LA-6744-MS

Informal Report

UC-34

Issued: March 1977

Stabilization of Short-Wavelength Disturbances in the Rayleigh-Taylor Instability of Plastic Solids by a Surface Layer of High Yield Strength

by

R. C. Mjolsness

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87545

UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
CONTRACT W-7405-ENG. 36

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161
Price: Printed Copy $3.50 Microfiche $3.00

STABILIZATION OF SHORT-WAVELENGTH DISTURBANCES IN THE RAYLEIGH-TAYLOR INSTABILITY OF PLASTIC SOLIDS BY A SURFACE LAYER OF HIGH YIELD STRENGTH

by

R. C. Mjolsness

ABSTRACT

The mechanical effects of a thin surface layer in reducing the growth of the Rayleigh-Taylor instability of a plastic solid plate is investigated by representing the surface layer as an elastic plate of high yield strength and high bonding strength to the plastic. This plate may be either the surface layer itself or a composite material formed by interactions of the surface layer with the solid surface layers. In this approximation it is found under wide ranges of conditions that short-wavelength disturbances are stabilized in linear approximation. In this limit the 'gravitational' energy increment of the displaced plastic solid is overcome by the elastic shear energy of the stretched plate. Long-wavelength disturbances are very weakly affected by this mechanism. For a plastic plate of ~1-cm thickness subjected to a pressure difference of ~100 kbar we expect wavelengths shorter than $\lambda_c \sim 10^{-1}$ cm to be effectively stabilized by this mechanism.

I. INTRODUCTION

In this report we examine whether the mechanical properties of a thin surface layer can be expected to stabilize, or partially stabilize, the observed[1] Rayleigh-Taylor instability of a plastic solid under strong acceleration. The surface layer may be just the material added to the surface or it may be composite material formed by interactions with the surface layers of the plastic, but it is treated here as a single material of uniform composition. The plastic is assumed to be an ordinary metal plate (such as aluminum or copper) which undergoes plastic transition when extreme ($\gtrsim$100 kbar) pressures cause stresses larger than the yield stress in the material.

If the surface layer has a sufficiently high yield strength to maintain its elasticity under the induced lateral compression and the instability induced longitudinal extension, and if the layer remains bonded to the plastic under the instability shear stresses at the interface, then the surface layer will stabilize moderately short wavelengths. We conjecture that very short wavelengths would be similarly stabilized. If the surface layer becomes plastic or does not remain bonded to the plastic plate, we do not see how it will be able to significantly affect the growth of the instability.

II. ENERGY ANALYSIS FOR PLASTIC SOLID

Here we summarize the main points made in an analysis of the kinetic, gravitational, and plastic energies of the plastic plate alone by Miles.[2] Before doing this we would like to discuss an apparent difficulty that has been found[1] in the Miles analysis.

Miles gives essentially two analyses of the problem. Case (1): The first is valid at <u>very</u> small surface displacements

$$ka(t) \ll s_1/G \ , \qquad (1)$$

where k is the wavenumber of the perturbation, a(t) is the amplitude of the surface displacement, s_1 is the yield stress in shear, and G is the shear modulus. Miles assumes the Prandtl-Reuss constitutive equations, assumes the material makes a transition to plastic flow uniformly throughout the material and calculates the stresses by a particular symmetry assumption. In this regime the plastic energy can be comparable to the other energies of the problem. It makes a significant contribution to the damping of the instability, having a mathematical effect similar to the energy of surface tension at a fluid interface in cutting off short-wavelength instabilities. Specifically, he finds[2]

$$\alpha = \left[kg - \frac{13}{4} \frac{G}{\rho} k^2 \right]^{1/2} \ , \qquad (2)$$

where α is the growth rate of the mode and ρ is the density of the plastic.

