Magnification Ratios for Explosive-Driven Shocks

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ABSTRACT

Measurement of the particle-velocity history at an explosive-driven shock in an inert material is often used to qualitatively infer the velocity profile in the explosive existing at the time the shock enters the inert. A forward-facing characteristic joins each point of the explosive profile to a point on the shock path, cutting off related segments of the profile and path. The magnification ratio at a given shock position is defined as the distance of shock travel into the inert divided by the length of the corresponding explosive profile segment.

We have calculated the magnification ratio vs distance of shock travel, an almost linear function, for several systems. Over the range 0 to 2 times the length of the HE it increases from (about) 5 to 10 for PBX9404/aluminum and from 2 to 6 for Comp. B/Plexiglas.

When a block of explosive drives a shock into an inert plate, the shock history in the plate qualitatively images the state profile in the explosive. Figure 1 is an x-t diagram showing the characteristics and the magnification ratios m and m'. At each point on the shock path, the overall magnification ratio m is defined as the distance X of shock travel into the inert divided by the length of the segment of the explosive profile cut off by the (+) characteristic from the shock point. The overall magnification ratio m has two components: (1) the refraction of the forward characteristics across the explosive/inert interface, and (2) the magnification in the inert itself. The second component is characterized by the local (infinitesimal) magnification ratio m' at the shock

\[ m' = (M - 1)^{-1} \]

\[ M = \frac{u + c}{U} \]

where U is the shock velocity, and u, c, and M are the particle velocity, sound speed, and Mach number evaluated immediately behind the shock. This local magnification ratio m' is like the overall magnification ratio m except that the profile segment in question is of infinitesimal length (and of course lies entirely in the inert); it measures the local rate at which characteristics overtake the shock.

For a given explosive/inert pair, and for the constitutive relations used here, the magnification ratios depend on a single variable, the dimensionless ratio X/L, where L is the length of the explosive. (We assume that the reaction in the explosive is instantaneous and that the detonation propagates at CJ velocity throughout.) Using the computer codes SIN\textsuperscript{2} (a Lagrangian mesh code) and RICSHAW\textsuperscript{3} (a method-of-characteristics code) and the results of an exact solution,\textsuperscript{1} we have calculated magnification ratios as a function of X/L for the systems listed in Table I. The constitutive relations are given in the Appendix.

The results for m and m' are shown in Fig. 2. Curves 1 and 2 compare the same system calculated with the two codes SIN and RICSHAW. Curve 3 is
the same as curve 2 except that the aluminum is given material strength instead of being treated as a fluid. Here the tail of the elastic rarefaction overtakes the shock at about $X/L = 0.5$, where $m(X/L)$ is discontinuous. We have not determined $m$ for values of $X/L$ less than 0.5. Curves 2 or 3 and curve 4 show the effect of changing the inert. Curves 4 and 5 show the effect of changing the explosive, although we caution that the equations of state used in the exact solution are necessarily of a special form and not completely realistic.

**TABLE I**

<table>
<thead>
<tr>
<th>No.</th>
<th>Source</th>
<th>Explosive</th>
<th>Inert</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RICSHAW</td>
<td>PBX9404</td>
<td>Aluminum (fluid)</td>
</tr>
<tr>
<td>2</td>
<td>SIN</td>
<td>PBX9404</td>
<td>Aluminum (fluid)</td>
</tr>
<tr>
<td>3</td>
<td>SIN</td>
<td>PBX9404</td>
<td>Aluminum (elastic-plastic)</td>
</tr>
<tr>
<td>4</td>
<td>SIN</td>
<td>PBX9404</td>
<td>Plexiglas (fluid)</td>
</tr>
<tr>
<td>5</td>
<td>Exact solution</td>
<td>Comp. B</td>
<td>Plexiglas (fluid)</td>
</tr>
</tbody>
</table>

One imaging is shown in Fig. 3, which compares the shock-path image of the HE velocity profile with the original. To make the horizontal lengths of the two curves the same, we have divided the shock-path...
The shock-path image of the HE profile compared with the original. From calculation 1. The point at the end of the shock path shown is at $X/L = 0.779$, and is reduced by the magnification ratio at this point, $m^* = 6.76$.

To summarize, the explosive profile image as read from shock measurements (in turn usually inferred from initial free-surface velocity measurements) is magnified 5 to 10 times in aluminum and 3 to 6 times in Plexiglas, with the total magnification ratio increasing nearly linearly with distance of shock travel. Of the two main components of the magnification ratio, the local magnification ratio in the inert $m'$ increases only slightly with distance of shock travel; most of the increase in the overall magnification ratio comes from the interface-refraction component.

APPENDIX

CONSTITUTIVE RELATIONS

The constitutive relations used are given in Table A-I. The $\gamma$-law equation of state for the explosive detonation products is

$$p = (\gamma - 1)\rho e$$

where $p$, $\rho$, and $e$ are pressure, density, and energy, and $\gamma$ is a constant, together with a CJ state defined by the CJ pressure and detonation velocity $D$. The Grüneisen equation of state for the inerts is

$$p = p_H(v) + \rho_f(e - e_H(v))$$

where $f$ is a constant and $p_H$ and $e_H$ are the pressure and energy on the principal Hugoniot implied by the linear relation

$$U = c + su$$

between Hugoniot shock and particle velocities $U$ and $u$, with constants $c$ and $s$. Material strength is added with the linearly elastic perfectly plastic model described in Ref. 2. It is specified by a Lamé constant $\mu$ and a yield stress $Y_o$. The hydrostat is obtained by a simple correction to the Grüneisen equation of state used for the fluid.

The exact solution of calculation 5 is obtained through a special choice of the equation of state: a $\gamma$-law with $\gamma = 3$ for the explosive, and a more complicated "Walsh" form for the inert. A realistic set of parameters for Comp. B is

<table>
<thead>
<tr>
<th>Explosives (γ-law)</th>
<th>D</th>
<th>$\gamma$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PBX9404</td>
<td>1.84</td>
<td>3</td>
<td>35.6</td>
</tr>
<tr>
<td>Comp. B</td>
<td>1.6</td>
<td>3</td>
<td>28.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inerts</th>
<th>$\rho_0$</th>
<th>$c$</th>
<th>$s$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plexiglas</td>
<td>1.18</td>
<td>2.43</td>
<td>1.58</td>
<td>1</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.785</td>
<td>5.328</td>
<td>1.338</td>
<td>2</td>
</tr>
</tbody>
</table>

Elastic constants: $Y_o = 0.55$, $\mu = 25$

Units: coherent mm-mg-μs set: density in Mg/m$^3$ ($= g/cm^3$), velocity in km/s ($= mm/μs$), pressure in GPa ($= 0.01$ Mb). The elastic constants $Y_o$ and $\mu$ both have dimensions of pressure.
\[ \rho_0 = 1.714, \quad D = 7.991, \quad \gamma = 2.77, \quad p = 29.1 \]

The \( \gamma = 3 \) equation of state for the exact solution was obtained from this by changing \( D \) and \( \rho_0 \) in such a way as to keep the \( p-u \) rarefaction locus (including the CJ state) essentially unchanged. The Walsh equation of state for the inert is based on essentially the same principal Hugoniot as that in Table A-I, but has slightly different rarefaction isentropes, whose \( p-u \) loci are identical with that of the Hugoniot.

REFERENCES