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APPROXIMATE SPHERICAL BLAST THEORY AND LASER-INITIATED PELLET MICROEXPLOSIONS

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ABSTRACT

Equations are presented which describe the shock speed and overpressure as a function of radius for strong spherical blasts in cases where the source mass is not negligible. Approximations similar to those used in the Taylor-Sedov approximate theory are employed. Sample calculations are presented for a laser-initiated 1-MJ (debris energy) pellet microexplosion, and compared to Taylor-Sedov approximate theory solutions.

I. INTRODUCTION

Both exact and approximate Taylor-Sedov (T-S) theory[1-4] have long been used to describe spherical blast waves emanating from strong point explosions. The assumptions that have been used in developing the T-S theory make it valid only in a midfield regime as outlined below.

Here we address the near-field regime where the source mass is not negligible. This is of interest for certain explosives work and for other problems such as laser-initiated pellet microexplosions.[5-6] The one-dimensional planar near-field case has already been treated.[7] Here we address the near-field spherical case.

Below we first briefly review the T-S approximate theory to review methodologies and limitations. Then the alternative approximate theory is developed by using similar methodology. Finally, results of calculations for a 1-MJ pellet-debris microexplosion are presented for both theories and compared.

II. REVIEW OF TAYLOR-SEDOV APPROXIMATE THEORY

In deriving the T-S theory it was assumed that: (1) a large amount of energy E_s is released instantaneously (Δt is 0) from a small (negligible) volume; (2) a spherically expanding strong hydroshock emanates; and (3) the mass of the energy source m_s is negligible; thus either $m_s = 0$, or the shock must sweep over a mass of ambient gas m_1 such that $m_1 >> m_s$.

Assumption (3) enables definition of a near-field region wherein $m_1 \leq m_s$, and thus where T-S theory is not valid. A midfield may be defined wherein $m_1 >> m_s$, but only out to a radius where strong shock theory is valid, i.e., $P_2 >> P_1$, where P_2 is the peak pressure behind the shock and P_1 is the ambient gas pressure. In this region the T-S theory is valid. The far-field region, where the assumption $P_2 >> P_1$ is not valid and where a weak shock wave and finally an acoustic wave exists, will not be treated in this report.

Given the above assumptions and certain similarity arguments,[3-4] the T-S theory gives the shock position R by

$$R = \xi_o \left(\frac{E_s}{\rho_1}\right)^{1/5} t^{2/5}, \qquad (1)$$

where t is time, ρ_1 is ambient gas density, and ξ_o is a constant involving the specific-heat ratio γ. Spherical symmetry is assumed.

The constant ξ_o is evaluated as follows. The energy of the system is assumed constant and equal to E_s. Then, after energy release,

$$E_s = \int_0^R \left(\frac{1}{2} u^2 + e\right)\rho \; 4\pi r^2 dr \;, \qquad (2)$$

where u is flow speed, e is specific internal energy, and ρ is density.

From strong-shock theory ($P_2 >> P_1$) the flow speed, density, and peak pressure immediately behind the shock, denoted by Subscript (2), are given by

$$U_2 = \frac{2}{\gamma + 1} \dot{R} , \qquad (3)$$

$$\rho_2 = \frac{\gamma + 1}{\gamma - 1} \rho_1 , \qquad (4)$$

$$P_2 = \frac{2}{\gamma + 1} \dot{R}^2 \rho_1 , \qquad (5)$$

where $\dot{R} = (dR/dt)$ from Eq. (1) is

$$\dot{R} = \frac{2}{5} \xi_o \left(\frac{E_s}{\rho_1}\right)^{1/5} t^{-3/5} . \qquad (6)$$

Also

$$e_2 = \frac{1}{\gamma - 1} \frac{P_2}{\rho_2} = \frac{2}{(\gamma + 1)^2} \dot{R}^2 . \qquad (7)$$