Case (2): The second analysis does not assume very small amplitudes but assumes the von Mises rigid-plastic constitutive equations and is only valid when

$$s_1 << G. \tag{3}$$

In this regime the plastic energy does not increase as rapidly as the other energies as the amplitude of the displacement increases. Thus the plastic energy makes a comparatively minor contribution to the damping of instabilities. The growth rates are modified considerably at very early times but the modification becomes very small before very long. The theory is not particularly likely to give a good estimate of the small effects of the plastic energy in slowing the growth of the disturbances, since at the very early time at which the effects are significant the material is likely to be in evolution toward rigid-plastic behavior and not well represented by the von Mises equations.

One of the contributions of Barnes et al.[1] was to recognize that precise, two-dimensional calculations of the nonlinear theory underlying Miles' case (1) yield results in a particular set of runs which disagree markedly with the growth rate of Eq. (2). In fact, supplemental work[3] showed that a formula fairly similar to Eq. (2), but with G replaced by s_1, represented their results fairly well. Additionally, their results agree with the available experiments.

An inference from these results which one might consider drawing is that something must be basically wrong with the Miles analysis, since it fails so badly to agree with either calculations or experiment. Actually the disagreement results only from the fact that a comparison of the numerical and experimental results with Eq. (2) never should have been made. The initial amplitude of the displacement machined into the sample and assumed in the calculation is already large enough to violate Eq. (1), the criterion that must be satisfied if Eq. (2) is to follow as a valid prediction. Thus, Miles made no major theoretical error. He merely was able to treat analytically a very small portion of the parameter space of the problem, and Ref. 1 indicates how substantial are the corrections to be expected from a full theoretical treatment of the problem. We can thus adopt substantial portions of Miles' analysis without making gross errors in the physics.

In particular, we work in the accelerated reference frame in which the unperturbed plastic plate of thickness h is at rest and its unperturbed unstable surface is at z = 0. We consider surface perturbations of the form $z = \eta(x,t)$. where

$$\eta(x) = a(t)\cos kx, \tag{4}$$

and assume that the motion of the plastic is incompressible and adequately described by a single mode of a scalar potential. We work in the regime in which

$$kh \gg 1. \tag{5}$$

Then[2] to good approximation, the kinetic energy in the plastic motion per unit length in the y-direction and per wavelength in the x-direction, E_{kin}, is specified by

$$E_{kin} = \frac{1}{8\pi}\rho\lambda^2\dot{a}^2(t), \tag{6}$$

where λ denotes the wavelength of the disturbance. Similarly, the gravitational energy generated by the displacement of the plastic is given[2] to equal accuracy by

$$E_{grav} = -\frac{1}{4}\rho\lambda g a^2(t), \tag{7}$$

where E_{grav} is the gravitational energy per unit length in the y-direction and per wavelength in the x-direction. All of the energies computed in this report are normalized to the same dimension in the x-and y-directions and all carry formally the dimensions of energy-per-unit length.

In the regime that we are considering (i.e., not restricted to very small amplitude) the plastic energy will normally be a fairly small influence in stabilizing the instability although it is fairly difficult to calculate its precise role. We will therefore make a conservative analysis of the stabilization of the instability by neglecting the plastic energy entirely. We can thus anticipate that the instability will grow less than is indicated by the analysis given here. We emphasize that we are not assuming that the plastic energy of the plate is negligible--frequently it will be comparable to the other energies of the problem. We merely seek a conservative estimate of the stabilizing effects of the surface layer which is also as mathematically tractable as is feasible.

III. ELASTIC ENERGY OF THE SURFACE LAYER

We treat the surface layer as an elastic plate of thickness ℓ, where

$$\ell \ll \lambda \, , \tag{8}$$

and thus $\ell \ll h$ as well. In addition, we assume that the layer is stress-free when $a(t) = 0$. The restriction of Eq. (8) simplifies the mathematics considerably, in that we may obtain a first approximation to the elastic free energy by treating the surface layer as a flat elastic plate under tension and subject to the strong equivalent gravitational field

$$g = \frac{1}{\rho h} \Delta p \, , \tag{9}$$

generated by the pressure difference across the plastic solid, Δp, and by neglecting the kinetic and gravitational energies of the surface layer with respect to the kinetic and gravitational energies, respectively, of the plastic plate. But the formalism thereby loses the capacity to deal with very short wavelength disturbances. However, since we find a linear stabilization mechanism that is the more effective the shorter the wavelength (within the validity domain of Eq. (8)) we will argue that the very short wavelengths must be stabilized too, although we will not compute their behavior.

