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LOS ALAMOS SCIENTIFIC LABORATORY

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THE UNIVERSITY OF CALIFORNIA

8 November 1951 LA-1308

This document consists of 52 pages

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THE RADIATION DOSE TO BE EXPECTED FROM AERIAL PENETRATION OF AN ATOMIC CLOUD

Report written by:

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REFERENCES

I. "The Radiation Hazard From An Atomic Bomb Cloud" - Rand Report R-139, 18 April 1949.

II. "The Effects of Atomic Weapons", Art. 8.80 - 8.83 inclusive.

ABSTRACT

A particular solution is presented for the following problem:

What is the probability that straight line path aerial penetration of an atomic cloud will lead to a normalized gamma radiation dose in excess of a given value - where the normalized dose is defined as the dose in roentgens divided by a parameter λ, having the dimensions of roentgens, which includes the penetration speed, the size of the weapon which produced the cloud, and the characteristics of the resultant gamma radiation.

The solution contained herein is only qualitative, as adequate means for evaluating the above mentioned parameter experimentally are not yet available. A supplement to this report is contemplated upon receipt of the Greenhouse gamma radiation experimental program results. Such a supplement will supply the information necessary to an evaluation of λ.

1. The radiation dose to be expected from aerial penetration of an atomic cloud is clearly a matter of interest to the Department of Defense. Unfortunately this problem is too complex to admit an exact solution, so various simplifying assumptions and approximate methods must be used. One such method will be found in reference (I), the final results of which are presented in brief in reference (II). In the following paragraphs, we shall attempt to develop a somewhat different method. The basic difference in these two methods is that they employ different assumptions as to the distribution of the source of radioactivity throughout the cloud. Reference (I) assumes that the available source is distributed uniformly over the entire cloud volume. We shall assume that the available source is concentrated at n points, and that these n points are distributed randomly throughout the cloud volume. The latter has the advantage of allowing for "hot spots" in the cloud, and presents its result as an expectation of dose.

2. The One Point Case.

Consider an aircraft flying through an atomic cloud, with the following assumptions:

(a) The aircraft flies on a straight line path ψ at constant speed.

(b) The motion of the cloud, in both ascent and expansion, is small compared to the speed of the aircraft.

(c) The level of radioactivity in the cloud is constant during the time of exposure of the aircraft.

(d) The source of radioactivity is assumed to be concentrated at one point, this point to be located within the cloud in a random fashion.

(e) The only radiological hazard comes from exposure to gamma radiation emitted by the fission products in the cloud and not from any direct contact with or inhalation of these products.

(f) Sufficient time has elapsed after formation of the cloud so that it has risen to an altitude where the cloud density is essentially that of the surrounding atmosphere.

The probability of a point p lying within x and x + dx of ψ is given by

$$p_1 = g\,(x)\,dx \qquad (2.1)$$

and the dose experienced by a point ρ, due to the gamma ray source at p, while traversing ψ is given by

$$D = f\,(x) \qquad (2.2)$$

We can then write:

$$x = \varphi\,(D) \quad ;\text{where } \varphi = f^{-1} \qquad (2.3)$$

and

$$p_1 = g\left[\varphi\,(D)\right]\,J\,(x,\,D)\;dD \qquad (2.4)$$

Where J (x,D) is the Jacobian of the transformation from the x domain to the D domain. For this case, $J = \frac{dx}{dD} = \varphi'$, hence

$$p_1 = \Sigma(D)\ dD \qquad (2.5)$$

where $\Sigma = g\,\varphi\cdot\varphi'$. $\Sigma(D)$ is thus the probability density function for the one point case.

3. The Extension to the n Point Case.

Let us now replace assumption (d) in paragraph 2 with the following:

(d') The source of radioactivity is assumed to be concentrated at n points, each of these points to be located within the cloud in a random fashion.

For simplicity of notation, we assume that each point carries $\frac{1}{n}$ of the total available source. This, however, imposes little restriction on our solution.

Equation (2.1) is now replaced by the system of equations:

$$p_n = n! \prod_{i=1}^{n} \left[g(x_i)\, dx_i \right] \qquad (3.1)$$

and equation (2.2) is replaced by the system:

$$D_n = \frac{1}{n} \sum_{i=1}^{n} f(x_i) \qquad (3.2)$$

which leads to

$$f(x_i) = i\, D_i - (i - 1)\, D_{i-1} \qquad (3.2a)$$

and hence

$$x_i = \varphi\left[i D_i - (i-1) D_{i-1} \right] \qquad (3.3)$$

To obtain the analogy to equation (2.4) we combine and write:

$$p_n = n! \int_{\alpha_i \big|_{i=1}^{i=n}} \prod_{i=1}^{n} \left\{ g \left[\varphi \left(iD_i - [i-1]\, D_{i-1} \right) \right\} \right. J\,(X_i,\, D_i) \prod_{i=1}^{n} dD_i \tag{3.4}$$

Where the symbol $\alpha_i \big|_{i=1}^{i=n}$ below the integral sign indicates that an n dimensional integration must be performed. In each case the range of integration α_i is that dose range obtainable from the i^{th} point source of gamma radiation.

To simplify the above, we note that:

$$J(x_i, D_i) \equiv \begin{pmatrix} \frac{\partial x_1}{\partial D_1} & \frac{\partial x_1}{\partial D_2} & \cdots & \frac{\partial x_1}{\partial D_n} \\ \frac{\partial x_2}{\partial D_1} & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \\ \frac{\partial x_n}{\partial D_1} & \cdots & & \frac{\partial x_n}{\partial D_n} \end{pmatrix}$$

and from (3.3)

$$\frac{\partial x_i}{\partial D_j} = \varphi' \frac{\partial}{\partial D_j} \left[i\, D_i - (i-1)\, D_{i-1} \right]$$

Hence:

$$j = i \Rightarrow \frac{\partial x_i}{\partial D_j} = i\,\varphi'$$

$$j = i-1 \Rightarrow \frac{\partial x_i}{\partial D_j} = -(i-1)\,\varphi'$$

$$j \neq i \text{ or } (i-1) \Rightarrow \frac{\partial x_i}{\partial D_j} = 0$$

Thus:

$$J(x_i, D_i) = \begin{pmatrix} \varphi' & 0 & 0 & \cdots & 0 \\ -\varphi' & 2\varphi' & & & 0 \\ 0 & -2\varphi' & 3\varphi' & & 0 \\ 0 & & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & n\varphi' \end{pmatrix}$$

Substituting in (3.4), we obtain:

$$P_n = (n!)^2 \int_{\sigma()_{i=1}^{i=n}} \prod_{i=1}^{n} \left\{ g\left[\varphi(iD_i - [i-1]D_{i-1})\right] \cdot \varphi'(iD_i - [i-1]D_{i-1})\, dD_i \right\} \tag{3.5}$$

$$= (n!)^2 \int_{\sigma()_{i=1}^{i=n}} \prod_{i=1}^{n} \left\{ \Sigma(\lambda_i)\, dD_i \right\}$$

Where,

$$\Sigma \equiv g\varphi \cdot \varphi'$$

and

$$\lambda_i \equiv iD_i - (i-1)D_{i-1} = f(x_i)$$

To set the limits of integration we first note that D_n ranges from D to $D + \delta D$, hence the integral is truly of dimension (n - 1), and we write:

$$p_n = (n!)^2 \int_{\alpha_i]_{i=1}^{i=n-1}} \prod_{i=1}^{n-1} \left\{ \sum (\lambda_i)\, dD_i \right\} \sum \left[nD - (n-1) D_{n-1} \right] dD$$

The lowest dose obtainable from any point source we shall denote by $f(R) = D_0$, where R is the length of the normal from ψ to the most distant point of the cloud. Hence, every lower limit is equal to D_0. To obtain the $(n-1)^{st}$ upper limit, we note that:

$$i D_i - (i-1) D_{i-1} \geq D_0$$

or

$$D_{i-1} \leq \frac{i D - D_0}{i-1} \quad ; \quad i \geq 2$$

Hence,

$$p_n = (n!)^2 \int_{D_0}^{\frac{nD - D_0}{n-1}} \cdots \int_{D_0}^{\frac{(n-m)D_{n-m} - D_0}{n-m-1}} \cdots \int_{D_0}^{2D_2 - D_0} \prod_{i=1}^{n-1} \left\{ \sum (\lambda_i)\, dD_i \right\} \sum \left[nD - (n-1) D_{n-1} \right] dD \tag{3.6}$$

which obviously reduces to equation (2.5) for n = 1.

To reduce this to a more usable form, let

$$Z_k = K (D_k - D_0)$$

obtaining:

$$\frac{p_n}{n \cdot n!} = \int_0^{n(D-D_0)} \cdots \int_0^{z_{n-m}} \cdots \int_0^{z_2} \prod_{i=1}^{n-1} \left\{ \left[\Sigma^*(z_i - z_{i-1}) \right] dz_i \right\} \Sigma^*\left[n(D-D_0) - z_{n-1} \right] \tag{3.7}$$

where

$$\Sigma^*(X) \equiv \Sigma(X + D_0)$$

It is now clear that we are dealing with a multiple Faltung integral. In fact,

$$\mathcal{L}\left\{\frac{p_n}{n \cdot n!}\right\} = \left[\mathcal{L}\left\{\Sigma[n(D-D_0)+D_0]\right\}\right]^n$$

Where $\mathcal{L}\{F(t)\} \equiv \int_0^{\infty} e^{-st} F(t)\, dt$, the Laplace transform of F(t).[1]

Hence, finally:

$$p_n = n \cdot n! \, \mathcal{L}^{-1}\left\{\left[\mathcal{L}\left\{\Sigma[n(D-D_0)+D_0]\right\}\right]^n\right\} \tag{3.8}$$

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(1). See "Modern Operational Mathematics in Engineering" by R. V. Churchill, Article 14.

4. Thus far we have made no assumptions about the shape of the atomic cloud, or about the intersection of ψ with the cloud. The form of our solution for the one point case will obviously be dependent upon both. Also, for the one point solution to be useful to us, its Laplace transform must exist. Further, we must be able to raise that transform to the n^{th} power and find the inverse transform of the result. Lastly, the assumptions we make about the shape of the cloud, and the intersection of ψ with it, must be consistent with previously observed facts about atomic cloud development and with reasonable and interesting aircraft flight plans.

The remainder of this paper will be devoted to one particular solution which satisfies the above.

5. Let us modify assumption (c) as follows:

(c') The aircraft flies through the centroid of the cloud on a straight line path, ψ , at constant speed.

and add the assumption

(g) The cloud is a sphere of radius R .

Our complete list of assumptions will then be (a), (b), (c'), (d), (e), (f), and (g).

The $g(x)$ of equation (2.1) is then given by:

$$g(x) = \frac{3x\sqrt{R^2 - x^2}}{R^3} \tag{5.1}$$

and the $f(x)$ of equation (2.2) is given by:

$$f(x) = \lambda \frac{I_1(\mu x)}{\mu x} \qquad (5.2)$$

where:

$$\lambda \equiv \frac{S\mu}{2\pi v}$$

$$I_1(\mu x) \equiv \int_0^{\pi/2} e^{-\frac{\mu x}{\sin\theta}}\, d\theta$$

S = The strength of the gamma ray source at p, in units of $\frac{\text{roentgens cm}^2}{\text{KT sec}}$

v = The velocity of the aircraft in units of $\frac{\text{cm}}{\text{sec}}$

μ = The effective total absorption coefficient for the gamma rays under consideration, in units of $\frac{1}{\text{cm}}$.

It is convenient to consider all distances in units of mean free paths, hence we make use of the following definitions:

$$\alpha \equiv \mu x$$

$$r \equiv \mu r$$

and equations (5.1) and (5.2) become:

$$g(\alpha) \equiv \frac{3\alpha \sqrt{r^2 - \alpha^2}}{r^3} \qquad (5.1a)$$

$$f(\alpha) \equiv \lambda \frac{I_1(\alpha)}{\alpha} \qquad (5.2a)$$

Also, in order that our solution will be applicable directly to a wide variety of cases, we replace D by $\bar{D}$, where

$$\bar{D} \equiv \frac{D}{\lambda}$$

The reader can easily convince himself that this will in no way alter our use of the equations developed in paragraphs 2 and 3. For any given situation, λ will be a constant. The analogy of equation (2.5) is then:

$$P_1 \equiv \sum(\bar{D})\, d\bar{D} \qquad (5.3)$$

Where:

$$\sum(\bar{D}) = \frac{3\alpha^2 \sqrt{r^2 - \alpha^2}}{r^3} \cdot \frac{1}{I_2 + \frac{I_1}{\alpha}} \cdot \qquad (5.4)$$

$$I_2 \equiv \int_0^{\pi/2} \frac{e^{-\frac{\alpha}{\sin\theta}}}{\sin\theta}\, d\theta$$

and p_1 is the probability that the dose experienced will be between the limits $\bar{D}$ and $\bar{D} + \delta\bar{D}$. Finally, the probability of experiencing a dose at least as great as $\bar{D}$ is given by:

$$P(\bar{\bar{D}}) = \int_{\infty}^{\bar{\bar{D}}} \Sigma(\tau)\, d\tau$$

Figures 1 and 2 are plots of $I_1(\alpha)$ and $I_2(\alpha)$ respectively.[2] Table 1 contains the results of computing $\bar{D}$ and p as functions of α , for various values of the parameter r .[3] Figure 3 was prepared from this table, and contains the resultant six plots of p versus $\bar{\bar{D}}$. Figure 4 exhibits the corresponding six plots of p versus $\bar{D}$. Figure 5 is an enlarged linear plot of p versus $\bar{D}$ for $r = 1$, and includes plots of two interesting simple approximations - a truncated power law and a truncated exponential law. Figure 6 is self-explanatory.

