculational purposes, (2.22) may be more conveniently written in terms of a new quantity

$$x = 1 - 2\frac{p_{1/2} - p_o}{p_1 - p_o} = \frac{(p_1 - p_{1/2}) - (p_{1/2} - p_o)}{p_1 - p_o} \qquad (2.23)$$

where $p_{1/2}$ is the pressure on the adiabat $S = S_1$ at a volume $(v_1 + v_o')/2$. (This quantity is a measure of the curvature of the adiabat, being zero for a straight line since then $p_1 - p_{1/2} = p_{1/2} - p_o$.) For the quadratic curve under discussion,

$$x = 1 - \frac{p'(v_1 - v_o') + p''(v_1 - v_o')^2/4}{p'(v_1 - v_o') + p''(v_1 - v_o')^2/2}$$

$$= \frac{p''(v_1 - v_o')/4}{p' + p''(v_1 - v_o')/2}$$

$$= \frac{3}{2}\left(\frac{A_1}{A_o}\right) \qquad (2.24)$$

Thus in terms of this variable, (2.22) may be written

$$\frac{U_r}{U_{p'}} = 1 - \frac{x^2}{6} - \frac{x^4}{8} - \ldots \tag{2.25}$$

A more accurate expression has been calculated by assuming that the pressure-volume relation along the adiabat can be represented by the following γ-law expression:

$$\frac{p - p^o}{p_o - p^o} = \left(\frac{v}{v_{o'}}\right)^{-\gamma} \tag{2.26}$$

where p^o is a suitable constant (zero for a perfect gas, generally negative for materials in a condensed phase). The algebra is quite tedious because of the numerous series expansions required and will not be repeated here. The results (obtained by following the same procedure as that outlined above) are

$$\frac{A_1}{A_o} = \frac{2x}{3} + \frac{2x^3}{45}\left\{1 + \frac{3\gamma + 5}{(\gamma + 1)^2}\right\} + \ldots\ldots \tag{2.27}$$

$$\frac{U_r}{U_{p'}} = 1 - \frac{3}{8}\left(\frac{A_1}{A_o}\right)^2 - \frac{117}{640}\left(\frac{A_1}{A_o}\right)^4\left\{1 - \frac{16\gamma}{13(\gamma + 1)^2}\right\} - \ldots$$

$$= 1 - \frac{x^2}{6} - \frac{7x^4}{120}\left\{1 + \frac{8(\gamma + 5)}{21(\gamma + 1)^2}\right\} - \ldots \tag{2.28}$$

The values of the factors in braces are not very sensitive to the value of γ assumed, and the commonly used value $\gamma = 3$ is sufficiently accurate.* This gives approximately

$$\frac{A_1}{A_o} = \frac{2x}{3} + \frac{x^3}{12} \tag{2.29}$$

$$\frac{U_r}{U_{p'}} = 1 - \frac{x^2}{6} - \frac{x^4}{14} \tag{2.30}$$

*For γ = 1, 2, 3, 4, and ∞, the values of the coefficient of x^4 are about 1/11, 1/13, 1/14, 1/15, and 1/17, respectively. The value of γ which best fitted one of the aluminum adiabats (calculated as described in the following chapter) was 3.0 ± 0.2.

and, from (2.18),

$$\frac{U_r}{U_p} = \left\{ 1 + \frac{x + \frac{x^3}{8}}{\frac{3}{p_1 - p_o}\left\{\left(\frac{\partial E}{\partial v}\right)_{p_o} + p_o\right\} + \frac{3}{2} - \left(x + \frac{x^3}{8}\right)} \right\}^{1/2} \left\{ 1 - \frac{x^2}{6} - \frac{x^4}{14} \right\} \tag{2.31}$$

Applications of this expression will be discussed in Chapter 4. For the present, it is interesting to examine the form of U_r/U_p for relatively weak shocks. This may be done by expanding the square root in (2.31), multiplying the two factors together, and neglecting powers of x above the second; however, it is simpler to use the expressions (2.15) and (2.22) which involve A_1 rather than x. These give

$$\frac{U_r}{U_p} \cong 1 + \frac{A_1/2\,(v_o - v_1)}{\left(\frac{\partial E}{\partial v}\right)_{p_o} + \frac{p_1 + p_o}{2}} - \frac{3}{8}\left(\frac{A_1}{A_o}\right)^2$$

It is evident from (2.17) that $(v_o' - v_o)/(v_o - v_1)$ tends to zero for weak shocks, so that then $v_o' - v_1 \cong v_o - v_1$. Using this approximation in (2.20) and (2.21), we obtain

$$\frac{U_r}{U_p} \cong 1 + \frac{p''(v_o - v_1)^2/24}{\left(\frac{\partial E}{\partial v}\right)_{p_o} + \frac{p_1 + p_o}{2}} - \frac{1}{24}\left\{\frac{p''(v_o - v_1)}{-2p' + p''(v_o - v_1)}\right\}^2$$

$$\cong 1 + \frac{p''}{24}\left\{\frac{1}{\left(\frac{\partial E}{\partial v}\right)_{p_o} + p_o} - \frac{p''}{4(p')^2}\right\}(v_o - v_1)^2 \tag{2.32}$$

Using the thermodynamic relations

$$\left(\frac{\partial E}{\partial v}\right)_p + p = T\left(\frac{\partial S}{\partial v}\right)_p = -T\left(\frac{\partial S}{\partial p}\right)_v p'$$

and $$\left(\frac{\partial p}{\partial S}\right)_v = -\left(\frac{\partial T}{\partial v}\right)_S \equiv -T' > 0$$

(the second of these following from (2.8) and the reciprocity relation for a perfect differential), (2.32) may also be written in the more symmetrical form

[redacted]

$$\frac{U_r}{U_p} \cong 1 + \frac{p''}{24p'}\left\{\frac{T'}{T} - \frac{p''}{4p'}\right\}(v_o - v_1)^2 \qquad (2.33)$$

[It has been pointed out to us by W. W. Wood that to terms correct to the second order in $(v_o - v_1)$, the expression

$$\frac{U_r}{U_p} = 1 + \frac{1}{p'}\left\{-\frac{T'}{12}\left(\frac{\partial^3 S}{\partial v^3}\right)_H - \frac{(p'')^2}{96p'}\right\}(v_o - v_1)^2 \qquad (2.34)$$

may be derived directly from a Taylor-series expansion for the equation of state $p(v,S)$. This is equivalent to (2.33) since the relation

$$T\left(\frac{\partial^3 S}{\partial v^3}\right)_H = -\frac{1}{2}\left(\frac{\partial^2 p}{\partial v^2}\right)_H = -\frac{1}{2}\left(\frac{\partial^2 p}{\partial v^2}\right)_S$$

may be readily obtained by successive differentiation of (2.9) at (p_o, v_o). The expression (2.31), derived by a non-series method, is much more accurate than (2.34) since it in effect includes various higher-order terms.]

It is evident from (2.32) and (2.33) that for weak shocks the departure from the approximation $U_{fs}/U_p = 2$ (cf. equation 2.3) is proportional to the square* of the shock strength $(v_o - v_1) = (p_1 - p_o)/|p'|$. Moreover, (remembering that p' and T' are negative),

$$U_{fs}/U_p \gtrless 2$$

according as

$$T\left(\frac{\partial S}{\partial v}\right)_{p_o} = \left(\frac{\partial E}{\partial v}\right)_{p_o} + p_o \lessgtr \frac{4(p')^2}{p''} \qquad (2.35)$$

or according as

$$\left|\frac{T'}{T}\right| \gtrless \left|\frac{p''}{4p'}\right|$$

Which of these alternatives holds depends on the equation of state of the material concerned. As examples, the following cases may be considered:

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*Since U_p is of the first order in shock strength, it follows that the difference $U_r - U_p$ is of the third order in $(v_o - v_1)$. This conclusion is obvious insofar as the volume effect is concerned, since $(v_o' - v_o)$ is proportional to the entropy increase across the shock front, and ΔS is of the third order in $(v_o - v_1)$.