In the approximate T-S theory, it is assumed that all the gas swept over by the shock is concentrated in a thin isotropic shell of thickness Δ, wherein all gas properties are uniform with values given by Eqs. (3) - (5) and Eq. (7). Thus, for conservation of mass (source mass neglected),

$$\frac{4}{3} \pi R^3 \rho_1 = 4\pi R^2 \Delta \rho_2 , \qquad (8)$$

whence

$$\rho_2 = \frac{\rho_1 R}{3\Delta} . \qquad (8a)$$

Using Eq. (1) and Eqs. (3) - (8) in Eq. (2) and solving for ξ_o gives

$$\xi_o = \left(\frac{5}{2}\right)^{2/5} \left(\frac{3}{4\pi}\right)^{1/5} \left(\frac{\gamma + 1}{2}\right)^{2/5} . \qquad (9)$$

Combining Eq. (1) and Eq. (6) yields

$$\dot{R} = \frac{2}{5} \xi^{5/2} \left(\frac{E_s}{\rho_1}\right)^{1/2} R^{-3/2}, \qquad (10)$$

giving $\dot{R}$ as a function of R for a given E_s, ρ_1, and γ.

Combining Eq. (5) and Eq. (10) yields

$$P_2 = \frac{2}{\gamma + 1} \left(\frac{2}{5}\right)^2 \xi^5 E_s R^{-3} , \qquad (11)$$

giving P_2 as a function of R for a given E_s and γ.

The above equations give solutions accurate to within a few percent [3-4] of the so-called exact solutions.

III. APPROXIMATE BLAST THEORY WITH SOURCE MASS

We now consider the case where the source mass is not negligible. We assume, as in the T-S theory, that the source energy E_s is delivered in a very short time ($\Delta t \to 0$). We also assume spherical symmetry.

We start with conservation of energy

$$E_s = E_d + E_g = \int_o^r \left(\frac{1}{2} u_d^2 + e_d\right)\rho_d \, 4\pi r^2 dr$$

$$+ \int_r^R \left(\frac{1}{2} u_g^2 + e_g\right)\rho_g \, 4\pi r^2 dr, \qquad (12)$$

where the subscript (d) refers to the source debris, the subscript (g) refers to the shock-heated ambient gas, and r refers to the radius of the debris/driven-gas interface (contact surface); the other terms have the same meaning as in the last section.

In the spirit of the T-S approximate theory the integral for the driven gas in Eq. (12) reduces to

$$E_g = \frac{4}{3} \pi R^3 \rho_1 \left(\frac{1}{2} u_g^2 + e_g\right). \qquad (12a)$$

Using Eqs. (3), (4), and (7) in Eq. (12a) gives

$$E_g = \frac{16}{3} \pi R^3 \rho_1 \frac{1}{(\gamma + 1)^2} \dot{R}^2 . \qquad (12.a.1)$$

For the debris mass, pressure and flow speed must be conserved across the contact surface, i.e.,

$$u_d = u_g = \frac{2}{\gamma + 1} \dot{R} , \qquad (13)$$

$$P_d = P_g = \frac{2}{\gamma + 1} \dot{R}^2 \rho_1, \qquad (14)$$

where P_g is given by Eq. (5) .

We define the average debris mass density, $\bar{\rho}_d$, as:

$$\bar{\rho}_d \equiv \rho_s \frac{R_s^3}{R^3} \qquad (15)$$

where the subscript s refers to the source before explosion. Equation (15) implies the assumption that $R \simeq r$, i.e., $(R - r)/r << 1$.

Using Eqs. (13) - (15) and employing the functional form of Eq. (7), the approximate expression for E_d becomes

$$E_d \simeq \frac{4}{3} \pi R^3 \bar{\rho}_d \left[\frac{1}{2} u_d^2 + e_d\right] = \ldots$$

$$\simeq \frac{8}{3} \frac{1}{(\gamma + 1)^2} \rho_s R_s^3 \dot{R}^2 + \frac{8}{3} \frac{\pi}{(\gamma^2 - 1)} \rho_1 R^3 \dot{R}^2 . \qquad (16)$$

Using Eq. (12.a.1) and Eq. (16), the approximate expression for Eq. (12) is

$$E_s \approx \frac{16}{3}\frac{\pi}{(\gamma+1)^2}\rho_1 R^3\dot{R}^2 + \frac{8}{3}\frac{\pi}{(\gamma+1)^2}\rho_s R_s^3\dot{R}^2 + \frac{8}{3}\frac{\pi}{(\gamma^2-1)}\rho_1 R^3\dot{R}^2 . \qquad (17)$$

Given E_s, γ, ρ_1, ρ_s, and R_s, Eq. (17) can be solved for $\dot{R}$ as a function of R. Corresponding P_2's can be found from Eq. (5), and flow speeds from Eq. (3).

