Burn Characteristics of Marginal Deuterium-Tritium Microspheres

by

Dale B. Henderson
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ABSTRACT

Existing studies of the thermonuclear burn of DT microspheres have assumed complete Maxwellian distributions in the reactivity \( \langle \sigma v \rangle \). Under marginal conditions, \( \rho R < 10^{-4} \), we find long mean-free-paths for ions in the tail of the distribution may quench the burn. This missing factor appears to explain the lack of success in laser-fusion experiments conducted to date.

The thermonuclear burn of laser-fusion pellets may be usefully studied by consideration of the burn of hot compressed microspheres. Such microspheres may be thought of as being the inner core of DT droplets which have been compressed and heated by laser-driven ablation of the outer region of the pellet. From such studies we know that the fractional burn-up \( f_{ro} \) may be expected to follow from the fusion reactivity \( \sigma \) as

\[
f_{ro} = \left( \frac{\langle \sigma v \rangle}{8G M_i} \right) \rho R
\]

for conditions in which \( f_{ro} < 1 \). In evaluating the expressions in equation 1, the density \( \rho \), radius \( R \), and sound speed \( C = (2\gamma T/M_i)^{1/2} \) are evaluated at the initial conditions. We use \( \gamma = 5/3 \) and \( M_i \) is the ion mass. For \( \langle \sigma v \rangle \) we use the average over the Maxwellian ion distribution \( f(v) \), integrated out to infinity

\[
\langle \sigma v \rangle = \int_0^\infty v \sigma(v) f(v) v^2 dv.
\]

Justification of the integration out to infinity requires that the mean-free-path for test ions at speed \( v \), \( \lambda(v) \), be smaller than dimensions of interest over ranges of \( v \) important to the integration. Unfortunately the cross section, \( \sigma \), rises very rapidly with increasing \( v \) (below the maximum near 100 keV, deuteron energy) so that important ranges of \( v \) in the integration may correspond to energies well above the thermal energy in \( f(v) \) and to very long mean-free-paths \( \lambda(v) \).

At low energies the cross-section behavior is due mostly to Gamow’s barrier penetration formula:

\[
\sigma \sim \exp(-v^*/\hbar),
\]

where \( v^* = (2\pi)^2 e^2/\hbar = 1.375 \times 10^9 \) cm/sec. It is convenient to actually perform the integrations with a center-of-mass energy variable \( E = M_i \frac{v^2}{2} \), while experimental data is usually reported in deuteron energy \( E_D = M_i \frac{v^2}{2} \). Thus \( E_D^* = 1.97 \times 10^6 \) eV = (1.403 \times 10^3) eV, while an empirical best fit is \( (E_D^*)^{1/2} = 1.453 \times 10^3 \) (eV) \( 1/2 \). Using this last value and \( E_D^* = (M_i/M_R) E \), we obtain

\[
\sigma \sim \exp(-\sqrt{3/5} \ A/\sqrt{E})/E.
\]

Then changing integration variable from \( dv \) to \( dE \), the integrand in equation (2) is proportional to

\[
K(E) = \exp(-\sqrt{3/5} A/\sqrt{E}) \exp(-E/kT)
\]

which has its maximum at

\[
E_M = [0.5 \sqrt{3/5} A kT]^{2/3}.
\]
which for \( kT = 1 \text{ keV} \), is 6.8 keV.

The mean-free-path for 90-degree deflection for a test ion, on the other hand, is

\[
\lambda = M_\perp \frac{v^2}{8\pi n e^4} \ln A.
\]

Substituting \( E_c = M_\perp \frac{v^2}{2} \) and \( n = \rho/M_\perp \), this expression can be evaluated as

\[
E_c = 559. \text{ keV} \left( \ln \frac{\lambda}{10} \right)^{1/2} (pA)^{1/2},
\]

where \( \ln A = 10 \) is typical, applying to a 1.0 \( \mu g \) microsphere of DT at \( \rho R = 10^{-4} \), and \( kT = 1 \text{ keV} \).

Thus if we consider that test ions with \( \lambda \sim R \) ought to quickly diffuse out of the hot compressed core and be lost to the thermonuclear burn, we would expect, in this case to lose ions above about 5.6 keV, while the maximum in the integral for \( < ov > \) is found to be at 6.8 keV. Thus, significant quenching is expected.