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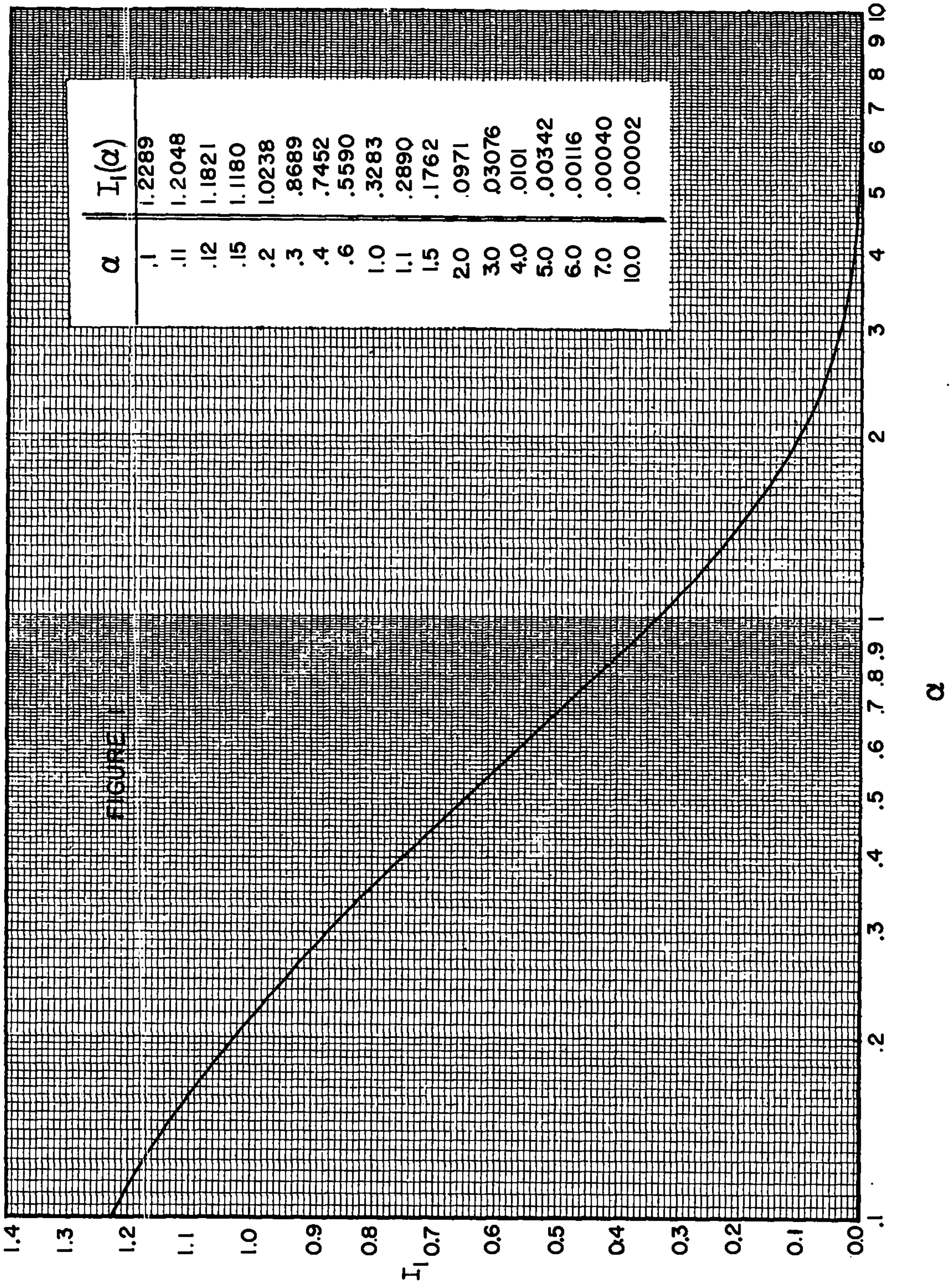
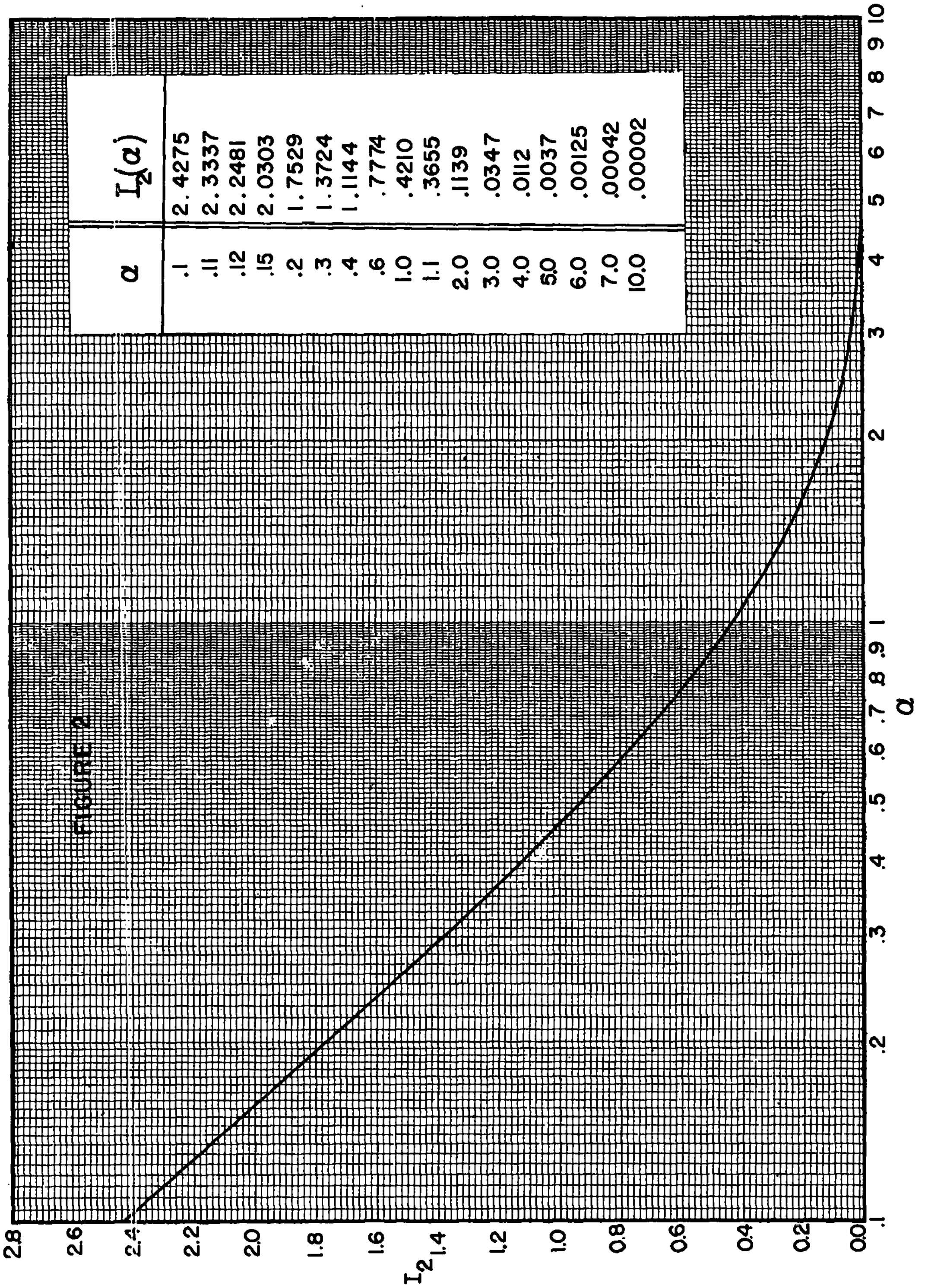
(2) These plots were made from machine calculations supplied the author by the Computing Section of the LASL Theoretical Division. The small tables on each contain the numerical results of those calculations.

(3) The decision to use integral values of r from $r = 1$ to $r = 6$ was based upon a perusal of Annex 4.1B of the Greenhouse Report - "The Development of the Atomic Cloud" by Kellogg, McKown, and McPherson.



α	$I_1(\alpha)$
.1	1.2289
.11	1.2048
.12	1.1821
.15	1.1180
.2	1.0238
.3	.8689
.4	.7452
.6	.5590
1.0	.3283
1.1	.2890
1.5	.1762
2.0	.0971
3.0	.03076
4.0	.0101
5.0	.00342
6.0	.00116
7.0	.00040
10.0	.00002



α	$I_2(\alpha)$
.1	2.4275
.11	2.3337
.12	2.2481
.15	2.0303
.2	1.7529
.3	1.3724
.4	1.1144
.6	.7774
1.0	.4210
1.1	.3655
2.0	.1139
3.0	.0347
4.0	.0112
5.0	.0037
6.0	.00125
7.0	.00042
10.0	.00002

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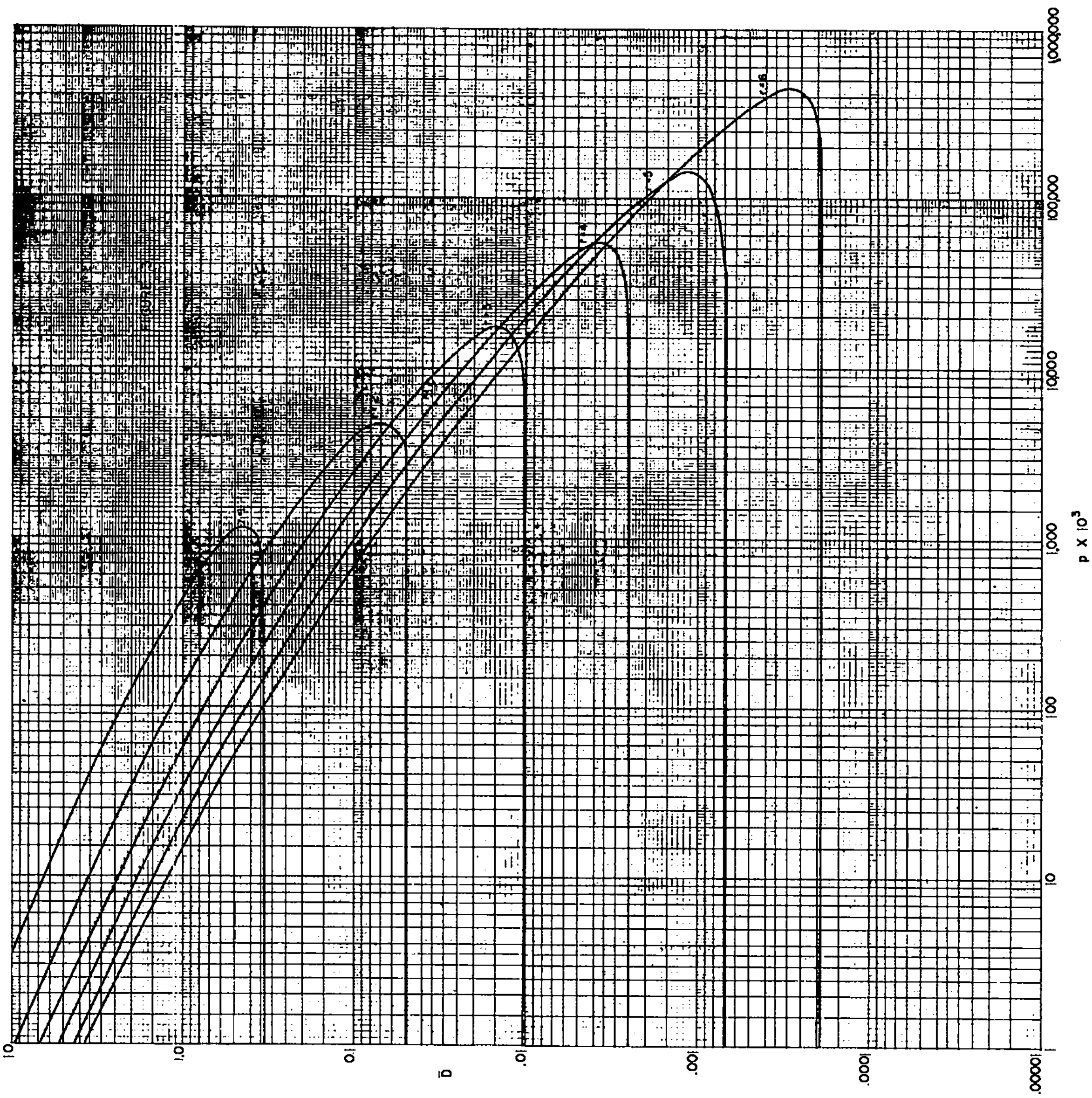
TABLE 1