We denote, as usual,[4] the internal stresses of the surface layer by σ_{ij} and the strains by u_{ij}. We assume that the surface layer occupies the region $-\ell \leqslant z \leqslant 0$ and that stresses appear on the surfaces x = const. and y = const. in order that: (1) no strains occur in the y-direction (the elastic surface layer is bounded to the surface of the plastic and this surface does not elongate in the y-direction) and (2) the extension in the x-direction is a simple homogeneous extension (and not sheared due to gravitationally induced strains; also, shears due to the curvature of the plastic surface are treated as a higher order perturbation). The surface layer elongates in the z-direction under the action of the 'gravitational' field and stresses occur in the z-direction to support the weight of the layer. The equations of elastic equilibrium[4] then have the solution

$$\sigma_{ij} = 0 \quad i \neq j ,$$

$$\sigma_{xx} = \frac{\sigma}{(1-\sigma)} \left(\frac{\rho_e}{\rho}\right)\left(\frac{\ell}{h}\right)\left(1 + \frac{z}{\ell}\right) \Delta p + \frac{1}{4} \frac{E}{(1-\sigma^2)} k^2 a^2(t) ,$$

$$\sigma_{yy} = \frac{\sigma}{(1-\sigma)} \left(\frac{\rho_e}{\rho}\right)\left(\frac{\ell}{h}\right)\left(1 + \frac{z}{\ell}\right) \Delta p + \frac{1}{4} \frac{\sigma E}{(1-\sigma^2)} k^2 a^2(t) ,$$

and

$$\sigma_{zz} = \left(\frac{\rho_e}{\rho}\right)\left(\frac{\ell}{h}\right)\left(1 + \frac{z}{\ell}\right) \Delta p , \tag{10}$$

where ρ_e, σ, and E are the density, Poisson ratio and Young's modulus, respectively, of the elastic surface layer, and

$$u_{ij} = 0 \quad i \neq j ,$$

$$u_{xx} = \frac{1}{4} k^2 a^2 ,$$

$$u_{yy} = 0 , \tag{11}$$

and

$$u_{zz} = \frac{(1+\sigma)(1-2\sigma)}{(1-\sigma)} \left(\frac{\rho_e}{\rho}\right)\left(\frac{\ell}{h}\right)\left(1 + \frac{z}{\ell}\right)\left(\frac{\Delta p}{E}\right) - \frac{1}{4} \frac{\sigma}{(1-\sigma)} k^2 a^2(t) .$$

In the above, the relations

$$\frac{\delta L}{L} = u_{xx} = \frac{1}{4} k^2 a^2(t) \tag{12}$$

follow for the small amplitude extension of the plastic surface undergoing sinusoidal displacement. The remainder follow from equations of elastic equilibrium, the boundary conditions, and the stress-strain relations.

By substituting the above expressions into the defining equation[4] for the free energy E_{free} associated with the elastic displacements of the surface layer under tension we obtain

$$\mathcal{E}_{\text{tension}} = \frac{1}{6}\,\frac{(1+\sigma)(1-2\sigma)}{(1-\sigma)}\left(\frac{\rho_e}{\rho}\right)^2\left(\frac{\ell}{h}\right)^2\left(\frac{\Delta p}{E}\right)\Delta p\ell\lambda + \frac{1}{32}\,\frac{E\ell\lambda}{(1-\sigma^2)}\,k^4a^4(t)\ . \tag{13}$$

The first term of Eq. (13) is a constant energy associated with the straining of the elastic by the gravitational fluid. The second term gives rise to nonlinear corrections to the growth of the Rayleigh-Taylor instability.