[redacted]

(a) For a perfect gas, described by $pv = nRT$ and the expression (2.26), it is easily seen that $p''/p' = -(\gamma + 1)/v$ and $T'/T = -(\gamma - 1)/v$, so that $U_{fs}/U_p \gtrless 2$ according as $\gamma \gtrless 5/3$. Thus for a perfect monotonic gas ($\gamma = 5/3$), U_{fs}/U_p equals 2 to terms of the order $(v_o - v_1)^2$; for all other perfect gases ($\gamma < 5/3$), U_{fs}/U_p is less than 2 (to the same approximation).

(b) For some substances, such as water, the pressure depends primarily on v and very little on S;[7] i.e., $(\partial v/\partial S)_p \cong 0$. In the limiting case, $(\partial S/\partial v)_p = \infty$ so that the volume effect is nil and $U_{fs}/U_p < 2$. (Values of U_r and U_p have been calculated for shock pressures up to 80 kilobars in water, with $U_r/U_p \cong 0.96$ for the stronger shocks.[8])

(c) For the metals considered below (using the LA-385 equation of state[9]), it seems to be generally true that U_{fs}/U_p is greater than 2.

Chapter 3

CALCULATIONS WITH THE LA-385 EQUATION OF STATE

Detailed calculations of U_r/U_p were carried out for two metals, aluminum and tuballoy, using equations (2.4) and (2.5) and the equation of state given by Keller and Metropolis[9] (LA-385). The equation of state was slightly modified (see Appendix) and none of the Hugoniots or adiabats calculated by Keller and Metropolis was actually used in the present calculations.

A Hugoniot and a series of adiabats were calculated for each metal. Values of $\left(\frac{\partial p}{\partial v}\right)_S$ for evaluation of the Riemann integral, equation (2.5), were obtained from the thermodynamic relation

$$\left(\frac{\partial p}{\partial v}\right)_S = \left(\frac{\partial p}{\partial v}\right)_T - \frac{T}{C_v}\left[\left(\frac{\partial p}{\partial T}\right)_v\right]^2 \qquad (3.1)$$

All of the quantities on the right can be evaluated directly from the analytic form of the equation of state.[9] Values of $\left(\frac{\partial p}{\partial v}\right)_S$ were determined at five equally spaced points on the adiabat and the integral was then evaluated by integration of the corresponding Lagrange polynomial.

Details concerning the use of the analytic form of the equation of state and the minor modifications that were made will be found in the Appendix.

The results of the calculation are given in Figs. 3.1 to 3.4, and in Tables 3.1 and 3.2. These results are for Hugoniot curves starting at $T_o = 300^oK$, $p_o = 1$ atm, $v_o =$ normal volume.* In Tables 3.1 and 3.2, the first five columns after the independent variable refer to the shock only. A comparison of the last three columns will give an idea of the relative magnitude of contributions of the volume and curvature effects to U_r/U_p: From the columns headed $\frac{v_o'}{v_o}$ (v_o' is the adiabatic volume for which $p = p_o$) and the column headed $\frac{U_p'}{U_p} = \frac{1}{U_p}\sqrt{(p_1 - p_o)(v_o' - v_1)}$ the relative contributions of the "volume" and "curvature" effects can be estimated. Figures 3.1 through 3.3 correspond to Tables 3.1 and 3.2. In view of interest in the possibility of phase changes, Fig. 3.4 has been included; it shows the final temperature after expansion as a function of v_1.

*The values used for $\rho_o = 1/v_o$ were 2.699 and 19.00 g/cc for aluminum and tuballoy, respectively.

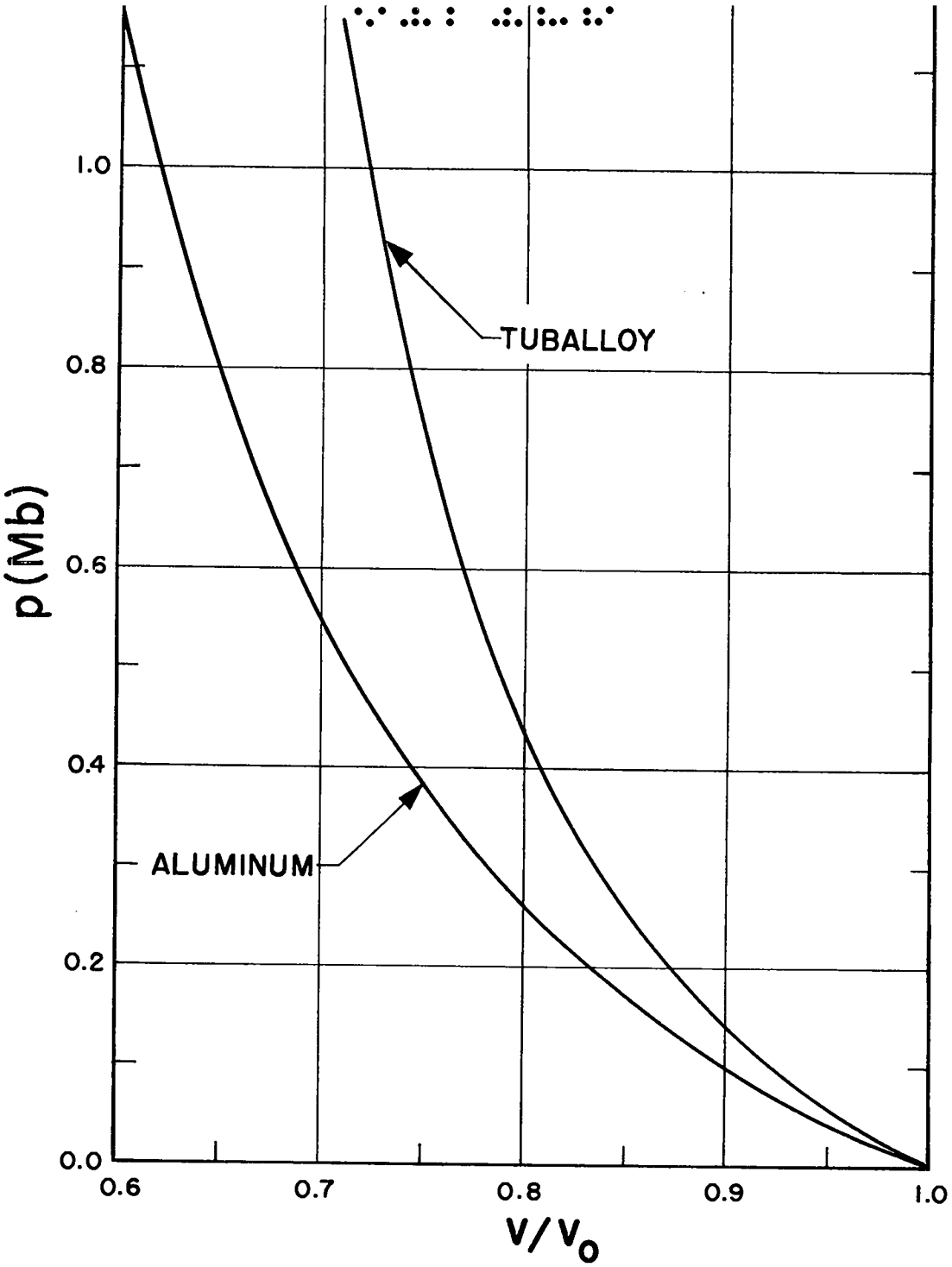

Fig. 3.1 Shock Hugoniots for aluminum and tuballoy (T_o = 300°K, p_o = 1 atm, v_o = 1).