α	D̄	p r = 1	p r = 2	p r = 3	p r = 4	p r = 5	p r = 6
.1	12.289	.002028	.00051	.00023			
.12	9.8508	.003544	.00089				
.15	7.4533	.007069	.00178				
.2	5.119	.017109	.00434	.00194	.00109	.0006985	.00048458
.25	3.76	.034285	.00879				
.3	2.896	.060340	.01563	.00699			
.4	1.863	.147748	.03950	.01775	.01002	.006427	.0044669
.5	1.288	.29376	.08209	.03716			
.6	.9317	.505528	.15071	.06879	.03904	.02486	.017465
.7	.6957	.77579	.25437	.11738			
.8	.5325	1.05156	.40157	.18768	.10455	.0692	.048252
.9	.4144	1.1777	.60315	..28636			
.95	.3684	1.03129					
.975	.3477	.80944					
.99	.3354	.5434					
1.0	.3283	0	.86681	.41940	.24221	.15692	.10965
1.1	.2676		1.1789				
1.2	.2193		1.5642	.79640	.46614	.30369	.21285
1.3	.1789		2.0341				
1.4	.1488		2.5685	1.41403	.84218	.55256	.38865
1.5	.1231		3.1778				
1.6	.1019		3.8185	2.39262	1.45775	.96473	.68149
1.7	.08454		4.41372				
1.8	.07018		4.77652	3.89610	2.44562	1.63569	1.16144
1.9	.05833		4.45332				
2.0	.04855		0	6.11739	3.99698	2.70809	1.93450
2.1	.04050			7.53125			
2.2	.03387			9.17397	6.29341	4.33230	3.11693
2.3	.02842			11.00739			
2.4	.02397			12.95973	9.71662	6.82146	4.94915
2.5	.02035			14.88612			
2.6	.01740			16.55153	14.17667	10.20156	7.47497
2.7	.01502			17.56600			
2.8	.01310			17.3064	19.35916	14.37834	10.65929
2.9	.01156			14.52934			
3.0	.01027			0	24.81299	19.21281	14.44341
3.1	.00880				29.03877		
3.2	.00753				33.62433	27.56722	21.07498
3.3	.00647				38.47665		
3.4	.00557				43.41651	38.68848	30.19247
3.5	.00482				47.99784		
3.6	.00419				51.67024	52.64780	42.14633
3.7	.00367				53.33863		
3.8	.00323				51.47077	68.58587	56.71498
3.9	.00288				42.60256		
4.0	.00252				0	83.96502	72.43368
4.1	.002236					90.86730	
4.2	.001947					103.31294	94.43015
4.3	.001700					114.48432	
4.4	.001490					124.96829	124.21970
4.5	.001311					133.71923	
4.6	.001160					139.18322	158.34266
4.7	.001031					139.33138	
4.8	.000927					130.17320	193.71112
4.9	.000838					104.28337	
5.0	.000684					0	262.68020
5.1	.000596						294.58205
5.2	.000519						326.88572
5.3	.000455						360.35139
5.4	.000398						391.13737
5.5	.000349						418.20257
5.6	.000307						437.00046
5.7	.000270						438.04145
5.8	.000240						412.49425
5.9	.000214						332.96717
6.0	.000193						0


p X 10^3
D
1,000,000
100,000
10,000
1,000
100
10
1.0
.1
.01
.001
.0001
.00001

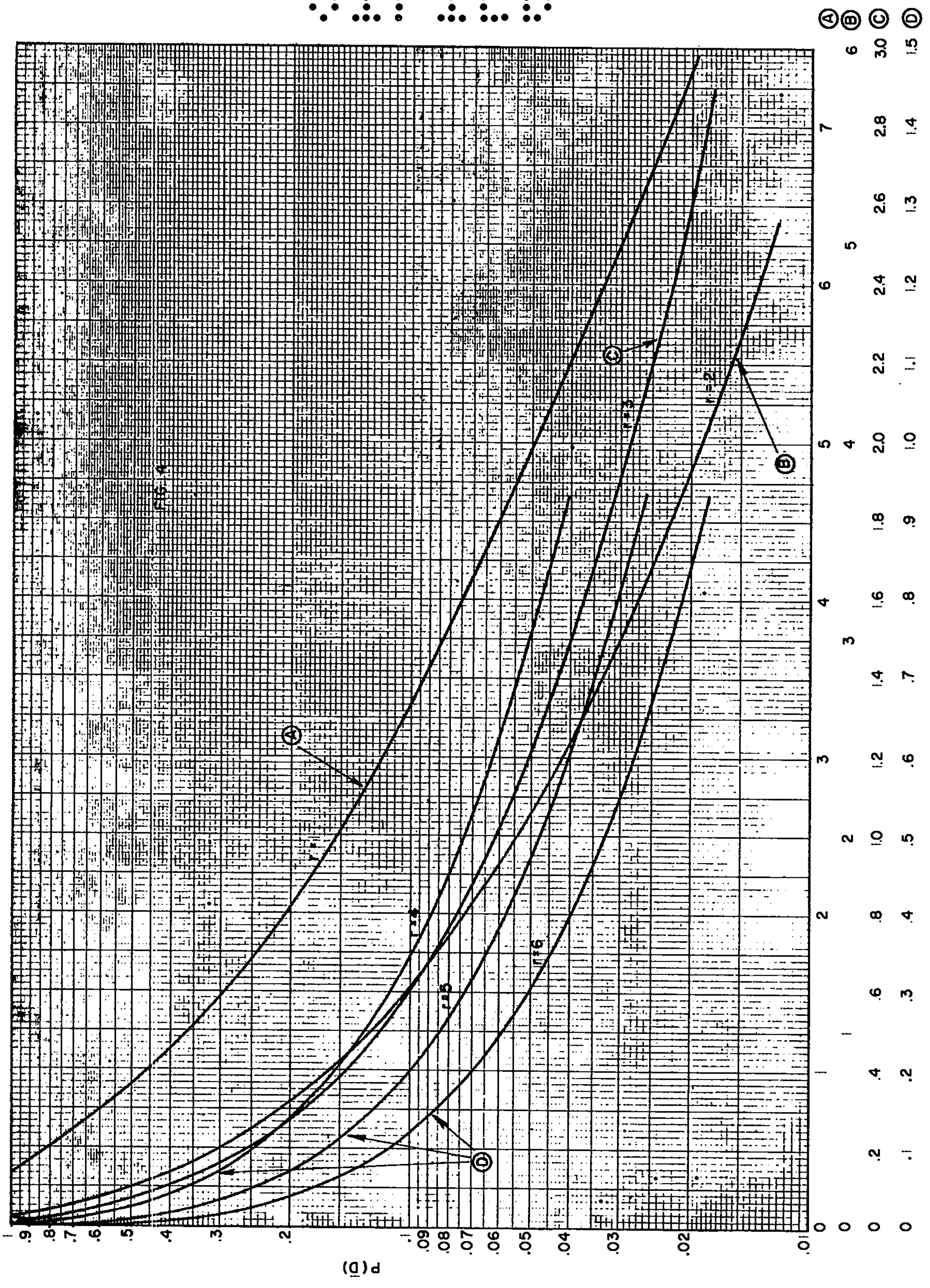


FIG. 4
P(D)
D

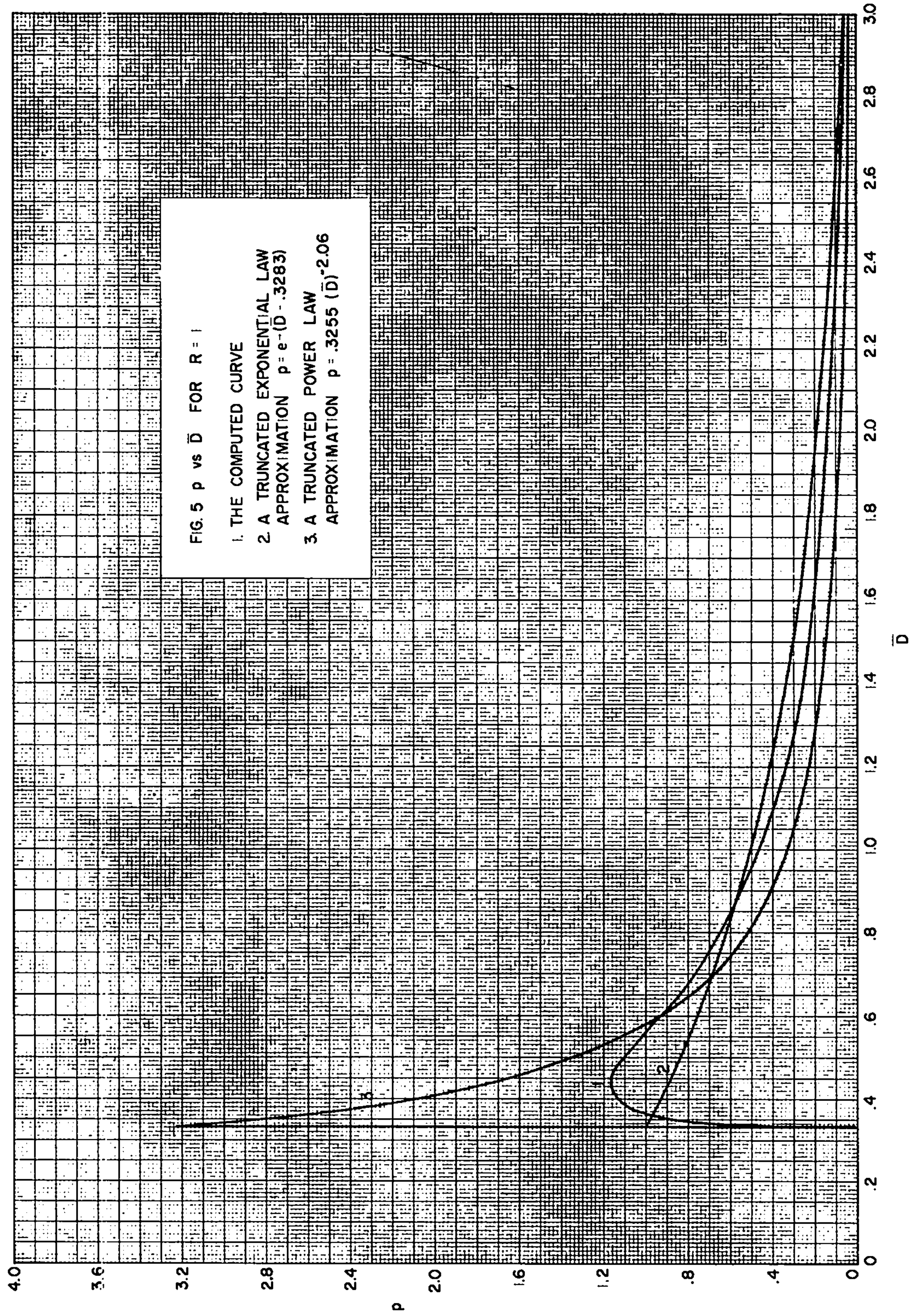


FIG. 5 p vs $\bar{D}$ FOR R = 1
1. THE COMPUTED CURVE
2. A TRUNCATED EXPONENTIAL LAW APPROXIMATION $p = e^{-(\bar{D} - .3283)}$
3. A TRUNCATED POWER LAW APPROXIMATION $p = .3255\,(\bar{D})^{-2.06}$
$\bar{D}$
p
0 .2 .4 .6 .8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3.0
0 .4 .8 1.2 1.6 2.0 2.4 2.8 3.2 3.6 4.0
1
2
3

[redacted]

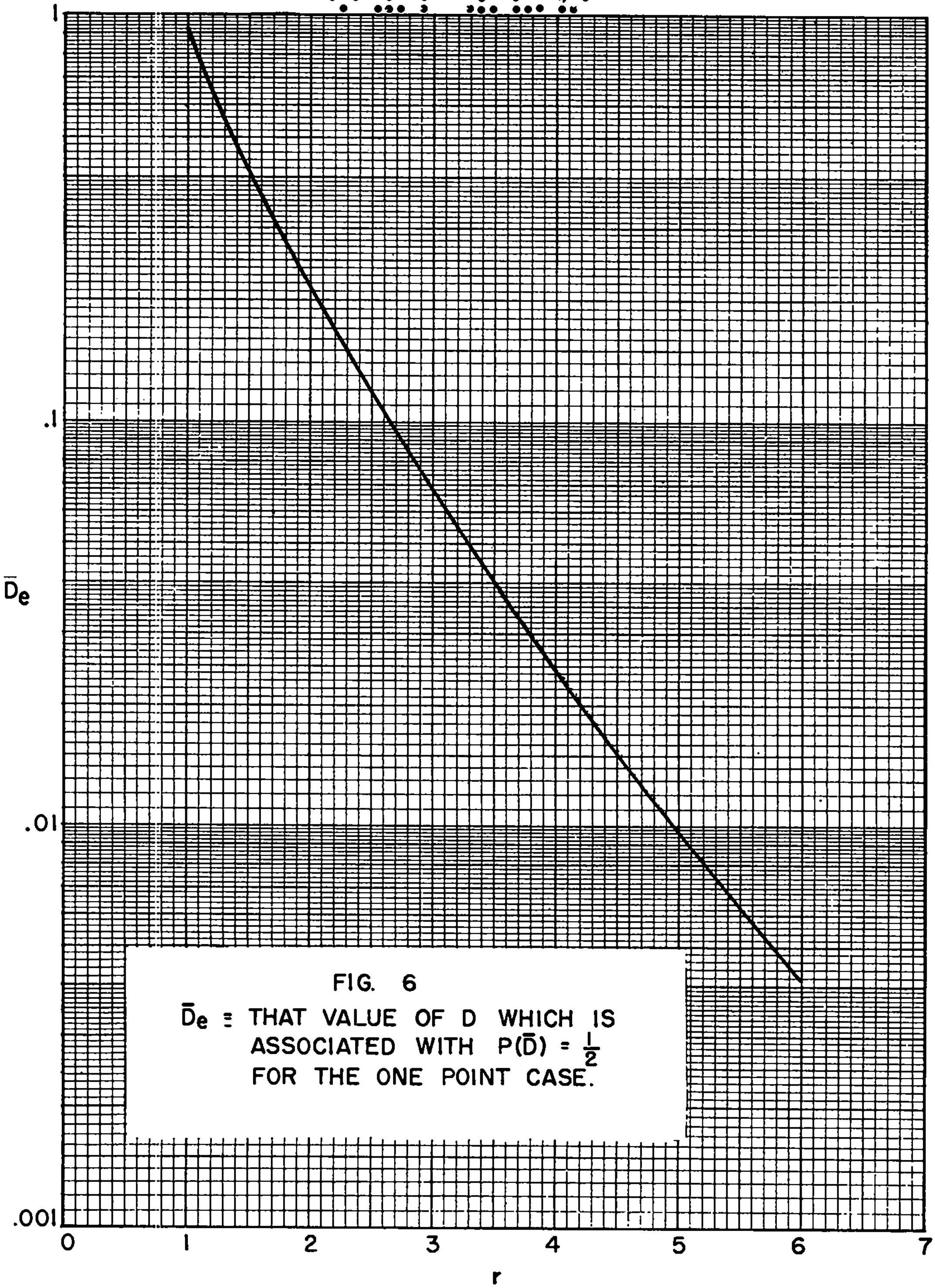


FIG. 6

$\bar{D}_e \equiv$ THAT VALUE OF D WHICH IS ASSOCIATED WITH $P(\bar{D}) = \frac{1}{2}$ FOR THE ONE POINT CASE.

[redacted]

6. From paragraph 3, we see that our next step must be to find $\mathcal{L}\{\Sigma\}$ and raise it to the n^{th} power. This of course can be done numerically, as an inspection of Figure 3 makes it obvious that $\mathcal{L}\{\Sigma\}$ exists. The labor involved seems hardly justified, however, when one re-examines assumption (g) in the light of existing atomic cloud development photographs. A more reasonable approach is to approximate $\Sigma(\bar{D})$ by some function whose Laplace transform is easily raised to the n^{th} power, in such a way that a minimum of violence is done to the corresponding integral $P(\bar{D})$. This is equivalent to changing the cloud shape from a sphere to some other distribution of points in space.