We have so far presumed that the elastic surface layer is a flat plate subject to displacements δr_x and δr_y in the x- and y-directions, where

$$\delta r_x = \frac{1}{4}\,k^2a^2(t)x \tag{14}$$

and

$$\delta r_z = \frac{(1+\sigma)(1-2\sigma)}{(1-\sigma)}\left(\frac{\rho_e}{\rho}\right)\left(\frac{\ell}{h}\right)\left(\frac{\Delta p}{E}\right)\left(z + \frac{1}{2}\,\frac{z^2}{\ell}\right) - \frac{1}{4}\,\frac{\sigma}{(1-\sigma)}\,k^2a^2(t)z\ .$$

To this must be added the shear displacement

$$(\delta r_z)_{\text{shear}} = -a(t)\cos kx \tag{15}$$

corresponding to the motion of the surface layer in adapting itself to the displacement of the surface of the plastic. This motion generates the strain

$$(u_{xz})_{\text{shear}} = \frac{1}{2}\,ka(t)\sin kx \tag{16}$$

and the stress

$$(\sigma_{xz})_{\text{shear}} = \frac{1}{2}\,\frac{E}{(1+\sigma)}\,ka(t)\sin kx\quad , \tag{17}$$

and thus gives rise to the additional free energy of shear displacement[4]

$$E_{shear} = \frac{1}{16} \frac{E\ell\lambda}{(1+\sigma)} k^2 a^2(t) \quad . \tag{18}$$

Now the second term of Eq. (13), for the elastic energy under tension, is clearly of order $k^2a^2(t)$ times the elastic energy under shear given by Eq. (18). Thus this term should be dropped in a calculation which only uses the dominant term, Eq. (18), of the elastic energy under shear. This we will do in the following development. It is clear physically that an additional stabilizing mechanism (for nonlinear stabilization) is thereby neglected. However, it turns out that the range of wavelengths and applied pressures for which the elastic fourth-order terms significantly exceed the fourth-order terms (of order $\sim k^2a^2(t)|E_{grav}|$) of the plastic plate roughly coincides with the range of conditions for which Eq. (18) stabilizes the Rayleigh-Taylor instability. Outside this range of conditions it is not known whether the total effect of the nonlinear terms is stabilizing or not. It was judged too difficult to attempt to produce a valid nonlinear theory of the effect here. Subject to the above restrictions we will adopt the expression

$$E_{elastic} = \frac{1}{6} \frac{(1+\sigma)(1-2\sigma)}{(1-\sigma)} \left(\frac{\rho_e}{\rho}\right)^2 \left(\frac{\ell}{h}\right)^2 \left(\frac{\Delta p}{E}\right) \Delta p\ell\lambda + \frac{1}{16} \frac{E\ell\lambda}{(1+\sigma)} K^2a^2(t) \tag{19}$$

for the elastic energy of the surface layer.

IV. STABILIZATION OF THE RAYLEIGH-TAYLOR INSTABILITY

The three energies E_{kin}, E_{grav}, and $E_{elastic}$ of Eqs. (6), (7), and (11), respectively, define a dynamical system for the generalized coordinate a(t). The equation of motion for a(t) may be put in Lagrangian form by substituting

$$T(\dot{a}) = E_{kin} \text{ and } V(a) = E_{grav} + E_{elastic} \; , \tag{20}$$

and resulting Euler-Lagrange equations take the form

$$\frac{1}{4\pi} \rho\lambda^2 \ddot{a}(t) = \left[\frac{1}{2} \rho\lambda g - \frac{1}{8} \frac{E\ell\lambda}{(1+\sigma)} k^2\right] a(t). \tag{21}$$