Rather than follow test-ions in a Monte Carlo process, we have adopted \( \lambda = R \) as the criteria for lost ions, and have truncated the Maxwellian tails at the corresponding cut-off \( E_c \)'s in performing the \( < ov > \) integration. A better treatment would also include reactions in-flight by the lost ions, but our criteria appear to be adequate to estimate this important and over-looked factor in standard treatments.

The cut-off \( E_c \)'s obtained are shown in Figure 1 for a 1.0 \( \mu g \) microsphere. The weak temperature dependence (in \( \ln A \)) is indicated; the still weaker mass (or density in \( \ln A \)) dependence is insignificant for 0.1 to 10.0 \( \mu g \) masses.

Fig. 1. Cut-off energy \( E_c \) for ion loss vs. areal mass density \( \rho R \).

Using a best fit to empirical data for the cross section \( \sigma \), we have numerically integrated \( < ov > \) up to various cut-off energies. The resulting \( < ov > \) curves are plotted in Figure 2. It is

Fig. 2. Fusion reactivity \( < ov > \) integrated to cut-off energy \( E_c \) for various values of temperature \( T \).

Fig. 3. Cut-off \( E_c \) required to reduce the reactivity \( < ov > \) to 0.5 and 0.1 of its asymptotic value vs. temperature \( T \).
observed that at high $E_c$ the several curves saturate to the traditional $\langle ov \rangle$ values, while at smaller $E_c$ they fall rapidly and become approximately independent of temperature. This approximate temperature independence is a remarkable and important result.

In order to get a better understanding of the important values of $E_c$, than simply that from Eq. (4), we have plotted the $E_c$ which reduces $\langle ov \rangle$ to 0.5 and to 0.1 of its asymptotic value in Figure 3. The $(kT)^{2/3}$ dependence for small temperatures is apparent.

In Reference 1, it is shown that the burn-up fraction $f_{\text{up}}$ may be expected to be one-half the characteristic expansion time $\tau_e = R/4c_s$ where $c_s = (2\gamma T/M_i)^{1/2}$ divided by the characteristic burn time $\tau_r = 1/\langle ov \rangle$. We have therefore plotted the ratio $(\tau_e/\tau_r)$, for 1.0 μg microspheres, in Fig. 4.

Fig. 4. Ratio of characteristic burn and expansion rates $(\tau_e/\tau_r)$ vs. areal mass density $\rho R$ for various temperatures $T$, for a 1.0 μg sphere of DT.

Fig. 5. Fraction burn-up $f_{\text{up}}$ of a 1.0 μg sphere of DT vs. areal mass density $\rho R$ for various temperatures $T$. This is a composite of Fig. 4 and one from Reference 1.

It is seen that the upper straight-line parts of the curves agree with Fig. 8 in Reference 1. The small $\rho R$ convergence to an approximate temperature-independent part is due to the lost ion cut-off.

Finally we have taken the data in Fig. 4 and joined it onto the complete burn study result from Fig. 10 of Reference 1. In some cases a small (1.0 to 1.1 x) multiplicative adjustment was made in order to join the two. (The computer simulation data in Reference 1 followed the dashed lines and terminated as shown.) In the composite figure, Fig. 5, we see that below $\rho R \approx 10^{-2}$, we have a weakly temperature-dependent, fast ion loss regime. From $\rho R \approx 10^{-2}$ to 1.0 gm cm$^{-2}$, we have scaling as predicted in Eq. (1). Above $\rho R \approx 1.0$ gm cm$^{-2}$, we have bootstrap heating to higher burn-up.

That the lost-ion regime is separated from the non-linear burn regime by a factor of 100 in $\rho R$ is...
important in the justification of the analysis done here.

To date there have been no true thermonuclear neutrons observed in experiments \(^9\), which in some cases has been hard to reconcile with Eq. (1). The faster-than-linear small \(\rho R\) fall-off of \(f_{10}\) in these marginal experiments is adequate and probably a correct explanation. We should note that above \(\rho R \sim 10^{-2}\) this difficulty goes away, and so our new results do not provide any obstacle in the path of useful laser-fusion applications. There is also a bright side to the present marginal \(\rho R\) experimental difficulty: when true thermonuclear neutrons are observed from small microspheres, they will serve as proof-of-compression; an important milestone to laser fusion.

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REFERENCES