We shall make use of a truncated exponential law to achieve the desired approximation. A law, that is, of the form:

$$-\Sigma(\bar{D}) = A\, e^{-B\bar{D}}\, u(\bar{D}-\bar{D}_o) \tag{6.1}$$

where: A, B are parameters depending only upon r, and therefore are constant for a given aircraft flight altitude and bomb.

$$u(\bar{D}-\bar{D}_o) = 0 \quad \text{for} \quad \bar{D} < \bar{D}_o$$
$$= 1 \quad \text{for} \quad \bar{D} \geq \bar{D}_o$$

The values of A and B will be determined later. For the present let us examine the extension of (6.1) to the n point case. From paragraph 3, we have

$$p_n = n\cdot n!\, \mathcal{L}^{-1}\left\{\left[\mathcal{L}\left\{\sum[n(\bar{D}-\bar{D}_0)+\bar{D}_0]\right\}\right]^n\right\} \tag{3.8}$$

Hence we can write:

$$\left[\mathcal{L}\left\{\frac{p_n}{n\cdot n!}\right\}\right]^{\frac{1}{n}} = \mathcal{L}\left\{A\ \exp\left[-B(n[\bar{D}-\bar{D}_0]+\bar{D}_0)\right]u(\bar{D}-\mu)\right\} \tag{6.2}$$

In which μ must be chosen in such a way that $p_n = 0$ when $\bar{D} = \bar{D}_0$.
Equation (6.2) can be written:

$$\left[\mathcal{L}\left\{\frac{p_n}{n\cdot n!}\right\}\right]^{\frac{1}{n}} = A\int_{\mu}^{\infty} e^{-st}e^{-Bt}\,dt \quad ; \quad t \equiv n(\bar{D}-\bar{D}_0)+\bar{D}_0$$

$$= \frac{A}{S+B}\,e^{-\mu(S+B)}$$

and:

$$\mathcal{L}\left\{\frac{p_n}{n\cdot n!}\right\} = \frac{A^n}{(S+B)^n}\,e^{-n\mu(S+B)} \tag{6.3}$$

Hence:

$$p_n = A^n n^2 e^{-n\mu B}\left[n(\bar{D}-\bar{D}_o)+\bar{D}_o-n\mu\right]^{n-1} \exp\left\{-B\left[n(\bar{D}-\bar{D}_o)+\bar{D}_o-n\mu\right]\right\} \tag{6.4}$$

From which we see that the correct choice of μ is obviously

$$\mu = \frac{\bar{D}_o}{n}$$

Substituting, we finally obtain:

$$p_n = A^n n^2 e^{-B\bar{D}_o}\left[n(\bar{D}-\bar{D}_o)\right]^{n-1} e^{-B\,n(\bar{D}-\bar{D}_o)} \tag{6.5}$$

7. Values of the parameter B (r) can be obtained in any number of ways, depending upon how one wishes to fit straight lines to the curves of Figure 4. The only justification for the method used herein is that it is simple and seems reasonable. Consider the curve labeled I in Figure 7. Each point of this curve was obtained empirically by measuring the slope of a line drawn from the P = 0.1 point to the P = 0.9 point of the corresponding curve in Figure 4. We shall define B by the expression:

$$B \equiv e^{0.6\,(r-1)} \tag{7.1}$$

(7.1) is the equation of the curve labeled II in Figure 7, which was fitted to curve I by eye.

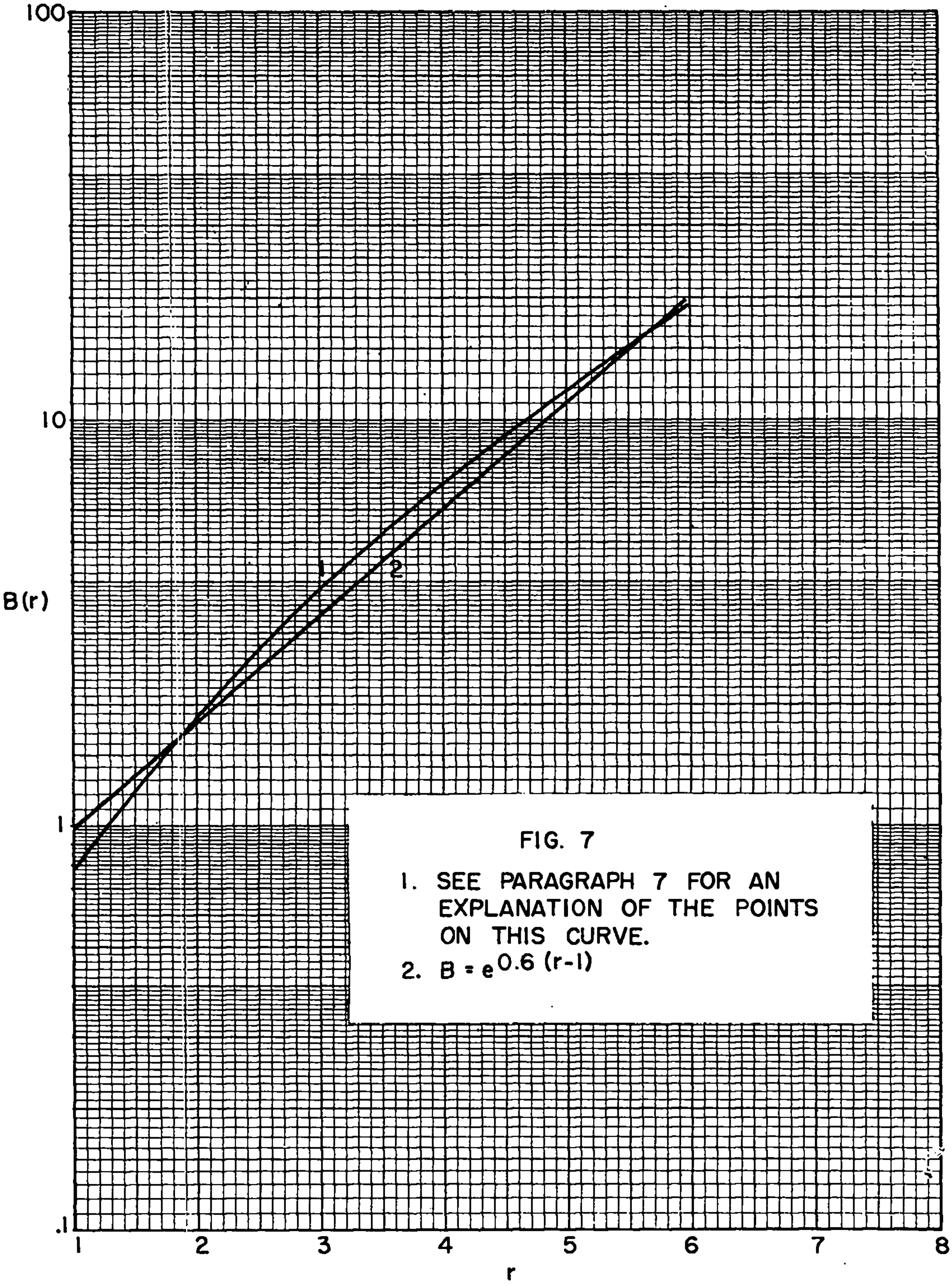


FIG. 7

1. SEE PARAGRAPH 7 FOR AN EXPLANATION OF THE POINTS ON THIS CURVE.
2. $B = e^{0.6\,(r-1)}$

To obtain an expression for A , we recall the restriction that:

$$\int_{\bar{D}_0}^{\infty} p_n(\tau)\, d\tau = 1$$

Hence:

$$\frac{1}{A^n\, n\, e^{-B\bar{D}_0}} = \int_0^{\infty} Z^{n-1} e^{-BZ}\, dZ = \frac{(n-1)\,!}{B^n}$$

and

$$A = \left[\frac{B^n e^{B\bar{D}_0}}{n\,!}\right]^{\frac{1}{n}} \tag{7.2}$$

Our final expression for $p_n(\bar{D}, r)$ can then be written as:

$$p_n(\bar{D}, r) = \frac{B^n n^2}{n\,!} \left[n(\bar{D} - \bar{D}_0)\right]^{n-1} e^{-Bn(\bar{D} - \bar{D}_0)} \tag{7.3}$$

To determine the behavior of (7.3) for very large n , we make use of Stirling's asymptotic expansion for $n!$, and write:

$$p_n(\bar{D}, r) \sim \frac{Bn^{\frac{1}{2}} e^n}{X\sqrt{2\pi}} X^n e^{-nX} \quad ; \text{ where } \quad X = B(\bar{D} - \bar{D}_0)$$

or

$$p_n\left(\frac{X}{B} + \bar{D}_0 , r\right) \sim \frac{Bn^{\frac{1}{2}}}{X\sqrt{2\pi}} e^{n\left[1 + \ln X - X\right]} \quad ; \quad X \geq 0 \tag{7.4}$$

Let us now consider the expression

$$y = 1 - X + \ln X \qquad ; \; X \geq 0$$

which has an extremum, and is equal to zero, at $X = 1$. It is obvious that y is negative for $0 \leq X < 1$, hence the extremum is a maximum and $y \leq 0$ for all X. As a result we can write:

$$\lim_{n \to \infty} p_n(\bar{D}, r) = \infty \qquad \bar{D} = \frac{1}{B} + \bar{D}_o$$

$$= 0 \qquad \bar{D} \neq \frac{1}{B} + \bar{D}_o$$

Or, equivalently, $p_\infty(\bar{D}, r)$ is a δ function, the position of whose peak is determined by r. Since letting n become infinite is equivalent to assuming a homogeneous distribution of gamma ray source throughout the cloud, we see that the equation:

$$\bar{D}_e = e^{-0.6\,(r-1)} + \bar{D}_o \tag{7.5}$$

defines the expectation doses for homogeneous distributions in clouds of various sizes.

Since we are primarily interested in the probability of obtaining a dose, we shall make no computations of p_n, but rather shall proceed to a consideration of its running integral:

$$P_n(\bar{D}, r) = \frac{nB^n}{n!} \int_{n(\bar{D} - \bar{D}_o)}^{\infty} z^{n-1} e^{-Bz}\, dz$$

$$= \frac{1}{\Gamma(n)} \int_{Bn(\bar{D}-\bar{D}_0)}^{\infty} t^{n-1} e^{-t}\, dt \quad ;\ n \text{ integral}$$

$$= 1 - \frac{\Gamma_{Bn(\bar{D}-\bar{D}_0)}(n)}{\Gamma(n)} \quad ;\ n \text{ integral} \qquad (7.6)$$

where

$$\Gamma_x(n) \equiv \int_0^x e^{-t}\, t^{n-1}\, dt$$

and

$$\Gamma_\infty(n) \equiv \Gamma(n)$$

The reference cited in Footnote (4) contains tables of

$$I\ (u,\ p) \equiv \frac{\Gamma_u\ (p + 1)}{\Gamma\ (p + 1)}$$

where

$$u \equiv \frac{X}{\sqrt{p + 1}}$$

---

(4) See "Tables of the Incomplete Γ- Function" by Karl Pearson. This volume is printed by the Cambridge University Press and published by the Office of "Biometrika". 1934 Edition.

Hence we can use these tables directly by noting that:

$$p_{n+1}(\bar{D}, r) = 1 - I\left(B\sqrt{n+1}\left[\bar{D} - \bar{D}_o\right], n\right) \qquad (7.7)$$

The function $p_n(\bar{D}, r)$ is presented numerically in Table 2 and graphically in Figures 8 and 9. The six sections of Figure 8 depict the trend of the function as n is varied and r is held fixed. Since the $\bar{D}$ scale for these figures was changed to allow for more clarity in plotting, Figure 9 was included to indicate the trend of the function as n is held fixed and r is varied.

8. As we noted in paragraph 6, the particular solution exhibited in Figures 8 and 9 is not for a spherical cloud but is for "some other distribution of points in space". It is only logical that we should attempt to derive some of the characteristics of that distribution.

In doing this, let us first re-examine the order of arguments in paragraph 5. We postulated assumption (g), and from this we wrote equations (5.1) and (5.1a). A more general approach is to replace assumption (g) with the following:

(g') The cloud is such a configuration of points in space that the $g(x)$ of equation (2.1) is given by

$$g(x) = \frac{3x\sqrt{R^2 - x^2}}{R^3} \qquad (5.1)$$

where R is the length of the normal from ψ to the most distant point of the cloud. One such configuration of points is of course a sphere of radius R.