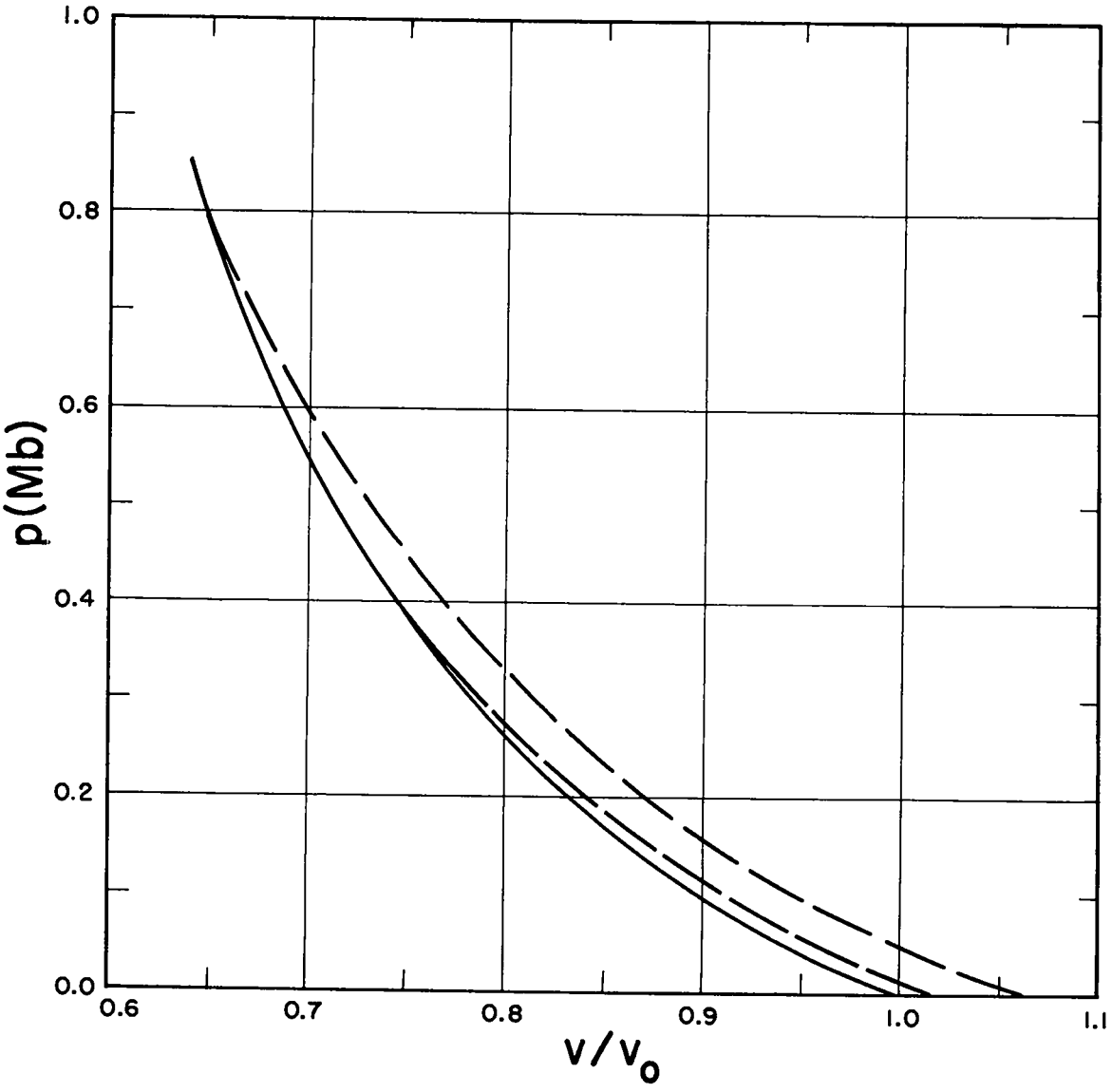

Fig. 3.2 Hugoniot and two adiabats for aluminum.

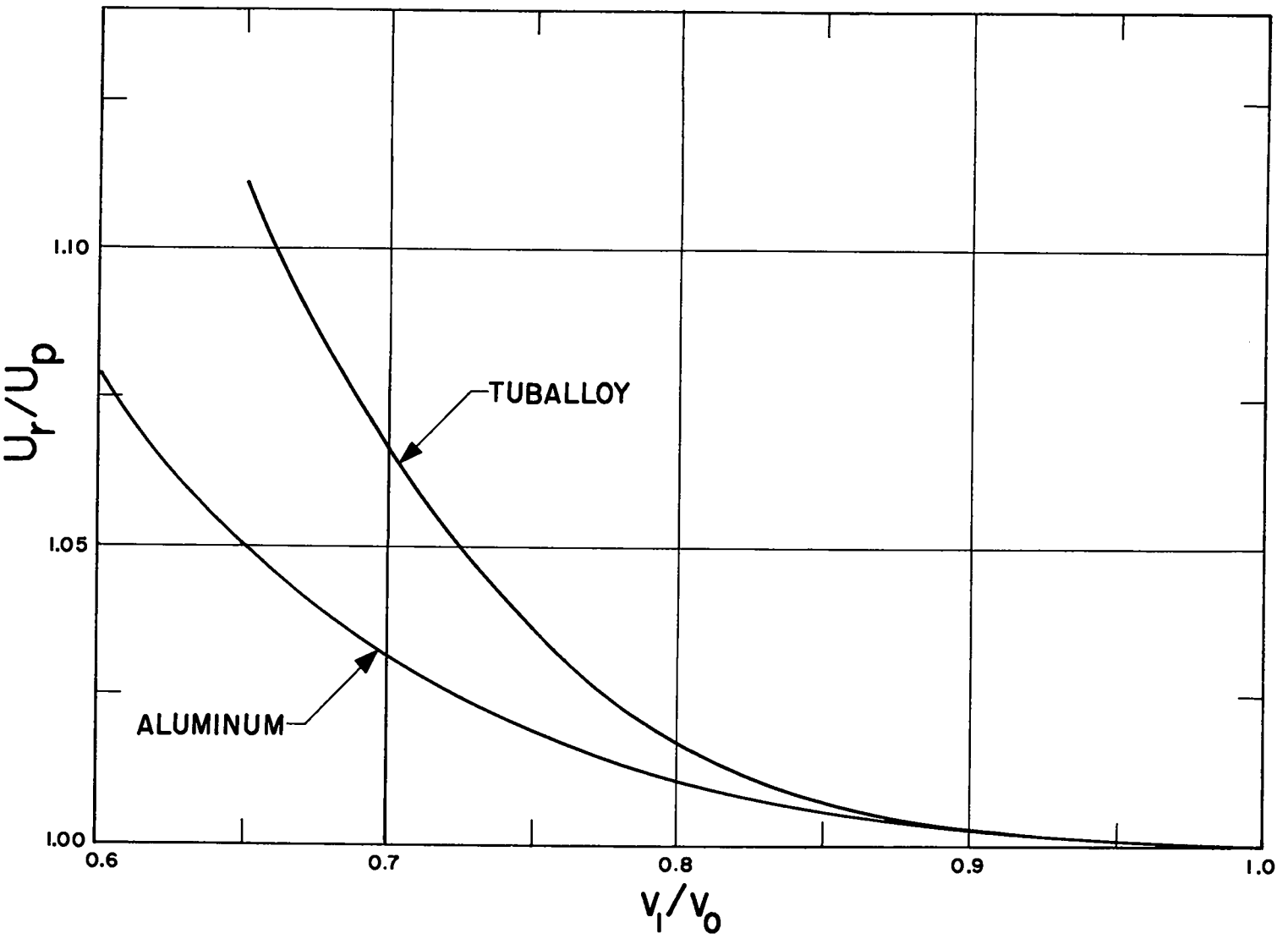

Fig. 3.3 U_r/U_p for aluminum and tuballoy.