IV. COMPUTATIONAL RESULTS

Shock speeds, $\dot{R}$, were calculated as a function of radius for both Eq. (10) and Eq. (17), using $E_s = 1 \times 10^6$ J, $\gamma = 1.67$, $\rho_s = 2 \times 10^2$ kg/m^3, $R_s = 10^{-3}$ m, and $\rho_1 = 10^{-4}$ to 10^{-9} kg/m^3; these are approximate conditions expected in certain laser-initiated fusion experiments involving a deuterium-tritium ice pellet, where ~ 1 MJ of the 7 MJ released would appear in the debris. Corresponding post-shock pressures, P_2, were calculated by using Eq. (5).

Computational results for the T-S approximate theory [Eq. (10) and Eq. (5)] are shown in Fig. 1. Note the very high shock speeds $\dot{R}$ for small radii R, namely speeds in excess of the speed of light. This result is obviously non-physical.

If within some very small radius (< 10^{-2} m) the source energy E_s is distributed equally over the source mass, and if all that energy is manifested in kinetic energy, then from $E_s = \frac{1}{2} m_s u^2$ a flow speed (quasi-free expansion) u of 1.54×10^6 m/s would be predicted. From Eq. (3), a shock speed, $\dot{R}$, of 2.06×10^6 m/s would result.

Computational results for the modified theory, Eq. (17), are shown in Fig. 2. For small radii the results show $\dot{R} \sim 2.06 \times 10^6$ m/s, in agreement with the speed predicted for small R for quasi-free expansion.

At higher densities and/or radii, the results of Fig. 2 show that the flow becomes a blast wave. This result would be expected as the expanding source-mass particles interact with the ambient gas and transfer energy to ambient gas particles, and is predictable when viewing Eq. (17) where the first and the third term on the right-hand side resemble Eq. (10), i.e., $E_s \sim \rho_1 R^3 \dot{R}^2$.

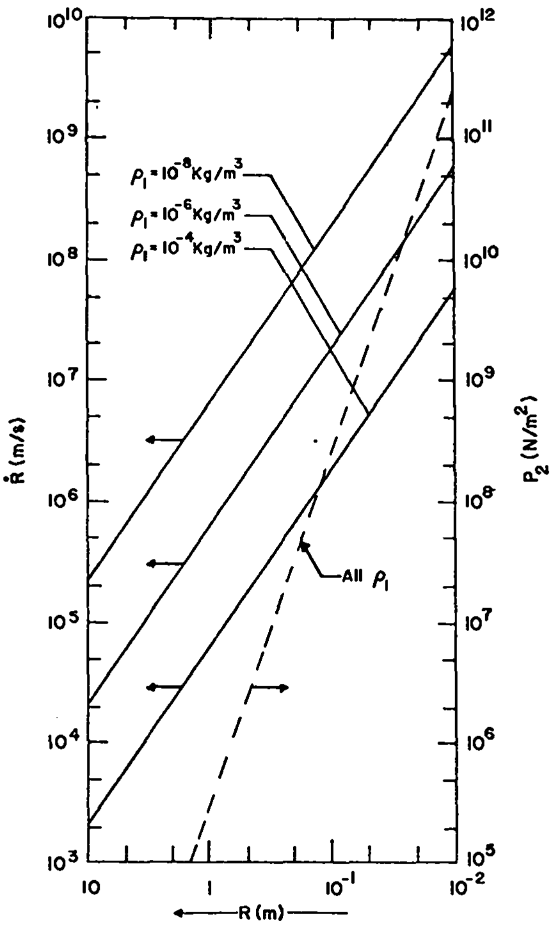

Fig. 1. Taylor-Sedov theory solutions for a 1-MJ pellet microexplosion.