TABLE 2A (r = 1)

n = 2		n = 5		n = 10	
$\overline{D}$	$P_2(\overline{D}, 1)$	$\overline{D}$	$P_5(\overline{D}, 1)$	$\overline{D}$	$P_{10}(\overline{D}, 1)$
.32830	1	.32830	1	.32830	1
.68185	.84172	.55191	.99419	.48641	.99994
1.03541	.58694	.77551	.92355	.92355	.99838
1.38896	.37416	.99912	.75268	.80264	.97655
1.74251	.22628	1.22272	.53740	.96076	.89193
2.09607	.13218	1.44633	.34364	1.11887	.72826
2.44962	.07534	1.66994	.20131	1.27698	.52354
2.80317	.04216	1.89355	.11002	1.43510	.33318
3.15673	.02326	2.11715	.05687	1.59321	.19026
3.51028	.01269	2.34076	.02810	1.75132	.09894
3.86383	.00686	2.56437	.01337	1.90944	.04749
4.21739	.00368	2.78797	.00617	2.06755	.02128
4.57084	.00196	3.01158	.00277	2.22567	.00899

n = 20		n = 30		n = 50	
$\overline{D}$	$P_{20}(\overline{D},1)$	$\overline{D}$	$P_{30}(\overline{D},1)$	$\overline{D}$	$P_{50}(\overline{D}, 1)$
.66371	.99998				
.77551	.99902	.69345	1		
.88732	.98909	.78473	.99991		
.99912	.94490	.87602	.99832		
1.11092	.83573	.96731	.98678	.82327	1
1.22273	.66077	1.05860	.94220	.89398	.99986
1.33453	.45922	1.14988	.83656	.96470	.99827
1.44633	.28017	1.24117	.66658	1.03541	.98835
1.55814	.15115	1.33246	.46668	1.10612	.95126
1.66994	.07288	1.42374	.28520	1.17683	.86053
1.78174	.03176	1.51503	.15262	1.24754	.70531
1.89355	.01265	1.60632	.07208	1.31825	.50962
2.00535	.00465	1.69761	.03034	1.38896	.32008
		1.78889	.01150	1.45967	.17418
		1.87988	.00396	1.53038	.08241
				1.60109	.03414
				1.67180	.01248
				1.74251	.00407

TABLE 2B (r = 2)

n = 2		n = 5		n = 10	
$\bar{\bar{D}}$	P_2 ($\bar{\bar{D}}$, 2)	$\bar{\bar{D}}$	P_5 ($\bar{\bar{D}}$, 2)	$\bar{\bar{D}}$	P_{10} ($\bar{\bar{D}}$, 2)
.04855	1	.04855	1	.04855	1
.24258	.84172	.17127	.99419	.13532	.99994
.43661	.58694	.29398	.92355	.22210	.99838
.63064	.37416	.41670	.75268	.30887	.97655
.82467	.22628	.53941	.53740	.39564	.89193
1.01870	.13218	.66212	.34364	.48241	.72826
1.21273	.07534	.78484	.20131	.56919	.52354
1.40676	.04216	.90756	.11002	.65596	.33318
1.60079	.02326	1.03027	.05687	.74273	.19026
1.79482	.01269	1.15299	.02810	.82950	.09894
1.98885	.00686	1.27571	.01337	.91628	.04749
2.18288	.00368	1.39842	.00617	1.00305	.02128
2.37691	.00196	1.52113	.00277	1.08983	.00899

n = 20		n = 30		n = 50	
$\bar{\bar{D}}$	P_{20} ($\bar{\bar{D}}$, 2)	$\bar{\bar{D}}$	P_{30} ($\bar{\bar{D}}$, 2)	$\bar{\bar{D}}$	P_{50} ($\bar{\bar{D}}$,2)
.23262	.99998				
.29398	.99902	.24894	1		
.35534	.98909	.29904	.99991		
.41670	.94490	.34914	.99832		
.47805	.83573	.39924	.98678	.32019	1
.53941	.66077	.44934	.94220	.35899	.99986
.60077	.45922	.49943	.83656	.39781	.99827
.66212	.28017	.54953	.66658	.43661	.98835
.72349	.15115	.59963	.46668	.47542	.95126
.78484	.07288	.64973	.28520	.51422	.86053
.84620	.03176	.69983	.15262	.55303	.70531
.90756	.01265	.74993	.07208	.59183	.50962
.96891	.00465	.80003	.03034	.63064	.32008
		.85012	.01150	.66945	.17418
		.90022	.00396	.70825	.08241
				.74706	.03414
				.78586	.01248
				.82467	.00407

TABLE 2C (r = 3)

n = 2		n = 5		n = 10	
$\bar{D}$	$P_2 (\bar{D}, 3)$	$\bar{D}$	$P_5 (\bar{D}, 3)$	$\bar{D}$	$P_{10} (\bar{D}, 3)$
.01027	1	.01027	1	.01027	1
.11676	.84172	.07762	.99419	.05789	.99994
.22325	.58694	.14497	.92355	.10551	.99838
.32974	.37416	.21232	.75268	.15314	.97655
.43623	.22628	.27967	.53740	.20076	.89193
.54272	.13218	.34702	.34364	.24839	.72826
.64921	.07534	.41437	.20131	.29601	.52354
.75570	.04216	.48172	.11002	.34364	.33318
.86219	.02326	.54907	.05687	.39126	.19026
.96868	.01269	.61642	.02810	.43888	.09894
1.07517	.00686	.68377	.01337	.48651	.04749
1.18166	.00368	.75112	.00617	.53413	.02128
1.28815	.00196	.81847	.00277	.58176	.00899

n = 20		n = 30		n = 50	
$\bar{D}$	$P_{20} (\bar{D}, 3)$	$\bar{D}$	$P_{30} (\bar{D}, 3)$	$\bar{D}$	$P_{50} (\bar{D}, 3)$
11129	.99998				
.14497	.99902	.12025	1		
.17865	.98909	.14775	.99991		
.21232	.94490	.17524	.99832		
.24599	.83573	.20274	.98678	.15935	1
.27967	.66077	.23024	.94220	.18065	.99986
.31335	.45922	.25773	.82656	.20195	.99827
.34702	.28017	.28523	.66658	.22325	.98835
.38070	.15115	.31272	.46668	.24455	.95126
.41437	.02788	.34022	.28520	.26585	.86053
.44805	.03176	.36771	.15262	.28715	.70531
.48172	.01265	.39521	.07208	.30844	.50962
.51540	.00465	.42271	.03034	.32974	.32008
		.45020	.01150	.35104	.17418
		.47770	.00396	.37234	.08241
				.39363	.03414
				.41493	.02148
				.43623	.00407

TABLE 2D (r = 4)

n = 2		n = 5		n = 10	
$\bar{D}$	$P_2(\bar{D}, 4)$	$\bar{D}$	$P_5(\bar{D}, 4)$	$\bar{D}$	$P_{10}(\bar{D}, 4)$
.00252	1	.00252	1	.00252	1
.06096	.84172	.03948	.99419	.02866	.99994
.11940	.58694	.07644	.92355	.05479	.99838
.17785	.37416	.11341	.75268	.08093	.97655
.23629	.22628	.15037	.53740	.10706	.89193
.29473	.13218	.18733	.34364	.13320	.72826
.35317	.07534	.22429	.20131	.15934	.52354
.41162	.04216	.26126	.11002	.18547	.33318
.47006	.02326	.29822	.05687	.21161	.19026
.52850	.01269	.33518	.02810	.23774	.09894
.58694	.00686	.37214	.01337	.26388	.04749
.64539	.00368	.40910	.00617	.29002	.02128
.70383	.00196	.44607	.00277	.31616	.00899

n = 20		n = 30		n = 50	
$\bar{D}$	$P_{20}(\bar{D}, 4)$	$\bar{D}$	$P_{30}(\bar{D}, 4)$	$\bar{D}$	$P_{50}(\bar{D}, 4)$
.05796	.99998				
.07644	.99902	.06288	1		
.09493	.98909	.07797	.99991		
.11341	.94490	.09306	.99832		
.13189	.83573	.10815	.98678	.08434	1
.15037	.66077	.12324	.94220	.09603	.99986
.16885	.45922	.13833	.83656	.10772	.99827
.18733	.28017	.15342	.66658	.11940	.98835
.20581	.15115	.16851	.46668	.13109	.95126
.22429	.07288	.18360	.28520	.14278	.86053
.24277	.03176	.19869	.15262	.15447	.70531
.26126	.01265	.21378	.07208	.16616	.50962
.27974	.00465	.22887	.03034	.17785	.32008
		.24396	.01150	.18954	.17418
		.25900	.00396	.20122	.08241
				.21291	.03414
				.22460	.01248
				.23629	.00407

TABLE 2E (r = 5)

n = 2		n = 5		n = 10	
$\overline{D}$	$P_2(\overline{D}, 5)$	$\overline{D}$	$P_5(\overline{D}, 5)$	$\overline{D}$	$P_{10}(\overline{D}, 5)$
.00068	1	.00068	1	.00068	1
.03275	.84172	.02096	.99419	.01502	.99994
.06481	.58694	.04124	.92355	.02936	.99838
.09688	.37416	.06152	.75268	.04370	.97655
.12895	.22628	.08180	.53740	.05804	.89193
.16102	.13218	.10208	.34364	.07238	.72826
.19308	.07534	.12237	.20131	.08672	.52354
.22515	.04216	.14265	.11002	.10107	.33318
.25272	.02326	.16293	.05687	.11541	.19026
.28928	.01269	.18321	.02810	.12975	.09894
.32135	.00686	.20349	.01337	.14409	.04749
.35342	.00368	.22377	.00617	.15843	.02128
.38549	.00196	.24405	.00277	.17277	.00899

n = 20		n = 30		n = 50	
$\overline{D}$	$P_{20}(\overline{D}, 5)$	$\overline{D}$	$P_{30}(\overline{D}, 5)$	$\overline{D}$	$P_{50}(\overline{D}, 5)$
.03110	.99998				
.04124	.99902	.03380	1		
.05138	.98909	.04208	.99991		
.06152	.94490	.05036	.99832		
.07166	.83573	.05864	.98678	.04557	1
.08180	.66077	.06692	.94220	.05199	.99986
.09194	.45922	.07520	.83656	.05840	.99827
.10208	.28017	.08348	.66658	.06481	.98835
.11223	.15115	.09176	.46668	.07123	.95126
.12237	.07288	.10004	.28520	.07764	.86053
.13251	.03176	.10832	.15262	.08406	.70531
.14265	.01265	.11660	.07208	.09047	.50962
.15279	.00465	.12488	.03034	.09688	.32008
		.13316	.01150	.10330	.17418
		.14144	.00396	.10971	.08241
				.11612	.03414
				.12254	.01248
				.12895	.00407

TABLE 2F (r = 6)

n = 2		n = 5		n = 10	
$\bar{D}$	P_2 ($\bar{D}$, 6)	$\bar{D}$	P_5 (D, 6)	$\bar{D}$	P_{10} ($\bar{D}$, 6)
.00019	1	.00019	1	.00019	1
.01780	.84172	.01133	.99419	.00806	.99994
.03540	.58694	.02246	.92355	.01594	.99838
.05301	.37416	.03360	.75268	.02381	.97655
.07062	.22628	.04473	.53740	.03169	.89193
.08822	.13218	.05587	.34364	.03956	.72826
.10583	.07534	.06700	.20131	.04743	.52354
.12344	.04216	.07814	.11002	.05531	.33318
.14105	.02326	.08927	.05687	.06318	.19026
.15865	.01269	.10041	.02810	.07106	.09894
.17626	.00686	.11155	.01337	.07893	.04749
.19387	.00368	.12268	.00617	.08680	.02128
.21147	.00196	.13382	.00277	.09468	.00899

n = 20		n = 30		n = 50	
$\bar{D}$	P_{20} ($\bar{D}$,6)	$\bar{D}$	P_{30} ($\bar{D}$,6)	$\bar{D}$	P_{50} ($\bar{D}$,6)
.01689	.99998				
.02246	.99902	.01837	1		
.02803	.98909	.02292	.99991		
.03360	.94490	.02747	.99832		
.03916	.83573	.03201	.98678	.02484	1
.04473	.66077	.03656	.94220	.02836	.99986
.05030	.45922	.04110	.83656	.03188	.99827
.05587	.28017	.04565	.66658	.03540	.98835
.06144	.15115	.05019	.46668	.03893	.95126
.06700	.07288	.05474	.28520	.04245	.86053
.07257	.03176	.05929	.15262	.04597	.70531
.07814	.01265	.06383	.07208	.04949	.50962
.08371	.00465	.06838	.03034	.05301	.32008
		.07293	.01150	.05653	.17418
		.07747	.00396	.06005	.08241
				.06357	.03414
				.06710	.01248
				.07062	.00407