Thus the system will be stable to small amplitude disturbances when

$$\frac{\pi^2}{2} \frac{E}{(1+\sigma)} \left(\frac{\ell}{\lambda}\right) > \frac{1}{2} \rho\lambda g \ , \tag{22}$$

or, independently of Poisson's ratio, when

$$\left(\frac{\lambda^2}{h\ell}\right)\left(\frac{\Delta p}{E}\right) < \pi^2 \ . \tag{23}$$

When the applied pressure difference Δp is small enough and the wavelengths λ are short enough that Eq. (23) is satisfied, we expect to find that the suface layer indeed stabilizes the disturbances, provided that the stresses in the surface layer are small enough that the layer remains elastic and remains bonded to the plastic. This involves principally a restriction on the initial perturbation size to very small amplitude.

It should be noted that, according to Eq. (23), the shorter wavelengths are farther from the stability boundary, once the wavelength is sufficiently short to stabilize at all. It is on this basis that we believe that very short wavelengths should be stable to the Rayleigh-Taylor instability. At very short wavelengths we do not lose the stabilizing mechanism of E_{shear}; it is merely that our particular approximations for computing E_{shear} are invalid in this limit.

V. WILL THE SURFACE LAYER REMAIN ELASTIC AND REMAIN BONDED TO THE PLASTIC?

To discuss the applicability of our model under the assumption that the surface layer is stress-free at a = 0, we calculate the several stresses in the surface layer and see whether any of them exceed the relevant yield strengths.

Y_t = yield strength in tension of surface layer

B_t = bond strength in tension between layer and plastic

B_s = bond strength in shear between layer and plastic. (24)

We calculate the stresses at $a = a_{max}$ to obtain a conservative estimate of whether yield strengths are exceeded.

The surface layer will remain elastic if the maximum of the shear stress σ_{xx} does not exceed Y_t. This yields the criterion

$$\frac{\sigma}{(1-\sigma)} \left(\frac{\rho_e}{\rho}\right)\left(\frac{\ell}{h}\right) \Delta p + \frac{1}{4} \frac{E}{(1-\sigma^2)} k^2 a^2(t) . \tag{25}$$

This criterion will be satisfied for a wide range of pressure pulses and instability wavelengths provided that Y_t is in the kilobar range. It should perhaps be emphasized that all the elastic parameters, yield strengths, and bond strengths of this report refer to material conditions after the system may have been shock heated by a shock that provides a portion of 'gravitational' field g. These can easily involve temperatures of several hundred degrees Celsius, and thus the correct values of parameters will often not be measured.

Similarly, the requirement that the forces required to support the surface layer in the large 'gravitational' field g should not exceed B_t may be stated as

$$\left(\frac{\rho_e}{\rho}\right)\left(\frac{\ell}{h}\right) \Delta p < B_t \quad . \tag{26}$$

This criterion is again easy to meet if B_t is in the kilobar range. But this, in turn, requires a strong bonding of the surface layer to the plastic.

The most severe requirement for the validity probably comes from the stipulation that the peak shear stress of Eq. (17) should not exceed B_s, namely,

$$\frac{1}{2} \frac{E}{(1+\sigma)} \; ka(t) < B_s \quad . \tag{27}$$

For attainable values of E and B_s this is likely to restrict the stabilization effect to very small amplitudes (small ka). For wavelengths stable according to Eq. (23), a(t) is of the order of the initial perturbation size for one e-folding period and then decays away. If the initial perturbation is sufficiently large it is quite possible that the bonds between the plastic and the surface layer will yield to the shear stress before the perturbation is damped to the range of safe values of the shear stress. If this happens, any subsequent stabilizing effect of the surface layer is probably minimal, since the surface layer will pull away from the plastic and tend to form a separate mechanical system of its own. It should also be noted that the requirements of Eq. (27) are largely opposed to the requirements of two of the other validity conditions for the stabilizing effect, namely, Eqs. (23) and (26). In the latter equations small Δp and large E increase the range of wavelengths satisfying the criteria. Also, small λ satisfies the criteria when other quantities are fixed. But in Eq. (27) small E is needed and for fixed parameters, large λ is needed.