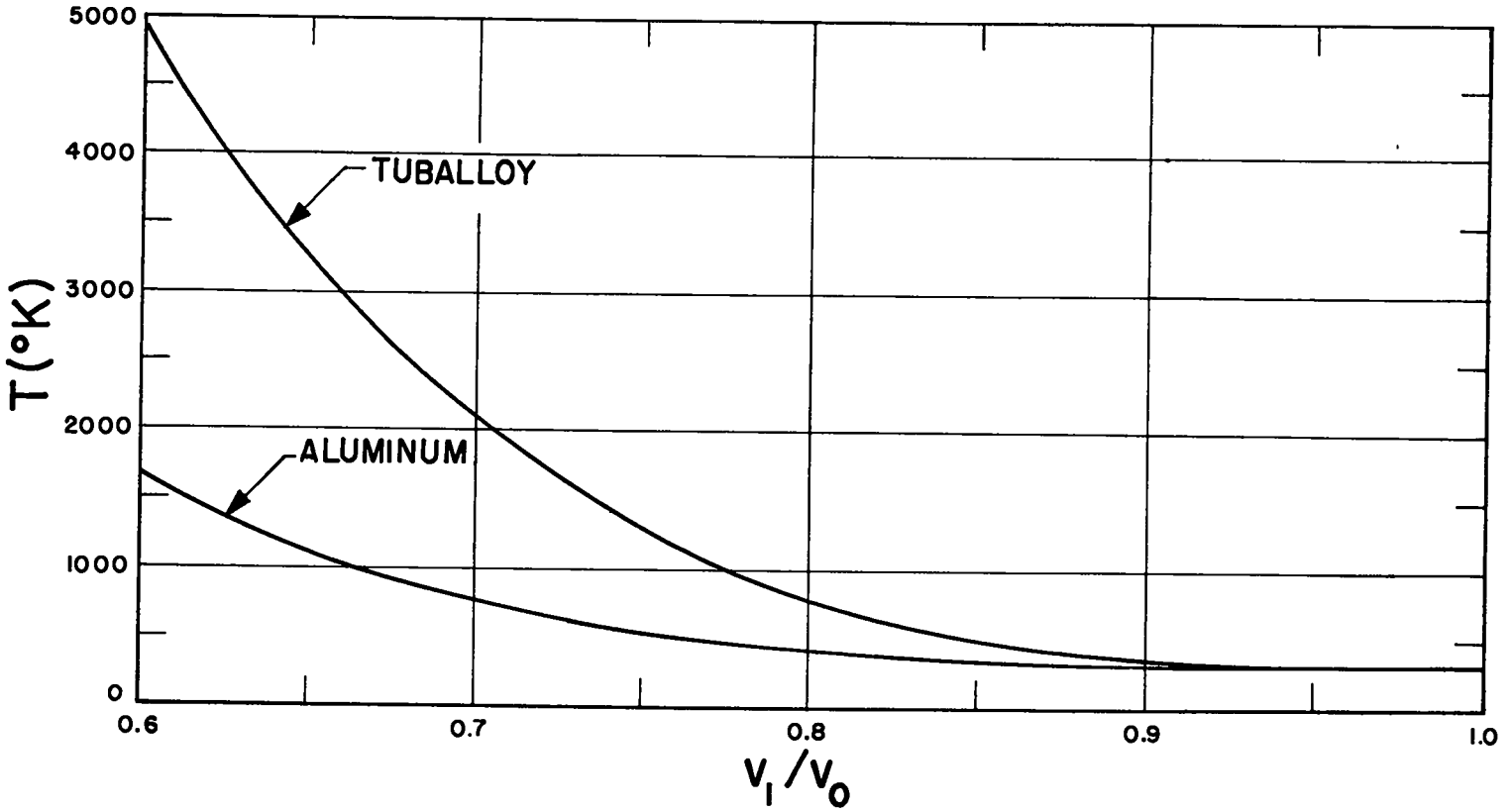

Fig. 3.4 Final temperature after expansion.

TABLE 3.1

SHOCK HUGONIOT AND VALUES OF $\frac{U_r}{U_p}$ FOR ALUMINUM

$\frac{v}{v_o}$	T ev	P Mb	U_p cm/μsec	D cm/μsec	$\rho_o \Delta S$ Mb/ev	$\frac{v_o'}{v_o} - 1$	$\frac{U_p'}{U_p} - 1$	$\frac{U_r}{U_p} - 1$
0.99	.026356	.007867	.005399	.539859	.000013	.000000	.000040	.000011
0.98	.026883	.016144	.010937	.54686	.000103	.000006	.000167	.000060
0.97	.027438	.024850	.016619	.55398	.000349	.000022	.000381	.000148
0.96	.028028	.034003	.022448	.56120	.000833	.000054	.000684	.000271
0.95	.028664	.043622	.028427	.56854	.001636	.000107	.001072	.000440
0.90	.032957	.099514	.060721	.60721	.013342	.000892	.004454	.002082
0.85	.040608	.17101	.097488	.64992	.044814	.003167	.010500	.005347
0.80	.054780	.26323	.13966	.69831	.10250	.008029	.019874	.010705
0.75	.081123	.38482	.18880	.75519	.18761	.017207	.033841	.018849
0.70	.13033	.55141	.24757	.82523	.29759	.033818	.054858	.031018
0.65	.22330	.79241	.32056	.91588	.42913	.064085	.087704	.049495
0.60	.40060	1.1640	.41534	1.0383	.58008	.122057	.142428	.078704
0.55	.73721	1.7738	.54382	1.2085	.74995	.252964	.249856	.130284

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TABLE 3.2

SHOCK HUGONIOT AND VALUES OF $\frac{U_r}{U_p}$ FOR TUBALLOY

$\frac{v}{v_o}$	T ev	P Mb	U_p cm/μsec	D cm/μsec	$\rho_o \Delta S$ Mb/ev	$\frac{v_o'}{v_o} - 1$	$\frac{U_p'}{U_p} - 1$	$\frac{U_r}{U_p} - 1$
.95	.028980	.057211	.012270	.24540	.0025199	.000136	.001354	.000473
.90	.035213	.13663	.026816	.26816	.023301	.001310	.006527	.002402
.8	.091087	.43835	.067928	.33964	.19813	.016455	.040323	.016921
.7	.38564	1.2492	.14044	.46814	.51533	.091877	.142916	.066167
.65	.76009	2.0980	.19659	.56169	.69118	.187486	.239223	.111067

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[redacted]

Modification of the Equation of State Coefficients to Improve Agreement with Experiment

The Hugoniot curve for 24ST Dural has recently been determined experimentally over the range p = 0.15 to 0.3 Mb from measurements of shock and free-surface velocity in metal plates driven by plane detonations in various explosives.[1] The pressures on this experimental Hugoniot (obtained by assuming $U_{fs} = 2U_p$) are about 20% higher than those calculated for the same compression from the LA-385 aluminum equation of state. An attempt was therefore made to alter the LA-385 aluminum equation of state in a simple way so as to have an approximately correct equation of state for 24ST Dural. Values of U_r/U_p calculated from this altered equation of state would be more appropriate for use in calculating the pressure and compression from equations (1.1) and (1.2).

The process of altering the equation of state follows. Since in the region of interest the isothermal and Hugoniot are not very different, a change in $p_o(v)$ will produce nearly the same change in the calculated Hugoniot pressure. Hence the difference, which we call Δp, between the calculated and experimental Hugoniots was first obtained (for the experimental curve, a curve was drawn by eye on a plot of the experimental points), and the new $p_o(v)$ was then taken to be $\left(p_o(v) + \Delta p(v)\right)$. To do this conveniently $\Delta p(v)$ was fitted at two points by a polynomial in μ:

$$\Delta p = \frac{\beta_1 \mu^2 + \delta_1 \mu^4}{1 + 1.5\mu^2}, \quad \mu = \frac{v_o}{v} - 1 \qquad (3.2)$$

This particular form was chosen because the original analytic expression for p_o had the form:

$$p_o(v) = \frac{\alpha \mu + \beta \mu^2 + \delta \mu^4}{1 + 1.5 \mu^2}$$

so that $(p_o(v) + \Delta p)$ would have the same analytic form as $p_o(v)$ with the coefficients β and δ changed.