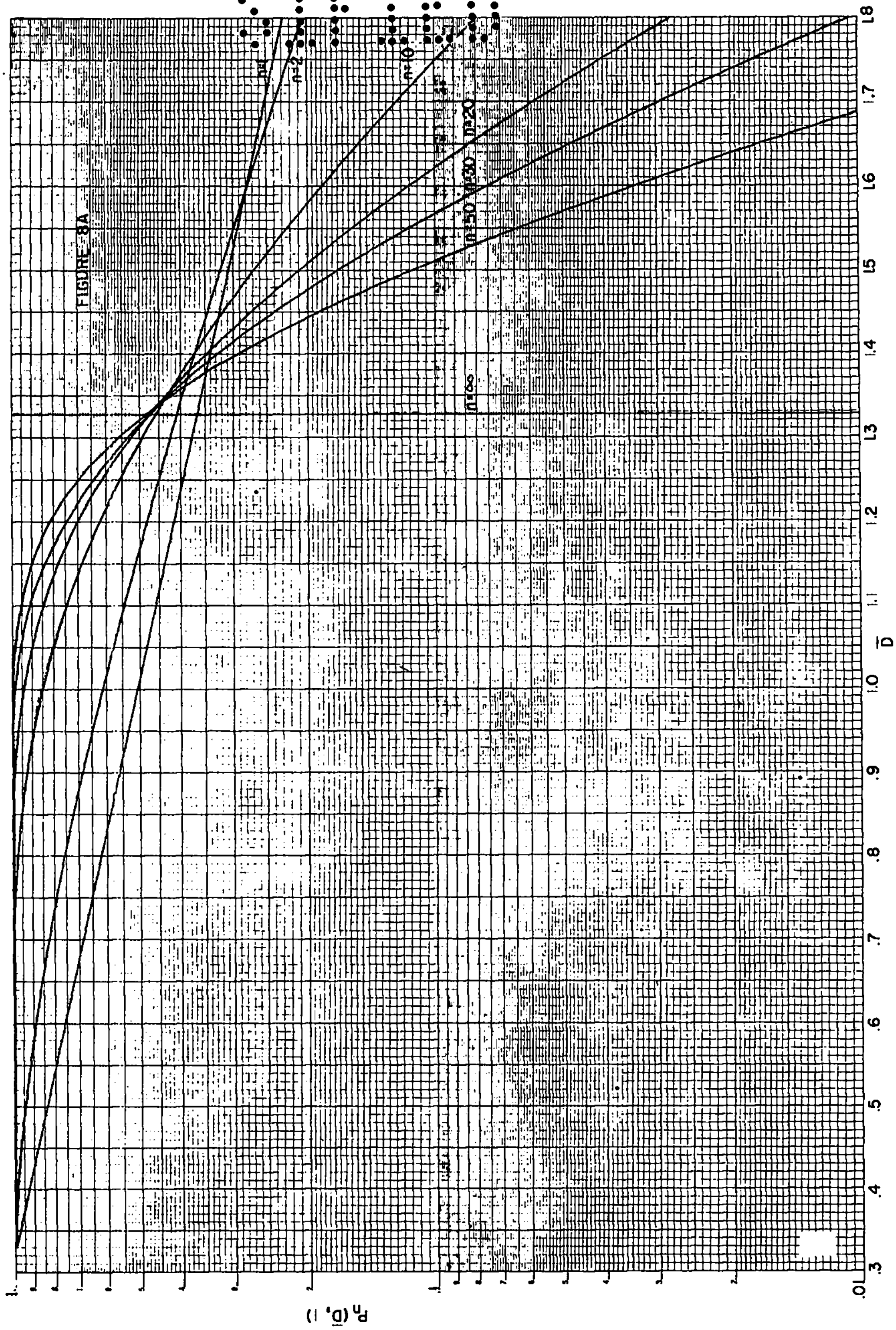
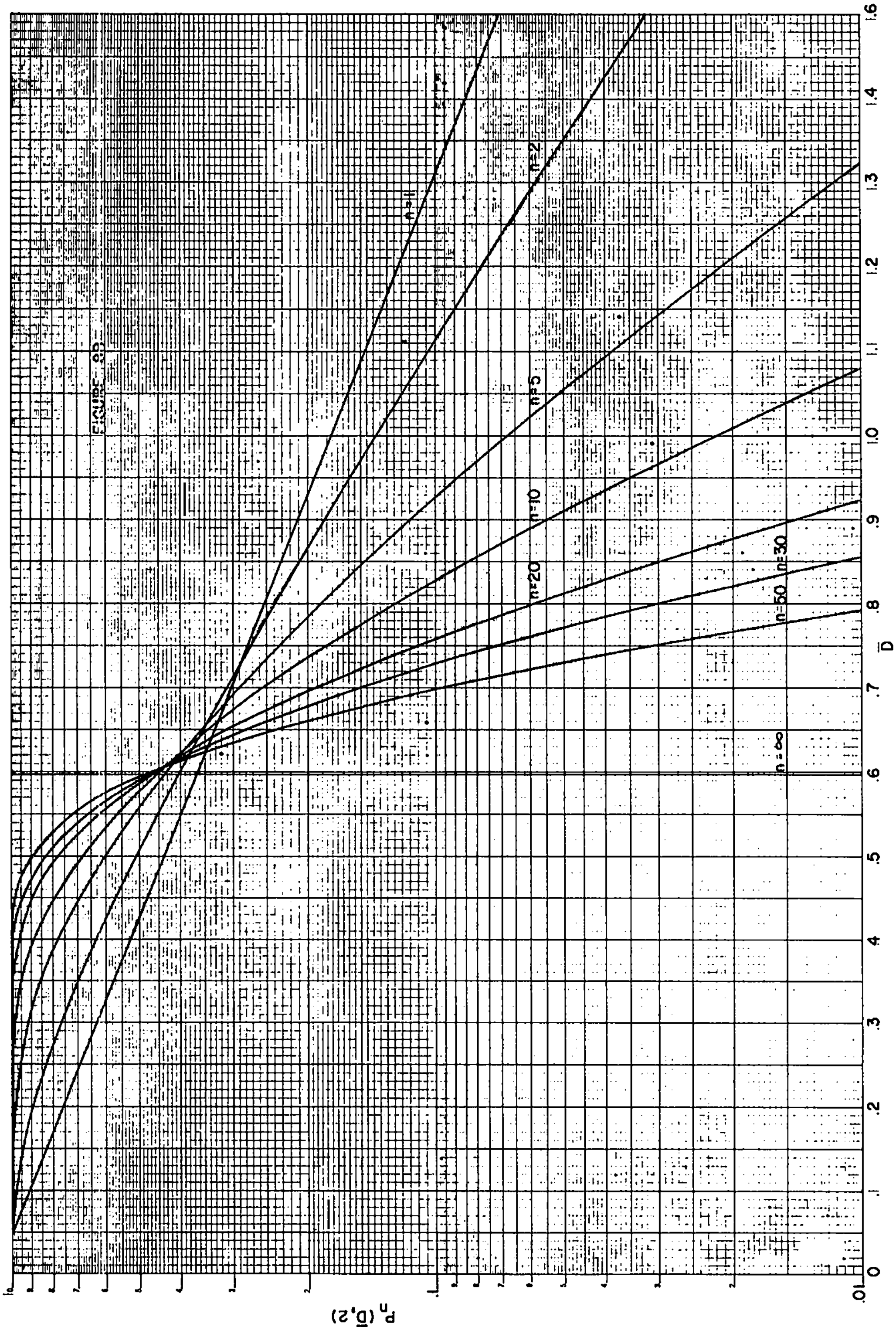


FIGURE 8A
n=1
n=2
n=10
n=20
n=30
n=50
n=∞
$\bar{D}$
$P_n(\bar{D}, 1)$
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
.01
.1
1



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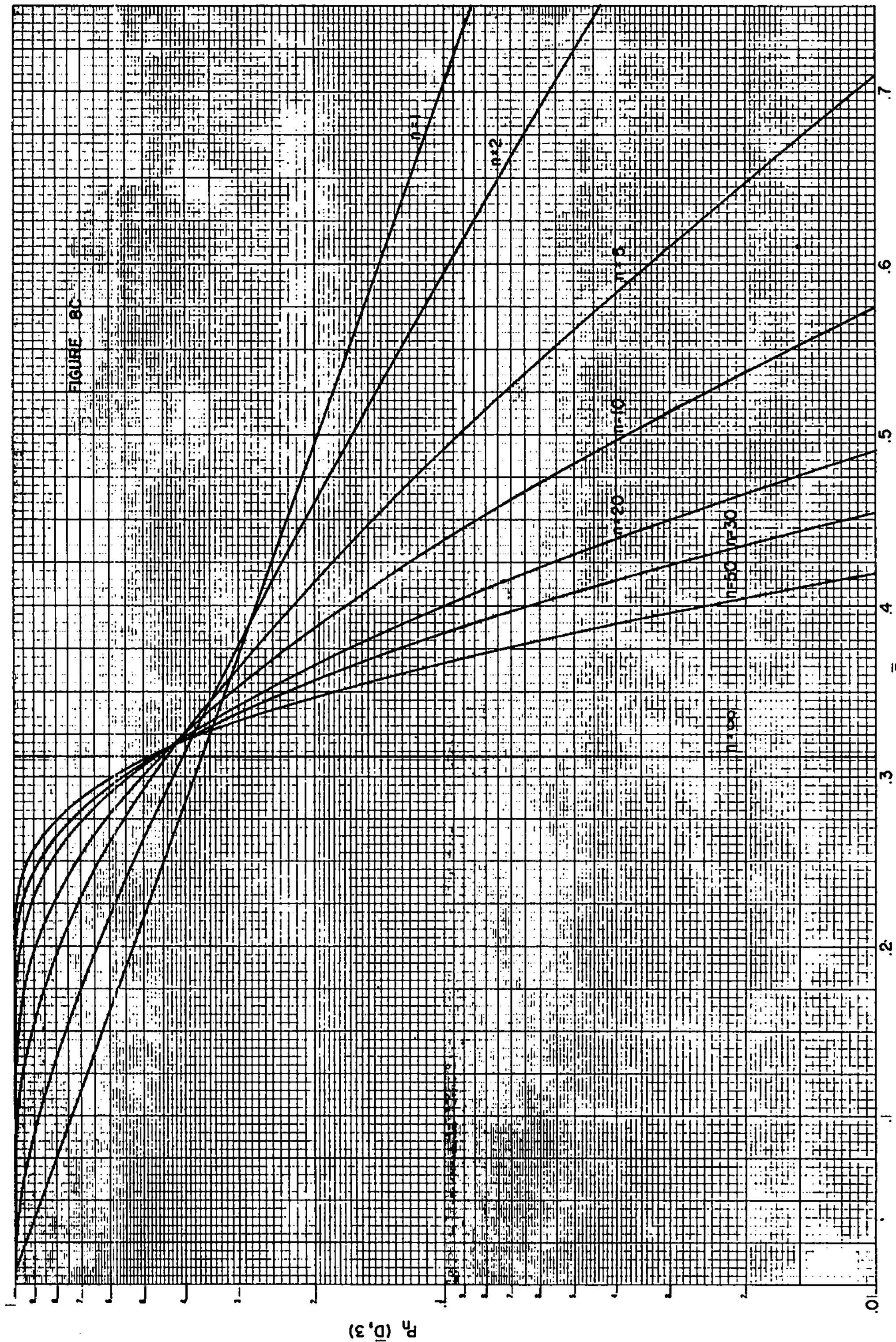
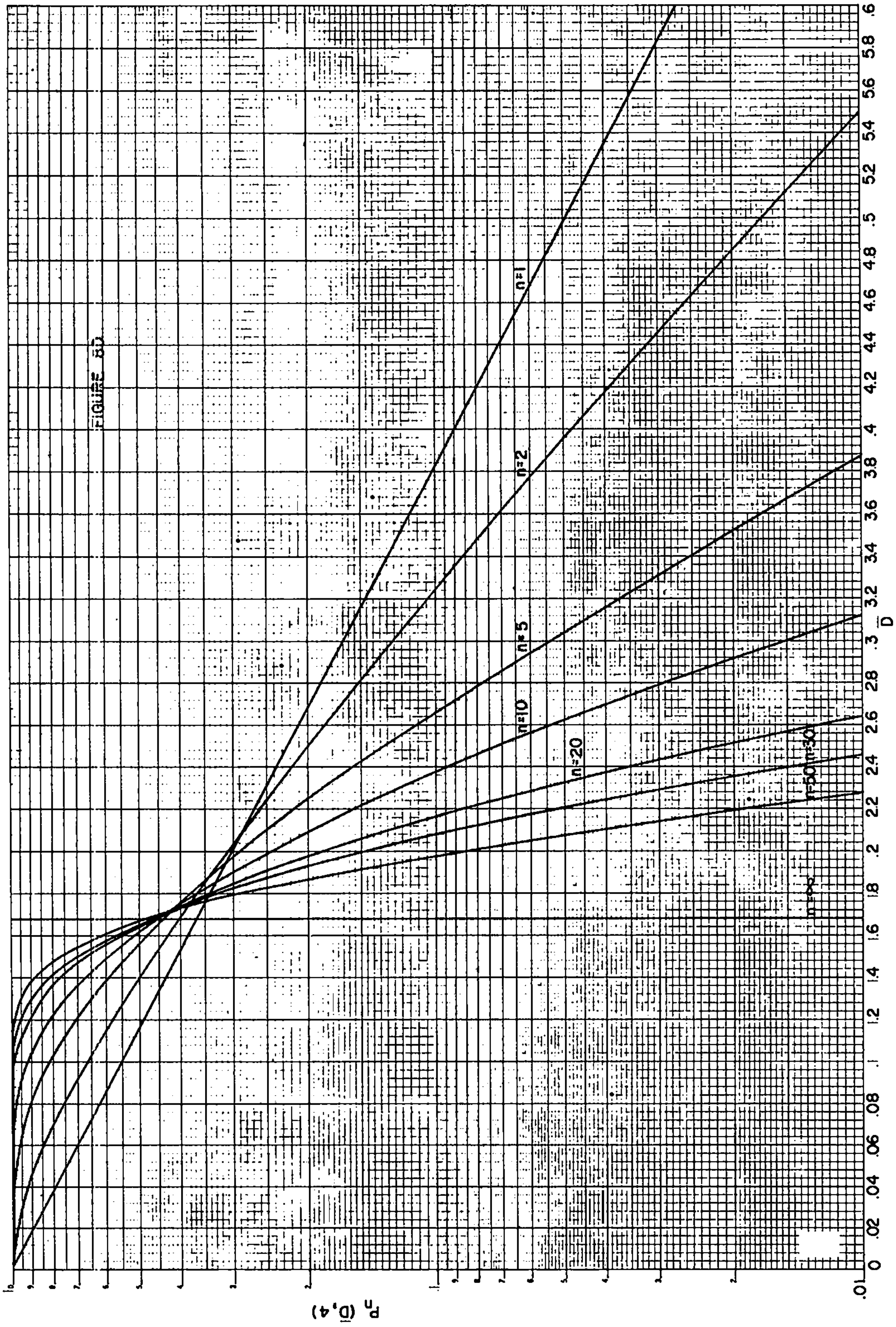
[redacted]