To fix ideas about what the several criteria imply, let us consider a numerical example in which parameters in the plastic are chosen to be similar to those adopted in previous numerical calculations.[1,3] We will estimate stresses as though the plastic were flat when the surface layer was added. In actual fact the sample would probably be prepared by machining in the initial perturbation and then adding the surface layer. In this case only the stability estimate is relevant when the initial perturbation is stable. Then the surface layer remains nearly stress-free throughout its history. We take

$$h = \frac{1}{4} \text{ cm},$$

$$\lambda = \frac{1}{2} \text{ cm},$$

$$ka_o = 0.1,$$

$$\Delta p = 100 \text{ kbar},$$

and

$$\rho = 7.6 \text{ cm}^{-3} , \tag{28}$$

where a_o is the initial amplitude displacement of the interface. Note that for these conditions $hk \sim 3$ and just barely satisfies our requirements, while we will see that the initial amplitude is almost too large for effective stabilization to be likely. For the surface layer we take

$$\frac{\rho_e}{\rho} = \frac{1}{4} ,$$

$$\ell = 2.5 \times 10^{-3} \text{ cm} ,$$

$$\sigma = \frac{1}{4} \quad , \tag{29}$$

and

$$E = 200 \text{ kbar} .$$

Then substituting into the preceding formulas we see that the stabilization mechanism is operative for

$$\lambda^2 < 8\pi^2 \times 10^{-2} \text{ cm}^2 \quad , \tag{30}$$

and so it is operative for our assumed perturbation. Also, the surface layer will remain elastic if

$$Y_t > 0.6 \text{ kbar}. \tag{31}$$

This is consistent with our general conclusion that significant stabilization of short-wavelength disturbances should occur for surface layers that are stress-free at a = 0 when yield strengths are in the kilobar range. However, the value of E adopted here is fairly small. Particular materials could display yield criteria for Y_t and B_s an order of magnitude larger than we calculate.
The surface layer plastic bonds will be able to resist the pull of the 'gravitational' field if

$$B_t > \frac{1}{4} \text{ kbar} \quad , \tag{32}$$

a condition that ought to be easy to meet. Finally, these bonds will be able to resist the shearing stresses provided that

$$B_s > 8 \text{ kbar} \quad . \tag{33}$$

This condition is obviously a more stringent one, but it might be possible to meet it. It is clear, though, that significantly larger values of ka_0 would probably lead to failure of the stabilization through rupturing of the surface layer plastic bonds.

When the initial perturbations are machined in before the surface layer is added to the plastic, the surface layer is initially stress-free and will remain so for stabilized perturbations. In many device configurations, where one is worried primarily about the fastest growing short-wavelength perturbations, this mechanism could be quite useful, since growth of the dangerous perturbations is inhibited. The unstabilized long wavelength instabilities will set up stresses that rupture the surface layer, but this may well happen on a slow enough time scale to be tolerable.

We emphasize again that we have made no specific assumption about the processes bonding the surface layer to the plastic. We assume that whatever processes are involved are likely to involve a spatial thickness of at most a few tens of Angstroms. Thus when we represent the composite surface layer (of pure surface layer material and whatever material that results in the bonding layer) through a uniform material with properties ρ_e, σ, and E, these properties are likely to be near those of the pure surface layer material. We merely characterize these bonds by the stresses B_t and B_s necessary to rupture them. We assume, but we don't know, that it is reasonable to find materials whose bond strengths are comparable to the yield stresses in ordinary materials (several to tens of kbar).

When the various stresses exceed Y_t, B_t, or B_s we expect that the surface layer will cease to be able to stabilize disturbances. This could only fail to be the case if the surface layer were able to alter the bulk properties of the plastic material in a stabilizing direction for a thickness of roughly a wavelength (that is, over an appreciable fraction of a centimeter). We do not know how likely this possibility is. Recent experiments[5] indicate that such a change in bulk yield strength can occur under static loading conditions.