This change of $p_o(v)$ resulted in an equation of state which reproduced the experimental Hugoniot quite well. This is shown in Fig. 3.5, in which the originally calculated aluminum Hugoniot and the experimental Dural Hugoniot are shown, together with a few points calculated from the aluminum equation of state with $p_o(v)$ modified as described above.

Values of U_r/U_p calculated with this modified aluminum equation of state are given in Table 3.3.

Table 3.3 also serves to give a rough idea of the changes to be expected in the calculated values of U_r/U_p resulting from reasonable-sized changes in the equation of state. The first two columns illustrate the effect of a fairly large change in $p_o(v)$, while the third column is calculated with the altered $p_o(v)$ and also with $a(v)$ changed to the expression

[redacted]

$$a(v) = \frac{a_1}{(v/v_o)} + a_2 = \frac{0.1449}{(v/v_o)} + 0.4671 \text{ Mb/ev}$$

(This expression is of a form suggested by J. R. Reitz,[10] with $a_1 = 3R/2v_o$ and a_2 chosen so that $a(v_o) = 3\alpha/K$, where α is the linear coefficient of thermal expansion and K is the isothermal compressibility.)

TABLE 3.3

U_r/U_p FROM DIFFERENT EQUATIONS OF STATE FOR ALUMINUM

	LA-385 Equation of State	(1) with $p_o(v)$ increased by $\Delta p(v)$	(2) with $a(v)$ changed to fit experimental data
v_1/v_o	(1)	(2)	(3)
0.85	1.00535	1.00623	
0.8	1.01070	1.01373	
0.75	1.01885	1.02695	1.02997

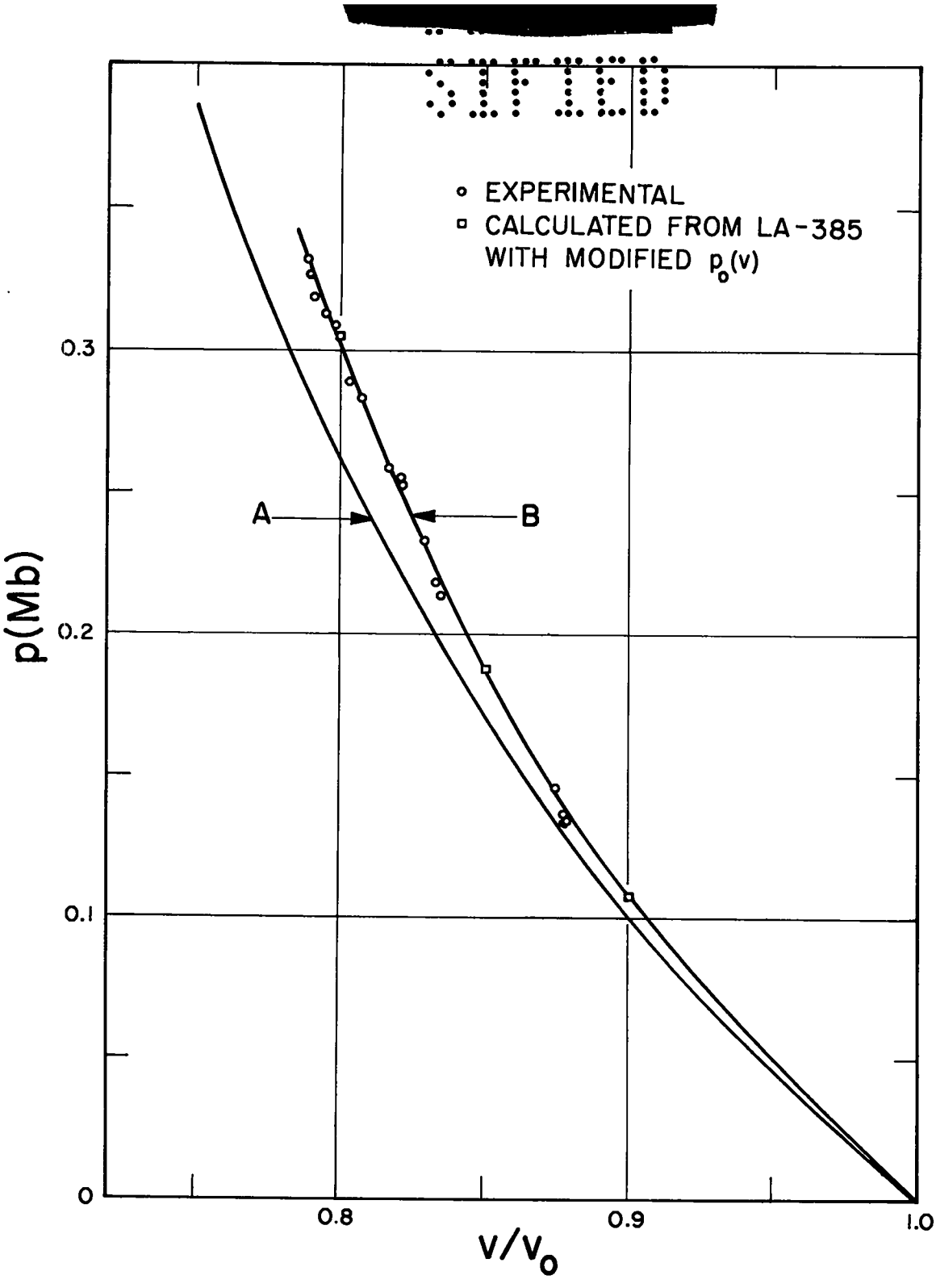

Fig. 3.5 Comparison of aluminum Hugoniots. Curve A, calculated from LA-385 equation of state for aluminum (T_O = 300°K, p_O = 1 atm, v_O = 1); B, curve drawn by eye through experimental points for 24ST Dural.

Chapter 4

USE OF THE SIMPLIFIED METHOD OF CALCULATION

The calculations described in the preceding section have shown how values of $U_{fs}/U_p = 1 + (U_r/U_p)$ can be obtained from equations of state given in LA-385. Such a method of determining U_r/U_p suffers from several drawbacks: (1) In the case of aluminum (Dural), at least, the LA-385 equation of state requires appreciable modification to give reasonable agreement with the experimental Hugoniot. (2) Calculation by hand is impractical, and the calculation of U_r/U_p for one compression requires 20 to 25 minutes even when performed on IBM-C. P. C. equipment. (3) LA-385 gives equations of state for only four elements, and derivation of corresponding equations for additional elements of interest is itself a lengthy process.

The ratio U_r/U_p can be found much more simply from equation (2.31) above, provided certain approximations are made. From the relation

$$\left\langle \left(\frac{\partial E}{\partial v}\right)_{p_o} + p_o \right\rangle_{av} = \left\langle \left(\partial(E + pv)/\partial T\right)_{p_o} \div (\partial v/\partial T)_{p_o} \right\rangle_{av}$$

$$= \left\langle C_p/v_o 3\alpha \right\rangle_{av} \cong \rho_o \overline{C}_p/3\overline{\alpha} \qquad (4.1)$$

where C_p is the specific heat and 3α is the volume coefficient of thermal expansion of the metal (and hence α is approximately the linear expansion coefficient) at constant pressure p_o, (2.31) may be written

$$\frac{U_r}{U_p} = \left\{ 1 + \frac{x + \frac{x^3}{8}}{\frac{\rho_o C_p}{(p_1 - p_o)\overline{\alpha}} + \frac{3}{2} - (x + \frac{x^3}{8})} \right\}^{1/2} \left\{ 1 - \frac{x^2}{6} - \frac{x^4}{14} \right\} \qquad (4.2)$$

Values of the normal density ρ_o, specific heat C_p, and expansion coefficient can easily be found in standard reference works for almost any metal of interest. Values of C_p and α at room temperature T_o can be used, but Fig. 3.4 indicates that average values over a temperature range of 20-200°C or 20-300°C should give greater accuracy for moderate shock strengths (the difference between U_r/U_p and unity being almost negligible for weak shocks anyway). The quantity

$$x = \frac{p_1 - 2\,p_{1/2} + p_o}{p_1 - p_o} \cong \frac{p_1 - 2\,p_{1/2}}{p_1} \qquad (4.3)$$

(where $p_{1/2}$ is the pressure at a volume midway between the shock volume v_1 and the volume at pressure p_o) should strictly speaking be evaluated from pressures along the appropriate adiabat; however, the shape of the adiabat through (p_1, v_1) is approximately the same as that of the adiabat through the initial point (p_o, v_o), and the latter is in turn approximated by the Hugoniot curve. Thus, if experimental data in the form of U_{fs} as a function of D are available, an approximate Hugoniot curve can be calculated from equations (1.1) and (1.2) and the approximation (1.3). Then with values of x found from this Hugoniot and equation (4.3)--where $p_{1/2}$ is the Hugoniot pressure at a volume $(v_1 + v_o)/2$--equation (4.2) can be used in place of (1.3) for calculating a more accurate Hugoniot. (The corrected Hugoniot would of course give values of x different from those given by the approximate Hugoniot so that the above process should logically be iterated; however, the corrections are so small in practice that a single cycle is usually sufficient.)