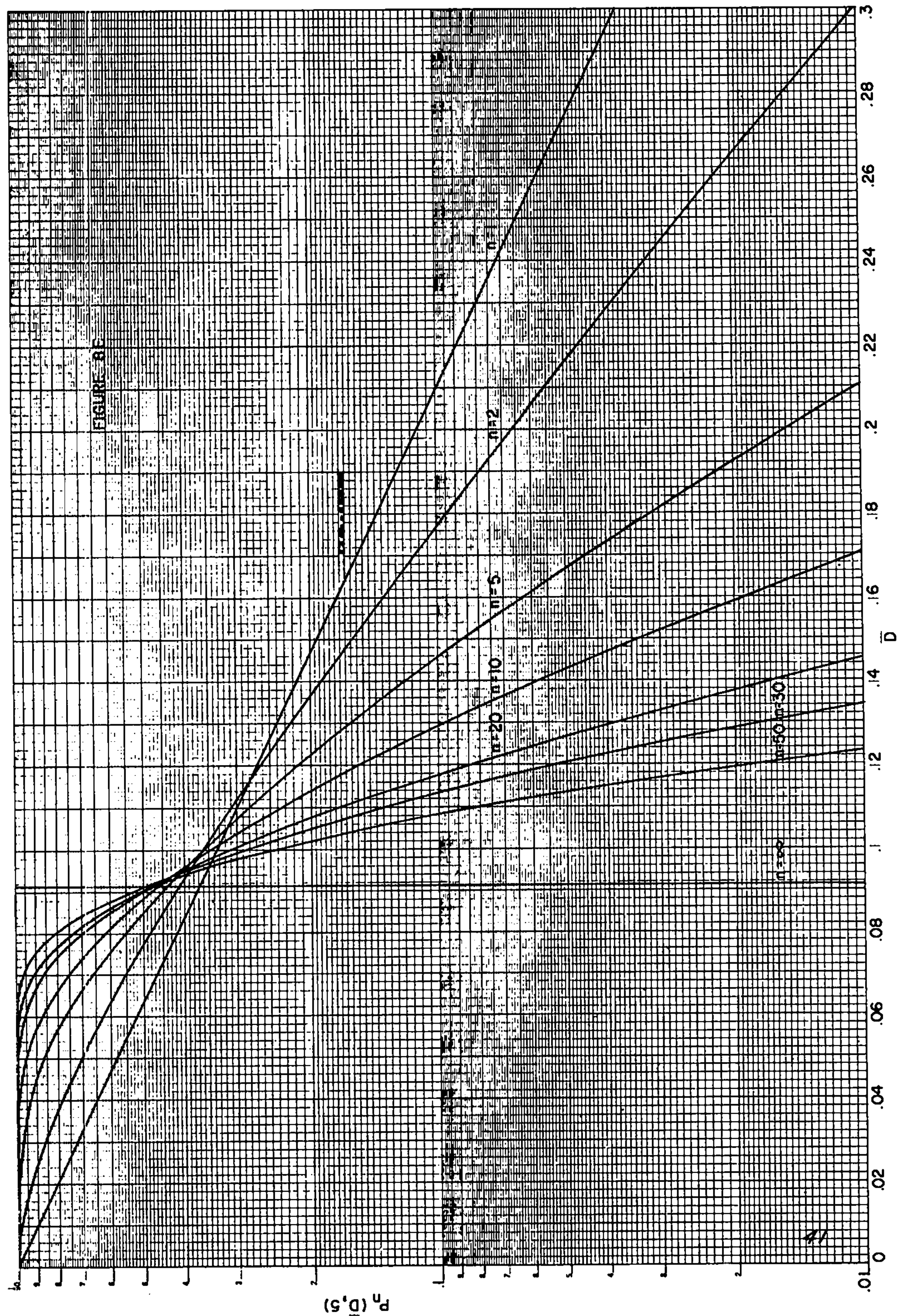
[redacted]

[redacted]



[redacted]

[redacted]



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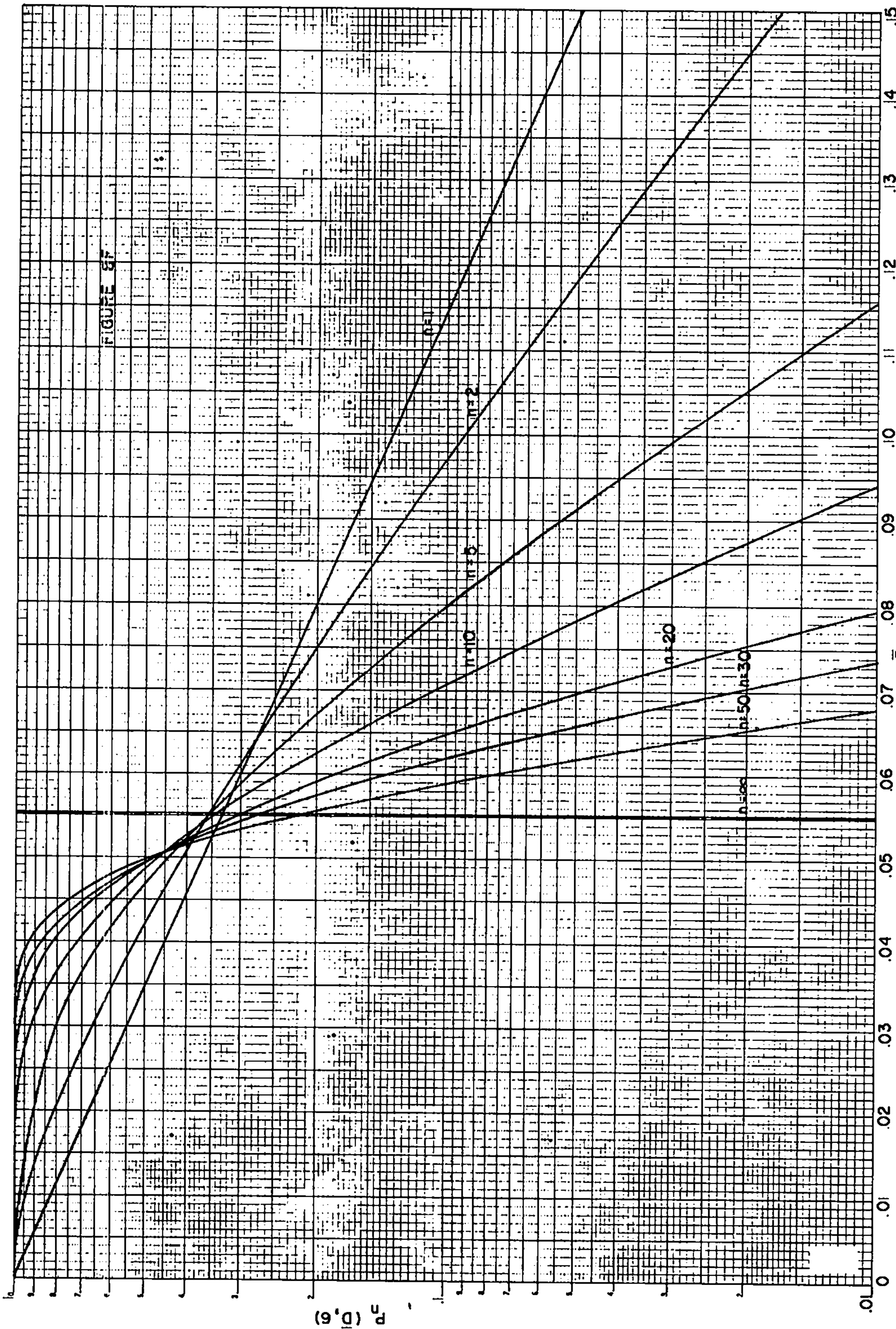

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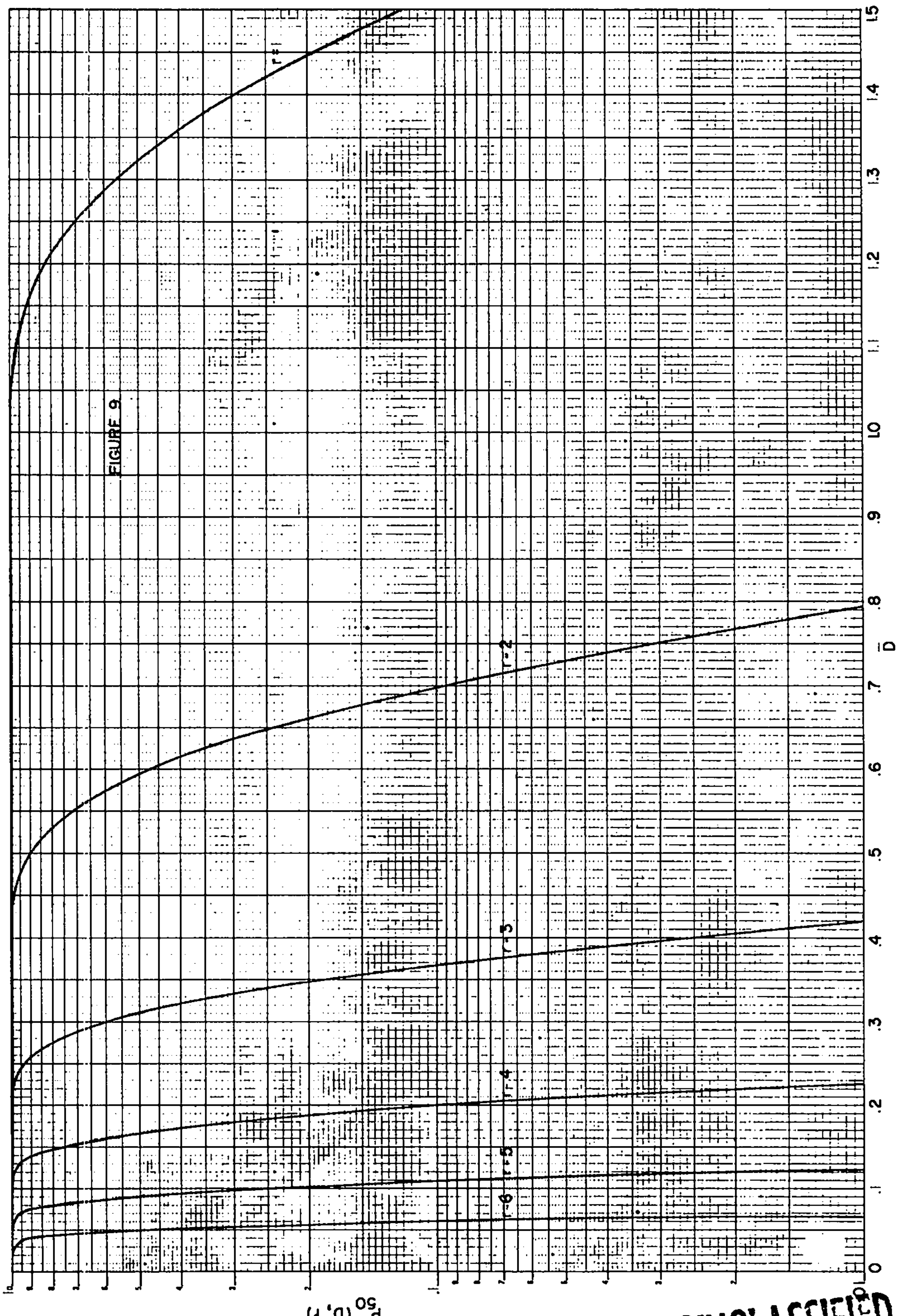
FIGURE 9
r=1
r=2
r=3
r=4
r=5
r=6
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
D̄
P50(D̄, r)

We could then have proceeded in exactly the same fashion as before, without having our thoughts limited by the "sphere" concept.

Our method for investigating the "cloud" associated with the solution of Figures 8 and 9 will be to work the problem in reverse. We want to determine the function g such that

$$-\Sigma_f(\bar{D}) \equiv -g\varphi\cdot\varphi'(\bar{D}) = B e^{-B(\bar{D}-\bar{D}_0)}\, u(\bar{D}-\bar{D}_0)$$

which, from paragraphs 2 and 5, can be written:

$$B e^{-B\left[\frac{I_1(\alpha)}{\alpha} - \frac{I_1(r)}{r}\right]}\, u(\bar{D}-\bar{D}_0) = -g(\alpha)\cdot\frac{1}{\frac{d}{d\alpha}\left[\frac{I_1}{\alpha}\right]}$$

or, more simply:

$$g(\alpha, r) = \frac{B}{\alpha}\left(I_2 + \frac{I_1}{\alpha}\right) e^{-B(\bar{D}-D_0)}\, u(\bar{D}-\bar{D}_0) \qquad (8.1)$$

where both B and $\bar{D}_0$ contain the dependence of g upon r . We thus see that our solution for P $(\bar{D}, r)$ is valid for any "cloud" such that g (α, r) is given by equation (8.1). Figure 10 exhibits the functional dependence of g upon α , for the interesting values of r , depicted by this equation. For purposes of comparison, Figure 11 is a similar graphical representation of equation (5.1a).

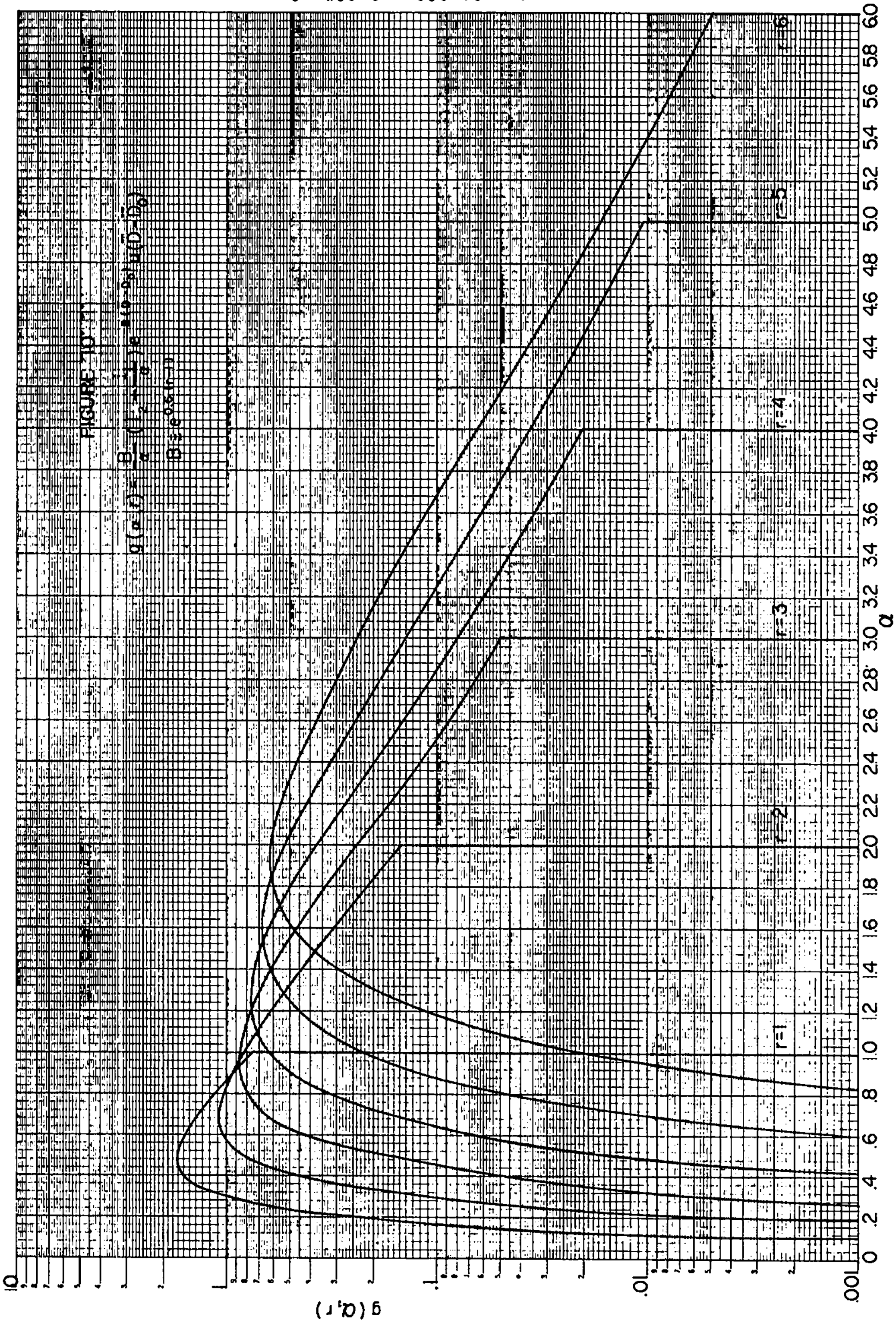
FIGURE 10
r=1
r=2
r=3
r=4
r=5
r=6
α
g(α,r)
0
.2
.4
.6
.8
1.0
2.0
3.0
4.0
5.0
6.0
10
1
.1
.01
.001