VI. STABILIZATION MECHANISM WHEN INITIAL PERTURBATION MACHINED INTO PLASTIC

When the surface layer is added on after the initial perturbation is machined into plastic (by unavoidable machining errors or deliberately, as in Barnes et al.[1]) the initial state of the surface layer (at $a(t) = a_0$) will be stress-free. This changes the elastic energy of the surface layer to

$$E_{elastic} = \frac{1}{6}\frac{(1+\sigma)(1-2\sigma)}{(1-\sigma)}\left(\frac{\rho_e}{\rho}\right)^2\left(\frac{\ell}{h}\right)^2\left(\frac{\Delta p}{E}\right)\Delta p\lambda\ell$$

$$+ \frac{1}{16}\frac{E\lambda\ell}{(1+\sigma)}\left[1 + \frac{2}{(1-\sigma)}k^2 a_o^{\,2}\right]k^2(a-a_o)^2 . \qquad (34)$$

Thus, there will be a slight shift in the range of stabilized wavelengths which we will ignore because normally we will have

$$k^2 a_o^{\,2} << 1 . \qquad (35)$$

Then the principal change in the dynamics of a(t) is that the stabilizing elastic potential well is now centered about a_o. In particular, the potential energy for the dynamical coordinate a(t) may be approximated as

$$V(a) = \frac{\pi^2}{\psi}\left(\frac{\ell}{\lambda}\right)E\left[(a-a_o)^2 - b^2 a^2\right] , \qquad (36)$$

where

$$b^2 = \frac{1}{\pi^2}\left(\frac{\lambda^2}{h\ell}\right)\left(\frac{\Delta p}{E}\right) .$$

The motion is then stable when

$$b^2 < 1 , \qquad (37)$$

as before, but now the system will oscillate about a new equilibrium position according to

$$a(t) = \frac{a_o}{(1-b^2)} - \frac{b^2 a_o}{(1-b^2)} \cos \omega t , \qquad (38)$$

where

$$\omega^2 = 4\pi^3\left(\frac{\ell}{\lambda}\right)\frac{E}{\rho\lambda^2}\;(1-b^2).$$

Thus when $b^2 \ll 1$ the stresses in the shear layer will be much smaller than those estimated in the previous sections. Even when b^2 is an appreciable fraction of unity the average stresses will be smaller. Only the peak stresses will be of the order of the previous stress levels. Thus the problem of yield strengths being exceeded is much reduced in this case. Only when the limit of stability ($b^2 \sim 1$) is approached do we lose the needed mechanical properties due to very high stresses. As noted previously, we believe this case is the closest model of situations that will occur in practice.

The utility of this mechanism in device configurations (weapons applications and laser-pellet and particle beam-pellet fusion devices) requires study before an assessment can be given. What is required is to find regimes in which the heavy material remains solid and a surface layer could maintain its material properties for a time of significant interest.

ACKNOWLEDGMENT

It is a pleasure to thank D. H. Sharp and R. Menikoff for discussions and critical analyses of the material presented here.

REFERENCES

1. J. F. Barnes, P. J. Blewett, R. G. McQueen, R. A. Meyer, and D. Venable, "Taylor Instability in Solids," J. Applied Phys., 45, 727 (1974).

2. J. W. Miles, "Taylor Instability of a Flat Plate," General Atomics report GAMD-7335 (1960).

3. K. A. Meyer and P. J. Blewett, "Some Preliminary Numerical Studies of Taylor Instability Which Include Effects of Material Strength," Los Alamos Scientific Laboratory Report LA-4754-MS (1971).

4. L. D. Landau and E. M. Lifshitz, Theory of Elasticity (Pergamon Press, New York, 1970), Ch. 1, p.p.4-15.

5. T. Takasugi and O. Izumi, "Surface Strengthening in Aluminum Single Crystals Coated with Electro-deposited Nickel Film," Acta Metallurgica 23, 1111 (1975).