Errors in the Approximate Calculation

An estimate of the errors involved in the various approximations has been made by calculating values of U_r/U_p from (4.2) and comparing with the values obtained from the calculations described in the preceding chapter.

(A) To check equation (2.31) insofar as the series expansions in x are concerned, we first calculated x from the appropriate adiabat and also used the appropriate value of $\overline{(\partial E/\partial v)}_{p_o}$ as given by the LA-385 calculations themselves. The differences between the values of U_r/U_p calculated from equation (2.31) and those calculated directly from the LA-385 equation of state are shown in the middle column of Table 4.1, and are seen to be less than 5% for $v_1/v_o \geq 0.6$.

On the other hand, the approximate equation (2.33) used with derivatives evaluated from the LA-385 equation of state is accurate for very weak shocks, but for $v \cong 0.8$ gives a value of $(U_r/U_p - 1)$ which is only half the correct value. In this connection, it may be pointed out that Mayer's expression[3]

$$U_r/U_p = 1 + \frac{(3\gamma - 5)(\gamma + 1)}{96\,\gamma^2\,\pi}\,p_1^2 \qquad (4.4)$$

may be obtained from (2.33) by substituting the appropriate derivatives of his equation of state and using the relation

$$p_1 \cong p'\,(v_1 - v_o) \qquad (4.5)$$

It is thus to be expected that Mayer's expression is accurate only for very weak shocks.*

(B) The error in the value of x introduced by calculating from the Hugoniot instead of the appropriate adiabat was never greater than 7%. For weak shocks, the Hugoniot value was of course greater than the adiabat values, but, for the LA-385 equations of state at least, the adiabat curvature increased for higher adiabats sufficiently fast to more than compensate for the greater curvature of the Hugoniot over the adiabat S_o; thus for a 3.5 Mb shock in tuballoy, the Hugoniot x was actually 5% smaller than the corresponding adiabat value. In addition, an error in x has such an effect on the volume and curvature factors in (4.2) that the errors introduced into U_r/U_p partially subtract out (e.g., for the 1.2 Mb shock in aluminum, the Hugoniot x is 2.9% high, but makes $(U_r/U_p) - 1$ and U_{fs}/U_p only 1.0% and 0.04% high, respectively). The error resulting from using the Hugoniot to calculate x seems to be generally small compared to other errors.

(C) The total error made by using values of x calculated from the Hugoniot and by using the value of $(\partial E/\partial v)_p$ at (p_o, v_o) is shown in the final column of Table 4.1. The largest error in $(U_r/U_p) - 1$ is 10.5%, and the largest error in U_{fs}/U_p is 0.6% (the latter for a 3.5 Mb shock in tuballoy). It appears to be generally true that the errors involved in using (4.2) are smaller than present experimental error in measuring U_{fs}, and certainly also smaller than the uncertainties inherent in the assumptions that the rarefaction takes place reversibly and that the effects of any phase changes are negligible--not to mention the inherent uncertainties in the LA-385 equations of state and the neglect of the effects of rigidity.

Example: 24ST Dural

As a specific example, equation (4.2) has been applied to the experimental U_{fs} vs D data for 24ST Dural which were referred to earlier.[1]

Using the approximation (1.3), $U_p = U_{fs}/2$, equations (1.1) and (1.2) give approximate values $v^{(1)}$ and $p^{(1)}$ for the Hugoniot volume and pressure from the experimental values of D and U_{fs}:

$$\begin{cases} 1 - v^{(1)}/v_o = U_{fs}/2\ D \\ p^{(1)} = \rho_o\ D\ U_{fs}/2 \end{cases}$$

The values $(v^{(1)}, p^{(1)})$ were graphed to give an approximate Hugoniot curve, from which the

*The fact that Mayer's value $U_r/U_p - 1 = 0.046$ for $p_1 = 0.332$ Mb (in aluminum) is larger than the value we obtain from (2.31) rather than smaller is due primarily to the curvature of the adiabat, the value of p_1 indicated by (4.5) being only about half the actual shock pressure of 0.332 Mb.

[redacted]

[redacted]

TABLE 4.1

ERRORS IN $\frac{U_r}{U_p}$ - 1 FROM SERIES CALCULATION

	v_1/v_o	p_1(Mb)	% Error: Values of x from Adiabat, and $(\partial E/\partial v)_p$	% Error: Values of x from Hugoniot, and $(\partial E/\partial v)_p$ evaluated at origin
Tuballoy:	0.90	0.137	+ 0.29	- 3.58
	0.80	0.438	+ 0.18	- 3.72
	0.70	1.25	+ 1.45	- 2.36
	0.65	2.10	+ 3.51	- 3.15
	0.60	3.49	+ 4.87	- 7.71
Aluminum:	0.90	0.100	0.48	10.47
	0.80	0.263	0.75	0.28
	0.70	0.551	1.00	- 1.48
	0.65	0.792	1.68	- 3.86
	0.60	1.16	2.47	- 8.16

TABLE 4.2

VALUES OF U_{fs}/U_p CALCULATED FROM EXPERIMENTAL DATA FOR DURAL

v_1/v_o	p_1(Mb)	x	$U_{fs}/U_p = 1 + U_r/U_p$
0.90	0.1062	0.124	2.0028
0.86	0.1689	0.187	2.0062
0.82	0.253	0.256	2.0118
0.78	0.355	0.330	2.0194

[redacted]

(smoothed) values of x given in Table 4.2 were obtained with the aid of equation (4.3). Values of U_{fs}/U_p were then calculated from (4.2) using*

$$\rho_o = 2.785 \text{ g/cc}$$
$$\bar{\alpha} = 24.7 \cdot 10^{-6}(K^o)^{-1}$$
$$\bar{C}_p = 0.2276 \text{ cal/g-}K^o = 9.523 \cdot 10^{-6} \text{ Mb-cc/g-}K^o$$
$$\rho_o \bar{C}_p/\bar{\alpha} = 1.074 \text{ Mb}$$

These values of U_{fs}/U_p (given in Table 4.2) were then plotted against $v^{(1)}/v_o$ in Fig. 4.1, so that values of U_{fs}/U_p could be obtained for any value of $v^{(1)}$. Corrected values (v,p) of Hugoniot volume and pressure were then found from these values and the expressions

$$\begin{cases} \dfrac{1 - v^{(1)}/v_o}{1 - v/v_o} = \dfrac{U_{fs}}{2U_p} \\ \dfrac{p^{(1)}}{p} = \dfrac{U_{fs}}{2U_p} \end{cases}$$

The original and corrected Hugoniot data are given in Table 4.3. It may be noted that since the value of D is not changed in the correction process, the corrections to pressure and volume are so related that the point (p,v) lies on the straight line joining $(p^{(1)}, v^{(1)})$ and (p_o, v_o), and this line intersects the Hugoniot curve at a rather acute angle.