V. DISCUSSION

The derivation of Eq. (17) admittedly involves several approximations. The derivation assumes constant γ, assumes an ideal gas, and neglects plasma phenomena such as fast-electron clouds accelerating ions by Coulomb forces.[8] Radiative losses from ionizing shocks are also neglected. Inclusion of such phenomena in the derivation would yield more accurate space- and time-dependent solutions, though numerical rather than analytic solutions would probably be required.

The above analytic results should be useful in providing estimates of cavity physics phenomena for systems analysis work on conceptual laser-fusion reactor concepts.

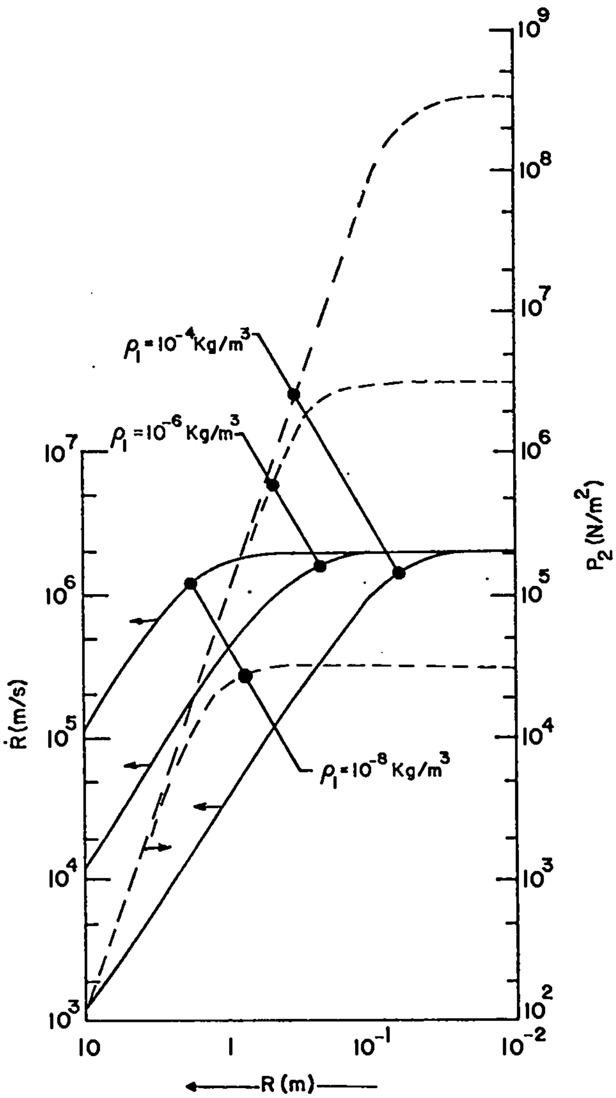

Fig. 2. Solutions with source-mass included for a 1 MJ pellet microexplosion

For example, in some reactor concepts the prompt-x-ray burst from the pellet microexplosion is absorbed in lithium-wetted first walls of a spherical cavity.[9] The subsequent lithium vaporization produces an impulse on the first wall (resulting in transient stresses). Subsequent inward expansion of the lithium vapor will, at some time, interact with the outward-expanding blast driven by the pellet debris. The time, position, and nature of such interaction will depend on the ambient-gas density in the cavity. Outward transmitted waves from the interaction will subsequently interact with the cavity walls.

Another scenario for dry-wall cavity concepts[10] again involves cavity dynamics. Absorption of the neutron burst in a lithium blanket surrounding the cavity may result in some cavity wall ringing. The arrival of the pellet debris-driven blast on the wall may amplify or negate an oscillation, depending on cavity dimensions, ambient gas density, and pellet characteristics.

The time of arrival of the pellet debris-driven blast at various radii is one important factor. For $\rho_1 = 10^{-8}$ kg/m^3 and the pellet characteristics stated above, the T-S theory, Eq. (1), predicts that the blast will reach one meter radius at $t \sim 6.23 \times 10^{-8}$ s. The results of Fig. 2 indicate that at one meter radius the arrival time would be $t \sim 4.85 \times 10^{-7}$ s. Thus, altered small-radius flow histories can greatly alter large-radius arrival times.

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