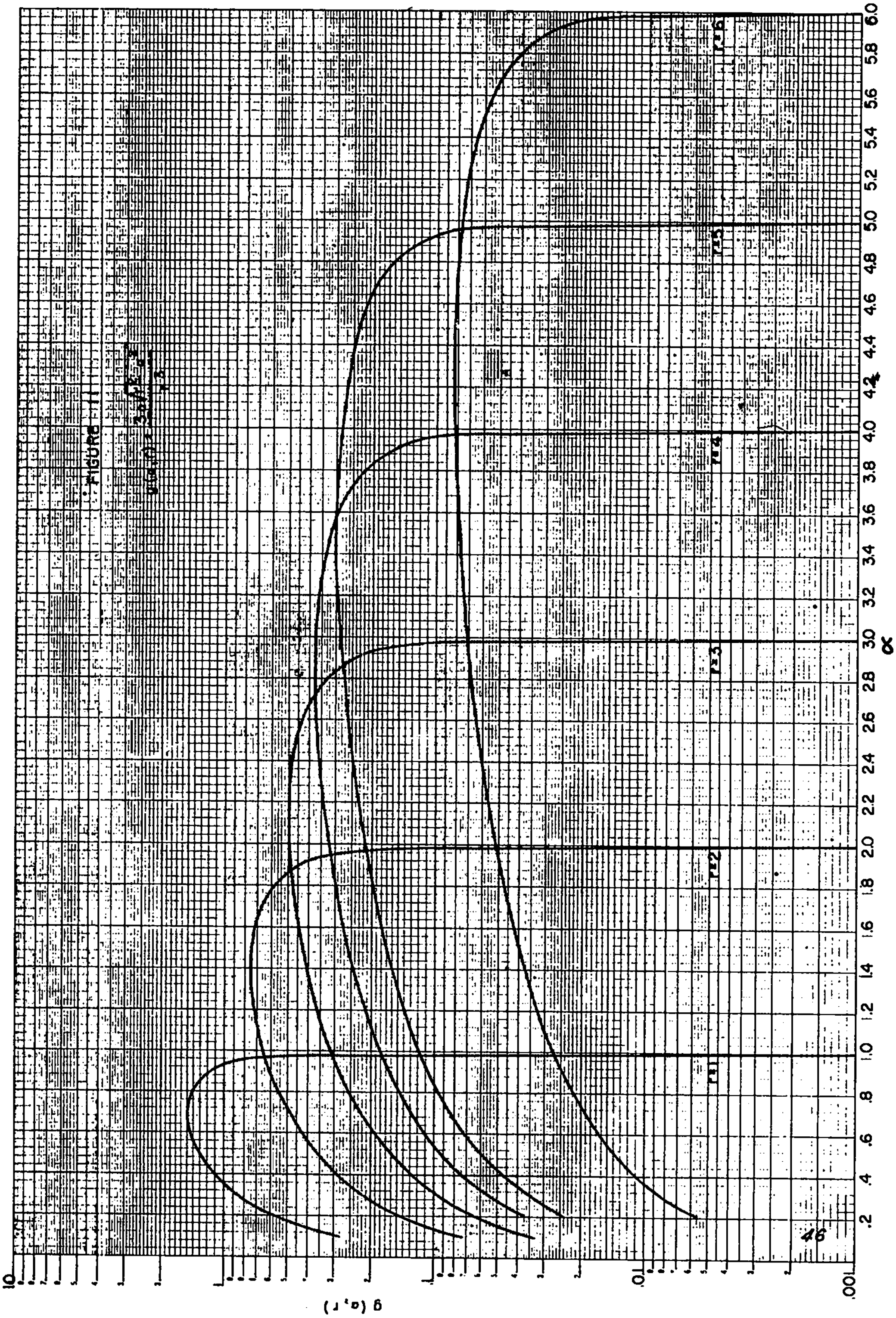
FIGURE 11
r=1
r=2
r=3
r=4
r=5
r=6
α
g(α,r)
46

Let us now define equivalence of clouds. Two clouds shall be said to be equivalent if the probable distribution of radiation dose obtained from aerial penetration of them, along a straight line path through the cloud centroid with constant velocity, is identical. Then to further describe the derived cloud, we take advantage of the fact that the entire class of point configurations described by (8.1) is equivalent to a particular solid of revolution about ψ. But every such solid of revolution can be described by

$$g(\alpha, r) = \frac{2\pi\alpha\beta(\alpha)}{V(r)} \tag{8.2}$$

where $V(r)$ is the volume of the solid and $\beta(\alpha)$ is the intersection of the solid with a line parallel to ψ at a distance α. Hence

$$\frac{\beta(\alpha)}{V(r)} = \frac{g(\alpha, r)}{2\pi\alpha} \tag{8.3}$$

is an equation which enables us to compare the physical contours of the derived cloud with those of any cloud for which the function $g(\alpha, r)$ can be calculated. Figure 12 contains one quadrant of such a comparison with the sphere of equation (5.1a) for the case of $r = 4$, which is representative of an atomic cloud at an altitude of approximately 40 thousand feet. In this figure, the axis of θ can be thought of as the flight path ψ. Also note that the volumes of revolution indicated are equivalent to the volumes of half-revolu-

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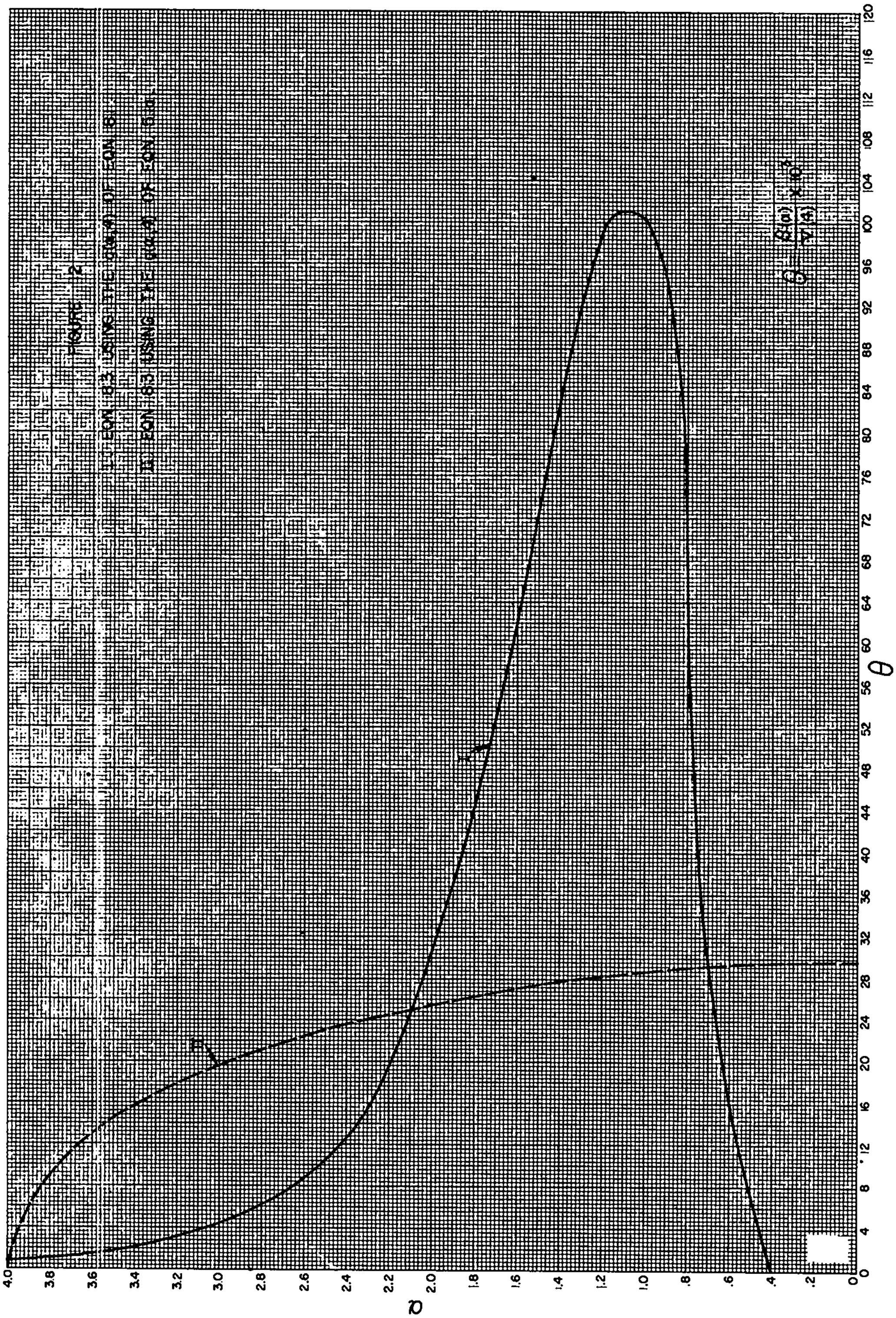

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tion obtained by folding the portion below the line $\alpha = 0$ into its image above that line, and that the configurations resulting from any sheer parallel to ψ are again equivalent.

9. In the writer's opinion, the most realistic source for evaluation of $\lambda = \frac{S\mu}{2\pi v}$ is the gamma ray experimental program report from Operation Greenhouse. That report is not yet available, but it is felt that the qualitative results contained herein are of sufficient interest in themselves to warrant publication at this time. When the above mentioned experimental results become available, a supplement to this report will be published, to include an evaluation of λ. If, subsequent to that time, the reader wishes to make quantitative estimates of $D = \lambda \bar{D}$, reference I is suggested as containing information from which crude values can be obtained for S and μ.

There is a possibility, of course, that the above mentioned Greenhouse report will not contain sufficient information for an experimental determination of λ. In this event, the proposed supplement will contain theoretical estimates of S and μ, based in part upon reference I.

10. No attempt will be made to draw conclusions as to the relative merits of our derived cloud as a model for tactical planning. In the writer's opinion, the derived model is at least as good as the spherical model, but such a statement is impossible to prove at the present time. A reasonable procedure is to delay such conclusions

until λ is evaluated - then comparisons can be made from the viewpoint of dose received.

It should also be noted that the procedure used herein is easily capable of developing other, and perhaps better, derived clouds. One needs only to select a different physical model from which to obtain the running integral curves of Figure 4. For instance, a simple starting model which appears highly interesting is the "ice cream cone" shape.

APPENDIX I

The reader may well be perplexed at the failure of the writer to approximate $p_1\ (\bar{D})$ by something other than a truncated exponential law. As a matter of fact, a truncated power law approximation would be highly desirable to augment this discussion because (see Figure 4) a combination of the power and exponential laws would allow the computed function to be bracketed in a certain sense. The following argument is designed to indicate the complexity of using such an approximation.

Let us assume that $p_1\ (\bar{D})$ can be approximated by:

$$p_1(\bar{D}) = A\ (\bar{D})^{-\beta}\ u\ (\bar{D} - \bar{D}_o) \qquad (I.1)$$

for the one point case, where both A and β are parameters dependent upon R. The value of A must then be determined by:

$$A \int_{\bar{D}_o}^{\infty} x^{-\beta}\, dx = 1$$

or

$$\frac{1}{A} = \frac{x^{1-\beta}}{1-\beta}\Bigg]_{\bar{D}_o}^{\infty} = \frac{1}{\beta-1}(\bar{D}_o)^{1-\beta} \ ; \ \beta > 1$$

We thus see that a meaningful definition of A is dependent upon β being greater than 1.

To extend this to the n point case, we write:

$$p_n = n \cdot n!\, \mathcal{L}^{-1}\left[\sigma_n(s)\right]^n$$

where

$$\sigma_n(s) \equiv \mathcal{L}\left\{A\, t^{-\beta} u(t-\mu)\right\} \qquad \text{(I.2)}$$

in which

$$t = n(\overline{D} - \overline{D}_0) + \overline{D}_0$$

and μ must be chosen in such a way that $p_n = 0$ for $\overline{D} = \overline{D}_0$

Hence:

$$\sigma_n(s) = A \int_{\mu}^{\infty} t^{-\beta} e^{-st}\, dt \quad ; \ \beta > 1$$

$$= \frac{A\, e^{-\mu s}}{s^n} \int_0^{\infty} (y + \mu s)^{n-1} e^{-y}\, dy \quad ; \ n < 0$$

$$= \frac{A\, \Gamma(n)}{s^n} \left\{ 1 - \frac{\Gamma_{\mu s}(n)}{\Gamma(n)} \right\} ; \ n \neq \text{neg. integer} \qquad \text{(I.3)}$$

Need we go further? The restriction of $n \neq$ a negative integer is hardly serious, but the formal problem of raising $\sigma_n(s)$ to the n^{th} power and finding the Laplace transform of the result is somewhat frightening. The truncated power law approximation was abandoned at this point.

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