*The data of $\bar{C}_p$ is a weighted value for 95.5% Al and 4.5% Cu based on specific heat data for the range 0-300°C given in the Handbook of Chemistry and Physics (33rd ed., p. 1892); the value of $\bar{\alpha}$ is for 24ST Dural over the range 20-300°C as given in Alcoa Aluminum and its Alloys, Aluminum Company of America, (1947), p. 87.

[redacted]

TABLE 4.3

CORRECTION OF EXPERIMENTAL DURAL HUGONIOT

	with $\frac{U_{fs}}{U_p} = 2$		with $\frac{U_{fs}}{U_p}$ from Fig. 4.1		
	$v^{(1)}/v_o$	$p^{(1)}$	$\frac{U_{fs}}{U_p}$	v/v_o	p
GMX-4*	0.78864	0.332	2.0177	.79049	.3291
	0.79554	0.313	2.0163	.79718	.3105
	0.79808	0.309	2.0157	.79965	.3066
	0.82919	0.233	2.0103	.83007	.2318
	0.83542	0.214	2.0093	.83618	.2130
	0.87413	0.146	2.0049	.87444	.1456
GMX-6*	0.78970	0.3266	2.0174	.79152	.3238
	0.79133	0.3189	2.0171	.79310	.3162
	0.80354	0.2887	2.0147	.80497	.2866
	0.80795	0.2829	2.0138	.80927	.2810
	0.82041	0.2549	2.0117	.82145	.2534
	0.82095	0.2532	2.0116	.82198	.2517
	0.81686	0.2586	2.0123	.81798	.2570
	0.83375	0.2180	2.0096	.83454	.2170
	0.87765	0.1368	2.0046	.87793	.1365
	0.87966	0.1341	2.0044	.87993	.1338
	0.87812	0.1339	2.0045	.87839	.1336

* Unpublished data.

[redacted]

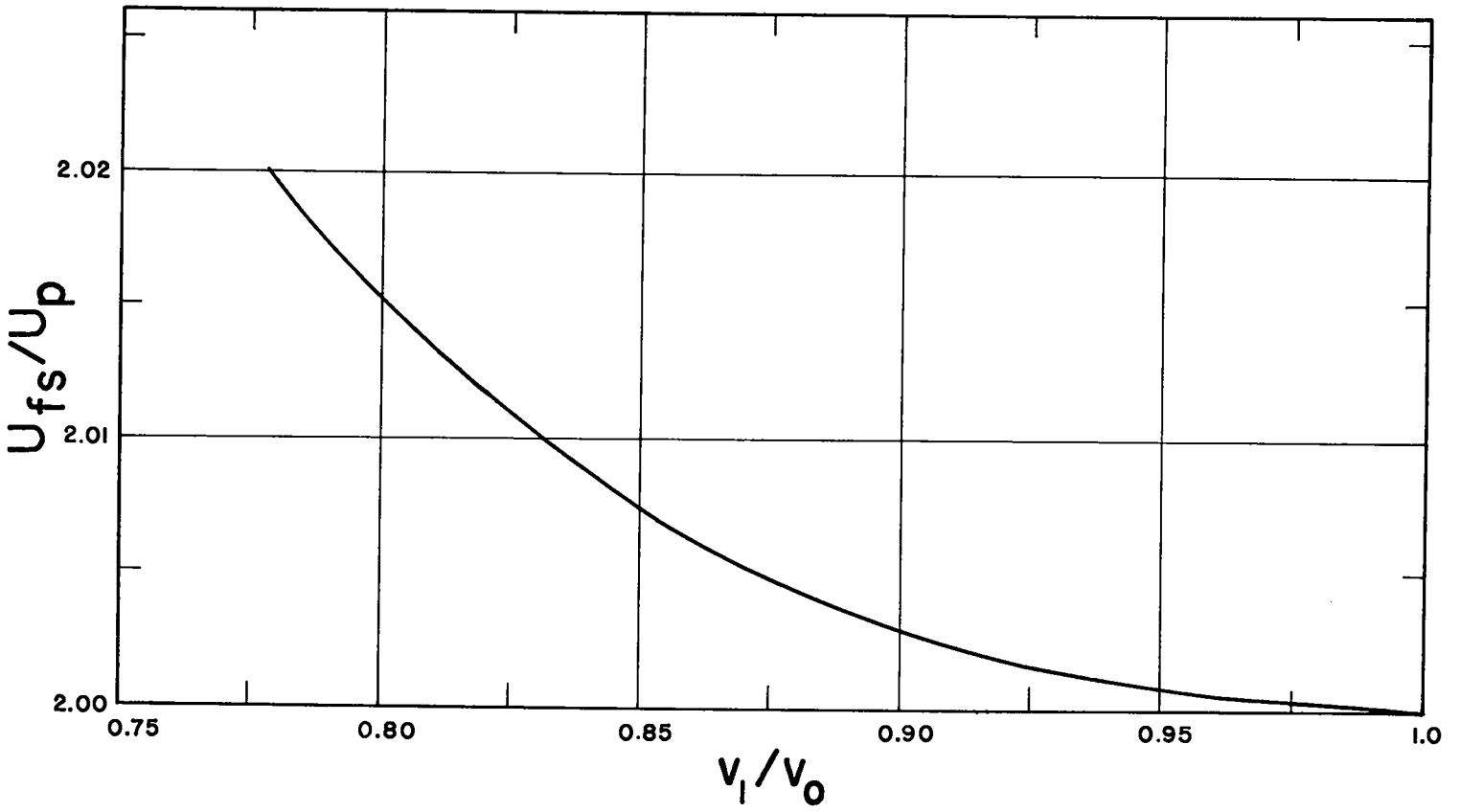

Fig. 4.1 U_{fs}/U_p from experimental Hugoniot for Dural

Chapter 5

EFFECT OF A PERMANENT COMPRESSION

The preceding chapter has shown how experimental data giving free-surface velocity as a function of shock velocity can be used (together with the normal density, specific heat, and coefficient of expansion) to calculate $(U_{fs}/U_p) - 1$ with an error probably less than 10% (and hence U_{fs}/U_p with an error of a few tenths of a percent) for shock compressions up to about 1-1/2 times normal density. However, an assumption of uncertain validity has been made throughout--namely, that the shocked material will upon cooling to the original temperature T_o also return to its original density ρ_o.

A case in which this assumption is expected to be strongly violated is that of sintered metals formed by pressing the powdered metal to a density much lower than that of the cast material. As an example, consider a sample of sintered aluminum (or Dural) having a density one-fourth that of the cast metal--such samples have been prepared and are being experimented with by GMX Division at the present time. It is not unreasonable to expect that such a substance would be compacted by a shock to such an extent that upon expanding to normal pressure and cooling to ambient temperature its density would be at least one-half that of the cast metal; the Hugoniot and adiabat might then appear somewhat as shown in Fig. 5.1, where v_o'' ($\ll v_o$) is the final specific volume after returning to (p_o, T_o). If in addition to U_p and U_p' (defined by equations 2.4 and 2.6, respectively), we define a quantity

$$U_p'' = \left\{(p_1 - p_o)\,(v_o'' - v_1)\right\}^{1/2} \qquad (5.1)$$

then

$$\frac{U_r}{U_p} = \frac{U_r}{U_p'} \cdot \frac{U_p'}{U_p''} \cdot \frac{U_p''}{U_p} \qquad (5.2)$$

Assuming that the curvature of the adiabat in Fig. 5.1 does not differ greatly from the curvature of a similar adiabat for the cast metal, then the value of U_r/U_p' is of the same order as the values calculated in Chapter 4--namely, about 0.99 or 0.98. The ratio U_p'/U_p'' corresponds to the quantity U_p'/U_p of the preceding chapters [cf. equation (2.18)]. Although $\overline{(\partial E/\partial v)}_p$ is doubtless much smaller for the sintered material than for the cast metal, U_p'/U_p'' is still probably no greater than, say, 1.10. The product $U_r/U_p' \cdot U_p'/U_p''$ thus does not differ greatly from the value of U_r/U_p for the cast metal. The remaining factor, however, may be written

$$\frac{U_p''}{U_p} = \left\{\frac{v_o'' - v_1}{v_o - v_1}\right\}^{1/2}$$

And if we assume that $v_o'' - v_1$ is even twice as great as the value of $v_o - v_1$ for the cast metal (i.e., about 0.6), then $U_p''/U_p \cong (0.6/2.6)^{1/2} = 0.48$. Evidently, the value of U_p''/U_p is of much greater importance in determining the value of U_r/U_p than are the values of the other two factors in (5.2).

Returning to the case of ordinary cast metal, it is evident that even a small permanent compression produced by the shock would be of importance. For example, a 1% compression resulting from a shock in which $v_1/v_o = 0.78$ would give

$$\frac{U_p''}{U_p} = \left(\frac{0.21}{0.22}\right)^{1/2} = 0.977$$

and this is sufficient to offset completely the value $U_r/U_p'' = 1.0194$ quoted in Table 4.2, and make $U_r/U_p < 1$. There seems to be no reliable experimental evidence as to whether such a permanent compression actually occurs for cast metals.

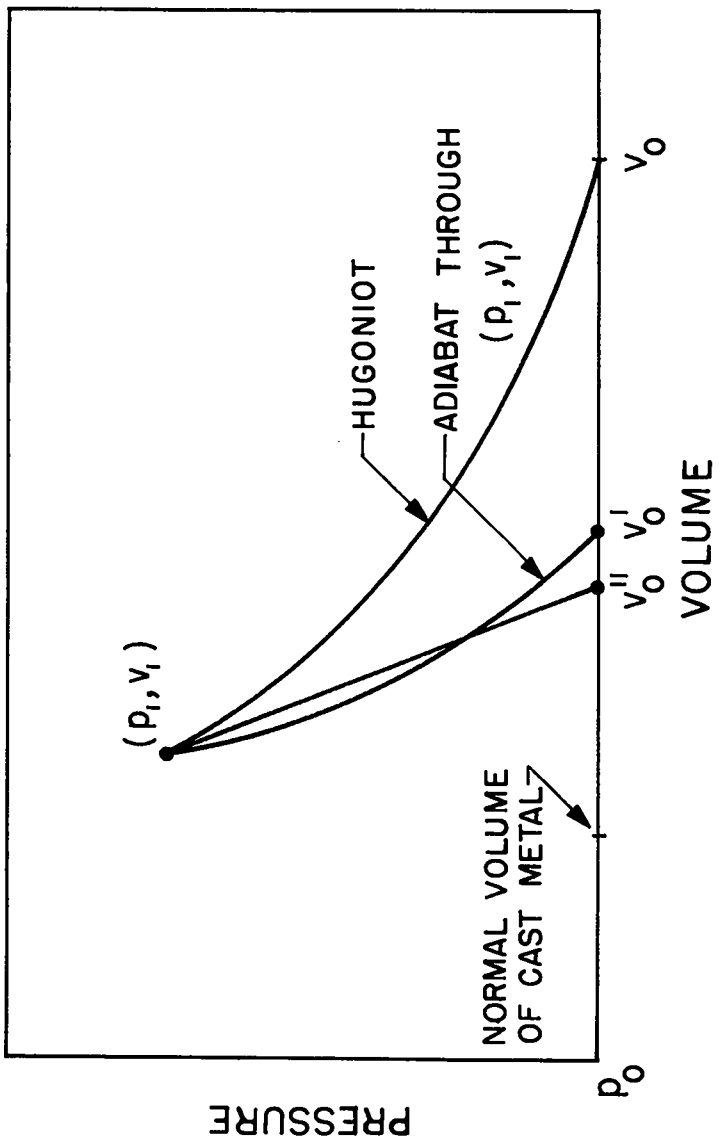

Fig. 5.1 Hugoniot and adiabat for a sintered metal (hypothetical).

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10. c.f. J. R. Reitz, LA-1454, Equation (8).

Appendix

DETAILS OF THE USE OF THE LA-385 EQUATION OF STATE

The equation of state as given in reference (9) has an annoying idiosyncrasy. At normal volume and room temperature the pressure is of the order of 10,000 atm. To have our Hugoniot curves start out at $T_o = 300^oK$, $p_o = 1$ atm, and $v_o = 1$, $p_o(v)$ was altered throughout by subtracting from it an amount

$$\delta = a(v_o)\, T_o + b(v_o)\, T_o^2$$

where

$$T_o = 300^oK$$

$$v_o = 1$$

so that the calculated pressure at $T_o = 300^oK$ and $v_o = 1$ was 1 atm.*

Both Hugoniots and adiabats were calculated by Keller and Metropolis,[9] but we did not make use of any of their numerical results. Their analytic fits to the adiabats could have been used with a simple correction to the pressure. However, small inconsistencies between the calculated Hugoniot and adiabat result in fairly large changes in U_r/U_p. Since the analytic fits of the adiabats were obtained graphically, it seemed likely that they might not be sufficiently accurate for our purposes.

Expressions for the internal energy and entropy are given in reference (9), as are analytic fits for $p_o(v)$, $a(v)$, and $b(v)$. The required integrals $\int a(v)\,dv$ and $\int b(v)\,dv$ were evaluated by integrating these analytic fits. Direct integration of the analytic fit for $p_o(v)$ appeared to give such a lengthy result that numerical quadrature with a sixteen point Gauss formula was used instead.

To simplify the calculations, two simplifications were made in the analytic fits: (1) The forms of all of the fits given in reference (9) for $v \leqslant 1$ were extended to $v > 1$. This simplification resulted in a change in the resulting values of $(U_r/U_p) - 1$ of about 2% and 5% for aluminum and tuballoy, respectively, at the highest shock strengths for which calculations were carried out. The change rapidly becomes less significant as shock strength decreases. (2) The form of the fit of $b(v)$ given for $v \geqslant 0.75$ for tuballoy was extended to $v < 0.75$. In the range over which calculations were made, the error resulting in this change was not appreciable; calculation showed that the two forms gave nearly the same results for $b(v)$ and $\int_{v_o}^{v} b(v)\,dv$, and in addition terms containing $b(v)$ affected the results only slightly.

*In conformity with LA-385, throughout this appendix the symbol v will stand for relative volume, v/v_o.

The values of $B = \int_0^1 b(v)\,dv$ were calculated as described in reference (9). The values obtained were:

Aluminum: $B = 0.061231$

Tuballoy: $B = 0.078658$

To check the Hugoniot calculation, Hugoniot curves were calculated for an initial point $T_o = 0^oK$, $p_o = 0$, $v_o = 1$. (In these calculations the correction to $p_o(v)$ to make the equation of state give $p = 1$ atm at $v_o = 1$, $T = 300^oK$ was of course not applied.) The results of these calculations should agree exactly with those given in reference (9). The results are compared in Table 3.1. The cause of the slight disagreement at the higher compressions is not known; it may be due to inaccuracy in the numerical calculation of $\int p_o(v)\,dv$, or to a difference in the value of B. The values of B used in reference (9) are not given. In the case of tuballoy, at least, a part of the disagreement may be caused by our use of the incorrect form for $b(v)$ for $v \leqslant 0.75$.

TABLE A.1

COMPARISON OF CALCULATED HUGONIOTS
WITH VALUES TABULATED IN LA-385
($T_o = 0^oK$, $p_o = 0$, $v_o = 1$)

	v	p(Mb) (LA-385)	p(Mb) (This report)
Aluminum	0.80	0.2539	0.25388
	0.75	0.3721	0.37194
	0.65	0.7710	0.76504
	0.55	1.7515	1.7402
	0.45	4.8751	4.7071
Tuballoy	0.80	0.430	0.42936
	0.75	0.730	0.72963
	0.70	1.237	1.2361
	0.65	2.089	2.0846
	0.60	3.487	3.4776
	0.55	5.753	5.7323
	0.50	9.446	9.400

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